

Lesson 1.1 **Imperial Measures of Length** **Exercises (pages 11–12)**

A

3. Answers may vary. For example:

- a) Foot; because my desk is higher than 1 ft., but not as high as 1 yd.
- b) Inch; because a mattress is thicker than 1 in., but not quite as thick as 1 ft.
- c) Foot; because the measurement would be more accurate in feet rather than yards
- d) Inch; because the measurement would be more accurate in inches rather than feet
- e) Mile; because a mile is closer to the distance from the school to my home than an inch, a foot, or a yard

4. a) Inch; because the length of a piece of paper is less than 1 ft.

- b) I used my thumb as a referent for one inch. I aligned the tip of my thumb with one corner of a piece of notepaper. I marked a point at my knuckle along the length of the paper. I moved my thumb so the tip aligned with the marked point. I continued until I got to the end of the length of the piece of paper. The paper was about eleven thumb units long. The actual measurement was 11 in., so my estimate was reasonable.

5. a) Foot; because this unit is large enough to measure the door, but small enough to get an accurate fraction if the measurement is not a whole number

- b) I used my shoe as a referent for one foot. I aligned the heel of my shoe with the bottom of the door. I marked a point at the tip of the shoe up the door with a piece of tape. I moved my shoe so the heel aligned with the marked point. I continued until I got to the top of the door. The door was about seven shoes high. The actual measurement was about $6\frac{1}{2}$ ft., so my estimate was reasonable.

6. Estimates may vary. For example:

- a) About 9 in.
- b) About 12 ft.
- c) About 10 yd.
- d) About 1 mi.

7. a) Since 1 ft. = 12 in., to convert feet to inches multiply by 12.
 3 ft. = 3(12 in.)
 3 ft. = 36 in.

- b) Since 1 yd. = 3 ft., to convert yards to feet multiply by 3.

$$63 \text{ yd.} = 63(3 \text{ ft.})$$

$$63 \text{ yd.} = 189 \text{ ft.}$$

- c) Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

$$48 \text{ in.} = \frac{48}{12} \text{ ft.}$$

$$48 \text{ in.} = 4 \text{ ft.}$$

B

8. a) Since 1 mi. = 5280 ft., to convert miles to feet multiply by 5280.

$$2 \text{ mi.} = 2(5280 \text{ ft.})$$

$$2 \text{ mi.} = 10\,560 \text{ ft.}$$

- b) Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

$$574 \text{ in.} = \frac{574}{12} \text{ ft.} \quad \text{Write the improper fraction as a mixed number.}$$

$$574 \text{ in.} = 47\frac{10}{12} \text{ ft.}$$

$$574 \text{ in.} = 47 \text{ ft. } 10 \text{ in.}$$

Since 3 ft. = 1 yd., divide 47 ft. by 3 to convert to yards.

$$47 \text{ ft.} = \frac{47}{3} \text{ yd.}$$

$$47 \text{ ft.} = 15\frac{2}{3} \text{ yd.}$$

$$47 \text{ ft.} = 15 \text{ yd. } 2 \text{ ft.}$$

$$\text{So, } 574 \text{ in.} = 15 \text{ yd. } 2 \text{ ft. } 10 \text{ in.}$$

- c) Since 1 mi. = 5280 ft., subtract 5280 ft. from 7390 ft.

$$7390 \text{ ft.} - 5280 \text{ ft.} = 2110 \text{ ft.}$$

$$\text{So, } 7390 \text{ ft.} = 1 \text{ mi. } 2110 \text{ ft.}$$

Since 3 ft. = 1 yd., to convert feet to yards divide by 3.

$$2110 \text{ ft.} = \frac{2110}{3} \text{ yd.} \quad \text{Write this improper fraction as a mixed number.}$$

$$2110 \text{ ft.} = 703\frac{1}{3} \text{ yd.}$$

$$2110 \text{ ft.} = 703 \text{ yd. } 1 \text{ ft.}$$

$$\text{So, } 7390 \text{ ft.} = 1 \text{ mi. } 703 \text{ yd. } 1 \text{ ft.}$$

9. First convert 165 in. to feet.

Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

$$165 \text{ in.} = \frac{165}{12} \text{ ft.}$$

Write the improper fraction as a mixed number.

$$\frac{165}{12} \text{ ft.} = 13\frac{9}{12} \text{ ft.}$$

$$\frac{165}{12} \text{ ft.} = 13 \text{ ft. } 9 \text{ in.}$$

Since 3 ft. = 1 yd., divide by 3 to convert feet to yards.

$$13 \text{ ft.} = \frac{13}{3} \text{ yd.}$$

Write the improper fraction as a mixed number.

$$\frac{13}{3} \text{ yd.} = 4\frac{1}{3} \text{ yd.}$$

$$\frac{13}{3} \text{ yd.} = 4 \text{ yd. } 1 \text{ ft.}$$

So, 165 in. = 4 yd. 1 ft. 9 in.

10. a) Let x represent the perimeter in yards.

The ratio of x yards to feet is equal to the ratio of 1 yd. to 3 ft.

Write a proportion.

$$\frac{x}{52} = \frac{1}{3}$$

$$52\left(\frac{x}{52}\right) = 52\left(\frac{1}{3}\right)$$

$$x = \frac{52}{3}$$

$$x = 17\frac{1}{3}$$

$$17\frac{1}{3} \text{ is } 17 \text{ yd. } 1 \text{ ft.}$$

So, the perimeter of the pen is 17 yd. 1 ft.

- b) The perimeter of the pen is greater than 17 yd., so Carolyn must buy 18 yd. of fencing material.

The cost, C , is:

$$C = 18(\$10.99)$$

$$C = \$197.82$$

Before taxes, the fencing material costs \$197.82.

11. a) Since the mats are measured in inches, convert the length of material to inches.

Convert 10 yd. to inches.

$$1 \text{ yd.} = 3 \text{ ft.}$$

$$10 \text{ yd.} = 10(3 \text{ ft.})$$

$$10 \text{ yd.} = 30 \text{ ft.}$$

$$1 \text{ ft.} = 12 \text{ in.}$$

$$30 \text{ ft.} = 30(12 \text{ in.})$$

$$30 \text{ ft.} = 360 \text{ in.}$$

The number of mats is: $\frac{360}{15} = 24$

David can make 24 mats.

- b)** To convert yards to inches, first convert yards to feet, then convert feet to inches.

Write a conversion factor for yards and feet, with feet in the numerator: $\frac{3 \text{ ft.}}{1 \text{ yd.}}$

Write a conversion factor for feet and inches, with inches in the numerator: $\frac{12 \text{ in.}}{1 \text{ ft.}}$

$$\begin{aligned} \text{Then, } 10 \text{ yd.} \times \frac{3 \text{ ft.}}{1 \text{ yd.}} \times \frac{12 \text{ in.}}{1 \text{ ft.}} &= \frac{10 \cancel{\text{ yd.}}}{1} \times \frac{3 \cancel{\text{ ft.}}}{1 \cancel{\text{ yd.}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ ft.}}} \\ &= (10 \times 3 \times 12) \text{ in.} \\ &= 360 \text{ in.} \end{aligned}$$

Since this measurement is equal to the measurement in part a, the conversion is verified.

- 12.** To check if Pierre-Marc's answer is correct, convert 21 ft. 9 in. into yards, feet, and inches.

3 ft. = 1 yd.

So, to convert feet to yards divide by 3.

$$21 \text{ ft.} = \frac{21}{3} \text{ yd.}$$

$$21 \text{ ft.} = 7 \text{ yd.}$$

So, 21 ft. 9 in. = 7 yd. 9 in.

Pierre-Marc's answer was incorrect.

- 13.** First, convert the heights to inches.

1 ft. = 12 in., so to convert feet to inches multiply by 12.

For Sandy Allen's height of 7 ft. 7 in.:

$$7 \text{ ft.} = 7(12 \text{ in.})$$

$$7 \text{ ft.} = 84 \text{ in.}$$

So, Sandy is 84 in. + 7 in. = 91 in. tall

For Leonid Stadnyk's height of 8 ft. 5 in.:

$$8 \text{ ft.} = 8(12 \text{ in.})$$

$$8 \text{ ft.} = 96 \text{ in.}$$

So, Leonid is 96 in. + 5 in. = 101 in. tall.

Subtract Sandy's height from Leonid's height to determine the difference:

$$101 \text{ in.} - 91 \text{ in.} = 10 \text{ in.}$$

So, Sandy is 10 in. shorter than Leonid.

- 14. a)** Determine the perimeter, P , of the bedroom minus the width of the doorway.

$$P = 2(12 \text{ ft. } 9 \text{ in.} + 8 \text{ ft. } 1 \text{ in.}) - 2 \text{ ft. } 6 \text{ in.}$$

$$P = 2(20 \text{ ft. } 10 \text{ in.}) - 2 \text{ ft. } 6 \text{ in.}$$

$$P = 40 \text{ ft. } 20 \text{ in.} - 2 \text{ ft. } 6 \text{ in.}$$

$$P = 38 \text{ ft. } 14 \text{ in.}$$

Since 1 ft. = 12 in.,

$$14 \text{ in.} = 1 \text{ ft. } 2 \text{ in.}$$

So, $P = 39 \text{ ft. } 2 \text{ in.}$

The total length of border needed is 39 ft. 2 in.

- b) The border is purchased in 12-ft. rolls.

The number of rolls is approximately $\frac{39 \text{ ft.}}{12 \text{ ft.}} = 3\frac{3}{12}$

Since only whole rolls can be bought, 4 rolls are needed.

- c) The cost, C , of 4 rolls is:

$$C = 4(\$12.49)$$

$$C = \$49.96$$

Before taxes, the border will cost \$49.96.

15. a) Determine the perimeter, P , of the room minus the width of the doorway.

Since the room is square,

$$P = 4(18 \text{ ft. } 4 \text{ in.}) - 3 \text{ ft.}$$

$$P = 72 \text{ ft. } 16 \text{ in.} - 3 \text{ ft.}$$

$$P = 69 \text{ ft. } 16 \text{ in.}$$

Since 1 ft. = 12 in.,

$$16 \text{ in.} = 1 \text{ ft. } 4 \text{ in.}$$

So, $P = 70 \text{ ft. } 4 \text{ in.}$

The perimeter of the room is greater than 70 ft., so the contractor must buy 71 ft. of trim.

The cost, C , is:

$$C = 71(\$1.69)$$

$$C = \$119.99$$

So, the cost of the trim for the room, before taxes, is \$119.99.

- b) Determine the perimeter, P , of the window.

$$P = 2(40 \text{ in.} + 26 \text{ in.})$$

$$P = 2(66 \text{ in.})$$

$$P = 132 \text{ in.}$$

Since the trim is measured in feet, convert 132 in. to feet.

12 in. = 1 ft., so to convert inches to feet, divide by 12.

$$132 \text{ in.} = \frac{132}{12} \text{ ft.}$$

$$132 \text{ in.} = 11 \text{ ft.}$$

The cost, C , is:

$$C = 11(\$1.69)$$

$$C = \$18.59$$

The cost of the trim for the window, before taxes, is \$18.59.

16. The puzzle scale is 1 in. represents 360 in.

$$35\frac{2}{5} \text{ in. represents } 35\frac{2}{5} (360 \text{ in.}) = 12\,744 \text{ in.}$$

$$1 \text{ ft.} = 12 \text{ in.}$$

Divide by 12 to convert 12 744 in. to feet:

$$\frac{12\,744}{12} = 1062$$

So, the height of the Eiffel Tower is 1062 ft.

17. The map scale is 1 in. represents 1 500 000.

$$2\frac{5}{8} \text{ represents } 2\frac{5}{8}(1\,500\,000 \text{ in.}) = 3\,937\,500 \text{ in.}$$

$$1 \text{ ft.} = 12 \text{ in.}$$

Divide by 12 to convert 3 937 500 in. to feet:

$$\frac{3\,937\,500}{12} = 328\,125$$

$$1 \text{ mi.} = 5280 \text{ ft.}$$

Divide by 5280 to convert 328 125 ft. to miles.

$$\frac{328\,125}{5280} = 62.144\dots$$

The distance between Trois-Rivières and Québec City is approximately 62 mi.

18. Convert 18 ft. to inches.

$$1 \text{ ft.} = 12 \text{ in.}$$

$$18 \text{ ft.} = 18(12 \text{ in.})$$

$$18 \text{ ft.} = 216 \text{ in.}$$

Since Erica is planting tulips 8 in. apart, divide 216 in. by 8.

$$\frac{216 \text{ in.}}{8 \text{ in.}} = 27$$

Erica begins and ends the row with tulip bulbs; so, she will need $27 + 1$, or 28 tulip bulbs.

C

19. 1 h = 60 min = 3600 s

$$\text{So, in 1 h, the student could walk } \frac{3600 \text{ s}}{10 \text{ s}}(30 \text{ ft.}) = 10\,800 \text{ ft.}$$

Convert 10 800 ft. to miles and yards.

$$5280 \text{ ft.} = 1 \text{ mi.}$$

$$10\,800 \text{ ft.} = \frac{10\,800}{5280} \text{ mi.}$$

$$10\,800 \text{ ft.} = 2\frac{240}{5280} \text{ mi.}$$

$$10\,800 \text{ ft.} = 2 \text{ mi. } 240 \text{ ft.}$$

Convert 240 ft. to yards.

$$3 \text{ ft.} = 1 \text{ yd.}$$

$$240 \text{ ft.} = \frac{240}{3} \text{ yd.}$$

$$240 \text{ ft.} = 80 \text{ yd.}$$

So, the student could walk 2 mi. 80 yd. in 1 h.

20. Convert 95 mi. to inches.

$$1 \text{ mi.} = 5280 \text{ ft. and } 1 \text{ ft.} = 12 \text{ in.}$$

$$\text{So, } 95 \text{ mi.} = 95(5280)(12) \text{ in.}$$

$$95 \text{ mi.} = 6\,019\,200 \text{ in.}$$

To determine the map scale, divide 6 019 200 in. by $2\frac{9}{16}$ in., or $\frac{41}{16}$ in.

$$\text{So, } \frac{6\,019\,200}{\frac{41}{16}} = 2\,348\,956.098\dots$$

The map scale, to the nearest thousand, is 1:2 349 000.

21. a) Since 20 reams of paper form a stack 40 in. high, then the height of 1 ream is:

$$\frac{40 \text{ in.}}{20} = 2 \text{ in.}$$

Convert the height of Mount Logan to inches.

$$1 \text{ ft.} = 12 \text{ in.}$$

$$19\,500 \text{ ft.} = 19\,500(12 \text{ in.})$$

$$19\,500 \text{ ft.} = 234\,000 \text{ in.}$$

Each ream is 2 in. high, so the height of Mount Logan is:

$$\frac{234\,000}{2} = 117\,000 \text{ reams}$$

Each ream costs \$3, so the value of the stack is: $\$3(117\,000) = \$351\,000$

The value of the stack is \$351 000.

- b) I estimate by rounding the height of Mount Logan to 20 000 ft.

Then, I use mental math to convert the height to inches:

$$20\,000 \text{ ft.} = 20\,000(12 \text{ in.})$$

$$20\,000 \text{ ft.} = 240\,000 \text{ in.}$$

Each ream is 2 in. high, so I use mental math again to determine the approximate number of reams in the height:

$$\frac{240\,000}{2} = 120\,000$$

Since each ream is \$3, I use mental math to determine the approximate cost:

$$\$3(120\,000) = \$360\,000$$

Since \$351 000 is close to \$360 000, the answer is reasonable.

22. Convert 100 mi. to inches.

$$1 \text{ mi.} = 5280 \text{ ft. and } 1 \text{ ft.} = 12 \text{ in.}$$

$$100 \text{ mi.} = 100(5280)(12) \text{ in.}$$

$$100 \text{ mi.} = 6\,336\,000 \text{ in.}$$

Think: How many stacks of 5 toonies would fit in this distance?

Divide by $\frac{2}{5}$. This is the same as multiplying by $\frac{5}{2}$.

$$6\,336\,000 \left(\frac{5}{2} \right) = 15\,840\,000$$

Each stack is 5 toonies, or $5(\$2) = \$10$.

So, the value of the stack is: $\$10(15\,840\,000) = \$158\,400\,000$

Lesson 1.2	Math Lab: Measuring Length and Distance	Assess Your Understanding (page 15)
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1. Referents may vary. For example:
 - a) The height of a door knob from the ground
 - b) The width of a finger
 - c) The thickness of a pencil lead
2. Referents may vary. For example: The distance walked in 15 min.
3. Answers may vary. For example: Calipers cannot be used for large measures, or to measure irregular objects accurately.
4. Referents may vary.
 - a) I would walk around the can placing my feet heel to toe, to get an estimate in feet.
I would wrap a piece of string around the garbage can, then measure the length of the string.
 - b) I would use the width of a finger to estimate the thickness of my hand, in centimetres.
I would use a caliper to measure the thickness on my hand.
 - c) I would use a pedometer to count the number of steps. Since each step is about 3 ft., I'd multiply the number of steps by 3 to get an estimate in feet.
I would measure the distance walked in 10 s, then see how long it takes me to walk from home to the store. Then I'd divide the number of seconds by 10 and multiply by the distance I can walk in 10 s, to get the distance from home to the store.
 - d) I would use the width of a finger to estimate the distance on the map, in centimetres;
I would use a ruler to measure the distance between the two cities on the map, then I could use the map scale to calculate the distance.
 - e) I would measure the distance walked in 10 s, then see how long it takes me to walk around the track. Then I'd divide the number of seconds by 10 and multiply by the distance I can walk in 10 s, to get the distance around the track.
 - f) I would use the thickness of a pencil lead as a referent to estimate the length in millimetres.
I would use a ruler to measure the length in millimetres.
5. To estimate the length in SI units, I know that one of my strides is about 1 m long. I would stride across the bridge and count my strides to get an estimate in metres.
To estimate the length in imperial units, I know that one of my steps is about 1 yd. long. I would walk across the bridge and count my steps to get an estimate in yards.
To measure the length in metres, I would ask my parents to drive across the bridge and use the car odometer to measure the distance in decimals of a kilometre.
To measure the length in yards, I would use a trundle wheel that measures in imperial units.

Lesson 1.3 Relating SI and Imperial Units

Exercises (pages 22–23)

A

Answers will vary, depending on the conversion ratios used.

4. a) $1 \text{ in.} = 2.54 \text{ cm}$

So, $16 \text{ in.} = 16(2.54 \text{ cm})$

$16 \text{ in.} = 40.64 \text{ cm}$

$16 \text{ in.} \doteq 40.6 \text{ cm}$

b) $1 \text{ ft.} \doteq 0.3 \text{ m}$

So, $4 \text{ ft.} \doteq 4(0.3 \text{ m})$

$4 \text{ ft.} \doteq 1.2 \text{ m}$

c) $1 \text{ yd.} \doteq 0.9 \text{ m}$

So, $5 \text{ yd.} \doteq 5(0.9 \text{ m})$

$5 \text{ yd.} \doteq 4.5 \text{ m}$

d) $1 \text{ yd.} \doteq 0.9 \text{ m}$

So, $1650 \text{ yd.} \doteq 1650(0.9 \text{ m})$

$1650 \text{ yd.} \doteq 1485 \text{ m}$

Divide by 1000 to convert metres to kilometres.

$$1650 \text{ yd.} \doteq \frac{1485 \text{ km}}{1000}$$

$1650 \text{ yd.} \doteq 1.485 \text{ km}$

$1650 \text{ yd.} \doteq 1.5 \text{ km}$

e) Use exact conversions.

$1 \text{ yd.} = 91.44 \text{ cm}$

So, $1 \text{ yd.} = 0.9144 \text{ m}$ Divide by 1000 to convert metres to kilometres.

$$1 \text{ yd.} = \frac{0.9144}{1000} \text{ km}$$

$1 \text{ yd.} = 0.000\,914\,4 \text{ km}$

$1 \text{ mi.} = 1760 \text{ yd.}$

So, $6 \text{ mi.} = 6(1760 \text{ yd.})$

$6 \text{ mi.} = 10\,560 \text{ yd.}$

From above, $1 \text{ yd.} = 0.000\,914\,4 \text{ km}$

So, $6 \text{ mi.} = 10\,560(0.000\,914\,4 \text{ km})$

$6 \text{ mi.} \doteq 9.656 \text{ km}$

$6 \text{ mi.} \doteq 9.7 \text{ km}$

f) $1 \text{ in.} = 2.54 \text{ cm}$

$1 \text{ cm} = 10 \text{ mm}$; so, $1 \text{ in.} = 25.4 \text{ mm}$

$2 \text{ in.} = 2(25.4 \text{ mm})$

$2 \text{ in.} = 50.8 \text{ mm}$

5. a) $25 \text{ mm} = 2.5 \text{ cm}$

$2.5 \text{ cm} \doteq 1 \text{ in.}$

b) $1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$

So, $2.5 \text{ m} \doteq 2.5(3\frac{1}{4} \text{ ft.})$

$2.5 \text{ m} \doteq 8.125 \text{ ft.}$

$2.5 \text{ m} \doteq 8 \text{ ft.}$

c) $0.9 \text{ m} \doteq 1 \text{ yd.}$

So, $10 \text{ m} \doteq \frac{10}{0.9} \text{ yd.}$

$10 \text{ m} \doteq 11.\bar{1} \text{ yd.}$

$10 \text{ m} \doteq 11 \text{ yd.}$

d) $1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$

So, $150 \text{ km} \doteq 150(\frac{6}{10} \text{ mi.})$

$150 \text{ km} \doteq 90 \text{ mi.}$

6. a) $1 \text{ ft.} = 12 \text{ in.}$

So, $1 \text{ ft. } 10 \text{ in.} = 12 \text{ in.} + 10 \text{ in.}$

$1 \text{ ft. } 10 \text{ in.} = 22 \text{ in.}$

$1 \text{ in.} = 2.54 \text{ cm}$

So, $22 \text{ in.} = 22(2.54 \text{ cm})$

$22 \text{ in.} = 55.88 \text{ cm}$

$22 \text{ in.} \doteq 55.9 \text{ cm}$

b) $1 \text{ yd.} = 36 \text{ in.}$

So, $2 \text{ yd.} = 2(36 \text{ in.})$

$2 \text{ yd.} = 72 \text{ in.}$

And, $1 \text{ ft.} = 12 \text{ in.}$

So, $2 \text{ ft.} = 2(12 \text{ in.})$

$2 \text{ ft.} = 24 \text{ in.}$

So, $2 \text{ yd. } 2 \text{ ft. } 5 \text{ in.} = 72 \text{ in.} + 24 \text{ in.} + 5 \text{ in.}$

$2 \text{ yd. } 2 \text{ ft. } 5 \text{ in.} = 101 \text{ in.}$

$1 \text{ in.} = 2.54 \text{ cm}$

So, $101 \text{ in.} = 101(2.54 \text{ cm})$

$101 \text{ in.} = 256.54 \text{ cm}$

$101 \text{ in.} \doteq 256.5 \text{ cm}$

c) $1 \text{ yd.} = 36 \text{ in.}$

So, $10 \text{ yd.} = 10(36 \text{ in.})$

$10 \text{ yd.} = 360 \text{ in.}$

And, $1 \text{ ft.} = 12 \text{ in.}$

So, $10 \text{ yd. } 1 \text{ ft. } 7 \text{ in.} = 360 \text{ in.} + 12 \text{ in.} + 7 \text{ in.}$

$10 \text{ yd. } 1 \text{ ft. } 7 \text{ in.} = 379 \text{ in.}$

$$\begin{aligned} 1 \text{ in.} &= 2.54 \text{ cm} \\ 1 \text{ cm} &= 0.01 \text{ m; so, } 1 \text{ in.} = 0.0254 \text{ m} \\ 379 \text{ in.} &= 379(0.0254 \text{ m}) \\ 379 \text{ in.} &= 9.6266 \text{ m} \\ 379 \text{ in.} &\approx 9.6 \text{ m} \end{aligned}$$

B

7. a) i) $2.54 \text{ cm} = 1 \text{ in.}$

$$75 \text{ cm} = \frac{75}{2.54} \text{ in.}$$

$$75 \text{ cm} = 29.527\dots \text{ in.}$$

$$75 \text{ cm} \approx 30 \text{ in.}$$

$$12 \text{ in.} = 1 \text{ ft.}$$

$$\text{So, } 30 \text{ in.} = \frac{30}{12} \text{ ft.}$$

$$30 \text{ in.} = 2\frac{6}{12} \text{ ft.}$$

$$\text{So, } 75 \text{ cm} \approx 2 \text{ ft. } 6 \text{ in.}$$

ii) $91.44 \text{ cm} = 1 \text{ yd.}$

$$\text{So, } 274 \text{ cm} = \frac{274}{91.44} \text{ yd.}$$

$$274 \text{ cm} = 2.996\dots \text{ yd.}$$

$$274 \text{ cm} \approx 3 \text{ yd.}$$

iii) $10\,000 \text{ m} = 10 \text{ km}$

$$1 \text{ km} \approx \frac{6}{10} \text{ mi.}$$

$$\text{So, } 10 \text{ km} \approx 10\left(\frac{6}{10} \text{ mi.}\right)$$

$$10 \text{ km} \approx 6 \text{ mi.}$$

b) Answers may vary.

i) To check:

$$1 \text{ ft.} \approx 30 \text{ cm}$$

$$2 \text{ ft.} \approx 60 \text{ cm}$$

$$3 \text{ ft.} \approx 90 \text{ cm}$$

Since 75 cm is halfway between 60 cm and 90 cm, then 75 cm is halfway between 2 ft. and 3 ft., which is 2 ft. 6 in.

So, the answer is reasonable.

ii) To check:
 $274 \text{ cm} = 2.74 \text{ m}$
 Since $1 \text{ m} \doteq 1 \text{ yd.}$,
 then $2.74 \text{ m} \doteq 2.74 \text{ yd.}$
 So, the answer is reasonable.

iii) To check:
 $1 \text{ mi.} \doteq 1.6 \text{ km}$
 Since $5 \text{ mi.} = (5 \times 1.6 \text{ km})$
 $\quad \quad \quad = 9 \text{ km}$
 then 6 mi. is a bit more than 9 km.
 So, the answer is reasonable.

8. Convert 110 yd. and 60 yd. to metres.

$1 \text{ yd.} = 91.44 \text{ cm, or } 0.9144 \text{ m}$

So, $110 \text{ yd.} = 110(0.9144 \text{ m})$

$110 \text{ yd.} = 100.584 \text{ m}$

To the nearest tenth of a metre, $110 \text{ yd.} \doteq 100.6 \text{ m}$

Then, $60 \text{ yd.} = 60(0.9144 \text{ m})$

$60 \text{ yd.} = 54.864 \text{ m}$

To the nearest tenth of a metre, $60 \text{ yd.} \doteq 54.9 \text{ m}$

The dimensions of a lacrosse field are approximately 100.6 m by 54.9 m.

9. To compare distances, convert one measurement so the units are the same.

$1 \text{ mi.} \doteq 1.6 \text{ km}$

So, $886 \text{ mi.} \doteq 886(1.6 \text{ km})$

$886 \text{ mi.} \doteq 1417.6 \text{ km}$

Since $1417.6 \text{ km} > 1375 \text{ km}$, the Tennessee River is longer than the Fraser River.

10. Convert 87 mi. to kilometres.

$1 \text{ mi.} \doteq 1.6 \text{ km}$

So, $87 \text{ mi.} \doteq 87(1.6) \text{ km}$

$87 \text{ mi.} \doteq 139.2 \text{ km}$

139.2 km is close to 142 km , so the odometer is probably accurate.

11. a) Convert one measurement so the units are the same.

Use the conversion: $1 \text{ yd.} = 91.44 \text{ cm, or } 0.9144 \text{ m.}$

$$\frac{\$0.89}{1 \text{ yd.}} = \frac{\$0.89}{0.9144 \text{ m}} \quad \text{Divide numerator and denominator by } 0.9144.$$

$$\frac{\$0.89}{1 \text{ yd.}} \doteq \frac{\$0.9733...}{1 \text{ m}}$$

$$\frac{\$0.89}{1 \text{ yd.}} \doteq \$0.97/\text{m}$$

Since $\$0.93/\text{m} < \$0.97/\text{m}$, the warehouse has the better price.

b) 1 yd. is approximately 90 cm , or 0.9 m.

$\$0.89/\text{m}$ is close to $\$0.90/\text{m.}$

So, the cost per metre is approximately: $\frac{\$0.90}{0.90 \text{ m}} = \$1/\text{m}$

\$1/m is close to \$0.97/m; so, my answer is reasonable.

- 12. a)** To compare distances, convert one measurement so the units are the same.

Convert the distance Jean-Luc ran to metres.

Use the conversion: 1 yd. = 0.9144 m

So, 400 yd. = 400(0.9144 m)

400 yd. = 365.76 m

Jean-Luc ran two laps, or: 2(365.76 m) = 731.52 m

Michael ran: 7(110 m) = 770 m

Since 770 m > 731.52 m, Michael ran farther.

- b)** Write a conversion factor for yards and metres, with metres in the numerator: $\frac{0.9144 \text{ m}}{1 \text{ yd.}}$

$$\text{So, } 400 \text{ yd.} \times \frac{0.9144 \text{ m}}{1 \text{ yd.}} = \frac{400 \cancel{\text{ yd.}}}{1} \times \frac{0.9144 \text{ m}}{1 \cancel{\text{ yd.}}}$$

$$= 400(0.9144 \text{ m})$$

$$= 365.76 \text{ m}$$

Since this measurement is equal to the measurement in part a, the conversion is verified.

- 13. a)** To determine the height of the CN Tower in feet, convert metres to inches, then convert inches to feet.

1 in. = 2.54 cm, or 0.0254 m

So, to convert 553.3 m to inches, divide:

$$553.3 \text{ m} = \frac{553.3}{0.0254} \text{ in.}$$

$$553.3 \text{ m} = 21\,783.464\dots \text{ in.}$$

Then, convert inches to feet. Use the conversion: 12 in. = 1 ft.

So, to convert 21 783.464... in. to feet, divide:

$$21\,783.464\dots \text{ in.} = \frac{21\,783.464\dots}{12} \text{ ft.}$$

$$21\,783.464\dots \text{ in.} \div 12 = 1815.288\dots \text{ ft.}$$

$$21\,783.464\dots \text{ in.} \div 12 = 1815 \text{ ft.}$$

To determine the height of the Willis Tower in metres, convert feet to inches, then inches to metres.

1 ft. = 12 in.

1451 ft. = 12(1451) in.

1451 ft. = 17 412 in.

Since 1 in. = 2.54 cm, 1 in. = 0.0254 m

So, 17 412 in. = 17 412(0.0254 m)

17 412 in. = 442.264... m

17 412 in. $\div$ 12 = 442.3 m

- b) Compare the heights in the same units.
 Since $553.3 \text{ m} > 442.3 \text{ m}$, the CN Tower is taller.
 Or, since $1815 \text{ ft.} > 1451 \text{ ft.}$, the CN Tower is taller.
- c) To determine the difference in heights, subtract.
 $553.3 \text{ m} - 442.3 \text{ m} = 111 \text{ m}$
 $1815 \text{ ft.} - 1451 \text{ ft.} = 364 \text{ ft.}$
 So, the difference in heights is approximately 111 m, or 364 ft.

14. Determine the length of a section of casing in metres.

To convert feet to metres, first convert feet to inches, then convert inches to metres.

Use the conversion: $1 \text{ ft.} = 12 \text{ in.}$

So, $32 \text{ ft.} = 32(12 \text{ in.})$

$32 \text{ ft.} = 384 \text{ in.}$

Then, use the conversion: $1 \text{ in.} = 2.54 \text{ cm}$

So, $384 \text{ in.} = 384(2.54 \text{ cm})$

$384 \text{ in.} = 975.36 \text{ cm}$ Divide by 100 to convert centimetres to metres.

$$384 \text{ in.} = \frac{975.36}{100} \text{ m}$$

$384 \text{ in.} = 9.7536 \text{ m}$

To determine how many sections of casings are needed, divide the distance to the oil reserve by the length of one casing.

$$\frac{1400 \text{ m}}{9.7536 \text{ m}} = 143.5367\dots$$

So, 144 sections of casing are needed.

C

15. Determine the height of the basketball net in metres.

Convert feet to inches, then inches to metres.

Use the conversions: $1 \text{ ft.} = 12 \text{ in.}$ and $1 \text{ in.} = 2.54 \text{ cm}$

So, $10 \text{ ft.} = 10(12)(2.54 \text{ cm})$

$10 \text{ ft.} = 304.8 \text{ cm}$ Divide by 100 to convert centimetres to metres.

$$10 \text{ ft.} = \frac{304.8 \text{ cm}}{100}$$

$10 \text{ ft.} = 3.048 \text{ m}$

Since the player has a maximum reach of 2.5 m, he has to jump $3.048 \text{ m} - 2.5 \text{ m} = 0.548 \text{ m}$ to reach the rim.

To reach 6 in. above the rim, the player has to jump an extra 6 in.

Convert 0.548 m to inches.

Use the conversion: $1 \text{ in.} = 2.54 \text{ cm}$, or 0.0254 m

So, to convert 0.548 m to inches, divide:

$$0.548 \text{ m} = \frac{0.548}{0.0254} \text{ in.}$$

$0.548 \text{ m} = 21.5748\dots \text{ in.}$

So, the player has to jump: $21.5748\dots + 6 \text{ in.} = 27.5748\dots$

The player has to jump approximately 28 in. to reach 6 in. above the rim.

16. The electrician purchased $2(4 \text{ m}) = 8 \text{ m}$ of wire.

The total length needed is: $2(2 \text{ ft.}) + 2(11 \text{ ft.}) = 4 \text{ ft.} + 22 \text{ ft.}$
 $= 26 \text{ ft.}$

Convert the length of wire needed to metres.

To convert feet to metres, first convert feet to inches, then convert inches to metres.

Use the conversion: $1 \text{ ft.} = 12 \text{ in.}$ and $1 \text{ in.} = 2.54 \text{ cm}$

So, $26 \text{ ft.} = 26(12)(2.54 \text{ cm})$

$26 \text{ ft.} = 792.48 \text{ cm}$

Divide by 100 to convert centimetres to metres.

$26 \text{ ft.} = \frac{792.48}{100} \text{ m}$

$26 \text{ ft.} = 7.9248 \text{ m}$

Since $8 \text{ m} > 7.9248 \text{ m}$, the electrician will have enough wire.

To determine the amount of wire left over, subtract:

$8 \text{ m} - 7.9248 \text{ m} = 0.0752 \text{ m}$, or 7.52 cm

So, the electrician will have approximately 8 cm of wire left over.

17. Each mobile home requires a width of: $14 \text{ ft.} + 8 \text{ ft.} = 22 \text{ ft.}$

Think: How many mobile homes would fit in 50 m?

Convert 50 m to feet. Use the conversion: $1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$

So, $50 \text{ m} \doteq 50(3\frac{1}{4} \text{ ft.})$

$50 \text{ m} \doteq 162.5 \text{ ft.}$

Divide to determine how many mobile homes would fit in 162.5 ft.

$\frac{162.5}{22} = 7.386\dots$

So, the maximum number of homes the developer can fit on the land is 7.

18. a) Use the conversion: $1 \text{ hectare} \doteq 2.471 \text{ acres}$

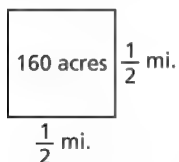
To convert 160 acres to hectares, divide:

$160 \text{ acres} \doteq \frac{160}{2.471} \text{ hectares}$

$160 \text{ acres} \doteq 64.751\dots \text{ hectares}$

Each settler received approximately 65 hectares.

- b) Sketch a square with side length $\frac{1}{2} \text{ mi.}$



Sketch one square mile:

	$\frac{1}{2}$ mi.	$\frac{1}{2}$ mi.
$\frac{1}{2}$ mi.	160 acres	160 acres
$\frac{1}{2}$ mi.	160 acres	160 acres

So, one square mile has $4(160 \text{ acres}) = 640 \text{ acres}$.

Convert 640 acres to hectares. Use the conversion: $1 \text{ hectare} \doteq 2.471 \text{ acres}$

To convert 640 acres to hectares, divide.

$$640 \text{ acres} \doteq \frac{640}{2.471} \text{ hectares}$$

$$640 \text{ acres} \doteq 259.004... \text{ hectares}$$

So, there are approximately 259 hectares in one square mile.

Checkpoint 1 Assess Your Understanding

(page 25)

1.1

1. Answers will vary.

- a) Inch; I used my eraser, which is approximately 2 in. long. The distance from my elbow to my wrist is about 5 erasers, or about 10 in.
- b) Inch; I used the length of my thumb to the first knuckle, which is approximately 1 in. The length of a cell phone is about 4 thumb lengths, or about 4 in.
- c) Foot; I used the length of my slipper, which is approximately 1 ft. long. The width of the bed is about $3\frac{1}{2}$ slippers, or about 3 ft. 6 in.
- d) Foot; I used the height of the hockey net as a referent, which is about 3 ft. I estimated that the gymnasium is about 6 hockey nets high, which is about 18 ft.

2. Answers will vary.

a) $9\frac{3}{16}$ in.

b) $4\frac{1}{16}$ in.

3. a) 3 ft. = 1 yd.

To convert feet to yards, divide:

$80 \text{ ft.} = \frac{80}{3} \text{ yd.}$ Write this improper fraction as a mixed number.

$80 \text{ ft.} = 26\frac{2}{3} \text{ yd.}$

$80 \text{ ft.} = 26 \text{ yd. } 2 \text{ ft.}$

b) 1 mi. = 1760 yd.

So, 3 mi. = 3(1760 yd.)

3 mi. = 5280 yd.

c) First, convert yards to inches. Use the conversion: 1 yd. = 36 in.

So, 2 yd. = 2(36 in.)

2 yd. = 72 in.

Then, convert feet to inches. Use the conversion: 1 ft. = 12 in.

So, 1 ft. = 12 in.

Add. 2 yd. 1 ft. = 72 in. + 12 in.

2 yd. 1 ft. = 84 in.

4. To compare the heights, convert one measurement so the units are the same.

Convert 3 ft. 11 in. to inches.

Use the conversion: 1 ft. = 12 in.



So, 3 ft. = 3(12 in.)
 3 ft. = 36 in.
 So, 3 ft. 11 in. = 36 in. + 11 in.
 3 ft. 11 in. = 47 in.
 Since 47 in. < 51 in., Sidney is shorter.

1.2

5. Answers and measurements will vary. For example:

a) 25.4 m; 27 yd. 2 ft. 4 in.

b) 143 cm; 4 ft. 8 in.

c) 77 cm; 30 in.

6. I would use calipers. I would use the unit marked on the calipers.

1.3

7. Answers will vary depending on the conversion ratios used.

a) 1 yd. = 91.44 cm, or 0.9144 m

To convert metres to yards, divide:

$$13 \text{ m} = \frac{13}{0.9144} \text{ yd.}$$

$$13 \text{ m} = 14.2169... \text{ yd.}$$

$$13 \text{ m} \approx 14.2 \text{ yd.}$$

$$1 \text{ yd.} = 3 \text{ ft.}$$

$$\text{So, } 0.2 \text{ yd.} = 0.2(3 \text{ ft.})$$

$$0.2 \text{ yd.} = 0.6 \text{ ft.}$$

$$\text{So, to the nearest foot, } 13 \text{ m} = 14 \text{ yd. } 1 \text{ ft.}$$

b) 1 ft. = 12 in. and 1 in. = 2.54 cm

$$4 \text{ ft.} = 4(12 \text{ in.})$$

$$4 \text{ ft.} = 48 \text{ in.}$$

$$48 \text{ in.} = 48(2.54 \text{ cm})$$

$$\text{So, } 48 \text{ in.} = 121.92 \text{ cm}$$

$$4 \text{ ft.} \approx 122 \text{ cm}$$

c) 2 km = 2000 m

Use the conversion: 1 yd. = 91.44 cm, or 0.9144 m

$$\text{So, } 2000 \text{ m} = \frac{2000}{0.9144} \text{ yd.}$$

$$2000 \text{ m} = 2187.226... \text{ yd.}$$

$$1 \text{ mi.} = 1760 \text{ yd.}$$

$$\text{Subtract: } 2187.226... \text{ yd.} - 1760 \text{ yd.} = 427.226... \text{ yd.}$$

$$\text{So, } 2 \text{ km} \approx 1 \text{ mi. } 427 \text{ yd.}$$

d) 91.44 cm = 1 yd.

$$\text{So, } 25\,000 \text{ cm} = \frac{25\,000}{91.44} \text{ yd.}$$

$$25\,000\text{ cm} = 273.403\dots\text{ yd.}$$

$$1\text{ yd.} = 3\text{ ft.}$$

$$\text{So, } 0.403\dots\text{ yd.} = (0.403\dots)(3)\text{ ft.}$$

$$0.403\dots\text{ yd.} = 1.209\dots\text{ ft.}$$

$$1\text{ ft.} = 12\text{ in.}$$

$$0.209\dots\text{ ft.} = (0.209\dots)(12)\text{ in.}$$

$$0.209\dots\text{ ft.} = 2.519\dots\text{ in.}$$

$$\text{So, } 25\,000\text{ cm} = 273\text{ yd. } 1\text{ ft. } 3\text{ in.}$$

e) $1\text{ in.} = 2.54\text{ cm, or } 0.0254\text{ m}$

$$\text{So, } 13\,000\text{ in.} = (13\,000)(0.0254\text{ m})$$

$$13\,000\text{ in.} = (13\,000)(0.0254\text{ m})$$

$$13\,000\text{ in.} = 330.2\text{ m}$$

f) $1750\text{ mm} = 175\text{ cm}$

$$2.54\text{ cm} = 1\text{ in.}$$

$$\text{So, } 175\text{ cm} = \frac{175}{2.54}\text{ in.}$$

$$175\text{ cm} = 68.897\dots\text{ in.}$$

$$12\text{ in.} = 1\text{ ft.}$$

$$\text{So, } 68.897\dots\text{ in.} = \frac{68.897\dots}{12}\text{ ft.}$$

$$68.897\dots\text{ in.} = 5.741\dots\text{ ft.}$$

$$0.741\dots\text{ ft.} = 12(0.741\dots)\text{ in.}$$

$$0.741\dots\text{ ft.} = 0.889\dots\text{ ft.}$$

$$\text{So, } 1750\text{ mm} \doteq 5\text{ ft. } 9\text{ in.}$$

8. The perimeter of the table is:

$$P = 2(30\text{ cm}) + 2(115\text{ cm})$$

$$P = 60\text{ cm} + 230\text{ cm}$$

$$P = 290\text{ cm}$$

$$2.54\text{ cm} = 1\text{ in.}$$

$$\text{So, } 290\text{ cm} = \frac{290}{2.54}\text{ in.}$$

$$290\text{ cm} = 114.173\dots\text{ in.}$$

$$1\text{ ft.} = 12\text{ in.}$$

$$114.173\dots\text{ in.} = \frac{114.173\dots}{12}\text{ ft.}$$

$$114.173\dots\text{ in.} = 9.514\dots\text{ ft.}$$

$$0.514\dots\text{ ft.} = 12(0.514\dots)\text{ in.}$$

$$0.514\dots\text{ ft.} = 6.168\dots\text{ in.}$$

So, $290 \text{ cm} \doteq 9 \text{ ft. } 6 \text{ in.}$
10 ft. of laminate is needed.

Lesson 1.4 **Surface Areas of Right Pyramids and
Right Cones**

Exercises (pages 34–35)

A

- 4. a)** Use the formula for the lateral area, A_L , of a right pyramid with a regular polygon base and slant height, s :

$$A_L = \frac{1}{2} s(\text{perimeter of base})$$

When the base is a square with side length l :

$$A_L = \frac{1}{2} s(4l) \quad \text{Substitute: } s = 11, l = 6$$

$$A_L = \frac{1}{2} (11)(4)(6)$$

$$A_L = (5.5)(24)$$

$$A_L = 132$$

The lateral area of the square pyramid is 132 square inches.

- b)** Use the formula for the lateral area, A_L , of a right pyramid with a regular polygon base and slant height, s :

$$A_L = \frac{1}{2} s(\text{perimeter of base})$$

When the base is an equilateral triangle with side length l ,

$$A_L = \frac{1}{2} s(3l) \quad \text{Substitute: } s = 11.3, l = 13$$

$$A_L = \frac{1}{2} (11.3)(3)(13)$$

$$A_L = (5.65)(39)$$

$$A_L = 220.35$$

The lateral area of the regular tetrahedron is approximately 220 cm².

- 5. a)** Use the formula for surface area of a right pyramid with a regular polygon base and slant height s :

$$\text{Surface area} = \frac{1}{2} s(\text{perimeter of base}) + (\text{base area})$$

When the base is a square with side length l ,

$$\text{Surface area} = \frac{1}{2} s(4l) + l^2 \quad \text{Substitute: } s = 11, l = 6$$

$$\text{Surface area} = \frac{1}{2} (11)(4)(6) + (6)^2$$

$$\text{Surface area} = 132 + 36$$

$$\text{Surface area} = 168$$

The surface area of the square pyramid is 168 square inches.

- b)** Use the formula for surface area of a right pyramid with a regular polygon base and slant height s :

$$\text{Surface area} = \frac{1}{2} s(\text{perimeter of base}) + (\text{base area})$$

When the base is an equilateral triangle with side length l ,

$$\text{Surface area} = \frac{1}{2}s(3l) + \frac{1}{2}s(l) \quad \text{Substitute: } s = 11.3, l = 13.0$$

$$\text{Surface area} = \frac{1}{2}(11.3)(3)(13.0) + \frac{1}{2}(13)(11.3)$$

$$\text{Surface area} = 220.35 + 73.45$$

$$\text{Surface area} = 293.8$$

The surface area of the regular tetrahedron is approximately 294 cm^2 .

6. a) Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s .

$$A_L = \pi rs \quad \text{Substitute: } r = 4, s = 8$$

$$A_L = \pi(4)(8)$$

$$A_L = 100.5309\dots$$

The lateral area of the right cone is approximately 101 square inches.

- b) Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s .

$$A_L = \pi rs \quad \text{Substitute: } r = 15, s = 35$$

$$A_L = \pi(15)(35)$$

$$A_L = 1649.3361\dots$$

The lateral area of the right cone is approximately 1649 cm^2 .

7. a) Use the formula for the surface area of a right cone with base radius r and slant height s .

$$\text{Surface area} = \pi rs + \pi r^2 \quad \text{Substitute: } r = 4, s = 8$$

$$\text{Surface area} = \pi(4)(8) + \pi(4)^2$$

$$\text{Surface area} = 150.7964\dots$$

The surface area of the right cone is approximately 151 square inches.

- b) Use the formula for the surface area of a right cone with base radius r and slant height s .

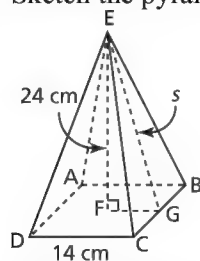
$$\text{Surface area} = \pi rs + \pi r^2 \quad \text{Substitute: } r = 15, s = 35$$

$$\text{Surface area} = \pi(15)(35) + \pi(15)^2$$

$$\text{Surface area} = 2356.1944\dots$$

The surface area of the right cone is approximately 2356 cm^2 .

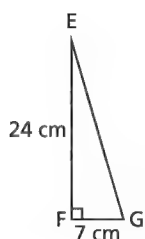
8. a) Sketch the pyramid and label its vertices.



Determine the slant height, s .

In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is 7 cm.

EF is the height of the pyramid, which is 24 cm.



Use the Pythagorean Theorem in right $\triangle EFG$.

$$EG^2 = EF^2 + FG^2 \quad \text{Substitute: } EF = 24, FG = 7$$

$$EG^2 = 24^2 + 7^2$$

$$EG^2 = 625$$

$$EG = \sqrt{625}$$

$$EG = 25$$

So, the slant height, s , is 25 cm.

Use the formula for the surface area of a right square pyramid with base side length l and slant height s .

$$\text{Surface area} = \frac{1}{2}s(4l) + l^2 \quad \text{Substitute: } s = 25, l = 14$$

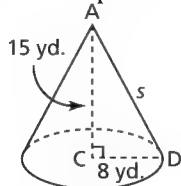
$$\text{Surface area} = \frac{1}{2}(25)(4)(14) + (14)^2$$

$$\text{Surface area} = 700 + 196$$

$$\text{Surface area} = 896$$

The surface area of the right square pyramid is 896 cm^2 .

- b) Let s represent the slant height. Label the cone.



Use the Pythagorean Theorem in right $\triangle ACD$.

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 15^2 + 8^2$$

$$s^2 = 289$$

$$s = \sqrt{289}$$

$$s = 17$$

Use the formula for the surface area of a right cone with base radius r and slant height s .

$$\text{Surface area} = \pi rs + \pi r^2 \quad \text{Substitute: } r = 8, s = 17$$

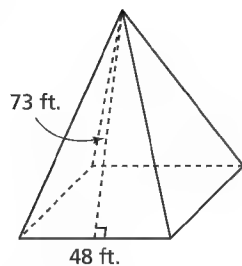
$$\text{Surface area} = \pi(8)(17) + \pi(8)^2$$

$$\text{Surface area} = 628.3185\dots$$

The surface area of the right cone is approximately 628 square yards.

B

9. a)



- b) Use the formula for the lateral area, A_L , of a right square pyramid with base side length l and slant height s .

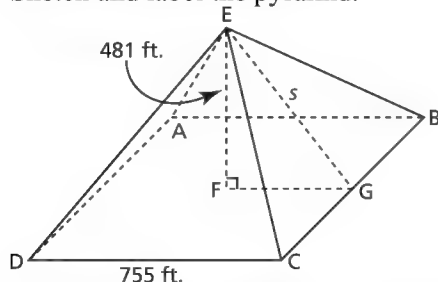
$$A_L = \frac{1}{2}s(4l) \quad \text{Substitute: } s = 73, l = 48$$

$$A_L = \frac{1}{2}(73)(4)(48)$$

$$A_L = 7008$$

The lateral area of the pyramid is 7008 square feet.

10. Sketch and label the pyramid.



Determine the slant height, s . Use the Pythagorean Theorem in right $\triangle EFG$.

FG is $\frac{1}{2}$ the length of DC . So, FG is $\frac{1}{2}(755 \text{ ft.})$, or $377\frac{1}{2} \text{ ft.}$

$$EG^2 = EF^2 + FG^2$$

$$s^2 = 481^2 + 377.5^2$$

$$s^2 = 373\,867.25$$

$$s = \sqrt{373\,867.25}$$

$$s = 611.4468\dots$$

Use the formula for surface area. Do not include the base, as it is not exposed.

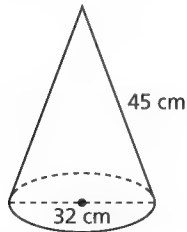
$$\text{Surface area} = \frac{1}{2}s(\text{perimeter of base})$$

$$\text{Surface area} = \frac{1}{2}(611.4468\dots)(4)(755)$$

$$\text{Surface area} = 923\,284.7430\dots$$

The original surface area of the Great Pyramid at Giza was approximately 923 285 square feet.

11. Sketch the cone.



- a) Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s :

$$A_L = \pi rs \quad \text{Substitute: } r = 16, s = 45$$

$$A_L = \pi(16)(45)$$

$$A_L = 2261.9467\dots$$

The lateral area of the volcano is approximately 2261.9 cm^2 .

- b) To determine the number of jars needed, divide the total area by the area covered by one jar:

$$\frac{2261.9 \text{ cm}^2}{400 \text{ cm}^2} \doteq 5.7$$

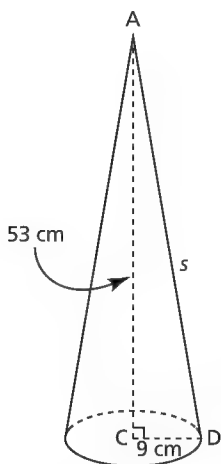
Since Aiden cannot buy part of a jar of paint, 6 jars are needed.

The total cost is: $6(\$1.99)$, or $\$11.94$

The paint will cost $\$11.94$.

12. Sketch and label a diagram. Let s represent the slant height of the cone.

The radius of the cone is 9 cm.



Use the Pythagorean Theorem in right $\triangle ACD$ to determine the slant height.

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 53^2 + 9^2$$

$$s^2 = 2890$$

$$s = \sqrt{2890}$$

Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s :

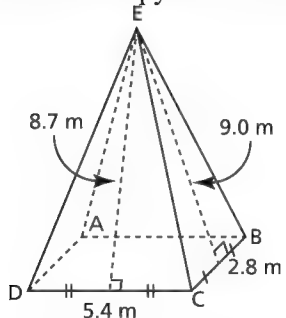
$$A_L = \pi rs \quad \text{Substitute: } r = 9, s = \sqrt{2890}$$

$$A_L = \pi(9)(\sqrt{2890})$$

$$A_L = 1519.9920\dots$$

The area to be painted is approximately 1520 cm^2 .

13. a) Sketch the pyramid and label its vertices.



Area, A , of $\triangle EDC$ is:

$$A = \frac{1}{2}(5.4)(8.7)$$

$$A = 23.49$$

Since $\triangle EDC$ and $\triangle EAB$ are congruent, the area of $\triangle EAB$ is 23.49 m^2 .

Area, A , of $\triangle EBC$ is:

$$A = \frac{1}{2}(2.8)(9.0)$$

$$A = 12.6$$

Since $\triangle EBC$ and $\triangle EAD$ are congruent, the area of $\triangle EAD$ is 12.6 m^2 .

Area, B , of the base of the pyramid is:

$$B = (2.8)(5.4)$$

$$B = 15.12$$

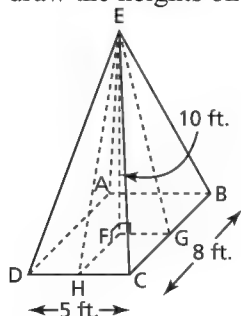
The surface area, SA , of the pyramid is:

$$SA = 2(23.49) + 2(12.6) + 15.12$$

$$SA = 87.3$$

The surface area of the right rectangular pyramid is approximately 87 m^2 .

- b) Sketch the pyramid and label its vertices. Since opposite triangular faces are congruent, draw the heights on two adjacent triangles. Determine these heights.



In $\triangle EFH$, FH is $\frac{1}{2}$ the length of BC , so FH is 4 ft.

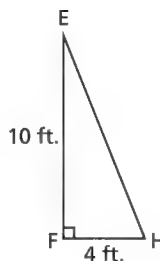
Use the Pythagorean Theorem in right $\triangle EFH$.

$$EH^2 = EF^2 + FH^2$$

$$EH^2 = 10^2 + 4^2$$

$$EH^2 = 116$$

$$EH = \sqrt{116}$$



Area, A , of $\triangle EDC$ is:

$$A = \frac{1}{2}(5)(\sqrt{116})$$

$$A = 2.5(\sqrt{116})$$

Since $\triangle EDC$ and $\triangle EAB$ are congruent, the area of $\triangle EAB$ is $2.5(\sqrt{116})$.

In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is $2\frac{1}{2}$ ft.

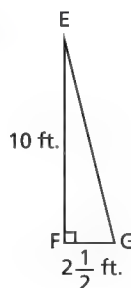
Use the Pythagorean Theorem in right $\triangle EFG$.

$$GE^2 = EF^2 + FG^2$$

$$GE^2 = 10^2 + 2.5^2$$

$$GE^2 = 106.25$$

$$GE = \sqrt{106.25}$$



Area, A , of $\triangle EBC$ is:

$$A = \frac{1}{2}(8)(\sqrt{106.25})$$

$$A = 4(\sqrt{106.25})$$

Since $\triangle EBC$ and $\triangle EAD$ are congruent, the area of $\triangle EAD$ is $4(\sqrt{106.25})$.

Area, B , of the base of the pyramid is:

$$B = (8)(5)$$

$$B = 40$$

Surface area, SA , of the pyramid is:

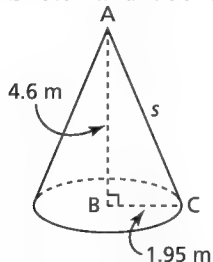
$$SA = 2(2.5)(\sqrt{116}) + 2(4)(\sqrt{106.25}) + 40$$

$$SA = 176.3137\dots$$

The surface area of the right rectangular pyramid is approximately 176 square feet.

14. The tipi approximates a right cone with radius 1.95 m.

Sketch and label a diagram. Let s represent the slant height.



Use the Pythagorean Theorem in right $\triangle ABC$.

$$AC^2 = AB^2 + BC^2$$

$$s^2 = 4.6^2 + 1.95^2$$

$$s^2 = 24.9625$$

$$s = \sqrt{24.9625}$$

Since the base is not covered, use the formula for the lateral area of a right cone with base radius r and slant height s .

$$A_L = \pi rs \quad \text{Substitute: } r = 1.95, s = \sqrt{24.9625}$$

$$A_L = \pi(1.95)(\sqrt{24.9625})$$

$$A_L = 30.6075\dots$$

Assume the bison hides have equal areas. Then,

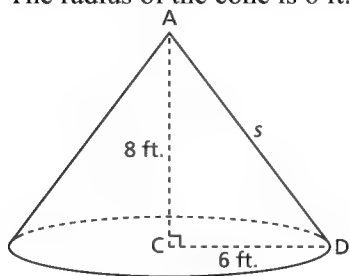
$$\text{Area of 1 hide} = \frac{\text{total area}}{\text{number of hides}}$$

$$\text{Area of 1 hide} = \frac{30.6075...}{15}$$

$$\text{Area of 1 hide} = 2.0405...$$

Each bison hide covers approximately 2.0 m^2 .

15. Sketch and label a diagram. Let s represent the slant height.
The radius of the cone is 6 ft.



Use the Pythagorean Theorem in right $\triangle ACD$.

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 8^2 + 6^2$$

$$s^2 = 64 + 36$$

$$s^2 = 100$$

$$s = \sqrt{100}$$

$$s = 10$$

Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s :

$$A_L = \pi rs \quad \text{Substitute: } r = 6, s = 10$$

$$A_L = \pi (6)(10)$$

$$A_L = 188.4955...$$

The surface area of the exposed grain is approximately 188 square feet.

16. a) Use the formula for the surface area, SA , of a right cone with base radius r and slant height s .

The radius of the cone is 24 mm.

$$SA = \pi rs + \pi r^2$$

$$\text{Substitute: } SA = 7012 \text{ and } r = 24$$

$$7012 = \pi (24)s + \pi (24)^2$$

$$\text{Solve for } s.$$

$$7012 = 24\pi s + 576\pi$$

$$\text{Subtract } 576\pi \text{ from each side.}$$

$$7012 - 576\pi = 24\pi s$$

$$\text{Divide each side by } 24\pi.$$

$$\frac{7012 - 576\pi}{24\pi} = \frac{24\pi s}{24\pi}$$

$$s = \frac{7012 - 576\pi}{24\pi}$$

$$s = 68.9995...$$

s is approximately 69.0 mm.

- b) Use the formula for the surface area, SA , of a right square pyramid with base side length l and slant height s .

$$SA = \frac{1}{2}s(4l) + l^2 \quad \text{Substitute: } SA = 65.5, l = 3.5$$

$$65.5 = \frac{1}{2}s(4)(3.5) + 3.5^2 \quad \text{Solve for } s.$$

$$65.5 = 7s + 12.25$$

$$65.5 - 12.25 = 7s$$

$$53.25 = 7s$$

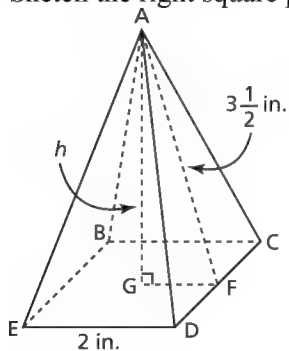
$$\frac{53.25}{7} = \frac{7s}{7}$$

$$s = \frac{53.25}{7}$$

$$s = 7.6071\dots$$

s is approximately 7.6 m.

17. a) Determine the height, h , of each block.
Sketch the right square pyramid and label its vertices.



In $\triangle AGF$, GF is $\frac{1}{2}$ the length of ED , so GF is 1 in.

Use the Pythagorean Theorem in right $\triangle AGF$.

$$AG^2 = AF^2 - GF^2$$

$$h^2 = 3.5^2 - 1^2$$

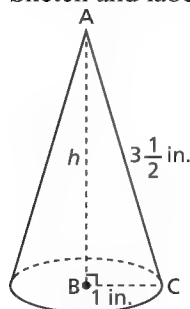
$$h^2 = 11.25$$

$$h = \sqrt{11.25}$$

$$h = 3.3541\dots$$

The height of the right square pyramid is approximately $3\frac{4}{10}$ in., or $3\frac{2}{5}$ in.

Sketch and label the right cone.



Use the Pythagorean Theorem in right $\triangle ABC$.

$$AB^2 = AC^2 - BC^2$$

$$h^2 = 3.5^2 - 1^2$$

$$h^2 = 11.25$$

$$h = \sqrt{11.25}$$

$$h = 3.3541\dots$$

The height of the right cone is the same as the height of the right square pyramid,

approximately $3\frac{4}{10}$ in., or $3\frac{2}{5}$ in.

Compare the heights.

The right square pyramid and right cone are the tallest blocks.

- b) Determine the surface area of each block.

Use the formula for the surface area, SA , of a right square pyramid with base side length l and slant height s :

$$SA = \frac{1}{2}s(4l) + l^2 \quad \text{Substitute: } s = 3.5, l = 2$$

$$SA = \frac{1}{2}(3.5)(4)(2) + 2^2$$

$$SA = 14 + 4$$

$$SA = 18$$

The surface area of the right square pyramid is 18 square inches.

Use the formula for the surface area, SA , of a right cone with base radius r and slant height s :

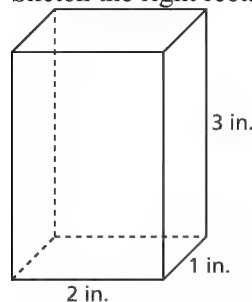
$$SA = \pi rs + \pi r^2 \quad \text{Substitute: } r = 1, s = 3.5$$

$$SA = \pi(1)(3.5) + \pi(1)^2$$

$$SA = 14.1371\dots$$

The surface area of the right cone is approximately 14 square inches.

Sketch the right rectangular prism:



Its surface area, SA , is:

$$SA = 2 \times \text{area of top face} + 2 \times \text{area of front face} + 2 \times \text{area of side face}$$

$$SA = 2(2 \times 1) + 2(2 \times 3) + 2(3 \times 1)$$

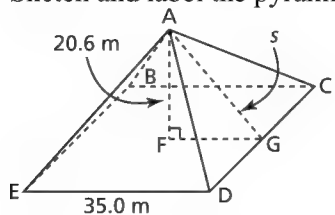
$$SA = 4 + 12 + 6$$

$$SA = 22$$

The surface area of the right rectangular prism is 22 square inches.

Compare the surface areas. The right rectangular prism has the greatest surface area. So, it requires the most paint.

18. Determine the lateral area of each pyramid.
Sketch and label the pyramid at the Louvre.



In $\triangle AFG$, FG is $\frac{1}{2}$ the length of ED , so FG is 17.5 m.

Use the Pythagorean Theorem in right $\triangle AFG$.

$$AG^2 = AF^2 + FG^2$$

$$s^2 = 20.6^2 + 17.5^2$$

$$s^2 = 730.61$$

$$s = \sqrt{730.61}$$

Use the formula for the lateral area, A_L , of a right square pyramid with base side length l and slant height s . The base is not included, because it is not a glass face.

$$A_L = \frac{1}{2} s(4l) \quad \text{Substitute: } s = \sqrt{730.61}, l = 35.0$$

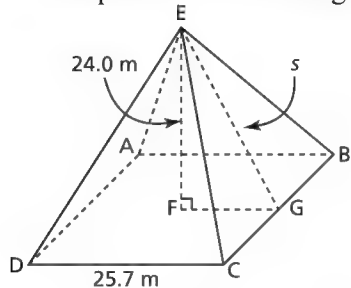
$$A_L = \frac{1}{2} (\sqrt{730.61})(4)(35.0)$$

$$A_L = 1892.0858\dots$$

The lateral area of the pyramid at the Louvre is approximately 1892 m².

Sketch the pyramid at the Muttart Conservatory.

Let s represent the slant height.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is 12.85 m.

Use the Pythagorean Theorem in right $\triangle EFG$.

$$s^2 = 24.0^2 + 12.85^2$$

$$s^2 = 741.1225$$

$$s = \sqrt{741.1225}$$

Use the formula for the lateral area, A_L , of a right square pyramid with base side length l and slant height s .

$$A_L = \frac{1}{2} s(4l) \quad \text{Substitute: } s = \sqrt{741.1225}, l = 25.7$$

$$A_L = \frac{1}{2}(\sqrt{741.1225})(4)(25.7)$$

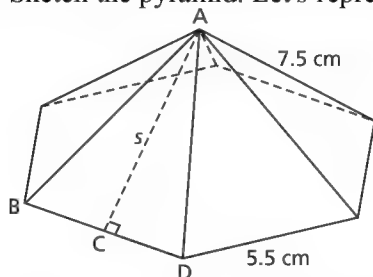
$$A_L = 1399.2912\dots$$

The lateral area of the pyramid at the Muttart Conservatory is approximately 1399 m^2 .

Compare the lateral areas. The pyramid at the Louvre requires more glass to enclose its space.

C

19. a) Sketch the pyramid. Let s represent the slant height.



Sketch and label one triangular face.

In $\triangle ACD$, CD is $\frac{1}{2}$ the length of BD , so CD is 2.75 cm.

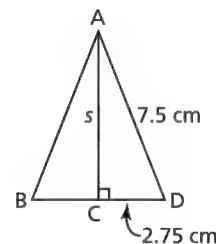
Use the Pythagorean Theorem in right $\triangle ACD$.

$$AC^2 = AD^2 - CD^2$$

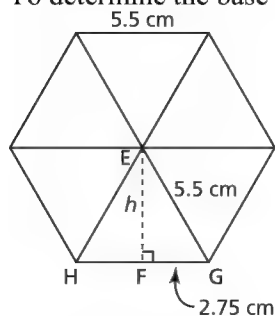
$$s^2 = 7.5^2 - 2.75^2$$

$$s^2 = 48.6875$$

$$s = \sqrt{48.6875}$$



To determine the base area, divide the hexagon into 6 equilateral triangles.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of HG , so FG is 2.75 cm.

Determine the area of one equilateral triangle. Use the Pythagorean Theorem in right $\triangle EFG$ to determine the height, h .

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 5.5^2 - 2.75^2$$

$$h^2 = 22.6875$$

$$h = \sqrt{22.6875}$$

So, the area, A , of one triangle:

$$A = \frac{1}{2}(\text{base})(\text{height})$$

$$A = \frac{1}{2}(5.5)(\sqrt{22.6875})$$

Multiply the area of one triangle by 6 to get the total area:

$$6 \times \frac{1}{2}(5.5)(\sqrt{22.6875}) = 3(5.5)(\sqrt{22.6875})$$

Use the formula for the surface area, SA , of a right regular pyramid:

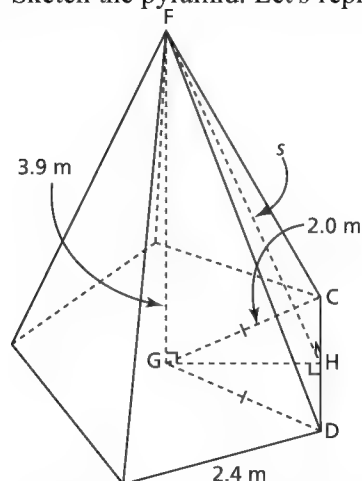
$$SA = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$$

$$SA = \frac{1}{2}(\sqrt{48.6875})(6)(5.5) + 3(5.5)(\sqrt{22.6875})$$

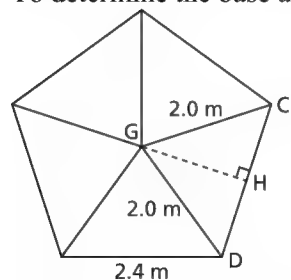
$$SA = 193.7229\dots$$

The surface area of the right pyramid is approximately 193.7 cm^2 .

- b) Sketch the pyramid. Let s represent the slant height.



To determine the base area, divide the pentagon into 5 isosceles triangles.



To determine the area of one of these triangles, we need to know its height GH .

In $\triangle GHC$, HC is $\frac{1}{2}$ the length of DC , so HC is 1.2 m .

Use the Pythagorean Theorem in $\triangle GHC$.

$$GH^2 = GC^2 - HC^2$$

$$GH^2 = 2.0^2 - 1.2^2$$

$$GH^2 = 2.56$$

$$GH = \sqrt{2.56}$$

$$GH = 1.6$$

The area, A , of one triangle is:

$$A = \frac{1}{2}(\text{base})(\text{height})$$

$$A = \frac{1}{2}(2.4)(1.6)$$

$$A = 1.92$$

So, the area of the base is: $5(1.92) = 9.6$

To determine the slant height, use the Pythagorean Theorem in right $\triangle FGH$ in the diagram of the pyramid.

$$FH^2 = FG^2 + GH^2$$

$$s^2 = 3.9^2 + 1.6^2$$

$$s^2 = 17.77$$

$$s = \sqrt{17.77}$$

Use the formula for the surface area of a right regular pyramid:

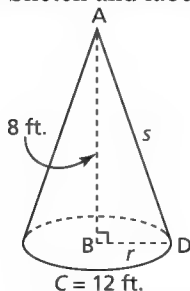
$$SA = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$$

$$SA = \frac{1}{2}(\sqrt{17.77})(5)(2.4) + 9.6$$

$$SA = 34.8926\dots$$

The surface area of the right pyramid is approximately 34.9 m^2 .

20. Sketch and label the cone. Let r represent the base radius.



Use the formula for the circumference, C , of a circle.

$$C = 2\pi r \quad \text{Substitute: } C = 12$$

$$12 = 2\pi r \quad \text{Solve for } r.$$

$$\frac{12}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{12}{2\pi}$$

$$r = 1.9098\dots$$

Use the Pythagorean Theorem in right $\triangle ABD$.

$$AD^2 = AB^2 + BD^2$$

$$s^2 = 8^2 + (1.9098\dots)^2$$

$$s^2 = 67.6475\dots$$

$$s = \sqrt{67.6475\dots}$$

$$s = 8.2248\dots$$

Use the formula for the surface area of a right cone with base radius r and slant height s :

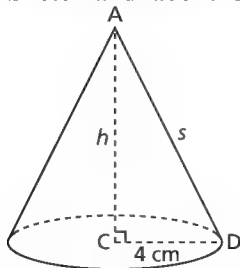
$$SA = \pi rs + \pi r^2 \quad \text{Substitute: } r = 1.9098\dots, s = 8.2248\dots$$

$$SA = \pi (1.9098\dots)(8.2248\dots) + \pi (1.9098\dots)^2$$

$$SA = 60.8080\dots$$

The surface area of the cone is approximately 61 square feet.

21. Sketch and label the cone. Let h represent the height.



Use the formula for the surface area of a right cone with base radius r and slant height s .

$$SA = \pi rs + \pi r^2$$

$$\text{Substitute: } SA = 258, r = 4$$

$$258 = \pi (4)s + \pi (4)^2$$

Solve for s .

$$258 = 4\pi s + 16\pi$$

$$258 - 16\pi = 4\pi s$$

$$\frac{258 - 16\pi}{4\pi} = \frac{4\pi s}{4\pi}$$

$$s = \frac{258 - 16\pi}{4\pi}$$

$$s = 16.5309\dots$$

Use the Pythagorean Theorem in right $\triangle ACD$ to determine the height, h , of the cone.

$$AC^2 = AD^2 - CD^2$$

$$h^2 = (16.5309\dots)^2 - 4^2$$

$$h^2 = 257.2735\dots$$

$$h = \sqrt{257.2735\dots}$$

$$h = 16.0397\dots$$

The height of the cone is approximately 16.0 cm.

Lesson 1.5 Volumes of Right Pyramids and
Right Cones

Exercises (pages 42–44)

A

4. a) Use the formula for the volume of a right rectangular prism.

$$V = lwh$$

$$\text{Substitute: } l = 6, w = 6, h = 8$$

$$V = (6)(6)(8)$$

$$V = 288$$

The volume of the prism is 288 cubic yards.

- b) Use the formula for the volume of a right rectangular prism.

$$V = lwh$$

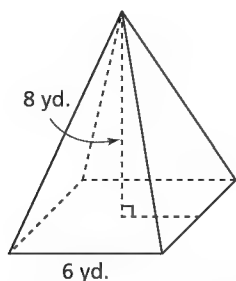
$$\text{Substitute: } l = 12, w = 10, h = 16$$

$$V = (12)(10)(16)$$

$$V = 1920$$

The volume of the prism is 1920 cubic feet.

5. a)



Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh$$

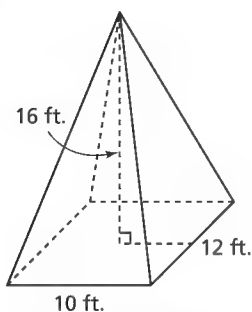
$$\text{Substitute: } l = 6, w = 6, h = 8$$

$$V = \frac{1}{3}(6)(6)(8)$$

$$V = 96$$

The volume of the pyramid is 96 cubic yards.

- b)



Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh$$

$$\text{Substitute: } l = 12, w = 10, h = 16$$

$$V = \frac{1}{3}(12)(10)(16)$$

$$V = 640$$

The volume of the pyramid is 640 cubic feet.

6. a) Use the formula for the volume of a right cylinder.

$$V = \pi r^2 h \quad \text{Substitute: } r = 5, h = 20$$

$$V = \pi (5)^2 (20)$$

$$V = 1570.7963\dots$$

The volume of the cylinder is approximately 1571 cm^3 .

- b) The radius, r , of the base of the cylinder is $\frac{1}{2}$ the diameter; so, r is 8 m.

Use the formula for the volume of a right cylinder.

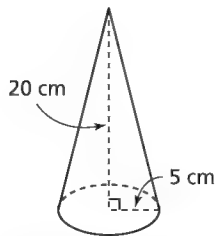
$$V = \pi r^2 h \quad \text{Substitute: } r = 8, h = 4$$

$$V = \pi (8)^2 (4)$$

$$V = 804.2477\dots$$

The volume of the cylinder is approximately 804 m^3 .

7. a)



Use the formula for the volume of a right cone.

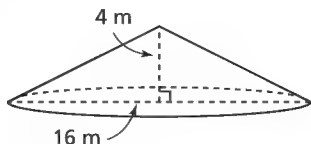
$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 5, h = 20$$

$$V = \frac{1}{3} \pi (5)^2 (20)$$

$$V = 523.5987\dots$$

The volume of the cone is approximately 524 cm^3 .

b)



The radius, r , of the base of the cone is $\frac{1}{2}$ the diameter; so, r is 8 m.

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 8, h = 4$$

$$V = \frac{1}{3} \pi (8)^2 (4)$$

$$V = 268.0825 \dots$$

The volume of the cone is approximately 268 m^3 .

8. a) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh \quad \text{Substitute: } l = 3, w = 3, h = 6$$

$$V = \frac{1}{3} (3)(3)(6)$$

$$V = 18$$

The volume of the pyramid is 18 m^3 .

- b) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh \quad \text{Substitute: } l = 7, w = 6, h = 12$$

$$V = \frac{1}{3} (7)(6)(12)$$

$$V = 168$$

The volume of the pyramid is 168 cubic yards.

9. a) Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 2.0, h = 9.0$$

$$V = \frac{1}{3} \pi (2.0)^2 (9.0)$$

$$V = 37.6991 \dots$$

The volume of the cone is approximately 37.7 m^3 .

- b) The radius, r , of the base of the cone is $\frac{1}{2}$ the diameter; so, r is 16 cm.

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 16, h = 11$$

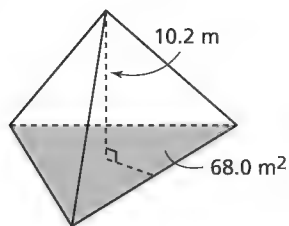
$$V = \frac{1}{3} \pi (16)^2 (11)$$

$$V = 2948.9083 \dots$$

The volume of the cone is approximately 2948.9 cm^3 .

B

10. a)



b) Use the formula for the volume of a right pyramid.

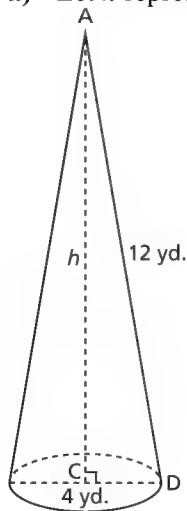
$$V = \frac{1}{3} (\text{base area})(\text{height})$$

$$V = \frac{1}{3} (68)(10.2)$$

$$V = 231.2$$

The volume of the tetrahedron is 231.2 m^3 .

11. a) Let h represent the height.



b) The radius, r , of the cone is $\frac{1}{2}$ the diameter; so, r is 2 yd.

Use the Pythagorean Theorem in right $\triangle ACD$ to determine the height, h , of the cone.

$$AC^2 = AD^2 - CD^2$$

$$h^2 = 12^2 - 2^2$$

$$h^2 = 140$$

$$h = \sqrt{140}$$

Use the formula for the volume of a right cone.

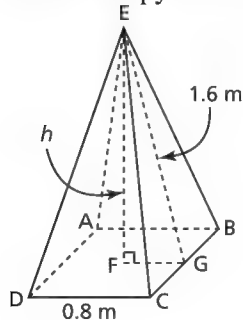
$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 2, h = \sqrt{140}$$

$$V = \frac{1}{3} \pi (2)^2 (\sqrt{140})$$

$$V = 49.5624 \dots$$

The volume of the cone is approximately 50 cubic yards.

12. Sketch the pyramid and label its vertices. Let h represent the height.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is 0.4 m.

Use the Pythagorean Theorem in right $\triangle EFG$.

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 1.6^2 - 0.4^2$$

$$h^2 = 2.4$$

$$h = \sqrt{2.4}$$

Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh \quad \text{Substitute: } l = 0.8, w = 0.8, h = \sqrt{2.4}$$

$$V = \frac{1}{3} (0.8)(0.8)(\sqrt{2.4})$$

$$V = 0.3304 \dots$$

The volume of the monument is approximately 0.3 m^3 .

13. a) Answers may vary. For example, Annika can use the formula for the volume of a right rectangular pyramid. She can multiply the three measurements, then divide by 3.

- b) Use the formula for the volume of a right rectangular pyramid.

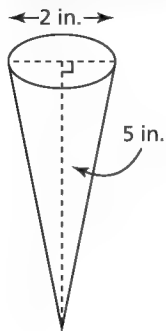
$$V = \frac{1}{3} lwh \quad \text{Substitute: } l = 10.4, w = 8.6, h = 14.8$$

$$V = \frac{1}{3} (10.4)(8.6)(14.8)$$

$$V = 441.2373 \dots$$

The volume of the pyramid is approximately 441.2 cm^3 .

14. a) Sketch the cone.



The radius, r , is $\frac{1}{2}$ the diameter; so, $r = 1$ in.

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 1, h = 5$$

$$V = \frac{1}{3} \pi (1)^2 (5)$$

$$V = 5.2359\dots$$

The cone can hold approximately 5 cubic inches of ice cream.

- b) Determine the total cost, C dollars, of an ice cream cone with whipped topping and sprinkles:

$$C = 0.55(5.2359\dots) + 0.35 + 0.10$$

$$C = 3.3297\dots$$

This dessert will cost \$3.33 to produce.

- c) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh \quad \text{Substitute: } l = 2, w = 2, h = 5$$

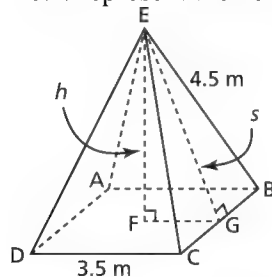
$$V = \frac{1}{3} (2)(2)(5)$$

$$V = 6.6666\dots$$

The pyramid would hold approximately 7 cubic inches of ice cream.

15. a) Sketch the pyramid and label its vertices.

Let h represent the height and let s represent the slant height.



- b) First determine the slant height, s .

In $\triangle EGB$, GB is $\frac{1}{2}$ the length of CB , so $GB = 1.75$ m.

Use the Pythagorean Theorem in right $\triangle EGB$.

$$EG^2 = EB^2 - GB^2$$

$$s^2 = 4.5^2 - 1.75^2$$

$$s^2 = 17.1875$$

$$s = \sqrt{17.1875}$$

In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so $FG = 1.75$ m.

To calculate the height, h , use the Pythagorean Theorem in right $\triangle EFG$.

$$EF^2 = EG^2 - FG^2$$

$$h^2 = (\sqrt{17.1875})^2 - 1.75^2$$

$$h^2 = 14.125$$

$$h = 3.7583\dots$$

The height of the pyramid is approximately 3.8 m.

- c) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh \quad \text{Substitute: } l = 3.5, w = 3.5, h = 3.7583\dots$$

$$V = \frac{1}{3}(3.5)(3.5)(3.7583\dots)$$

$$V = 15.3464\dots$$

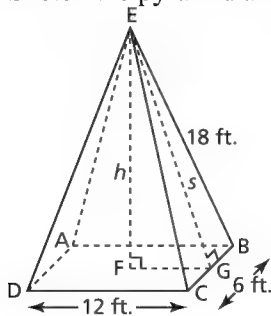
The volume of the pyramid is approximately 15.3 m^3 .

16. Convert the equal side length of each triangular face from yards to feet, so all measurements are in the same unit. Use the conversion: 1 yd. = 3 ft.

So, 6 yd. = 6(3 ft.)

6 yd. = 18 ft.

Sketch the pyramid and label its vertices. Let h represent its height.



First determine the slant height, s . In $\triangle EGB$, GB is $\frac{1}{2}$ the length of CB , so $GB = 3$ ft.

Use the Pythagorean Theorem in right $\triangle EGB$.

$$EG^2 = EB^2 - GB^2$$

$$s^2 = 18^2 - 3^2$$

$$s^2 = 315$$

$$s = \sqrt{315}$$

Now use the Pythagorean Theorem in right $\triangle EFG$ to determine h .

Side FG is $\frac{1}{2}$ the length of DC, so $FG = 6$ ft.

$$EF^2 = EG^2 - FG^2$$

$$h^2 = (\sqrt{315})^2 - 6^2$$

$$h^2 = 279$$

$$h = \sqrt{279}$$

Use the formula for the volume of a right rectangular pyramid.

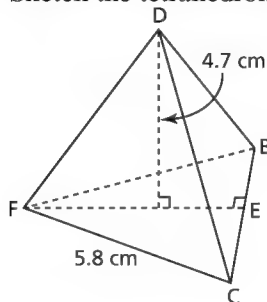
$$V = \frac{1}{3}lwh \quad \text{Substitute: } l = 12, w = 6, h = \sqrt{279}$$

$$V = \frac{1}{3}(12)(6)(\sqrt{279})$$

$$V = 400.8790\dots$$

The volume of the pyramid is approximately 401 cubic feet.

17. a) Sketch the tetrahedron and label its vertices.



In equilateral $\triangle FBC$, EB is $\frac{1}{2}$ the length of CB , so $EB = 2.9$ cm.

Use the Pythagorean Theorem in right $\triangle FEB$ to determine the height of the base, FE .

$$FE^2 = 5.8^2 - 2.9^2$$

$$FE^2 = 25.23$$

$$FE = \sqrt{25.23}$$

So, the area, A , of the base is:

$$A = \frac{1}{2}(5.8)(\sqrt{25.23})$$

$$A = 14.5665\dots$$

The area of the base is approximately 15 cm^2 .

- b) Use the formula for the volume of a right pyramid.

$$V = \frac{1}{3}(\text{base area})(\text{height})$$

$$V = \frac{1}{3}(14.5665\dots)(4.7)$$

$$V = 22.8209\dots$$

The volume of the tetrahedron is approximately 23 cm^3 .

- c) No, the volume of tea in the bag will be less than 23 cm^3 . There is some air inside the tea bag to allow the tea to expand when boiling water is poured on it.

18. a) Use the formula for the volume of a rectangular prism.

$$\begin{aligned}
 V &= lwh && \text{Substitute: } V = 88.8, l = 5.9, w = 3.2 \\
 88.8 &= (5.9)(3.2)h && \text{Solve for } h. \\
 88.8 &= 18.88h \\
 \frac{88.8}{18.88} &= \frac{18.88h}{18.88} \\
 h &= \frac{88.8}{18.88} \\
 h &= 4.7033... \\
 h &\text{ is approximately } 4.7 \text{ cm.}
 \end{aligned}$$

- b) Use the formula for the volume of a right rectangular pyramid.

$$\begin{aligned}
 V &= \frac{1}{3}lwh && \text{Substitute: } V = 554.9, l = x, w = x, h = 15.1 \\
 554.9 &= \frac{1}{3}(x)(x)(15.1) && \text{Multiply both sides by 3.} \\
 (3)(554.9) &= (x)(x)(15.1) \\
 1664.7 &= 15.1x^2 && \text{Solve for } x. \\
 \frac{1664.7}{15.1} &= \frac{15.1x^2}{15.1} \\
 x^2 &= \frac{1664.7}{15.1} \\
 x &= \sqrt{\frac{1664.7}{15.1}} \\
 x &= 10.4997... \\
 x &\text{ is approximately } 10.5 \text{ m.}
 \end{aligned}$$

- c) Use the formula for the volume of a right cylinder.

$$\begin{aligned}
 V &= \pi r^2 h && \text{Substitute: } V = 219.0, h = 6.4 \\
 219.0 &= \pi r^2 (6.4) && \text{Divide both sides by } 6.4\pi. \\
 \frac{219.0}{6.4\pi} &= \frac{6.4\pi r^2}{6.4\pi} && \text{Solve for } r. \\
 r^2 &= \frac{219.0}{6.4\pi} \\
 r &= \sqrt{\frac{219.0}{6.4\pi}} \\
 r &= 3.300... \\
 r &\text{ is approximately } 3.3 \text{ m.}
 \end{aligned}$$

- d) Use the formula for the volume of a right cone.

$$\begin{aligned}
 V &= \frac{1}{3}\pi r^2 h && \text{Substitute: } V = 164.9, h = 11.5
 \end{aligned}$$

$$164.9 = \frac{1}{3} \pi r^2 (11.5) \quad \text{Multiply both sides by 3.}$$

$$494.7 = 11.5 \pi r^2 \quad \text{Divide both sides by } 11.5\pi.$$

$$\frac{494.7}{11.5\pi} = \frac{11.5\pi r^2}{11.5\pi} \quad \text{Solve for } r.$$

$$r^2 = \frac{494.7}{11.5\pi}$$

$$r = \sqrt{\frac{494.7}{11.5\pi}}$$

The diameter, d , of the cone is twice the radius.

$$d = 2 \times \sqrt{\frac{494.7}{11.5\pi}}$$

$$d = 7.400\dots$$

d is approximately 7.4 cm.

19. a) Answers may vary. For example, Sunil could use the diameter to determine the radius, substitute the known data in the formula for the volume of a right cone, then solve for h .

- b) The radius, r , is $\frac{1}{2}$ the diameter; so, $r = 2.0$ cm

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h \quad \text{Substitute: } V = 33.5, r = 2.0$$

$$33.5 = \frac{1}{3} \pi (2.0)^2 h \quad \text{Multiply both sides by 3.}$$

$$100.5 = 4\pi h \quad \text{Divide both sides by } 4\pi.$$

$$\frac{100.5}{4\pi} = \frac{4\pi h}{4\pi} \quad \text{Solve for } h.$$

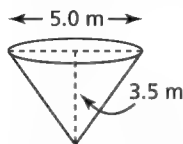
$$h = \frac{100.5}{4\pi}$$

$$h = 7.9975\dots$$

The height of the cone is approximately 8.0 cm.

C

20. a) The radius, r , is $\frac{1}{2}$ the diameter of the cone; so, $r = 2.5$ m



Use the formula for the volume of a right cone.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Substitute: } r = 2.5, h = 3.5$$

$$V = \frac{1}{3}\pi (2.5)^2 (3.5)$$

$$V = 22.9074\dots$$

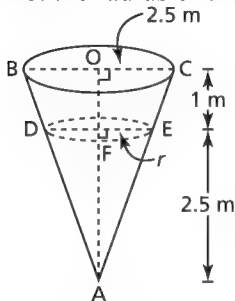
The volume is approximately 22.9 m^3 .

Since $1 \text{ m}^3 = 1 \text{ kL}$, the capacity of the tank is approximately 22.9 kL .

- b) Sketch and label a diagram.

When the water level is below the top of the tank, the water is 2.5 m deep.

Let the radius of the water surface be r .



Use similar triangles to determine r .

Right $\triangle AFE$ and $\triangle AOC$ are similar. So, the ratio of corresponding side lengths is equal.

$$\frac{AF}{AO} = \frac{EF}{CO} \quad \text{Substitute: } AF = 2.5, AO = 3.5, CO = 2.5, EF = r$$

$$\frac{2.5}{3.5} = \frac{r}{2.5} \quad \text{Solve for } r.$$

$$r = \frac{(2.5)(2.5)}{3.5}$$

$$r = 1.7857\dots$$

Use the formula for the volume of a right cone.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Substitute: } r = 1.7857\dots, h = 2.5$$

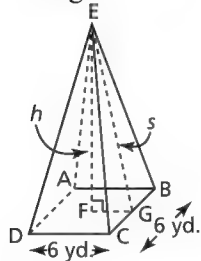
$$V = \frac{1}{3}\pi (1.7857\dots)^2 (2.5)$$

$$V = 8.3481\dots$$

The volume is approximately 8.3 m^3 .

Since $1 \text{ m}^3 = 1 \text{ kL}$, there is approximately 8.3 kL of water in the tank.

21. Sketch the pyramid and label its vertices. Let s represent the slant height and let h represent the height.



$$V = 111 \text{ cubic yards}$$

First determine the height, h . Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh$$

$$\text{Substitute: } V = 111, l = w = 6$$

$$111 = \frac{1}{3}(6)(6)h$$

Solve for h .

$$111 = 12h$$

$$\frac{111}{12} = \frac{12h}{12}$$

$$h = \frac{111}{12}$$

$$h = 9.25$$

To determine s , use the Pythagorean Theorem in right $\triangle EFG$. Side length FG is $\frac{1}{2}$ the length of DC , so $FG = 3$ yd.

$$EG^2 = EF^2 + FG^2$$

$$s^2 = 9.25^2 + 3^2$$

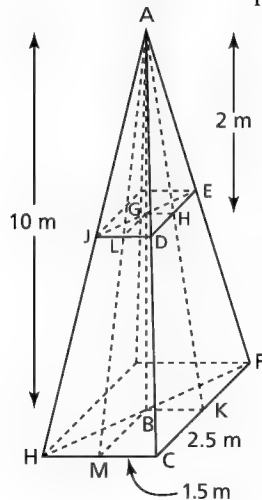
$$s^2 = 94.5625$$

$$s = \sqrt{94.5625}$$

$$s = 9.7243\dots$$

The slant height of the pyramid is approximately 10 yd.

22. Sketch and label the pyramid.



Use similar triangles to determine the dimensions of the base of the pyramid that is removed.

$\triangle AGH$ and $\triangle ABK$ are similar.

$$\text{So, } \frac{GH}{BK} = \frac{AG}{AB}$$

$$\frac{GH}{1.5} = \frac{2}{10}$$

$$GH = \frac{2(1.5)}{10}$$

$$GH = \frac{3}{10}, \text{ or } 0.3$$

$\triangle AGL$ and $\triangle ABM$ are similar.

$$\text{So, } \frac{AG}{AB} = \frac{GL}{BM}$$

$$\frac{2}{10} = \frac{GL}{2.5}$$

$$GL = \frac{2(2.5)}{10}$$

$$GL = \frac{5}{10}, \text{ or } 0.5$$

The dimensions of the base of the smaller pyramid are $2(0.3)$ m by $2(0.5)$ m,
or 0.6 m by 1.0 m

So, the volume of the smaller pyramid is: $\frac{1}{3}(0.6)(1.0)(2) = 0.4$

The volume of the larger pyramid is: $\frac{1}{3}(5)(3)(10) = 50$

So, the remaining piece has volume: $50 - 0.4 = 49.6$

The volume of the remaining piece is 49.6 m^3 .

Lesson 1.6 Surface Area and Volume of a Sphere

Exercises (pages 51–52)

A

3. a) Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 5$$

$$SA = 4\pi(5)^2$$

$$SA = 314.1592\dots$$

The surface area of the sphere is approximately 314 cm^2 .

- b) Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 1.6$$

$$SA = 4\pi(1.6)^2$$

$$SA = 32.1699\dots$$

The surface area of the sphere is approximately 32 m^2 .

- c) Use the formula for the surface area of a sphere.

The radius, r , is $\frac{1}{2}(8 \text{ ft.}) = 4 \text{ ft.}$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 4$$

$$SA = 4\pi(4)^2$$

$$SA = 201.0619\dots$$

The surface area of the sphere is approximately 201 square feet.

- d) Use the formula for the surface area of a sphere.

The radius, r , is $\frac{1}{2}(5.6 \text{ cm}) = 2.8 \text{ cm}$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 2.8$$

$$SA = 4\pi(2.8)^2$$

$$SA = 98.5203\dots$$

The surface area of the sphere is approximately 99 cm^2 .

4. a) Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 5$$

$$V = \frac{4}{3}\pi(5)^3$$

$$V = 523.5987\dots$$

The volume of the sphere is approximately 524 cm^3 .

- b) Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 1.6$$

$$V = \frac{4}{3}\pi(1.6)^3$$

$$V = 17.1572\dots$$

The volume of the sphere is approximately 17 m^3 .

- c) Use the formula for the volume of a sphere.

The radius, r , is $\frac{1}{2}(8 \text{ ft.}) = 4 \text{ ft.}$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 4$$

$$V = \frac{4}{3}\pi(4)^3$$

$$V = 268.0825\dots$$

The volume of the sphere is approximately 268 cubic feet.

- d) Use the formula for the volume of a sphere.

The radius, r , is $\frac{1}{2}(5.6 \text{ cm}) = 2.8 \text{ cm}$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 2.8$$

$$V = \frac{4}{3}\pi(2.8)^3$$

$$V = 91.9523\dots$$

The volume of the sphere is approximately 92 cm³.

5. a) SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 6$$

$$SA = 3\pi(6)^2$$

$$SA = 339.2920\dots$$

The surface area of the hemisphere is approximately 339 m².

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 6$$

$$V = \frac{2}{3}\pi(6)^3$$

$$V = 452.3893\dots$$

The volume of the hemisphere is approximately 452 m³.

- b) The radius, r , is $\frac{1}{2}(9 \text{ yd.}) = 4\frac{1}{2} \text{ yd.}$

SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 4.5$$

$$SA = 3\pi(4.5)^2$$

$$SA = 190.8517\dots$$

The surface area of the hemisphere is approximately 191 square yards.

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right)$$

$$V = \frac{2}{3} \pi r^3 \quad \text{Substitute: } r = 4.5$$

$$V = \frac{2}{3} \pi (4.5)^3$$

$$V = 190.8517\dots$$

The volume of the hemisphere is approximately 191 cubic yards.

B

6. Answers will vary depending on object chosen.

For example, I used a baseball. I measured its diameter to be 3 in.

The radius, r , of the baseball is:

$$r = \frac{1}{2} (3 \text{ in.})$$

$$r = 1\frac{1}{2} \text{ in.}$$

Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 1.5$$

$$SA = 4\pi (1.5)^2$$

$$SA = 28.2743\dots$$

The surface area of the baseball is approximately 28 square inches.

Use the formula for the volume of a sphere.

$$V = \frac{4}{3} \pi r^3 \quad \text{Substitute: } r = 1.5$$

$$V = \frac{4}{3} \pi (1.5)^3$$

$$V = 14.1371\dots$$

The volume of the baseball is approximately 14 cubic inches.

7. Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 8.4$$

$$SA = 4\pi (8.4)^2$$

$$SA = 886.6831\dots$$

The surface area of the sphere is approximately 886.7 m^2 .

Use the formula for the volume of a sphere.

$$V = \frac{4}{3} \pi r^3 \quad \text{Substitute: } r = 8.4$$

$$V = \frac{4}{3}\pi(8.4)^3$$

$$V = 2482.7127\dots$$

The volume of the sphere is approximately 2482.7 m^3 .

8. Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } SA = 127$$

$$127 = 4\pi r^2 \quad \text{Solve for } r.$$

$$\frac{127}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$r^2 = \frac{127}{4\pi}$$

$$r = \sqrt{\frac{127}{4\pi}}$$

$$r = 3.1790\dots$$

The radius of the tennis ball is approximately 3.2 cm.

9. Use the formula for the surface area of a sphere and determine r .

$$SA = 4\pi r^2 \quad \text{Substitute: } SA = 452$$

$$452 = 4\pi r^2 \quad \text{Solve for } r.$$

$$\frac{452}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$r^2 = \frac{452}{4\pi}$$

$$r = \sqrt{\frac{452}{4\pi}}$$

The diameter, d , is:

$$d = 2\left(\sqrt{\frac{452}{4\pi}}\right)$$

$$d = 11.9948\dots$$

The diameter of the sphere is approximately 12 in.

10. a) The radius, r , is $\frac{1}{2}(20 \text{ cm}) = 10 \text{ cm}$

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 10$$

$$V = \frac{2}{3}\pi(10)^3$$

$$V = 2094.3951\dots$$

Since $1000 \text{ cm}^3 = 1 \text{ L}$, the capacity of the bowl is: $\frac{2094.3951...}{1000} = 2.0943...$

The capacity of the bowl is approximately 2.1 L.

- b)** 1 cup = 250 mL, or 0.250 L

The number of cups is: $\frac{2.0943...}{0.250} = 8.3775...$

The bowl can hold approximately 8 cups of punch.

- 11. a)** Determine the surface areas of the sphere and hemisphere, then compare.
Use the formula for the surface area of a sphere.

The radius, r , of the sphere is: $\frac{1}{2}(12 \text{ cm}) = 6 \text{ cm}$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 6$$

$$SA = 4\pi(6)^2$$

$$SA = 452.3893...$$

The surface area of the sphere is approximately 452.4 cm^2 .

SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 8$$

$$SA = 3\pi(8)^2$$

$$SA = 603.1857...$$

The surface area of the hemisphere is approximately 603.2 cm^2 .

Since $603.2 > 452.4$, the hemisphere has the greater surface area.

- b)** Determine the volumes of the sphere and hemisphere, then compare.
Use the formula for the volume of a sphere.

The radius, r , of the sphere is: 6 cm

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 6$$

$$V = \frac{4}{3}\pi(6)^3$$

$$V = 904.7786...$$

The volume of the sphere is approximately 904.8 cm^3 .

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 8$$

$$V = \frac{2}{3}\pi(8)^3$$

$$V = 1072.3302\dots$$

The volume of the hemisphere is approximately 1072.3 cm^3 .

Since $1072.3 > 904.8$, the hemisphere has the greater volume.

12. a) Use the formula for the surface area of a sphere.

The radius, r , of the sphere is: $\frac{1}{2}(15.8 \text{ m}) = 7.9 \text{ m}$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 7.9$$

$$SA = 4\pi(7.9)^2$$

$$SA = 784.2671\dots$$

The surface area of the sphere is approximately 784 m^2 .

- b) Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 7.9$$

$$V = \frac{4}{3}\pi(7.9)^3$$

$$V = 2065.2369\dots$$

The volume of the sphere is approximately 2065 m^3 .

Since $1 \text{ kL} = 1 \text{ m}^3$, the capacity of the sphere is approximately 2065 kL .

13. a) Use the formula for the surface area of a sphere.

The radius, r , of Earth is: $\frac{1}{2}(12\,756 \text{ km}) = 6378 \text{ km}$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 6378$$

$$SA = 4\pi(6378)^2$$

$$SA = 511\,185\,932.5\dots$$

The surface area of Earth is approximately $511\,185\,933 \text{ km}^2$.

- b) Determine 70% of $511\,185\,933.5\dots$

$$0.70 \times 511\,185\,933.5\dots = 357\,830\,152.8\dots$$

Approximately $357\,830\,153 \text{ km}^2$ of Earth's surface is covered in water.

- c) Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 6378$$

$$V = \frac{4}{3}\pi(6378)^3$$

$$V = 1\,086\,781\,293\,000\dots$$

The volume of Earth is approximately $1\,086\,781\,293\,000 \text{ km}^3$.

- d) Determine the volume of the inner core of Earth.
Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 1278$$

$$V = \frac{4}{3}\pi (1278)^3$$

$$V = 8\,743\,416\,579\dots$$

So, the volume of Earth that is not part of the inner core, in cubic kilometres, is:

volume of Earth – volume of inner core

$$= 1\,086\,781\,293\,000\dots - 8\,743\,416\,579\dots$$

$$= 1\,078\,037\,876\,000\dots$$

The volume of Earth that is not part of the inner core is approximately

$$1\,078\,037\,876\,000\text{ km}^3.$$

14. The diameter of Earth through the poles is: $12\,756\text{ km} - 16\text{ km} = 12\,740\text{ km}$

$$\text{The radius, } r, \text{ at the poles is: } \frac{1}{2}(12\,740\text{ km}) = 6370\text{ km}$$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 6370$$

$$V = \frac{4}{3}\pi (6370)^3$$

$$V = 1\,082\,696\,932\,000\dots$$

The volume of Earth, using the polar radius, is approximately $1\,082\,696\,932\,000\text{ km}^3$.

The diameter of Earth at the equator is: $12\,756\text{ km} + 26\text{ km} = 12\,782\text{ km}$

$$\text{The radius, } r, \text{ at the equator is: } \frac{1}{2}(12\,782\text{ km}) = 6391\text{ km}$$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 6391$$

$$V = \frac{4}{3}\pi (6391)^3$$

$$V = 1\,093\,440\,264\,000\dots$$

The volume of Earth, using the equatorial radius, is approximately $1\,093\,440\,264\,000\text{ km}^3$.

15. Determine the surface area of one sphere of dough.

Use the formula for the surface area of a sphere.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(2.5\text{ cm}) = 1.25\text{ cm}$$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 1.25$$

$$SA = 4\pi (1.25)^2$$

$$SA = 19.6349\dots$$

The number of spheres that can be glazed is:

$$\frac{\text{total surface area}}{\text{area of one sphere}} = \frac{4710}{19.6349\dots}$$

$$= 239.8783\dots$$

So, 239 doughnut spheres can be glazed.

16. a) Use the formula for the circumference, C , of a circle to determine r .

For the volleyball:

$$C = 2\pi r$$

Substitute: $C = 66$

$$66 = 2\pi r$$

Solve for r . Divide both sides by 2π .

$$\frac{66}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{66}{2\pi}$$

$$r = 10.5042\dots$$

The radius of a volleyball is approximately 11 cm.

For the basketball:

$$C = 2\pi r$$

Substitute: $C = 29.5$

$$29.5 = 2\pi r$$

Solve for r . Divide both sides by 2π .

$$\frac{29.5}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{29.5}{2\pi}$$

$$r = 4.6950\dots$$

The radius of a basketball is approximately 5 in.

- b) Use the formula for the surface area of a sphere.

For the volleyball:

$$SA = 4\pi r^2$$

Substitute: $r = 10.5042\dots$

$$SA = 4\pi (10.5042\dots)^2$$

$$SA = 1386.5578\dots$$

The surface area of a volleyball is approximately 1387 cm^2 .

For the basketball:

$$SA = 4\pi r^2$$

Substitute: $r = 4.6950\dots$

$$SA = 4\pi (4.6950\dots)^2$$

$$SA = 277.0091\dots$$

The surface area of a basketball is approximately 277 square inches.

- c) Use the formula for the volume of a sphere.

For the volleyball:

$$V = \frac{4}{3}\pi r^3$$

Substitute: $r = 10.5042\dots$

$$V = \frac{4}{3}\pi (10.5042\dots)^3$$

$$V = 4854.9058\dots$$

The volume of a volleyball is approximately 4855 cm^3 .

For the basketball:

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 4.6950\dots$$

$$V = \frac{4}{3}\pi (4.6950\dots)^3$$

$$V = 433.5259\dots$$

The volume of a basketball is approximately 434 cubic inches.

- d) To determine which ball is larger, compare their radii. Convert the radius of the basketball to centimetres. Use the exact conversion: 1 in. = 2.54 cm

$$\text{So, } 29.5 \text{ in.} = 29.5(2.54 \text{ cm})$$

$$29.5 \text{ in.} = 74.93 \text{ cm}$$

Since $74.93 > 66$, the basketball is larger.

17. a) Use the formula for the volume of a sphere.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(3.15 \text{ m}) = 1.575 \text{ m}$$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 1.575$$

$$V = \frac{4}{3}\pi (1.575)^3$$

$$V = 16.3655\dots$$

The volume of the inside of the shell is approximately 16.4 m^3 .

- b) Determine the surface areas of the inside and outside shells. Use the formula for the surface area of a sphere.

For the outside shell:

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(3.20 \text{ m}) = 1.6 \text{ m}$$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 1.6$$

$$SA = 4\pi (1.6)^2$$

$$SA = 32.1699\dots$$

For the inside shell:

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 1.575$$

$$SA = 4\pi (1.575)^2$$

$$SA = 31.1724\dots$$

Subtract the two surface areas to determine the difference:

$$32.1699\dots - 31.1724\dots = 0.9974\dots$$

So, the difference between the outside and inside surface areas is approximately 1.0 m^2 .

18. Use the formula for the circumference, C , of a circle to determine r .

$$C = 2\pi r$$

$$\text{Substitute: } C = 47.1$$

$$47.1 = 2\pi r \quad \text{Solve for } r. \text{ Divide both sides by } 2\pi.$$

$$\frac{47.1}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{47.1}{2\pi}$$

$$r = 7.4961\dots$$

SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 7.4961\dots$$

$$SA = 3\pi (7.4961\dots)^2$$

$$SA = 529.6063\dots$$

The surface area of the hemisphere is approximately 529.6 m^2 .

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 7.4961\dots$$

$$V = \frac{2}{3}\pi (7.4961\dots)^3$$

$$V = 882.2298\dots$$

The volume of the sphere is approximately 882.2 m^3 .

19. Determine the volume of the ball. Use the formula for the volume of a sphere.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(28 \text{ cm}) = 14 \text{ cm}$$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 14$$

$$V = \frac{4}{3}\pi (14)^3$$

$$V = 11\,494.0403\dots$$

Divide the total volume by the volume per pump to determine the number of pumps needed:

$$\frac{11\,494.0403\dots}{280} = 41.0501\dots$$

So, 42 pumps are needed to inflate the ball.

20. Determine the volume of the pail of cookie dough and the volume of a scoop of dough.

For the pail, use the formula for the volume of a cylinder.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(17 \text{ cm}) = 8.5 \text{ cm}$$

$$V = \pi r^2 h \quad \text{Substitute: } r = 8.5, h = 13$$

$$V = \pi (8.5)^2 (13)$$

$$V \doteq 2950.7409\dots$$

For the scoop, use the formula for the volume of a sphere.

The radius, r , is: $\frac{1}{2}(5 \text{ cm}) = 2.5 \text{ cm}$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 2.5$$

$$V = \frac{4}{3}\pi (2.5)^3$$

$$V = 65.4498\dots$$

To determine how many cookies can be made, divide the volume of the pail by the volume of a scoop:

$$\frac{2950.7408\dots}{65.4498\dots} = 45.0840\dots$$

So, 45 cookies can be made from the pail of dough.

C

21. a) Determine the volume of the block of wood.

Use the formula for the volume of a right rectangular prism.

$$V = lwh \quad \text{Substitute: } l = 14, w = 12, h = 10$$

$$V = (14)(12)(10)$$

$$V = 1680$$

The volume of the block of wood is 1680 cm^3 .

Determine the volume of the sphere. Use the formula for the volume of a sphere.

The diameter of the sphere is the least measurement of the block, or 10 cm.

The radius, r , is: $\frac{1}{2}(10 \text{ cm}) = 5 \text{ cm}$

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 5$$

$$V = \frac{4}{3}\pi (5)^3$$

$$V = 523.5987\dots$$

The volume of the sphere is $523.5987\dots \text{ cm}^3$.

$$\text{The volume of wood wasted is: } 1680 \text{ cm}^3 - 523.5987\dots \text{ cm}^3 = 1156.4012\dots \text{ cm}^3$$

Write the volume of wood wasted as a fraction of the total amount of wood:

$$\frac{1156.4012\dots}{1680} = 0.6883\dots$$

Write this decimal as a percent: $0.6883\dots \times 100\% = 68.83\dots\%$

So, approximately 69% of the wood is wasted.

- b) I assume that the ball is created from one solid piece of wood and that it has the greatest possible diameter.

22. Use the formula for the surface area of a sphere.

The radius r , is one-half the diameter, d : $r = \frac{1}{2}d$

$$SA = 4\pi r^2 \quad \text{Substitute: } r = \frac{1}{2}d$$

$$SA = 4\pi \left(\frac{1}{2}d\right)^2$$

$$SA = 4\pi \left(\frac{1}{4}d^2\right)$$

$$SA = \pi d^2$$

To check, calculate the surface area of the sphere in question 3c.

$$SA = \pi d^2 \quad \text{Substitute: } d = 8$$

$$SA = \pi (8)^2$$

$$SA = 201.0619\dots$$

The surface area of the sphere is approximately 201 square feet. This is the same as the answer for question 3c; so, my formula is correct.

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = \frac{1}{2}d$$

$$V = \frac{4}{3}\pi \left(\frac{1}{2}d\right)^3$$

$$V = \frac{4}{3}\pi \left(\frac{1}{8}d^3\right)$$

$$V = \frac{4}{24}\pi d^3$$

$$V = \frac{1}{6}\pi d^3$$

To check, calculate the volume of the sphere from question 3c.

$$V = \frac{1}{6}\pi d^3 \quad \text{Substitute: } d = 8$$

$$V = \frac{1}{6}\pi (8)^3$$

$$V = 268.0825\dots$$

The volume of the sphere is approximately 268 cubic feet. This is the same as the answer for question 4c; so, my formula is correct.

23. 70% of the volume is: 420 cubic inches

So, 1% of the volume is: $\frac{420}{70}$ cubic inches

And, 100% of the volume is: $\frac{420(100)}{70}$ cubic inches = 600 cubic inches

The maximum volume of the beach ball is 600 cubic inches.

Use the formula for the volume of a sphere to determine r .

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } V = 600$$

$$600 = \frac{4}{3}\pi r^3 \quad \text{Solve for } r. \text{ Multiply both sides by 3.}$$

$$1800 = 4\pi r^3 \quad \text{Divide both sides by } 4\pi.$$

$$\frac{1800}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$r^3 = \frac{1800}{4\pi}$$

$$r = \sqrt[3]{\frac{1800}{4\pi}}$$

$$r = 5.2322\dots$$

At its maximum volume, the radius of the beach ball is approximately 5 in.

24. a) Determine the circumference, C , of the original balloon. Use the formula for the circumference of a circle.

$$C = 2\pi r \quad \text{Substitute: } r = 10$$

$$C = 2\pi(10)$$

$$C = 20\pi$$

Determine the circumference, C , of the inflated balloon. Use the formula for the circumference of a circle.

The radius, r , is three times the original radius:

$$r = 3(10 \text{ cm}) = 30 \text{ cm}$$

$$C = 2\pi r \quad \text{Substitute: } r = 30$$

$$C = 2\pi(30)$$

$$C = 60\pi$$

To compare the circumferences, divide: $\frac{20\pi}{60\pi} = \frac{1}{3}$

The circumference of the inflated balloon is 3 times as great as the circumference of the original balloon.

- b) Determine the surface area of the original balloon. Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 10$$

$$SA = 4\pi(10)^2$$

$$SA = 400\pi$$

Determine the surface area of the inflated balloon. Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 30$$

$$SA = 4\pi (30)^2$$

$$SA = 3600\pi$$

To compare the surface areas, divide: $\frac{3600\pi}{400\pi} = 9$

The surface area of the inflated balloon is 9 times as great as the surface area of the original balloon.

- c) Determine the volume of the original balloon. Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 10$$

$$V = \frac{4}{3}\pi (10)^3$$

$$V = \frac{4000\pi}{3}$$

Determine the volume of the inflated balloon. Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 30$$

$$V = \frac{4}{3}\pi (30)^3$$

$$V = 36\,000\pi$$

To compare the volumes, divide: $\frac{36\,000\pi}{\frac{4000\pi}{3}} = \frac{108}{4} = 27$

The volume of the inflated balloon is 27 times as great as the volume of the original balloon.

Checkpoint 2 Assess Your Understanding

(page 54)

1.4

1. a) Use the formula for surface area of a right square pyramid with slant height s and base side length l :

$$\text{Surface area} = \frac{1}{2}s(4l) + l^2 \quad \text{Substitute: } s = 8, l = 4$$

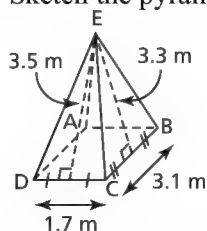
$$\text{Surface area} = \frac{1}{2}(8)(4)(4) + (4)^2$$

$$\text{Surface area} = 64 + 16$$

$$\text{Surface area} = 80$$

The surface area of the pyramid is 80 square feet.

- b) Sketch the pyramid and label its vertices.



Area, A , of $\triangle EDC$ is:

$$A = \frac{1}{2}(1.7)(3.5)$$

$$A = 2.975$$

Since $\triangle EDC$ and $\triangle EAB$ are congruent, the area of $\triangle EAB$ is 2.975 m^2 .

Area, A , of $\triangle EBC$ is:

$$A = \frac{1}{2}(3.1)(3.3)$$

$$A = 5.115$$

Since $\triangle EBC$ and $\triangle EAD$ are congruent, the area of $\triangle EAD$ is 5.115 m^2 .

Area, B , of the base of the pyramid is:

$$B = (1.7)(3.1)$$

$$B = 5.27$$

Each of two triangles has area 2.975 m^2 , and each of the other two triangles has area 5.115 m^2 .

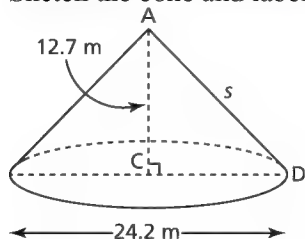
The surface area, SA , of the pyramid is:

$$SA = 2(2.975) + 2(5.115) + 5.27$$

$$SA = 21.45$$

The surface area of the right rectangular pyramid is approximately 21 m^2 .

- c) Sketch the cone and label its vertices. Let s represent the slant height.



The radius, CD , of the cone is one-half the diameter: $\frac{1}{2}(24.2 \text{ m}) = 12.1 \text{ m}$

Use the Pythagorean Theorem in right $\triangle ACD$ to determine s .

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 12.7^2 + 12.1^2$$

$$s^2 = 307.7$$

$$s = \sqrt{307.7}$$

Use the formula for the surface area of a right cone.

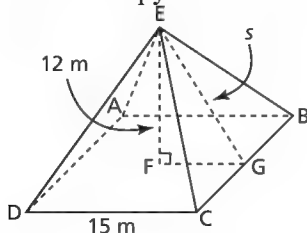
$$\text{Surface area} = \pi rs + \pi r^2 \quad \text{Substitute: } r = 12.1, s = \sqrt{307.7}$$

$$\text{Surface area} = \pi (12.1)(\sqrt{307.7}) + \pi (12.1)^2$$

$$\text{Surface area} = 1126.7658\dots$$

The surface area of the right cone is approximately 1127 m^2 .

2. Sketch the pyramid and label its vertices. Let s represent the slant height.



FG is $\frac{1}{2}$ the length of DC , so FG is 7.5 m .

Use the Pythagorean Theorem in right $\triangle EFG$ to determine s .

$$EG^2 = EF^2 + FG^2$$

$$s^2 = 12^2 + 7.5^2$$

$$s^2 = 200.25$$

$$s = \sqrt{200.25}$$

Determine the lateral area, A_L , of the right square pyramid with slant height s and base side length l .

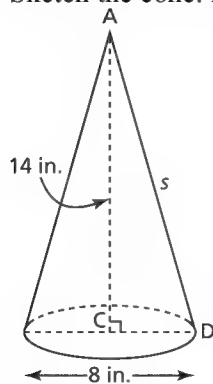
$$A_L = \frac{1}{2}s(4l) \quad \text{Substitute: } s = \sqrt{200.25}, l = 15$$

$$A_L = \frac{1}{2}(\sqrt{200.25})(4)(15)$$

$$A_L = 424.5291\dots$$

The area that needs to be re-covered is approximately 425 m^2 .

3. Sketch the cone. Let s represent the slant height.



CD is $\frac{1}{2}$ the diameter, so CD is 4 in.

Use the Pythagorean Theorem in right $\triangle ACD$ to determine s .

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 14^2 + 4^2$$

$$s^2 = 212$$

$$s = \sqrt{212}$$

Use the formula for the lateral area, A_L , of a right cone with base radius r and slant height s .

$$A_L = \pi rs \quad \text{Substitute: } r = 4, s = \sqrt{212}$$

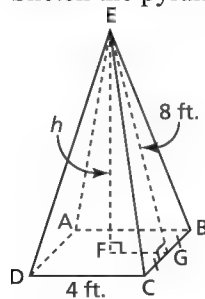
$$A_L = \pi (4)(\sqrt{212})$$

$$A_L = 182.9691\dots$$

The lateral area of the right cone is approximately 183 square inches.

1.5

4. a) Sketch the pyramid and label its vertices. Let h represent the height.



FG is $\frac{1}{2}$ the length of DC, so FG is 2 ft.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine h .

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 8^2 - 2^2$$

$$h^2 = 60$$

$$h = \sqrt{60}$$

Use the formula for the volume of a right rectangular pyramid.

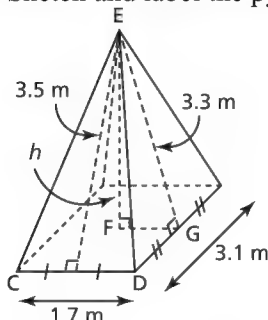
$$V = \frac{1}{3}lwh \quad \text{Substitute: } l = 4, w = 4, h = \sqrt{60}$$

$$V = \frac{1}{3}(4)(4)(\sqrt{60})$$

$$V = 41.3118\dots$$

The volume of the pyramid is approximately 41 cubic feet.

- b) Sketch and label the pyramid. Let h represent the height.



FG is $\frac{1}{2}$ the length of DC , so FG is 0.85 m.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine h .

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 3.3^2 - 0.85^2$$

$$h^2 = 10.1675$$

$$h = \sqrt{10.1675}$$

Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh \quad \text{Substitute: } l = 3.1, w = 1.7, h = \sqrt{10.1675}$$

$$V = \frac{1}{3}(3.1)(1.7)(\sqrt{10.1675})$$

$$V = 5.6013\dots$$

The volume of the pyramid is approximately 6 m^3 .

- c) The radius, r , of the base of the cone is $\frac{1}{2}$ the diameter: $\frac{1}{2}(24.2 \text{ m}) = 12.1 \text{ m}$

Use the formula for the volume of a right cone.

$$V = \frac{1}{3}\pi r^2 h \quad \text{Substitute: } r = 12.1, h = 12.7$$

$$V = \frac{1}{3}\pi (12.1)^2(12.7)$$

$$V = 1947.1664\dots$$

The volume of the cone is approximately 1947 m^3 .

5. a) Use the formula for the volume of a right rectangular pyramid to determine h .

$$V = \frac{1}{3}lwh \quad \text{Substitute: } l = w = 7.3, V = 168.8$$

$$168.8 = \frac{1}{3}(7.3)(7.3)h \quad \text{Multiply both sides by 3.}$$

$$506.4 = 53.29h$$

Solve for h . Divide both sides by 53.29.

$$h = \frac{506.4}{53.29}$$

$$h = 9.5027\dots$$

h is approximately 9.5 cm.

- b) Use the formula for the volume of a right cone to determine h .

$$V = \frac{1}{3}\pi r^2 h$$

Substitute: $r = 1.7$, $V = 8.2$

$$8.2 = \frac{1}{3}\pi (1.7)^2 h$$

Multiply both sides by 3.

$$24.6 = 2.89\pi h$$

Solve for h . Divide both sides by 2.89π .

$$\frac{24.6}{2.89\pi} = \frac{2.89\pi h}{2.89\pi}$$

$$h = 2.7094\dots$$

h is approximately 2.7 m.

- c) Use the formula for the volume of a right rectangular pyramid to determine x .

$$V = \frac{1}{3}lwh$$

Substitute: $V = 1594.5$, $l = w = x$, $h = 15.8$

$$1594.5 = \frac{1}{3}(x)(x)(15.8)$$

Multiply both sides by 3.

$$4783.5 = 15.8x^2$$

Solve for x . Divide both sides by 15.8.

$$\frac{4783.5}{15.8} = \frac{15.8x^2}{15.8}$$

$$x^2 = \frac{4783.5}{15.8}$$

$$x = \sqrt{\frac{4783.5}{15.8}}$$

$$x = 17.3998\dots$$

x is approximately 17.4 cm.

1.6

6. a) Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2$$

Substitute: $r = 8.8$

$$SA = 4\pi (8.8)^2$$

$$SA = 973.1397\dots$$

The surface area of the sphere is approximately 973.1 km^2 .

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

Substitute: $r = 8.8$

$$V = \frac{4}{3}\pi (8.8)^3$$

$$V = 2854.5432\dots$$

The volume of the sphere is approximately 2854.5 km^3 .

- b) The radius, r , is: $\frac{1}{2}(6.8 \text{ cm}) = 3.4 \text{ cm}$

SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 3.4$$

$$SA = 3\pi (3.4)^2$$

$$SA = 108.9504\dots$$

The surface area of the hemisphere is approximately 109.0 cm^2 .

Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 3.4$$

$$V = \frac{2}{3}\pi (3.4)^3$$

$$V = 82.3181\dots$$

The volume of the hemisphere is approximately 82.3 cm^3 .

7. The circumference is the length of the longest circle that can be drawn on the surface of the spherical globe.

Use the formula for the circumference, C , of a circle to determine r .

$$C = 2\pi r \quad \text{Substitute: } C = 158$$

$$158 = 2\pi r \quad \text{Solve for } r. \text{ Divide both sides by } 2\pi.$$

$$\frac{158}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{158}{2\pi}$$

$$r = 25.1464\dots$$

Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 25.1464\dots$$

$$SA = 4\pi (25.1464\dots)^2$$

$$SA = 7946.2879\dots$$

The area to be painted is approximately 7946 cm^2 .

Lesson 1.7

Solving Problems Involving Objects

Exercises (pages 59–61)

A

3. a) The surface area comprises the lateral area of the cone, one base of the cylinder, and the curved surface of the cylinder.

The cone and cylinder have equal radii.

Surface area of composite object is:

Lateral area of cone + area of base of cylinder + area of curved surface of cylinder

Use the algebraic formulas for surface area.

$$SA = \pi rs + \pi r^2 + 2\pi rh$$

$$\text{Substitute: } r = 3, s = 5, h = 5$$

$$SA = \pi(3)(5) + \pi(3)^2 + 2\pi(3)(5)$$

$$SA = 15\pi + 9\pi + 30\pi$$

$$SA = 54\pi$$

$$SA = 169.6460\dots$$

The surface area of the composite object is approximately 170 cm².

- b) The surface area comprises the lateral area of the right square pyramid, plus the areas of the four rectangular faces and base of the right square prism.

The base of the prism is a square.

Surface area of composite object is:

lateral area of right square pyramid + area of 4 rectangular faces + area of base of prism

Use the algebraic formulas for surface area.

$$SA = \frac{1}{2}s(4l) + 4hw + lw$$

$$\text{Substitute } s = 13, l = w = 10, h = 17$$

$$SA = \frac{1}{2}(13)(4)(10) + 4(17)(10) + (10)(10)$$

$$SA = 1040$$

The surface area of the composite object is 1040 square feet.

- c) The surface area comprises the curved surface of the cylinder, plus the surface area of the right square prism.

The top face of the cylinder is equal to the area of the square prism that is covered by the bottom face of the cylinder. So, do not include the area of the top face of the cylinder and do include the top face of the prism.

Surface area of composite object is:

Curved surface of cylinder + 2(area of top face of right square prism) + 2(area of front face of right square prism) + 2(area of side face of right square prism)

Use the algebraic formulas for surface area.

$$SA = 2\pi rh + 2lw + 2hw + 2hl$$

$$\text{Substitute: } r = 1, l = w = 5, h = 1$$

$$SA = 2\pi(1)(4) + 2(5)(5) + 2(1)(5) + 2(1)(5)$$

$$SA = 95.1327\dots$$

The surface area of the composite object is approximately 95 square inches.

- d) The surface area comprises the curved surface of the hemisphere, plus the lateral area of the right cone.

The hemisphere and cone have equal radii.

Surface area of composite object is:

Area of curved surface of hemisphere + lateral area of cone

Use the algebraic formulas for surface area.

$$SA = 2\pi r^2 + \pi rs$$

$$\text{Substitute: } r = 4, s = 17$$

$$SA = 2\pi(4)^2 + \pi(4)(17)$$

$$SA = 314.1592\dots$$

The surface area of the composite object is approximately 314 square inches.

4. a) I could calculate the volume of the object in part c without determining any further dimensions.
- b) Use the formula for the volume of a right cylinder and the formula for the volume of a right rectangular prism.

$$V = \pi r^2 H + lwh$$

$$\text{Substitute: } r = 1, H = 4, l = w = 5, h = 1$$

$$V = \pi(1)^2(4) + (5)(5)(1)$$

$$V = 37.5663\dots$$

The volume of the composite object is approximately 38 cubic inches.

B

5. a) The surface area comprises the curved surfaces of the two hemispheres, plus the curved surface of the cylinder.

The hemispheres and cylinder have equal radii.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(6.0 \text{ cm}) = 3.0 \text{ cm}$$

Surface area of composite object is:

Area of curved surface of hemisphere + area of curved surface of cylinder + area of curved surface of hemisphere

Use the algebraic formulas for surface area.

$$SA = 2\pi r^2 + 2\pi rh + 2\pi r^2$$

$$SA = 4\pi r^2 + 2\pi rh$$

$$\text{Substitute: } r = 3.0, h = 8.5$$

$$SA = 4\pi(3.0)^2 + 2\pi(3.0)(8.5)$$

$$SA = 273.3185\dots$$

The surface area of the composite object is approximately 273.3 cm².

Volume of composite object:

Volume of hemisphere + volume of cylinder + volume of hemisphere
= volume of sphere + volume of cylinder

Use the algebraic formulas for volume.

$$V = \frac{4}{3}\pi r^3 + \pi r^2 h \quad \text{Substitute: } r = 3.0, h = 8.5$$

$$V = \frac{4}{3}\pi (3.0)^3 + \pi (3.0)^2(8.5)$$

$$V = 353.4291\dots$$

The volume of the composite object is approximately 353.4 cm^3 .

- b) The surface area comprises the surface area of the right square prism, the lateral area of the right square pyramid, minus the overlap.

Surface area of composite object is:

surface area of right square prism + lateral area of right square pyramid – overlap

The overlap is the base of the right square pyramid.

Use the algebraic formulas for surface area.

$$SA = 4lH + 2lw + \frac{1}{2}s(4l) - lw \quad \text{Substitute: } l = 1.0, H = 2.0, w = 1.0, s = 1.5$$

$$SA = 4(1.0)(2.0) + 2(1.0)(1.0) + \frac{1}{2}(1.5)(4)(1.0) - (1.0)(1.0)$$

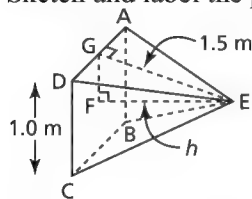
$$SA = 12.0$$

The surface area of the composite object is 12.0 m^2 .

The volume of the composite object is:

Volume of right square prism + volume of right square pyramid

Sketch and label the pyramid. Let h represent its height.



FG is $\frac{1}{2}$ the length of CD, so FG is 0.5 m.

Use the Pythagorean Theorem in right $\triangle EFG$.

$$FE^2 = GE^2 - FG^2$$

$$h^2 = 1.5^2 - 0.5^2$$

$$h^2 = 2$$

$$h = \sqrt{2}$$

Use the algebraic formulas for volume.

$$V = lwH + \frac{1}{3}lwh \quad \text{Substitute: } l = w = 1.0, H = 2.0, h = \sqrt{2}$$

$$V = (1.0)(1.0)(2.0) + \frac{1}{3}(1.0)(1.0)(\sqrt{2})$$

$$V = 2.4714\dots$$

The volume of the composite object is approximately 2.5 m^3 .

6. a) Use the formula for the curved surface area of a right cylinder to determine r .

$$SA = 2\pi rh$$

$$\text{Substitute: } SA = 219, h = 12$$

$$219 = 2\pi r(12)$$

$$219 = 24\pi r$$

Solve for r . Divide both sides by 24π .

$$\frac{219}{24\pi} = \frac{24\pi r}{24\pi}$$

$$r = \frac{219}{24\pi}$$

The diameter, d , is:

$$d = 2r \quad \text{Substitute: } r = \frac{219}{24\pi}$$

$$d = 2\left(\frac{219}{24\pi}\right)$$

$$d = 5.809\dots$$

d is approximately $5\frac{4}{5}$ in.

- b) Use the formula for the surface area of a right cylinder to determine h .

$$SA = 2\pi r^2 + 2\pi rh$$

$$\text{Substitute: } SA = 137.2, r = 2.4$$

$$137.2 = 2\pi(2.4)^2 + 2\pi(2.4)h$$

$$137.2 = 11.52\pi + 4.8\pi h$$

$$137.2 - 11.52\pi = 4.8\pi h$$

Solve for h . Divide both sides by 4.8π .

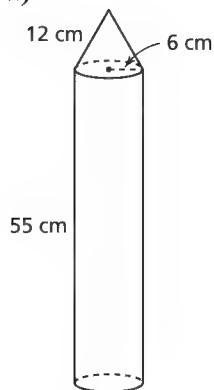
$$\frac{137.2 - 11.52\pi}{4.8\pi} = \frac{4.8\pi h}{4.8\pi}$$

$$h = \frac{137.2 - 11.52\pi}{4.8\pi}$$

$$h = 6.6983\dots$$

h is approximately 6.7 cm.

7. a)



- b) The surface area of the composite object comprises the curved surface area of the cylinder plus the area of one base of the cylinder, plus the lateral surface area of the cone.

The cone and cylinder have equal radii.

Since the area of the bottom base of the cylinder is equal to the area of the base of the cone, the surface area of composite object:

Curved surface area of cylinder + surface area of cone

Use the algebraic formulas for surface area.

$$SA = 2\pi rH + \pi rs + \pi r^2 \quad \text{Substitute: } r = 6, H = 55, s = 12$$

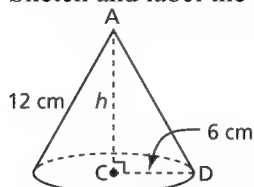
$$SA = 2\pi(6)(55) + \pi(6)(12) + \pi(6)^2$$

$$SA = 2412.7431\dots$$

The surface area of the rocket is approximately 2413 cm^2 .

- c) The volume of the rocket is the volume of the cone, plus the volume of the cylinder.

Sketch and label the cone. Let h represent its height.



Use the Pythagorean Theorem in right $\triangle ACD$ to determine h .

$$AC^2 = AD^2 - CD^2$$

$$h^2 = 12^2 - 6^2$$

$$h^2 = 108$$

$$h = \sqrt{108}$$

Use the algebraic formulas for volume.

$$V = \frac{1}{3}\pi r^2 h + \pi r^2 H \quad \text{Substitute: } r = 6, h = \sqrt{108}, H = 55$$

$$V = \frac{1}{3}\pi(6)^2(\sqrt{108}) + \pi(6)^2(55)$$

$$V = 6612.1341\dots$$

The volume of the rocket is approximately 6612 cm^3 .

- d) One-third of the interior space is one-third of the volume of the rocket.

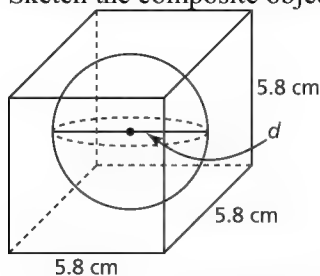
$$V_{\text{interior}} = \frac{1}{3}V_{\text{rocket}} \quad \text{Substitute: } V_{\text{rocket}} = 6612.1341\dots$$

$$V_{\text{interior}} = \frac{1}{3}(6612.1341\dots)$$

$$V_{\text{interior}} = 2204.0447\dots$$

The rocket can hold approximately 2204 cm^3 , or 2204 mL of fuel.

8. Sketch the composite object.



Determine the volume of the sphere and the volume of the cube.

The radius, r , of the sphere is: $\frac{1}{2}(5.8 \text{ cm}) = 2.9 \text{ cm}$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 2.9$$

$$V = \frac{4}{3}\pi (2.9)^3$$

$$V = 102.1604\dots$$

Use the formula for the volume of a cube.

$$V = lwh \quad \text{Substitute: } l = w = h = 5.8$$

$$V = (5.8)(5.8)(5.8)$$

$$V = 195.112$$

The volume of air in the cube is:

$$\begin{aligned} \text{Volume of cube} - \text{volume of sphere} &= 195.112 - 102.1604\dots \\ &= 92.9515\dots \end{aligned}$$

The volume of air in the cube is approximately 93 cm^3 .

9. a) Determine the volumes of the bins, then compare them.

The first bin comprises a right square prism and a right square pyramid.

Use the algebraic formulas for volume.

$$V = lwh + \frac{1}{3}lwh \quad \text{Substitute: } l = w = 15, h = 10, H = 4$$

$$V = (15)(15)(10) + \frac{1}{3}(15)(15)(4)$$

$$V = 2250 + 300$$

$$V = 2550$$

The volume of the first bin is 2550 cubic feet.

The second bin comprises a right cylinder and a right cone.

Use the algebraic formulas for volume.

$$V = \pi r^2 h + \frac{1}{3}\pi r^2 H \quad \text{Substitute: } r = 12, h = 8, H = 3$$

$$V = \pi (12)^2(8) + \frac{1}{3}\pi (12)^2(3)$$

$$V = 3619.1147... + 452.3893...$$

$$V = 4071.5040...$$

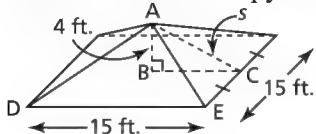
The volume of the second bin is approximately 4072 cubic feet.

Since $4072 > 2550$, the second bin holds more grain.

- b) Determine the surface areas of the bins, then compare them.

The surface area of the first bin is the sum of the areas of the 4 walls, plus the lateral area of the pyramid. First determine the slant height, s , of the right square pyramid.

Sketch and label the pyramid.



In $\triangle ABC$, BC is $\frac{1}{2}$ the length of DE , so BC is $7\frac{1}{2}$ ft.

Use the Pythagorean Theorem in $\triangle ABC$ to determine s .

The slant height, s , of the pyramid is:

$$AC^2 = AB^2 + BC^2$$

$$s^2 = 4^2 + 7.5^2$$

$$s^2 = 72.25$$

$$s = \sqrt{72.25}$$

Use the algebraic formulas for surface area.

$$SA = 4lh + \frac{1}{2}s(4l) \quad \text{Substitute: } l = 15, h = 10, s = \sqrt{72.25}$$

$$SA = 4(15)(10) + \frac{1}{2}(\sqrt{72.25})(4)(15)$$

$$SA = 855$$

The first bin has a surface area of 855 square feet.

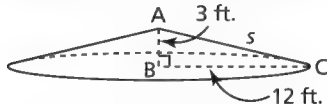
The cost to build this bin is:

$$855(\$10.49) = \$8968.95$$

The cost to build the first bin is approximately \$8970.

The surface area of the second bin is the area of the curved surface of the cylinder, plus the lateral area of the cone. First determine the slant height, s , of the cone.

Sketch and label the cone.



Use the Pythagorean Theorem in $\triangle ABC$ to determine s .

$$AC^2 = AB^2 + BC^2$$

$$s^2 = 3^2 + 12^2$$

$$s^2 = 153$$

$$s = \sqrt{153}$$

Use the algebraic formulas for surface area.

$$SA = 2\pi rh + \pi rs \quad \text{Substitute: } r = 12, h = 8, s = \sqrt{153}$$

$$SA = 2\pi(12)(8) + \pi(12)(\sqrt{153})$$

$$SA = 1069.4980\dots$$

The second bin has a surface area of approximately 1069 square feet.

The cost to build this bin is:

$$(1069.4980\dots)(\$9.25) = \$9892.8569\dots$$

The cost to build the second bin is approximately \$9890.

Since $\$8970 < \$9890$, it is cheaper to build the first bin.

10. a) The volume of the object is:
Volume of right square prism – volume of right square pyramid

Use the algebraic formulas for volume.

$$V = lwh - \frac{1}{3}lwh \quad \text{Substitute: } l = w = 10, H = 15, h = 6$$

$$V = (10)(10)(15) - \frac{1}{3}(10)(10)(6)$$

$$V = 1300$$

The volume of the object is 1300.0 cm^3 .

- b) The volume of the object is:
Volume of right cylinder – volume of hemisphere

The cylinder and hemisphere have equal radii.

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(1.5 \text{ m}) = 0.75 \text{ m}$$

Use the algebraic formulas for volume.

$$V = \pi r^2 h - \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 0.75, h = 4.0$$

$$V = \pi(0.75)^2(4.0) - \frac{2}{3}\pi(0.75)^3$$

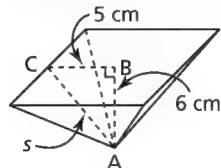
$$V = 6.1850\dots$$

The volume of the object is approximately 6.2 m^3 .

11. a) The surface area of the object is:
Area of 4 rectangular faces of prism + area of base of prism + lateral area of pyramid

First determine the slant height, s , of the pyramid.

Sketch and label the pyramid.



Use the Pythagorean Theorem in $\triangle ABC$.

$$AC^2 = AB^2 + BC^2$$

$$s^2 = 6^2 + 5^2$$

$$s^2 = 61$$

$$s = \sqrt{61}$$

Use the algebraic formulas for surface area.

$$SA = 4lh + lw + \frac{1}{2}s(4l) \quad \text{Substitute: } l = 10, h = 15, w = 10, s = \sqrt{61}$$

$$SA = 4(10)(15) + (10)(10) + \frac{1}{2}(\sqrt{61})(4)(10)$$

$$SA = 856.2049\dots$$

The surface area of the object is approximately 856.2 cm^2 .

- b)** The surface area of the object is:
Area of the curved surface of the cylinder + area of base of cylinder + area of curved surface of hemisphere

Use the algebraic formulas for surface area.

$$SA = 2\pi rh + \pi r^2 + 2\pi r^2$$

$$SA = 2\pi rh + 3\pi r^2 \quad \text{Substitute: } r = 0.75, h = 4$$

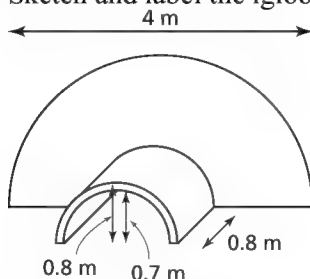
$$SA = 2\pi(0.75)(4) + 3\pi(0.75)^2$$

$$SA = 24.1509\dots$$

The surface area of the object is approximately 24.2 m^2 .

C

- 12.** Sketch and label the igloo.



The surface area is:

curved surface area of hemisphere $- \frac{1}{2}$ area of base of cylinder $+ \frac{1}{2}$ curved surface area of cylinder + surface area of the entrance

The curved surface area of hemisphere is: $2\pi r^2$, where $r = 2.0$

So, the curved surface area of hemisphere is: $2\pi(2)^2 = 8\pi$

$\frac{1}{2}$ area of base of cylinder is: $\frac{1}{2}\pi r^2$, where $r = 0.8$

So, $\frac{1}{2}$ area of base of cylinder is: $\frac{1}{2}\pi(0.8)^2$

$\frac{1}{2}$ curved surface area of cylinder is: πrh , where $r = 0.8$ and $h = 0.8$

So, $\frac{1}{2}$ curved surface area of cylinder is: $\pi(0.8)(0.8) = \pi(0.8)^2$

Surface area of the entrance is the difference in areas of those semicircles with radii 0.8 m and 0.7 m; that is, $\frac{1}{2} \pi (0.8)^2 - \frac{1}{2} \pi (0.7)^2$

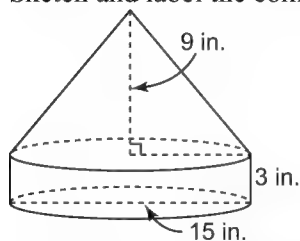
So, the total surface area SA is:

$$SA = 8\pi - \frac{1}{2} \pi (0.8)^2 + \pi (0.8)^2 + \frac{1}{2} \pi (0.8)^2 - \frac{1}{2} \pi (0.7)^2$$

$$SA = 26.3736\dots$$

The surface area of the outside of the igloo and tunnel is approximately 26.4 m^2 .

13. a) Sketch and label the composite object.



The volume of the composite object is:

Volume of right cylinder + volume of right cone

$$\text{The radius, } r, \text{ is: } \frac{1}{2}(15 \text{ in.}) = 7\frac{1}{2} \text{ in.}$$

Use the algebraic formulas for volume.

$$V = \pi r^2 H + \frac{1}{3} \pi r^2 h \quad \text{Substitute: } r = 7.5, H = 3, h = 9$$

$$V = \pi (7.5)^2 (3) + \frac{1}{3} \pi (7.5)^2 (9)$$

$$V = 1060.2875\dots$$

The volume of the sculpture is approximately 1060 cubic inches.

- b) The sculpture has diameter 15 in. and total height: $9 \text{ in.} + 3 \text{ in.} = 12 \text{ in.}$ So, the least possible dimensions for the right square prism are 15 in. by 15 in. by 12 in.

- c) Determine the volume of the right rectangular prism.

Use the algebraic formula for volume.

$$V = lwh \quad \text{Substitute: } l = 16, w = 15, h = 12$$

$$V = (16)(15)(12)$$

$$V = 2880$$

The volume of ice remaining is:

$$\begin{aligned} \text{Volume of prism} - \text{volume of sculpture} &= 2880 - 1060.2875\dots \\ &= 1819.7124\dots \end{aligned}$$

The volume of ice remaining is approximately 1820 cubic inches.

Review

(pages 64–66)

1.1

1. Answers may vary. For example:

- a) Inch; my arm is between 2 ft. and 3 ft. long, but for the most accurate measure I'd use inches.
- b) Foot; the measurement would be more accurate in feet than yards.
- c) Yard; the length of the gym is measured in yards, and since I ran the length of the gym many times, I'd measure my distance in yards.

2. Answers will vary. For example:

- a) The length of my arm: The referent was the length of my thumb to the first knuckle, which is approximately 1 inch. I estimated that the length of my arm from shoulder to fingertip is 27 in.
The width of my classroom: My referent was my foot length from heel to toe, which is a little less than 1 ft. I estimated that the width of my classroom is approximately 25 ft.
- b) I used a ruler to measure the width of the length of my arm: 26 in.
My estimate was reasonable.
I used a tape measure to measure the width of the classroom: 24 ft.
My estimate was reasonable.

3. a) Since 1 yd. = 3 ft., to convert yards to feet, multiply by 3.

$$14 \text{ yd.} = 14(3 \text{ ft.})$$

$$14 \text{ yd.} = 42 \text{ ft.}$$

- b) Since 1 mi. = 1760 yd., to convert miles to yards, multiply by 1760.

$$5 \text{ mi.} = 5(1760 \text{ yd.})$$

$$5 \text{ mi.} = 8800 \text{ yd.}$$

- c) Convert 6 ft. to inches.

Since 1 ft. = 12 in., to convert feet to inches, multiply by 12.

$$6 \text{ ft.} = 6(12 \text{ in.})$$

$$6 \text{ ft.} = 72 \text{ in.}$$

$$\begin{aligned} \text{So, } 6 \text{ ft. } 3 \text{ in.} &= 72 \text{ in.} + 3 \text{ in.} \\ &= 75 \text{ in.} \end{aligned}$$

- d) First convert inches to feet.

Since 12 in. = 1 ft., divide by 12.

$$123 \text{ in.} = \frac{123}{12} \text{ ft.}$$

$$123 \text{ in.} = 10\frac{3}{12} \text{ ft.}$$

$$123 \text{ in.} = 10 \text{ ft. } 3 \text{ in.}$$

Now convert feet to yards.

Since 3 ft. = 1 yd., divide by 3.

$$10 \text{ ft.} = \frac{10}{3} \text{ yd.}$$

$$10 \text{ ft.} = 3\frac{1}{3} \text{ yd.}$$

$$10 \text{ ft.} = 3 \text{ yd. } 1 \text{ ft.}$$

$$\text{So, } 123 \text{ in.} = 3 \text{ yd. } 1 \text{ ft. } 3 \text{ in.}$$

4. The scale of the model is 1 in. represents 40 in.
So, 8 in. represents $8(40 \text{ in.}) = 320 \text{ in.}$

First convert inches to feet.

Since $12 \text{ in.} = 1 \text{ ft.}$, divide by 12.

$$320 \text{ in.} = \frac{320}{12} \text{ ft.}$$

$$320 \text{ in.} = 26\frac{8}{12} \text{ ft.}$$

$$320 \text{ in.} = 26 \text{ ft. } 8 \text{ in.}$$

Now convert feet to yards.

Since $3 \text{ ft.} = 1 \text{ yd.}$, divide by 3.

$$26 \text{ ft.} = \frac{26}{3} \text{ yd.}$$

$$26 \text{ ft.} = 8\frac{2}{3} \text{ yd.}$$

$$26 \text{ ft.} = 8 \text{ yd. } 2 \text{ ft.}$$

So, the actual plane is 320 in., or 8 yd. 2 ft. 8 in.

1.2

5. Strategies may vary. For example:

a) I would use the width of my hand, which is about 4 in. or 10 cm, to estimate the diameter.
I would place one hand next to the other, beginning where the tire touches the ground, and measuring to the highest point on the tire.
I would use a tape measure to measure the diameter in imperial units or SI units.

b) I would use the length of my stride, which is about 0.6 m or about 2 ft., to estimate the length of a car. I would use a tape measure marked in imperial or SI units to measure the length.

c) I would use the width of my thumb as a referent, which is approximately 1.5 cm or $\frac{1}{2}$ in.
I would use calipers to measure the diameter in inches or centimetres, then divide by 2 to determine the radius.

1.3

6. Answers will vary depending on the conversion ratios used.

a) Use the exact conversion: $2.54 \text{ cm} = 1 \text{ in.}$

$$\text{So, } 261 \text{ cm} = \frac{261}{2.54} \text{ in.}$$

$$261 \text{ cm} = 102.755... \text{ in.}$$

$$261 \text{ cm} \doteq 103 \text{ in.}$$

$$12 \text{ in.} = 1 \text{ ft.}$$

$$\text{So, } 103 \text{ in.} = \frac{103}{12} \text{ ft.}$$

$$103 \text{ in.} = 8\frac{7}{12} \text{ ft.}$$

$$103 \text{ in.} = 8 \text{ ft. } 7 \text{ in.}$$

$$\text{So, } 261 \text{ cm} \doteq 8 \text{ ft. } 7 \text{ in.}$$

- b)** Use the exact conversion:

$$1 \text{ yd.} = 91.44 \text{ cm}$$

$$1 \text{ yd.} = 0.9144 \text{ m}$$

Divide to convert metres to yards.

$$125 \text{ m} = \frac{125}{0.9144} \text{ yd.}$$

$$125 \text{ m} = 136.7016... \text{ yd.}$$

$$1 \text{ yd.} = 3 \text{ ft.}$$

$$\text{So, } 0.7016... \text{ yd.} = 0.7016...(3 \text{ ft.})$$

$$0.7016... \text{ yd.} = 2.1049... \text{ ft.}$$

$$1 \text{ ft.} = 12 \text{ in.}$$

$$\text{So, } 0.1049... \text{ ft.} = 0.1049...(12 \text{ in.})$$

$$0.1049... \text{ ft.} = 1.2598... \text{ in.}$$

$$\text{So, } 125 \text{ m} \doteq 136 \text{ yd. } 2 \text{ ft. } 1 \text{ in.}$$

- c)** Use: $1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$

$$\text{So, } 6 \text{ km} = 6\left(\frac{6}{10}\right) \text{ mi.}$$

$$6 \text{ km} = 3.6 \text{ mi.}$$

$$\text{Use: } 1 \text{ mi.} = 1760 \text{ yd.}$$

$$\text{So, } 0.6 \text{ mi.} = 0.6(1760) \text{ yd.}$$

$$0.6 \text{ mi.} = 1056 \text{ yd.}$$

$$\text{So, } 6 \text{ km} \doteq 3 \text{ mi. } 1056 \text{ yd.}$$

- d)** Use: $10 \text{ mm} = 1 \text{ cm}$
So, $350 \text{ mm} = 35 \text{ cm}$

$$\text{Use: } 2.54 \text{ cm} = 1 \text{ in.}$$

$$\text{So, } 35 \text{ cm} = \frac{35}{2.54} \text{ in.}$$

$$35 \text{ cm} = 13.7795... \text{ in.}$$

$$35 \text{ cm} \doteq 14 \text{ in., or } 1 \text{ ft. } 2 \text{ in.}$$

$$\text{So, } 350 \text{ mm} \doteq 1 \text{ ft. } 2 \text{ in.}$$

7. a) First, convert 13 yd. 2 ft. to feet.

Use the conversion: 1 yd. = 3 ft.

$$\text{So, } 13 \text{ yd.} = 13(3 \text{ ft.})$$

$$13 \text{ yd.} = 39 \text{ ft.}$$

$$\text{So, } 13 \text{ yd. } 2 \text{ ft.} = 39 \text{ ft.} + 2 \text{ ft.}$$

$$13 \text{ yd. } 2 \text{ ft.} = 41 \text{ ft.}$$

Since 1 in. = 2.54 cm and 12 in. = 1 ft.,

$$1 \text{ ft.} = 12(2.54 \text{ cm})$$

$$1 \text{ ft.} = 30.48 \text{ cm}$$

$$\text{So, } 41 \text{ ft.} = 41(30.48 \text{ cm})$$

$$41 \text{ ft.} = 1249.68 \text{ cm}$$

$$41 \text{ ft.} = 12.49... \text{ m}$$

$$41 \text{ ft.} \approx 12.5 \text{ m}$$

$$\text{So, } 13 \text{ yd. } 2 \text{ ft.} \approx 12.5 \text{ m}$$

- b) First, convert 4 mi. 350 yd. to yards. Use the conversion 1 mi. = 1760 yd.

$$\text{Then, } 4 \text{ mi.} = 4(1760 \text{ yd.})$$

$$4 \text{ mi.} = 7040 \text{ yd.}$$

$$\text{So, } 4 \text{ mi. } 350 \text{ yd.} = 7040 \text{ yd.} + 350 \text{ yd.}$$

$$4 \text{ mi. } 350 \text{ yd.} = 7390 \text{ yd.}$$

Then, use the conversion: 1 yd. = 91.44 cm

$$7390 \text{ yd.} = 7390(91.44 \text{ cm})$$

$$7390 \text{ yd.} = 675\,741.6 \text{ cm}$$

$$7390 \text{ yd.} = 6.7574... \text{ km}$$

$$7390 \text{ yd.} \approx 6.8 \text{ km}$$

$$\text{So, } 4 \text{ mi. } 350 \text{ yd.} \approx 6.8 \text{ km}$$

- c) First, convert 1 ft. 7 in. to inches. Use the conversion: 1 ft. = 12 in.

$$\text{So, } 1 \text{ ft. } 7 \text{ in.} = 12 \text{ in.} + 7 \text{ in.}$$

$$1 \text{ ft. } 7 \text{ in.} \approx 19 \text{ in.}$$

Then, use the conversion: 1 in. = 2.54 cm

$$\text{So, } 19 \text{ in.} = 19(2.54 \text{ cm})$$

$$19 \text{ in.} = 48.26 \text{ cm}$$

$$19 \text{ in.} \approx 48.3 \text{ cm}$$

$$\text{So, } 1 \text{ ft. } 7 \text{ in.} \approx 48.3 \text{ cm}$$

- d) Use the conversion: 1 in. = 2.54 cm

$$8\frac{1}{2} \text{ in.} = 8.5 \text{ in.}$$

$$\text{So, } 8\frac{1}{2} \text{ in.} = 8.5(2.54 \text{ cm})$$

$$8\frac{1}{2} \text{ in.} = 21.59 \text{ cm}$$

$$\text{So, } 8\frac{1}{2} \text{ in.} = 215.9 \text{ mm}$$

8. First convert 460 km to centimetres.

$$460 \text{ km} = 460(1000) \text{ m}$$

$$460 \text{ km} = 460(1000)(100) \text{ cm}$$

$$460 \text{ km} = 46\,000\,000 \text{ cm}$$

Now convert 46 000 000 cm to inches.

Use: 2.54 cm = 1 in.

$$\text{So, } 46\,000\,000 \text{ cm} = \frac{46\,000\,000}{2.54} \text{ in.}$$

$$46\,000\,000 \text{ cm} = 18\,110\,236.22\ldots \text{ in.}$$

The length of Vancouver Island is 18 110 236.22... in. If Sarah's average stride is 27 in., divide to determine how many strides she takes:

$$\frac{18\,110\,236.22\ldots \text{ in.}}{27 \text{ in.}} = 670\,749.4896\ldots$$

Sarah takes approximately 670 750 strides.

1.4

9. a) Use the formula for the surface area of a right cone.

$$SA = \pi rs + \pi r^2 \quad \text{Substitute: } r = 3, s = 5$$

$$SA = \pi (3)(5) + \pi (3)^2$$

$$SA = 75.3982\ldots$$

The surface area of the cone is approximately 75 square feet.

- b) Use the formula for the surface area of a right pyramid with a regular triangle base.

$$SA = \frac{1}{2}s(3l) + \frac{1}{2}sl \quad \text{Substitute: } s = 6.1, l = 7.0$$

$$SA = \frac{1}{2}(6.1)(3)(7.0) + \frac{1}{2}(7.0)(6.1)$$

$$SA = 85.4$$

The surface area of the regular tetrahedron is approximately 85 cm².

- c) The radius, r , is: $\frac{1}{2}(22 \text{ mm}) = 11 \text{ mm}$

Use the formula for the surface area of a right cone.

$$SA = \pi rs + \pi r^2 \quad \text{Substitute: } r = 11, s = 15$$

$$SA = \pi (11)(15) + \pi (11)^2$$

$$SA = 898.4954\ldots$$

The surface area of the cone is approximately 898 mm².

- d) The surface area is the area of the 4 triangular faces, plus the area of the base. Opposite triangular faces are congruent.

The area, A , of the front triangular face is:

$$A = \frac{1}{2}(7.8)(11.2)$$

$$A = 43.68$$

Since the front and back triangular faces are congruent, the area of the back triangular face is 43.68.

The area, A , of the triangular face at the right is:

$$A = \frac{1}{2}(5.4)(11.6)$$

$$A = 31.32$$

Since the triangular faces at the right and left are congruent, the area of the triangular face at the left is 31.32.

The area, B , of the base of the pyramid is:

$$B = (7.8)(5.4)$$

$$B = 42.12$$

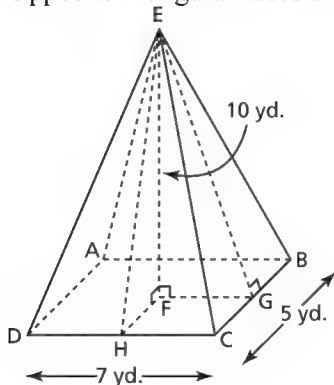
Surface area, SA , of the pyramid is:

$$SA = 2(43.68) + 2(31.32) + 42.12$$

$$SA = 192.12$$

The surface area of the pyramid is approximately 192 m^2 .

10. There are 4 triangular faces and a rectangular base. Sketch the pyramid and label its vertices. Opposite triangular faces are congruent. Draw the heights on two adjacent triangles.



In $\triangle EFH$, FH is $\frac{1}{2}$ the length of BC , so FH is $2\frac{1}{2}$ yd.

EF is the height of the pyramid, which is 10 yd.

Use the Pythagorean Theorem in right $\triangle EFH$ to determine the length of EH .

$$EH^2 = EF^2 + FH^2$$

$$EH^2 = 10^2 + 2.5^2$$

$$EH^2 = 106.25$$

$$EH = \sqrt{106.25}$$

Area, A , of $\triangle EDC$ is:

$$A = \frac{1}{2}(7)(\sqrt{106.25})$$

$$A = 3.5(\sqrt{106.25})$$

Since $\triangle EDC$ and $\triangle EAB$ are congruent, the area of $\triangle EAB$ is $3.5(\sqrt{106.25})$.

In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is $3\frac{1}{2}$ yd.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine the length of GE .

$$GE^2 = EF^2 + FG^2$$

$$GE^2 = 10^2 + 3.5^2$$

$$GE^2 = 112.25$$

$$GE = \sqrt{112.25}$$

Area, A , of $\triangle EBC$ is:

$$A = \frac{1}{2}(5)(\sqrt{112.25})$$

$$A = 2.5(\sqrt{112.25})$$

Since $\triangle EBC$ and $\triangle EAD$ are congruent, the area of $\triangle EAD$ is $2.5(\sqrt{112.25})$.

Area, B , of the base of the pyramid is:

$$B = (7)(5)$$

$$B = 35$$

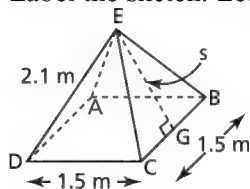
Surface area, SA , of the pyramid is:

$$SA = 2(3.5)(\sqrt{106.25}) + 2(2.5)(\sqrt{112.25}) + 35$$

$$SA = 160.1283\dots$$

The surface area of the pyramid is approximately 160 square yards.

11. a) Label the sketch. Let s represent the slant height of the tent.



- b) In $\triangle EGB$, GB is $\frac{1}{2}$ the length of CB , so GB is 0.75 m.

Use the Pythagorean Theorem in right $\triangle EGB$.

$$EG^2 = EB^2 - GB^2$$

$$s^2 = 2.1^2 - 0.75^2$$

$$s^2 = 3.8475$$

$$s = \sqrt{3.8475}$$

$$s = 1.9615\dots$$

The slant height of the tent is approximately 2.0 m.

- c) The lateral area, A_L , of the tent is:

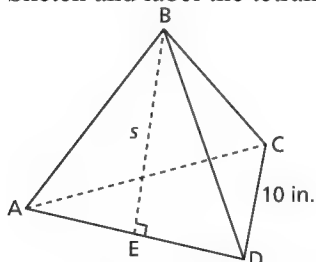
$$A_L = \left(\frac{1}{2}s\right)(4l) \quad \text{Substitute: } s = \sqrt{3.8475}, l = 1.5$$

$$A_L = \frac{1}{2}(\sqrt{3.8475})(4)(1.5)$$

$$A_L = 5.8845\dots$$

The lateral surface area of the tent is approximately 6 m^2 .

12. a) Sketch and label the tetrahedron. Let s represent the slant height.



In $\triangle BED$, ED is $\frac{1}{2}$ the length of AD , so ED is 5 in.

Use the Pythagorean Theorem in right $\triangle BED$.

$$BE^2 = BD^2 - ED^2$$

$$s^2 = 10^2 - 5^2$$

$$s^2 = 75$$

$$s = \sqrt{75}$$

$$s = 8.6602\dots$$

The slant height of the tetrahedron is approximately $8\frac{7}{10}$ in.

- b) The area, A , of each face of the tetrahedron is:

$$A = \frac{1}{2}(10)(\sqrt{75})$$

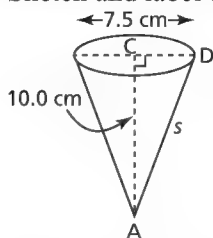
The surface area, SA , is:

$$SA = 4\left(\frac{1}{2}\right)(10)(\sqrt{75})$$

$$SA = 173.2050\dots$$

The surface area of the tetrahedron is approximately 173 square inches.

13. Sketch and label the cone. Let s represent the slant height.



The radius, r , is: $\frac{1}{2}(7.5 \text{ cm}) = 3.75 \text{ cm}$

Use the Pythagorean Theorem in right $\triangle ACD$ to determine s .

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 10^2 + 3.75^2$$

$$s^2 = 114.0625$$

$$s = \sqrt{114.0625}$$

Use the formula for the lateral area of a right cone.

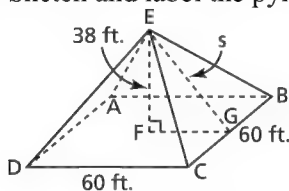
$$SA = \pi rs \quad \text{Substitute: } r = 3.75, s = \sqrt{114.0625}$$

$$SA = \pi (3.75)(\sqrt{114.0625})$$

$$SA = 125.8208\dots$$

The area to be coated is approximately 125.8 cm^2 .

14. Sketch and label the pyramid. Let s represent the slant height.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is 30 ft.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine s .

$$EG^2 = EF^2 + FG^2$$

$$s^2 = 38^2 + 30^2$$

$$s^2 = 2344$$

$$s = \sqrt{2344}$$

Determine the lateral area, A_L , of the pyramid.

$$A_L = \left(\frac{1}{2}s\right)(4l) \quad \text{Substitute: } s = \sqrt{2344}, l = 60$$

$$A_L = \frac{1}{2}(\sqrt{2344})(4)(60)$$

$$A_L = 5809.7848\dots$$

The area of limestone needed is approximately 5810 square feet.

1.5

15. a) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3}lwh$$

$$\text{Substitute: } l = 3.5, w = 2.0, h = 4.5$$

$$V = \frac{1}{3}(3.5)(2.0)(4.5)$$

$$V = 10.5$$

The volume of the pyramid is approximately 11 m^3 .

- b) Use the formula for the volume of a right cone.

$$V = \frac{1}{3}\pi r^2 h$$

$$\text{Substitute: } r = 18, h = 26$$

$$V = \frac{1}{3} \pi (18)^2 (26)$$

$$V = 8821.5921 \dots$$

The volume of the cone is approximately 8822 cubic inches.

- c) Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh$$

$$\text{Substitute: } l = 2, w = 2, h = 5$$

$$V = \frac{1}{3} (2)(2)(5)$$

$$V = 6.6666 \dots$$

The volume of the pyramid is approximately 7 cubic feet.

- d) The radius, r , is: $\frac{1}{2} (13 \text{ mm}) = 6.5 \text{ mm}$

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h$$

$$\text{Substitute: } r = 6.5, h = 5$$

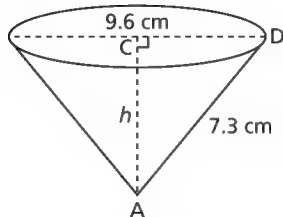
$$V = \frac{1}{3} \pi (6.5)^2 (5)$$

$$V = 221.2204 \dots$$

The volume of the cone is approximately 221 mm^3 .

16. No, Owen needs to determine the height of the cone to determine its volume.

Sketch and label the cone. Let h represent the height of the cone.



$$\text{The radius, } r, \text{ is: } \frac{1}{2} (9.6 \text{ cm}) = 4.8 \text{ cm}$$

Use the Pythagorean Theorem in right $\triangle ACD$ to determine h .

$$AC^2 = AD^2 - CD^2$$

$$h^2 = 7.3^2 - 4.8^2$$

$$h^2 = 30.25$$

$$h = \sqrt{30.25}$$

$$h = 5.5$$

Use the formula for the volume of a right cone.

$$V = \frac{1}{3} \pi r^2 h$$

$$\text{Substitute: } r = 4.8, h = 5.5$$

$$V = \frac{1}{3} \pi (4.8)^2 (5.5)$$

$$V = 132.7008\dots$$

The volume of the cone is approximately 132.7 cm^3 .

17. Use the formula for the volume of a right rectangular pyramid to determine h .

$$V = \frac{1}{3} lwh$$

$$\text{Substitute: } V = 400, l = w = 10$$

$$400 = \frac{1}{3} (10)(10)h$$

$$1200 = 100h$$

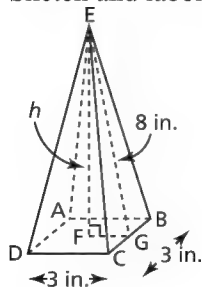
Solve for h .

$$\frac{1200}{100} = \frac{100h}{100}$$

$$h = 12$$

The height of the pyramid was 12 cm.

18. a) Sketch and label the pyramid. Let h represent its height.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC , so FG is $1\frac{1}{2}$ in.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine h .

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 8^2 - 1.5^2$$

$$h^2 = 61.75$$

$$h = \sqrt{61.75}$$

Use the formula for the volume of a right rectangular pyramid.

$$V = \frac{1}{3} lwh$$

$$\text{Substitute: } l = 3, w = 3, h = \sqrt{61.75}$$

$$V = \frac{1}{3} (3)(3)(\sqrt{61.75})$$

$$V = 23.5743\dots$$

The volume of the ornament is approximately 24 cubic inches.

- b) Let x inches represent the side length of the base.

Use the formula for the volume of a right rectangular pyramid to determine x .

$$V = \frac{1}{3} lwh$$

$$\text{Substitute: } V = 96, l = w = x, h = \sqrt{61.75}$$

$$96 = \frac{1}{3} (x)(x)(\sqrt{61.75})$$

Multiply both sides by 3.

$$\begin{aligned}
 288 &= \sqrt{61.75} x^2 && \text{Divide both sides by } \sqrt{61.75}. \\
 \frac{288}{\sqrt{61.75}} &= \frac{\sqrt{61.75} x^2}{\sqrt{61.75}} \\
 x^2 &= \frac{288}{\sqrt{61.75}} \\
 x &= \sqrt{\frac{288}{\sqrt{61.75}}} \\
 x &= 6.0539\dots
 \end{aligned}$$

The side length of the base is approximately 6 in.

19. a) Use the formula for the volume of a right cone to determine r .

$$\begin{aligned}
 V &= \frac{1}{3} \pi r^2 h && \text{Substitute: } V = 41.6, h = 9.0 \\
 41.6 &= \frac{1}{3} \pi r^2 (9.0) && \text{Multiply both sides by 3.} \\
 124.8 &= 9.0 \pi r^2 && \text{Solve for } r. \text{ Divide both sides by } 9.0\pi. \\
 \frac{124.8}{9.0\pi} &= \frac{9.0\pi r^2}{9.0\pi} \\
 r^2 &= \frac{124.8}{9.0\pi} \\
 r &= \sqrt{\frac{124.8}{9.0\pi}} \\
 r &= 2.1009\dots \\
 r &\text{ is approximately 2.1 m.}
 \end{aligned}$$

- b) Use the formula for the volume of a right rectangular pyramid to determine x .

$$\begin{aligned}
 V &= \frac{1}{3} lwh && \text{Substitute: } V = 68.4, l = 9.0, w = x, h = 10.0 \\
 68.4 &= \frac{1}{3} (9.0)(x)(10.0) && \text{Multiply both sides by 3.} \\
 205.2 &= 90.0x && \text{Solve for } x. \text{ Divide both sides by 3.} \\
 \frac{205.2}{90.0} &= \frac{90.0x}{90.0} \\
 x &= \frac{205.2}{90.0} \\
 x &= 2.28 \\
 x &\text{ is approximately 2.3 cm.}
 \end{aligned}$$

1.6

20. a) Use the formula for the surface area of a sphere.

$$\begin{aligned}
 SA &= 4\pi r^2 && \text{Substitute: } r = 4.5 \\
 SA &= 4\pi (4.5)^2 \\
 SA &= 254.4690\dots \\
 &\text{The surface area of the sphere is approximately 254 square inches.}
 \end{aligned}$$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 4.5$$

$$V = \frac{4}{3}\pi (4.5)^3$$

$$V = 381.7035\dots$$

The volume of the sphere is approximately 382 cubic inches.

b) The radius, r , is: $\frac{1}{2}(6.5 \text{ m}) = 3.25 \text{ m}$

Use the formula for the surface area of a sphere.

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 3.25$$

$$SA = 4\pi (3.25)^2$$

$$SA = 132.7322\dots$$

The surface area of the sphere is approximately 133 m².

Use the formula for the volume of a sphere.

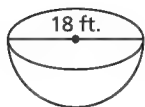
$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 3.25$$

$$V = \frac{4}{3}\pi (3.25)^3$$

$$V = 143.7933\dots$$

The volume of the sphere is approximately 144 m³.

21.



a) The radius, r , of the hemisphere is: $\frac{1}{2}(18 \text{ ft.}) = 9 \text{ ft.}$

SA of a hemisphere = SA of one-half a sphere + area of circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$SA = 2\pi r^2 + \pi r^2$$

$$SA = 3\pi r^2 \quad \text{Substitute: } r = 9$$

$$SA = 3\pi (9)^2$$

$$SA = 763.4070\dots$$

The surface area of the hemisphere is approximately 763 square feet.

b) Volume of a hemisphere = volume of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$V = \frac{2}{3}\pi r^3 \quad \text{Substitute: } r = 9$$

$$V = \frac{2}{3}\pi(9)^3$$

$$V = 1526.8140\dots$$

The volume of the hemisphere is approximately 1527 cubic feet.

22. Use the formula for the surface area of a sphere to determine r .

$$SA = 4\pi r^2$$

$$\text{Substitute: } SA = 66$$

$$66 = 4\pi r^2$$

Solve for r . Divide both sides by 4π .

$$\frac{66}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$r^2 = \frac{66}{4\pi}$$

$$r = \sqrt{\frac{66}{4\pi}}$$

The diameter, d , is:

$$d = 2r$$

$$d = 2\left(\sqrt{\frac{66}{4\pi}}\right)$$

$$d = 4.5834\dots$$

$$d \doteq 4.6$$

The diameter of the sphere is approximately $4\frac{6}{10}$ in., or $4\frac{3}{5}$ in.

23. Use the formula for circumference, C , to determine r .

$$C = 2\pi r$$

$$\text{Substitute: } C = 18$$

$$18 = 2\pi r$$

Solve for r . Divide both sides by 2π .

$$\frac{18}{2\pi} = \frac{2\pi r}{2\pi}$$

$$r = \frac{18}{2\pi}$$

$$r = 2.8647\dots$$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$\text{Substitute: } r = 2.8647\dots$$

$$V = \frac{4}{3}\pi(2.8647\dots)^3$$

$$V = 98.4841\dots$$

The volume of the sphere is approximately 98 cm^3 .

24. Use the formula for the surface area of a sphere to determine r .

$$SA = 4\pi r^2$$

$$\text{Substitute: } SA = 314$$

$$314 = 4\pi r^2$$

Solve for r . Divide both sides by 4π .

$$\frac{314}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$r^2 = \frac{314}{4\pi}$$

$$r = \sqrt{\frac{314}{4\pi}}$$

$$r = 4.9987\dots$$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3 \quad \text{Substitute: } r = 4.9987\dots$$

$$V = \frac{4}{3}\pi (4.9987\dots)^3$$

$$V = 523.2006\dots$$

The volume of the sphere is approximately 523 cubic inches.

1.7

25. a) The surface area comprises the lateral area of the right square pyramid, plus the areas of the four rectangular faces and base of the right square prism.

Surface area of composite object is:

Lateral area of pyramid + area of 4 rectangular faces + area of base

Use the algebraic formulas for surface area.

$$SA = \frac{1}{2}s(4l) + 4Hw + lw \quad \text{Substitute: } s = 11, l = w = 6, H = 13$$

$$SA = \frac{1}{2}(11)(4)(6) + 4(13)(6) + (6)(6)$$

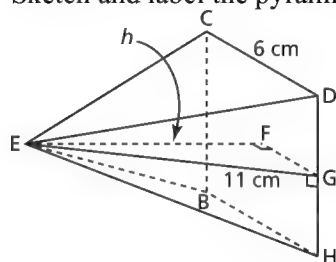
$$SA = 480$$

The surface area of the composite object is 480 cm^2 .

The volume of the composite object is:

Volume of right square pyramid + volume of right square prism

Sketch and label the pyramid. Let h represent its height.



In $\triangle EFG$, FG is $\frac{1}{2}$ the length of CD , so FG is 3 m.

Use the Pythagorean Theorem in right $\triangle EFG$ to determine h .

$$EF^2 = EG^2 - FG^2$$

$$h^2 = 11^2 - 3^2$$

$$h^2 = 112$$

$$h = \sqrt{112}$$

Use the algebraic formulas for volume.

$$V = \frac{1}{3}lwh + lwh \quad \text{Substitute: } l = w = 6, h = \sqrt{112}, H = 13$$

$$V = \frac{1}{3}(6)(6)(\sqrt{112}) + (6)(6)(13)$$

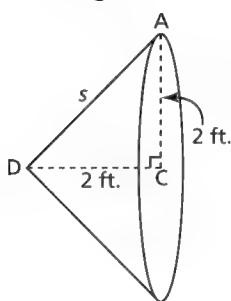
$$V = 594.9960\dots$$

The volume of the composite object is approximately 595 cm^3 .

- b) The surface area comprises the curved surface area of the cylinder, plus the lateral areas of the cones.

Surface area of composite object:

Lateral area of cone at the left + curved surface area of cylinder + lateral area of cone at the right



Determine the slant heights of the cones.

For the cone at the left, let s represent the slant height.

Use the Pythagorean Theorem in right $\triangle ACD$.

$$AD^2 = AC^2 + CD^2$$

$$s^2 = 2^2 + 2^2$$

$$s^2 = 8$$

$$s = \sqrt{8}$$

For the cone at the right, let x represent the slant height.

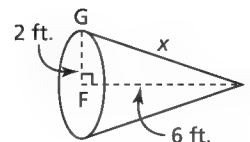
Use the Pythagorean Theorem in right $\triangle EFG$.

$$EG^2 = EF^2 + FG^2$$

$$x^2 = 6^2 + 2^2$$

$$x^2 = 40$$

$$x = \sqrt{40}$$



Use the algebraic formulas for surface area.

$$SA = \pi rs + 2\pi rh + \pi rx \quad \text{Substitute: } r = 2, s = \sqrt{8}, h = 4, x = \sqrt{40}$$

$$SA = \pi(2)(\sqrt{8}) + 2\pi(2)(4) + \pi(2)(\sqrt{40})$$

$$SA = 107.7753\dots$$

The surface area of the composite object is approximately 108 square feet.

The volume of the composite object is:

Volume of cone at the left + volume of right cylinder + volume of cone at the right

Use the algebraic formulas for volume.

$$V = \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{1}{3}\pi r^2 h_3 \quad \text{Substitute: } r = 2, h_1 = 2, h_2 = 4, h_3 = 6$$

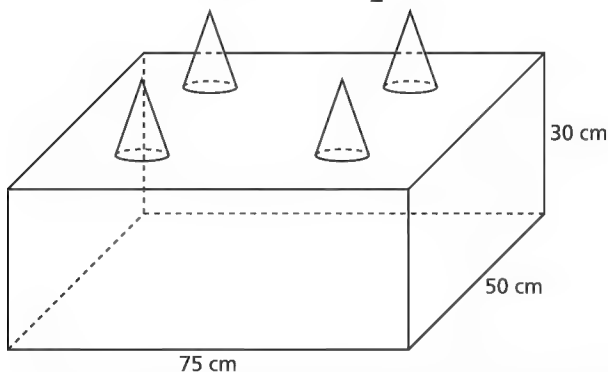
$$V = \frac{1}{3}\pi(2)^2(2) + \pi(2)^2(4) + \frac{1}{3}\pi(2)^2(6)$$

$$V = 83.7758\dots$$

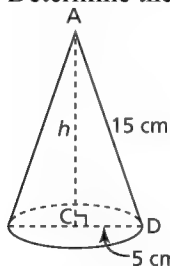
The volume of the composite object is approximately 84 cubic feet.

26. a) The volume of the sandcastle is:
Volume of right rectangular prism + volume of 4 right cones

The radius, r , of each cone is: $\frac{1}{2}(10 \text{ cm}) = 5 \text{ cm}$



Determine the height, h , of a cone. Use the Pythagorean Theorem in right $\triangle ACD$.



$$\begin{aligned} AC^2 &= AD^2 - CD^2 \\ h^2 &= 15^2 - 5^2 \\ h^2 &= 200 \\ h &= \sqrt{200} \end{aligned}$$

Use the algebraic formulas for volume.

$$V = lwh + 4\left(\frac{1}{3}\pi r^2 h\right) \quad \text{Substitute: } l = 75, w = 50, H = 30, r = 5, h = \sqrt{200}$$

$$V = (75)(50)(30) + 4\left(\frac{1}{3}\pi (5)^2(\sqrt{200})\right)$$

$$V = 113\,980.961$$

The volume of sand required to construct the castle is approximately $113\,981 \text{ cm}^3$.

- b) The surface area of the castle is:
Surface area of four side faces and top face of right rectangular prism + lateral area of 4 right cones – area of the bases of 4 cones

Use the algebraic formulas for surface area.

$$SA = 2(\text{area of front face}) + 2(\text{area of side face}) + \text{area of top face} + 4\pi rs - 4\pi r^2$$

$$SA = 2(75)(30) + 2(50)(30) + (50)(75) + 4\pi(5)(15) - 4\pi(5)^2$$

$$SA = 11\,878.3185\dots$$

The surface area of the sandcastle is approximately $11\,878 \text{ cm}^2$.

27. a) Use the formula for the surface area of a right square pyramid to determine s .

$$SA = \frac{1}{2}s(4l) + l^2 \quad \text{Substitute: } SA = 132, l = 6$$

$$132 = \frac{1}{2}s(4)(6) + (6)(6)$$

$$132 = 12s + 36$$

$$96 = 12s$$

$$\frac{96}{12} = \frac{12s}{12}$$

$$s = \frac{96}{12}$$

$$s = 8$$

s is 8 cm.

- b) Use the formula for the surface area of a right cone to determine s .

$$SA = \pi rs + \pi r^2 \quad \text{Substitute: } SA = 176, r = 4$$

$$176 = \pi(4)s + \pi(4)^2$$

$$176 = 4\pi s + 16\pi$$

$$176 - 16\pi = 4\pi s$$

$$\frac{176 - 16\pi}{4\pi} = s$$

$$s = 10.0056\dots$$

s is approximately 10 mm.

Practice Test

(page 67)

1. Use the conversion: $1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$

To convert centimetres to inches, multiply by $\frac{4}{10}$.

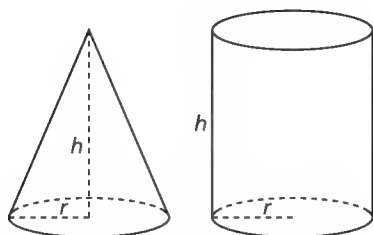
$$\text{So, } 3 \text{ cm} \doteq 3\left(\frac{4}{10} \text{ in.}\right)$$

$$3 \text{ cm} \doteq 3(0.4 \text{ in.})$$

So, the correct answer is B.

2. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, so the correct answer is C.

3.



Use the formula for the volume of a cylinder.

$$V_{\text{cylinder}} = \pi r^2 h$$

Use the formula for the volume of a cone.

$$V_{\text{cylinder}} = \frac{1}{3}\pi r^2 h$$

To compare the volumes, divide:

$$\frac{V_{\text{cylinder}}}{V_{\text{cone}}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = 3$$

So, the volume of a right cylinder is 3 times the volume of a right cone with the same base radius and the same height.

4. a) The volume of the composite object is:
Volume of rectangular prism + volume of rectangular pyramid

Use the algebraic formulas for volume.

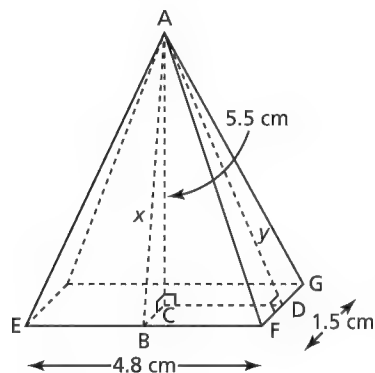
$$V = lwh + \frac{1}{3}(lwh) \quad \text{Substitute: } l = 4.8, w = 1.5, H = 1.5, h = 5.5$$

$$V = (4.8)(1.5)(2.1) + \frac{1}{3}(4.8)(1.5)(5.5)$$

$$V = 28.32$$

The volume of the composite object is approximately 28.32 cm^3 .

For the surface area, first determine the heights of adjacent triangular faces of the pyramid.



BC is $\frac{1}{2}(1.5 \text{ cm})$, or 0.75 cm.

Let x represent the height of $\triangle AEF$.

Use the Pythagorean Theorem in right $\triangle ABC$.

$$x^2 = 5.5^2 + 0.75^2$$

$$x^2 = 30.8125$$

$$x = \sqrt{30.8125}$$

Let y represent the height of $\triangle AFG$.

CD is $\frac{1}{2}(4.8 \text{ cm})$, or 2.4 cm.

Use the Pythagorean Theorem in right $\triangle ACD$.

$$y^2 = 5.5^2 + 2.4^2$$

$$y^2 = 36.01$$

$$y = \sqrt{36.01}$$

Surface area of the composite object is:

Area of 4 triangular faces + area of 4 rectangular faces + area of base

Use the algebraic formulas for surface area.

$$SA = 2\left(\frac{1}{2}xl\right) + 2\left(\frac{1}{2}yw\right) + 2(lh) + 2(wh) + lw$$

Substitute: $x = \sqrt{30.8125}$, $l = 4.8$, $y = \sqrt{36.01}$, $w = 1.5$, $h = 2.1$

$$SA = (\sqrt{30.8125})(4.8) + (\sqrt{36.01})(1.5) + 2(4.8)(2.1) + 2(1.5)(2.1) + (4.8)(1.5)$$

$$SA = 69.3055\dots$$

The surface area of the composite object is approximately 69.3 cm^2 .

- b) The radius of the cylinder is r : $\frac{1}{2}(10 \text{ m}) = 5 \text{ m}$

The radius of the cone is R : $\frac{1}{2}(6 \text{ m}) = 3 \text{ m}$

The volume of the composite object is:

Volume of cylinder + volume of cone

Use the algebraic formulas for volume:

$$V = \pi r^2 h + \frac{1}{3}\pi R^2 h \quad \text{Substitute: } r = 5, h = 15, R = 3, H = 4$$

$$V = \pi (5)^2(15) + \frac{1}{3}\pi (3)^2(4)$$

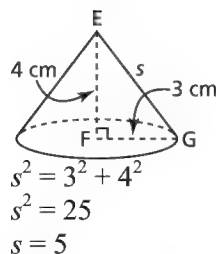
$$V = 1215.7963\dots$$

The volume of the composite object is approximately 1215.8 m^3 .

The surface area of the composite object is:

Surface area of cylinder + lateral area of cone – overlap

To determine the slant height, s , of the cone, use the Pythagorean Theorem in $\triangle EGF$.



Use the algebraic formulas for surface area.

$$SA = 2\pi r^2 + 2\pi rh + \pi Rs - \pi R^2$$

Substitute: $r = 5$, $h = 15$, $s = 5$, $R = 3$

$$SA = 2\pi(5)^2 + 2\pi(5)(15) + \pi(3)(5) - \pi(3)^2$$

$$SA = 647.1680\dots$$

The surface area of the composite object is approximately 647.2 m^2 .

5. Answers may vary.

- a) The student may have used a ruler marked in inches, because she recorded her measurements in inches.
- b) To calculate the volume, the student could determine the area of the square base, then use the formula for the volume of a right rectangular pyramid.
To calculate the surface area, the student could use the Pythagorean Theorem to determine the slant height of the pyramid, then use the formula for the surface area of a right square pyramid.

6. Use the formula for the surface area of a sphere, radius r .

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 5.0$$

$$SA = 4\pi(5.0)^2$$

$$SA = 100\pi$$

Use the formula for the surface area of a hemisphere, radius R .

$$SA = 2\pi R^2 + \pi R^2$$

$$SA = 3\pi R^2 \quad \text{Substitute: } SA = 100\pi$$

$$100\pi = 3\pi R^2 \quad \text{Solve for } R. \text{ Divide both sides by } 3\pi.$$

$$R^2 = \frac{100\pi}{3\pi}$$

$$R = \sqrt{\frac{100}{3}}$$

$$R = 5.7735\dots$$

The radius of the hemisphere is approximately 5.8 cm.

Lesson 2.1 The Tangent Ratio

Exercises (pages 75–77)

A

3. a) The acute angles are $\angle A$ and $\angle C$.

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

$$\tan A = \frac{BC}{AB}$$

BC is opposite $\angle A$.

AB is adjacent to $\angle A$.

$$\tan A = \frac{6}{7}$$

$$\tan C = \frac{\text{length of side opposite } \angle C}{\text{length of side adjacent to } \angle C}$$

$$\tan C = \frac{AB}{BC}$$

AB is opposite $\angle C$.

BC is adjacent to $\angle C$.

$$\tan C = \frac{7}{6}$$

- b) The acute angles are $\angle D$ and $\angle F$.

$$\tan D = \frac{\text{length of side opposite } \angle D}{\text{length of side adjacent to } \angle D}$$

$$\tan D = \frac{EF}{DE}$$

EF is opposite $\angle D$.

DE is adjacent to $\angle D$.

$$\tan D = \frac{6}{4}$$

$$\tan D = \frac{3}{2}$$

$$\tan F = \frac{\text{length of side opposite } \angle F}{\text{length of side adjacent to } \angle F}$$

$$\tan F = \frac{DE}{EF}$$

DE is opposite $\angle F$.

EF is adjacent to $\angle F$.

$$\tan F = \frac{4}{6}$$

$$\tan F = \frac{2}{3}$$

- c) The acute angles are $\angle H$ and $\angle J$.

$$\tan H = \frac{\text{length of side opposite } \angle H}{\text{length of side adjacent to } \angle H}$$

$$\tan H = \frac{GJ}{GH}$$

GJ is opposite $\angle H$.

GH is adjacent to $\angle H$.

$$\tan H = \frac{10}{8}$$

$$\tan H = \frac{5}{4}$$

$$\tan J = \frac{\text{length of side opposite } \angle J}{\text{length of side adjacent to } \angle J}$$

$$\tan J = \frac{GH}{GJ}$$

GH is opposite $\angle J$.

GJ is adjacent to $\angle J$.

$$\tan J = \frac{8}{10}$$

$$\tan J = \frac{4}{5}$$

- d) The acute angles are $\angle K$ and $\angle M$.

$$\tan K = \frac{\text{length of side opposite } \angle K}{\text{length of side adjacent to } \angle K}$$

$$\tan K = \frac{MN}{KN}$$

MN is opposite $\angle K$.

KN is adjacent to $\angle K$.

$$\tan K = \frac{5}{7}$$

$$\tan M = \frac{\text{length of side opposite } \angle M}{\text{length of side adjacent to } \angle M}$$

$$\tan M = \frac{KN}{MN}$$

KN is opposite $\angle M$.

MN is adjacent to $\angle M$.

$$\tan M = \frac{7}{5}$$

4. Use a calculator to determine the measure of each angle.

a) $\tan X = 0.25$

$$\angle X = \tan^{-1}(0.25)$$

$$= 14.0362\dots$$

So, $\angle X \doteq 14^\circ$

b) $\tan X = 1.25$

$$\angle X = \tan^{-1}(1.25)$$

$$= 51.3401\dots$$

So, $\angle X \doteq 51^\circ$

c) $\tan X = 2.50$

$$\angle X = \tan^{-1}(2.50)$$

$$= 68.1985\dots$$

So, $\angle X \doteq 68^\circ$

d) $\tan X = 20$

$$\angle X = \tan^{-1}(20)$$

$$= 87.1375\dots$$

So, $\angle X \doteq 87^\circ$

5. a) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{3}{6}$$

$$= \frac{1}{2}$$

Use a calculator to determine the measure of the angle.

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.5650\dots^\circ$$

So, the angle measure is approximately 27° .

- b) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{14}{14}$$

$$= 1$$

Use a calculator to determine the measure of the angle.

$$\tan^{-1}(1) = 45^\circ$$

So, the angle measure is 45° .

- c) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{9}{5}$$

Use a calculator to determine the measure of the angle.

$$\tan^{-1}\left(\frac{9}{5}\right) = 60.9453\dots^\circ$$

So, the angle measure is approximately 61° .

- d) Use the tangent ratio.

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{8}{3}$$

Use a calculator to determine the measure of the angle.

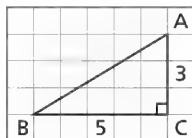
$$\tan^{-1}\left(\frac{8}{3}\right) = 69.4439\dots^\circ$$

So, the angle measure is approximately 69° .

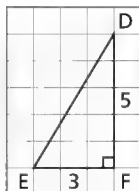
B

6. Sketches will vary. For example:

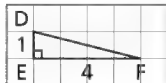
a)



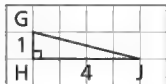
b)



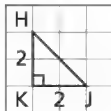
c)



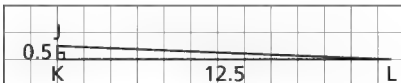
d)



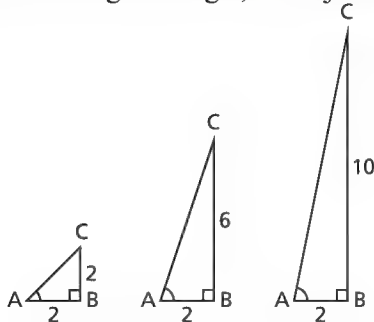
e)



f)

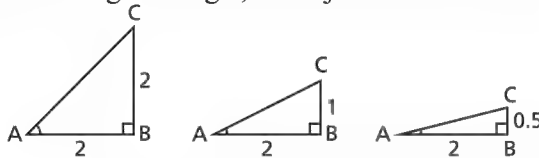


7. a) In this right triangle, the adjacent side is constant.



When the opposite and adjacent sides are equal, $\tan A = 1$ and $\angle A = 45^\circ$. As $\angle A$ increases, the opposite side increases, so the ratio $\frac{\text{opposite}}{\text{adjacent}}$ increases, and $\tan A$ increases. So, $\tan 60^\circ$ is greater than $\tan 45^\circ$, which is 1.

- b) In this right triangle, the adjacent side is constant.



As $\angle A$ decreases, the opposite side decreases, so the ratio $\frac{\text{opposite}}{\text{adjacent}}$ decreases, and $\tan A$ decreases. So, $\tan 30^\circ$ is less than $\tan 45^\circ$, which is 1.

8. a) First, use the Pythagorean Theorem in right $\triangle EFG$ to determine the length of the side adjacent to $\angle F$.

$$EF^2 = EG^2 + FG^2 \quad \text{Isolate the unknown.}$$

$$FG^2 = EF^2 - EG^2$$

$$FG^2 = 5.9^2 - 3.5^2$$

$$= 22.56$$

$$FG = \sqrt{22.56}$$

Then, use the tangent ratio in right $\triangle EFG$ to determine the length of the side opposite $\angle F$.

$$\tan F = \frac{EG}{FG}$$

EG is opposite $\angle F$.

FG is adjacent to $\angle F$.

$$\tan F = \frac{3.5}{\sqrt{22.56}}$$

$$\tan F = 0.7368\dots$$

$$\angle F \doteq 36.4^\circ$$

- b) First, use the Pythagorean Theorem in right $\triangle HJK$ to determine the length of the side opposite $\angle J$.

$$JK^2 = HJ^2 + HK^2 \quad \text{Isolate the unknown.}$$

$$HK^2 = JK^2 - HJ^2$$

$$HK^2 = 6.4^2 - 2.4^2$$

$$= 35.2$$

$$HK = \sqrt{35.2}$$

Then, use the tangent ratio in right $\triangle HJK$.

$$\tan J = \frac{HK}{HJ}$$

HK is opposite $\angle J$.

HJ is adjacent to $\angle J$.

$$\tan J = \frac{\sqrt{35.2}}{2.4}$$

$$\tan J = 2.4720\dots$$

$$\angle J \doteq 68.0^\circ$$

9. a) i) In right $\triangle ABC$,

$$\tan B = \frac{AC}{BC}$$

AC is opposite $\angle B$.

BC is adjacent to $\angle B$.

$$\tan B = \frac{8}{4}$$

$$\tan B = 2$$

- ii) In right $\triangle DEF$,

$$\tan D = \frac{EF}{DE}$$

EF is opposite $\angle D$.

DE is adjacent to $\angle D$.

$$\tan D = \frac{4}{2}$$

$$\tan D = 2$$

- iii) In right $\triangle GHJ$,

$$\tan G = \frac{HJ}{GJ}$$

HJ is opposite $\angle G$.

GJ is adjacent to $\angle G$.

$$\tan G = \frac{9.0}{4.5}$$

$$\tan G = 2$$

Each right triangle has an acute angle with a tangent ratio of 2.

So, $\triangle ABC$, $\triangle FDE$, and $\triangle HGJ$ are similar triangles.

- b) i) From the answer to part a i,

$$\tan B = 2$$

$$\angle B \doteq 63.4^\circ$$

The sum of the angles in a triangle is 180° , so:

$$\angle A \doteq 180^\circ - 63.4^\circ - 90^\circ$$

$$\doteq 26.6^\circ$$

- ii) $\triangle FDE$ is similar to $\triangle ABC$, so

$$\angle D = \angle B$$

$$\doteq 63.4^\circ$$

and

$$\angle F = \angle A$$

$$\doteq 26.6^\circ$$

- iii) $\triangle HGJ$ is similar to $\triangle ABC$, so

$$\angle G = \angle B$$

$$\doteq 63.4^\circ$$

and

$$\angle H = \angle A$$

$$\doteq 26.6^\circ$$

- c) No. First I determined the measures of the angles in one triangle. Then, I used the fact that corresponding angles in similar triangles have equal measures to determine the angle measures in the other triangles.

10. a) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{4.0}{5.5}$$

Use a calculator.

$$\angle A = \tan^{-1} \left(\frac{4.0}{5.5} \right)$$

$$= 36.0273\dots$$

The angle of inclination is approximately 36° .

- b) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{7.5}{6.5}$$

Use a calculator.

$$\angle A = \tan^{-1} \left(\frac{7.5}{6.5} \right)$$

$$= 49.0856\dots$$

The angle of inclination is approximately 49.1° .

- c) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{3.4}{9.2} \quad \text{Use a calculator.}\end{aligned}$$

$$\begin{aligned}\angle A &= \tan^{-1}\left(\frac{3.4}{9.2}\right) \\ &= 20.2825...\end{aligned}$$

The angle of inclination is approximately 20.3° .

- d) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{8.2}{1.1} \quad \text{Use a calculator.}\end{aligned}$$

$$\begin{aligned}\angle A &= \tan^{-1}\left(\frac{8.2}{1.1}\right) \\ &= 82.3595...\end{aligned}$$

The angle of inclination is approximately 82.4° .

11. a) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{20}{100} \quad \text{Use a calculator.}\end{aligned}$$

$$\begin{aligned}\angle A &= \tan^{-1}\left(\frac{20}{100}\right) \\ &= 11.3099...\end{aligned}$$

The angle of inclination is approximately 11° .

- b) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\begin{aligned}\tan A &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{25}{100} \quad \text{Use a calculator.}\end{aligned}$$

$$\begin{aligned}\angle A &= \tan^{-1}\left(\frac{25}{100}\right) \\ &= 14.0362...\end{aligned}$$

The angle of inclination is approximately 14° .

- c) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{10}{100} \quad \text{Use a calculator.}$$

$$\angle A = \tan^{-1}\left(\frac{10}{100}\right)$$

$$= 5.7105\dots$$

The angle of inclination is approximately 6° .

- d) Use the tangent ratio. Let the angle of inclination be $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{15}{100} \quad \text{Use a calculator.}$$

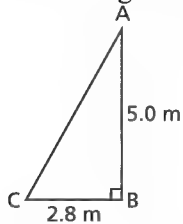
$$\angle A = \tan^{-1}\left(\frac{15}{100}\right)$$

$$= 8.5307\dots$$

The angle of inclination is approximately 9° .

12. The solar panel will work best when the angle of inclination of the roof is approximately equal to the latitude of the house.

Draw a right triangle to represent the cross-section of the roof and solar panel.



$\angle C$ is the angle of inclination. In right $\triangle ABC$,

$$\tan C = \frac{AB}{BC}$$

AB is opposite $\angle C$.

BC is adjacent to $\angle C$.

$$\tan C = \frac{5.0}{2.8}$$

$$\angle C = \tan^{-1}\left(\frac{5.0}{2.8}\right)$$

$$\doteq 60.8^\circ$$

The latitude of Whitehorse is approximately 60.8° . It is the only location whose latitude is within 1° of the angle of inclination of the solar panel.

13. The acute angles in the diagram are: $\angle P$, $\angle R$, $\angle PQS$, and $\angle RQS$.

In right $\triangle PQR$,

$$\tan P = \frac{QR}{PQ}$$

QR is opposite $\angle P$.

PQ is adjacent to $\angle P$.

$$\tan P = \frac{12}{5}$$

$$\angle P \doteq 67.4^\circ$$

Use the fact that the sum of the angles in a triangle is 180° .

In $\triangle PQR$, $\angle Q = 90^\circ$ and $\angle P \doteq 67.4^\circ$, so:

$$\angle R \doteq 180^\circ - 67.4^\circ - 90^\circ$$

$$\doteq 22.6^\circ$$

In $\triangle PQS$, $\angle S = 90^\circ$ and $\angle P \doteq 67.4^\circ$, so:

$$\angle PQS \doteq 180^\circ - 67.4^\circ - 90^\circ$$

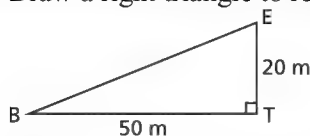
$$\doteq 22.6^\circ$$

In $\triangle RQS$, $\angle S = 90^\circ$ and $\angle R \doteq 22.6^\circ$, so:

$$\angle RQS \doteq 180^\circ - 22.6^\circ - 90^\circ$$

$$\doteq 67.4^\circ$$

14. Draw a right triangle to represent the birdwatcher, the base of the tree, and the eagle.



$\angle B$ is the angle of inclination. In right $\triangle BTE$,

$$\tan B = \frac{ET}{BT}$$

ET is opposite $\angle B$.

BT is adjacent to $\angle B$.

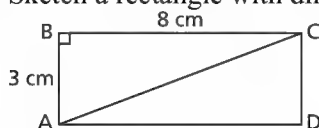
$$\tan B = \frac{20}{50}$$

$$\tan B = 0.4$$

$$\angle B = 21.8014\dots^\circ$$

So, the birdwatcher must incline his camera about 22° to take a photograph of the eagle.

15. Sketch a rectangle with dimensions 3 cm by 8 cm.



A diagonal of the rectangle divides it into two congruent right triangles. So, to determine the angles the diagonal makes with the sides of the rectangle, determine the measures of $\angle A$ and $\angle C$ in right $\triangle ABC$.

To determine the angle the diagonal makes with the shorter side, determine the measure of $\angle A$:

$$\tan A = \frac{BC}{AB}$$

BC is opposite $\angle A$.

AB is adjacent to $\angle A$.

$$\tan A = \frac{8}{3}$$

$$\angle A = 69.4439\dots^\circ$$

Use the fact that the sum of angles in a triangle is 180° .

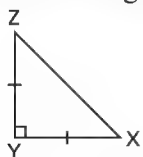
In $\triangle ABC$, $\angle B = 90^\circ$ and $\angle A \doteq 69.4^\circ$, so:

$$\angle C \doteq 180^\circ - 90^\circ - 69.4^\circ$$

$$\doteq 20.6^\circ$$

So, the diagonal makes an angle of approximately 69.4° with the shorter side of the rectangle and an angle of approximately 20.6° with the longer side.

16. Sketch a right isosceles triangle.



The acute angles in right $\triangle XYZ$ are $\angle X$ and $\angle Z$.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan X = \frac{YZ}{XY} \quad XY = YZ$$

$$\tan X = \frac{YZ}{YZ}$$

$$\tan X = 1$$

Similarly,

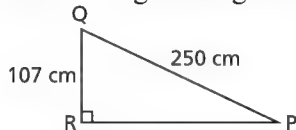
$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ} \quad XY = YZ$$

$$\tan Z = \frac{YZ}{YZ}$$

$$\tan Z = 1$$

17. Sketch a right triangle to represent the slide.



The length of the slide is the hypotenuse of the right triangle.

To use the tangent ratio to determine $\angle P$, we need to know the length of PR.

Use the Pythagorean Theorem.

$$PQ^2 = QR^2 + PR^2 \quad \text{Isolate the unknown.}$$

$$PR^2 = PQ^2 - QR^2$$

$$PR^2 = 250^2 - 107^2$$

$$= 51\,051$$

$$PR = \sqrt{51\,051}$$

Use the tangent ratio in right $\triangle PQR$.

$$\tan P = \frac{QR}{PR}$$

QR is opposite $\angle P$.

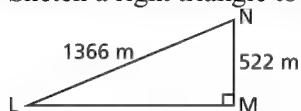
PR is adjacent to $\angle P$.

$$\tan P = \frac{107}{\sqrt{51\,051}}$$

$$\angle P \doteq 25^\circ$$

The angle the slide makes with the ground is approximately 25° .

18. Sketch a right triangle to represent the ski lift.



The length of the ski lift is the hypotenuse of the right triangle.

To use the tangent ratio to determine $\angle L$, we need to know the length of LM.

Use the Pythagorean Theorem.

$$LN^2 = LM^2 + MN^2 \quad \text{Isolate the unknown.}$$

$$LM^2 = LN^2 - MN^2$$

$$LM^2 = 1366^2 - 522^2$$

$$= 1\,593\,472$$

$$LM = \sqrt{1\,593\,472}$$

Use the tangent ratio in right $\triangle LMN$.

$$\tan L = \frac{MN}{LM}$$

MN is opposite $\angle L$.

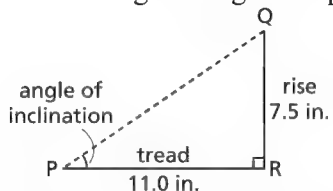
LM is adjacent to $\angle L$.

$$\tan L = \frac{522}{\sqrt{1\,593\,472}}$$

$$\angle L \doteq 22^\circ$$

The angle of inclination of the ski lift is approximately 22° .

19. Sketch a right triangle to represent the stairs.



Calculate the angle of inclination of the stairs.

$$\tan P = \frac{\text{rise}}{\text{tread}}$$

$$\tan P = \frac{QR}{PR}$$

QR is opposite $\angle P$.

PR is adjacent to $\angle P$.

$$\tan P = \frac{7.5}{11.0}$$

$$\angle P \doteq 34^\circ$$

The angle to be cut and the angle of inclination form a straight line.

So, the angle to be cut is approximately:

$$180^\circ - 34^\circ \doteq 146^\circ$$

The carpenter should cut the board at an angle of approximately 146° .

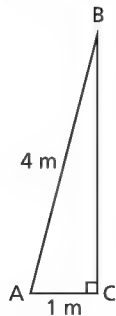
20. Sketch a right triangle to represent a ladder, AB, and its distance, AC, from the wall.

The distance AC must be less than or equal to $\frac{1}{4}$ AB.

That is, $AC \leq \frac{1}{4}AB$

So, $AB \geq 4AC$

Assume $AB = 4AC$. Let AC be 1 m and AB be 4 m.



To use the tangent ratio to determine $\angle A$, we need to know the length of BC.

Use the Pythagorean Theorem.

$$AB^2 = AC^2 + BC^2 \quad \text{Isolate the unknown.}$$

$$BC^2 = AB^2 - AC^2$$

$$BC^2 = 4^2 - 1^2$$

$$= 15$$

$$BC = \sqrt{15}$$

Use the tangent ratio in right $\triangle ABC$.

$$\tan A = \frac{BC}{AC}$$

BC is opposite $\angle A$.

AC is adjacent to $\angle A$.

$$\tan A = \frac{\sqrt{15}}{1}$$

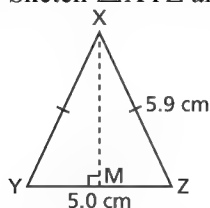
$$\tan A = \sqrt{15}$$

$$\angle A \doteq 76^\circ$$

The least allowed angle of inclination of a ladder is approximately 76° .

C

21. Sketch $\triangle XYZ$ and draw the perpendicular bisector of YZ through point M.



Line MX divides $\triangle XYZ$ into 2 congruent right triangles: $\triangle MXY$ and $\triangle MXZ$

To use the tangent ratio to determine $\angle Y$, first determine the length of MX .

Use the Pythagorean Theorem in right $\triangle MXY$.

$$XY^2 = MX^2 + MY^2 \quad \text{Isolate the unknown.}$$

$$MX^2 = XY^2 - MY^2$$

$$\begin{aligned} MX^2 &= 5.9^2 - \left(\frac{5.0}{2}\right)^2 \\ &= 28.56 \end{aligned}$$

$$MX = \sqrt{28.56}$$

Use the tangent ratio in right $\triangle MXY$.

$$\tan Y = \frac{MX}{MY}$$

MX is opposite $\angle Y$.

MY is adjacent to $\angle Y$.

$$\tan Y = \frac{\sqrt{28.56}}{2.5}$$

$$\angle Y = 64.9297\dots^\circ$$

$$\angle Y \doteq 64.9^\circ$$

Since $\triangle XYZ$ is isosceles and $XY = XZ$, $\angle Z = \angle Y$. So, $\angle Z \doteq 64.9^\circ$

The sum of the angles in a triangle is 180° , so in $\triangle XYZ$:

$$\angle X \doteq 180^\circ - 64.9^\circ - 64.9^\circ$$

$$\doteq 50.2$$

So, the angle measures in $\triangle XYZ$ are: $\angle X \doteq 50.2^\circ$, $\angle Y \doteq 64.9^\circ$, and $\angle Z \doteq 64.9^\circ$

22. a) There is no least possible value. The tangent can be arbitrarily close to 0. Consider a right triangle with acute $\angle A$ and suppose the side opposite $\angle A$ has length 1 unit. Then

$$\tan A = \frac{1 \text{ unit}}{\text{length of side adjacent to } \angle A}$$

When the side adjacent to $\angle A$ is much longer than the side opposite $\angle A$, $\tan A$ is very close to 0.

- b) There is no greatest possible value. The tangent can be arbitrarily large. Consider a right triangle with acute $\angle A$ and suppose the side adjacent to $\angle A$ has length 1 unit. Then

$$\tan A = \frac{\text{length of side opposite } \angle A}{1 \text{ unit}}$$

When the side opposite $\angle A$ is much longer than the side adjacent to $\angle A$, $\tan A$ is very large.

23. a) In each right triangle, the side opposite the angle at the centre of the spiral is 1 unit long. For the 1st triangle, the length, in units, of the adjacent side is: 1

So, the tangent of the angle for the 1st triangle is: $\frac{1}{1}$, or 1

The hypotenuse of each right triangle is the side adjacent to the angle at the centre of the spiral in the next right triangle. So, use the Pythagorean Theorem to determine the lengths of the adjacent sides in the 2nd to 5th triangles.

For the 2nd triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + 1^2} = \sqrt{2}$

So, the tangent of the angle for the 2nd triangle is: $\frac{1}{\sqrt{2}}$

For the 3rd triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$

So, the tangent of the angle for the 3rd triangle is: $\frac{1}{\sqrt{3}}$

For the 4th triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4}$, or 2

So, the tangent of the angle for the 4th triangle is: $\frac{1}{\sqrt{4}}$, or $\frac{1}{2}$

For the 5th triangle, the length, in units, of the adjacent side is: $\sqrt{1^2 + 1^2} = \sqrt{5}$

So, the tangent of the angle for the 5th triangle is: $\frac{1}{\sqrt{5}}$

- b) The pattern is the tangent of the angle at the centre of the spiral for the n th triangle is:

$$\frac{1}{\sqrt{n}}$$

So, the tangent of the angle at the centre of the spiral for the 100th triangle is:

$$\frac{1}{\sqrt{100}}, \text{ or } \frac{1}{10}$$

Lesson 2.2

Using the Tangent Ratio to Calculate Lengths

Exercises (pages 82–83)

A

3. a) In right $\triangle ABC$, AC is the side opposite $\angle B$ and BC is the side adjacent to $\angle B$.
Use the tangent ratio to write an equation.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AC}{BC}$$

$$\tan 27^\circ = \frac{b}{5.0} \quad \text{Solve for } b.$$

$$5.0 \times \tan 27^\circ = \frac{b}{5.0} \times 5.0$$

$$5.0 \tan 27^\circ = b$$

$$b = 2.5476\dots$$

So, AC is approximately 2.5 cm long.

- b) In right $\triangle DEF$, EF is the side opposite $\angle D$ and DE is the side adjacent to $\angle D$.
Use the tangent ratio to write an equation.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{EF}{DE}$$

$$\tan 35^\circ = \frac{d}{2.0} \quad \text{Solve for } d.$$

$$2.0 \times \tan 35^\circ = \frac{d}{2.0} \times 2.0$$

$$2.0 \tan 35^\circ = d$$

$$d = 1.4004\dots$$

So, EF is approximately 1.4 cm long.

- c) In right $\triangle GHJ$, HJ is the side opposite $\angle G$ and GH is the side adjacent to $\angle G$.
Use the tangent ratio to write an equation.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan G = \frac{HJ}{GH}$$

$$\tan 59^\circ = \frac{g}{3.0} \quad \text{Solve for } g.$$

$$3.0 \times \tan 59^\circ = \frac{g}{3.0} \times 3.0$$

$$3.0 \tan 59^\circ = g$$

$$g = 4.9928\dots$$

So, HJ is approximately 5.0 cm long.

- d) In right $\triangle KMN$, KN is the side opposite $\angle M$ and KM is the side adjacent to $\angle M$.
Use the tangent ratio to write an equation.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{KN}{KM}$$

$$\tan 43^\circ = \frac{m}{8.0} \quad \text{Solve for } m.$$

$$8.0 \times \tan 43^\circ = \frac{m}{8.0} \times 8.0$$

$$8.0 \tan 43^\circ = m$$

$$m = 7.4601\dots$$

So, KN is approximately 7.5 cm long.

4. a) In right $\triangle NPQ$, NP is the side opposite $\angle Q$ and NQ is the side adjacent to $\angle Q$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{NP}{NQ}$$

$$\tan 64^\circ = \frac{4.5}{p} \quad \text{Solve for } p.$$

$$p \times \tan 64^\circ = \frac{4.5}{p} \times p$$

$$p \times \tan 64^\circ = 4.5$$

$$\frac{p \times \tan 64^\circ}{\tan 64^\circ} = \frac{4.5}{\tan 64^\circ}$$

$$p = \frac{4.5}{\tan 64^\circ}$$

$$p = 2.1947\dots$$

So, NQ is approximately 2.2 cm long.

- b) In right $\triangle RST$, RS is the side opposite $\angle T$ and ST is the side adjacent to $\angle T$.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{RS}{ST}$$

$$\tan 72^\circ = \frac{8.7}{r} \quad \text{Solve for } r.$$

$$r \times \tan 72^\circ = \frac{8.7}{r} \times r$$

$$r \times \tan 72^\circ = 8.7$$

$$\frac{r \times \tan 72^\circ}{\tan 72^\circ} = \frac{8.7}{\tan 72^\circ}$$

$$r = \frac{8.7}{\tan 72^\circ}$$

$$r = 2.8268\dots$$

So, ST is approximately 2.8 cm long.

- c) In right $\triangle UVW$, VW is the side opposite $\angle U$ and UW is the side adjacent to $\angle U$.

$$\tan U = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan U = \frac{VW}{UW}$$

$$\tan 23^\circ = \frac{1.2}{v} \quad \text{Solve for } v.$$

$$v \times \tan 23^\circ = \frac{1.2}{v} \times v$$

$$v \times \tan 23^\circ = 1.2$$

$$\frac{v \times \tan 23^\circ}{\tan 23^\circ} = \frac{1.2}{\tan 23^\circ}$$

$$v = \frac{1.2}{\tan 23^\circ}$$

$$v = 2.8270\dots$$

So, UW is approximately 2.8 cm long.

5. a) In right $\triangle PQR$, QR is the side opposite $\angle P$ and PQ is the side adjacent to $\angle P$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{QR}{PQ}$$

$$\tan 15^\circ = \frac{1.5}{r} \quad \text{Solve for } r.$$

$$r \times \tan 15^\circ = \frac{1.5}{r} \times r$$

$$r \times \tan 15^\circ = 1.5$$

$$\frac{r \times \tan 15^\circ}{\tan 15^\circ} = \frac{1.5}{\tan 15^\circ}$$

$$r = \frac{1.5}{\tan 15^\circ}$$

$$r = 5.5980\dots$$

So, PQ is approximately 5.6 cm long.

- b) In right $\triangle STU$, SU is the side opposite $\angle T$ and ST is the side adjacent to $\angle T$.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{SU}{ST}$$

$$\tan 61^\circ = \frac{7.4}{u} \quad \text{Solve for } u.$$

$$u \times \tan 61^\circ = \frac{7.4}{u} \times u$$

$$u \times \tan 61^\circ = 7.4$$

$$\frac{u \times \tan 61^\circ}{\tan 61^\circ} = \frac{7.4}{\tan 61^\circ}$$

$$u = \frac{7.4}{\tan 61^\circ}$$

$$u = 4.1018\dots$$

So, ST is approximately 4.1 cm long.

- c) In right $\triangle VWX$, VX is the side opposite $\angle W$ and WX is the side adjacent to $\angle W$.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{VX}{WX}$$

$$\tan 32^\circ = \frac{2.4}{v} \quad \text{Solve for } v.$$

$$v \times \tan 32^\circ = \frac{2.4}{v} \times v$$

$$v \times \tan 32^\circ = 2.4$$

$$\frac{v \times \tan 32^\circ}{\tan 32^\circ} = \frac{2.4}{\tan 32^\circ}$$

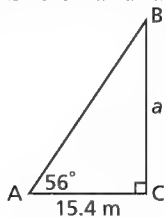
$$v = \frac{2.4}{\tan 32^\circ}$$

$$v = 3.8408\dots$$

So, WX is approximately 3.8 cm long.

B

6. Sketch and label a diagram to represent the information in the problem.



In right $\triangle ABC$, BC is the side opposite $\angle A$ and AC is the side adjacent to $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 56^\circ = \frac{a}{15.4} \quad \text{Solve for } a.$$

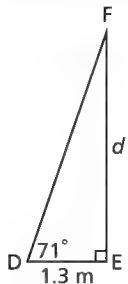
$$15.4 \times \tan 56^\circ = \frac{a}{15.4} \times 15.4$$

$$15.4 \tan 56^\circ = a$$

$$a = 22.8314\dots$$

The wire reaches approximately 22.8 m up the tower.

7. Sketch and label a diagram to represent the information in the problem.



In right $\triangle DEF$, EF is the side opposite $\angle D$ and DE is the side adjacent to $\angle D$.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{EF}{DE}$$

$$\tan 71^\circ = \frac{d}{1.3} \quad \text{Solve for } d.$$

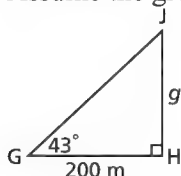
$$1.3 \times \tan 71^\circ = \frac{d}{1.3} \times 1.3$$

$$1.3 \tan 71^\circ = d$$

$$d = 3.7754 \dots$$

The ladder reaches approximately 3.8 m up the wall.

8. Sketch and label a diagram to represent the information in the problem.
Assume the ground is horizontal.



In right $\triangle GHJ$, HJ is the side opposite $\angle G$ and GH is the side adjacent to $\angle G$.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan G = \frac{HJ}{GH}$$

$$\tan 43^\circ = \frac{g}{200} \quad \text{Solve for } g.$$

$$200 \times \tan 43^\circ = \frac{g}{200} \times 200$$

$$200 \tan 43^\circ = d$$

$$d = 186.5030 \dots$$

The helicopter is approximately 187 m high.

9. Calculate the unknown length of the leg of the right triangle using the tangent ratio. Then use the Pythagorean Theorem to determine the length of the hypotenuse.

a) In right $\triangle FGH$, FG is the side opposite $\angle H$ and GH is the side adjacent to $\angle H$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{FG}{GH}$$

$$\tan 42^\circ = \frac{FG}{2.7} \quad \text{Solve for } FG.$$

$$2.7 \times \tan 42^\circ = \frac{FG}{2.7} \times 2.7$$

$$2.7 \tan 42^\circ = FG$$

$$FG = 2.4310\dots$$

Use the Pythagorean Theorem in right $\triangle FGH$.

$$FH^2 = FG^2 + GH^2$$

$$FH^2 = (2.4310\dots)^2 + 2.7^2$$

$$= 13.2002\dots$$

$$FH = \sqrt{13.2002\dots}$$

$$= 3.6332\dots$$

The length of the hypotenuse is approximately 3.6 cm.

b) In right $\triangle JKM$, KM is the side opposite $\angle J$ and JM is the side adjacent to $\angle J$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{KM}{JM}$$

$$\tan 36^\circ = \frac{5.9}{JM} \quad \text{Solve for } JM.$$

$$JM \times \tan 36^\circ = \frac{5.9}{JM} \times JM$$

$$JM \times \tan 36^\circ = 5.9$$

$$\frac{JM \times \tan 36^\circ}{\tan 36^\circ} = \frac{5.9}{\tan 36^\circ}$$

$$JM = \frac{5.9}{\tan 36^\circ}$$

$$JM = 8.1206\dots$$

Use the Pythagorean Theorem in right $\triangle JKM$.

$$JK^2 = JM^2 + MK^2$$

$$JK^2 = (8.1206\dots)^2 + 5.9^2$$

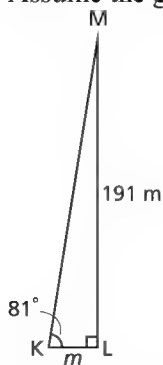
$$= 100.7550\dots$$

$$JK = \sqrt{100.7550\dots}$$

$$= 10.0376\dots$$

The length of the hypotenuse is approximately 10.0 cm.

10. Sketch and label a diagram to represent the information in the problem.
Assume the ground is horizontal.



In right $\triangle KLM$, LM is the side opposite $\angle K$ and KL is the side adjacent to $\angle K$.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{LM}{KL}$$

$$\tan 81^\circ = \frac{191}{m}$$

Solve for m .

$$m \times \tan 81^\circ = \frac{191}{m} \times m$$

$$m \times \tan 81^\circ = 191$$

$$\frac{m \times \tan 81^\circ}{\tan 81^\circ} = \frac{191}{\tan 81^\circ}$$

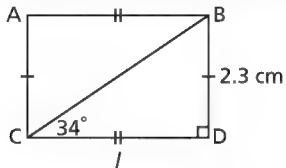
$$m = \frac{191}{\tan 81^\circ}$$

$$m = 30.2514\dots$$

Claire was approximately 30 m from the tower.

The distance is approximate because I assumed the ground was horizontal and I didn't consider the height of Claire's eye above the ground.

11. a) Sketch and label the rectangle. Let the length of CD be l .



- b) In right $\triangle BCD$, BD is the side opposite $\angle C$ and CD is the side adjacent to $\angle C$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BD}{CD}$$

$$\tan 34^\circ = \frac{2.3}{l} \quad \text{Solve for } l.$$

$$l \times \tan 34^\circ = \frac{2.3}{l} \times l$$

$$l \times \tan 34^\circ = 2.3$$

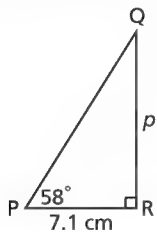
$$\frac{l \times \tan 34^\circ}{\tan 34^\circ} = \frac{2.3}{\tan 34^\circ}$$

$$l = \frac{2.3}{\tan 34^\circ}$$

$$l = 3.4098\dots$$

The length of the rectangle is approximately 3.4 cm.

12. Sketch and label a diagram to represent the information in the problem.



To determine the area of a triangle, we need to know its base and height.

In right $\triangle PQR$, QR is the side opposite $\angle P$ and PR is the side adjacent to $\angle P$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{QR}{PR}$$

$$\tan 58^\circ = \frac{p}{7.1} \quad \text{Solve for } p.$$

$$7.1 \times \tan 58^\circ = \frac{p}{7.1} \times 7.1$$

$$7.1 \tan 58^\circ = p$$

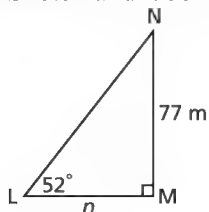
$$p = 11.3623\dots$$

The area of the triangle is:

$$\begin{aligned}\frac{1}{2}(\text{PR})(\text{QR}) &= \frac{1}{2}(7.1)(11.3623\dots) \\ &= 40.3364\dots\end{aligned}$$

The area of the triangle is approximately 40.3 cm^2 .

13. Sketch and label a diagram to represent the information in the problem.



In the diagram, n represents Liam's distance from a point on the ground vertically below the statue. In right $\triangle LMN$, MN is the side opposite $\angle L$ and LM is the side adjacent to $\angle L$.

$$\tan L = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan L = \frac{MN}{LM}$$

$$\tan 52^\circ = \frac{77}{n} \quad \text{Solve for } n.$$

$$n \times \tan 52^\circ = \frac{77}{n} \times n$$

$$n \times \tan 52^\circ = 77$$

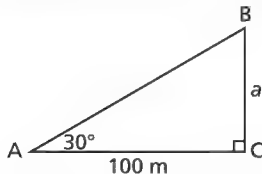
$$\frac{n \times \tan 52^\circ}{\tan 52^\circ} = \frac{77}{\tan 52^\circ}$$

$$n = \frac{77}{\tan 52^\circ}$$

$$n = 60.1589\dots$$

Liam is approximately 60 m from a point on the ground vertically below the statue.

14. Sketch and label a diagram to represent the information in the problem.



In right $\triangle ABC$, BC is the side opposite $\angle A$ and AC is the side adjacent to $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 30^\circ = \frac{a}{100} \quad \text{Solve for } a.$$

$$100 \times \tan 30^\circ = \frac{a}{100} \times 100$$

$$100 \tan 30^\circ = a$$

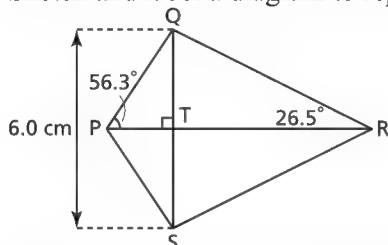
$$a = 57.7350\dots$$

The balloon is approximately 58 m high.

I am assuming that the ground between Janelle and the store is horizontal.

C

15. Sketch and label a diagram to represent the information in the problem.



The diagonal PR divides the kite into two congruent triangles. So:

$$\angle SPT = \angle QPT = 56.3^\circ$$

$$\angle SRT = \angle QRT = 26.5^\circ$$

$$\angle QPS = \angle QPT + \angle SPT$$

$$= 56.3^\circ + 56.3^\circ$$

$$= 112.6^\circ$$

$$\angle QRS = \angle QRT + \angle SRT$$

$$= 26.5^\circ + 26.5^\circ$$

$$= 53.0^\circ$$

In $\triangle PQR$, the sum of the angles is 180° . So:

$$\angle PQR = 180^\circ - \angle P - \angle R$$

$$= 180^\circ - 56.3^\circ - 26.5^\circ$$

$$= 97.2^\circ$$

$$\angle PSR = \angle PQR = 97.2^\circ$$

In $\triangle PQT$, the sum of the angles is 180° . So:

$$\angle PQT = 180^\circ - \angle QPT - \angle PTQ$$

$$= 180^\circ - 56.3^\circ - 90^\circ$$

$$= 33.7^\circ$$

$$\angle PST = \angle PQT = 33.7^\circ$$

$\angle PTQ$ and $\angle QTR$ form a straight angle. So:

$$\begin{aligned}\angle QTR &= 180^\circ - \angle PTQ \\ &= 180^\circ - 90^\circ \\ &= 90^\circ\end{aligned}$$

$$\angle PTS = \angle QTP = 90^\circ$$

$$\angle RTS = \angle QTR = 90^\circ$$

In $\triangle QRT$, the sum of the angles is 180° . So:

$$\begin{aligned}\angle RQT &= 180^\circ - \angle QRT - \angle QTR \\ &= 180^\circ - 26.5^\circ - 90^\circ \\ &= 63.5^\circ\end{aligned}$$

$$\angle RST = \angle RQT = 63.5^\circ$$

In right $\triangle PQT$, PT is the side opposite $\angle Q$ and QT is the side adjacent to $\angle Q$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{PT}{QT}$$

$$\tan 33.7^\circ = \frac{PT}{3.0}$$

Solve for PT .

$$3.0 \times \tan 33.7^\circ = \frac{PT}{3.0} \times 3.0$$

$$3.0 \tan 33.7^\circ = PT$$

$$PT = 2.0007\dots$$

Use the Pythagorean Theorem in right $\triangle PQT$.

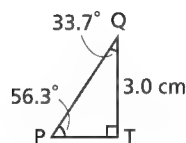
$$PQ^2 = PT^2 + QT^2$$

$$\begin{aligned}PQ^2 &= (2.0007\dots)^2 + 3.0^2 \\ &= 13.0030\dots\end{aligned}$$

$$\begin{aligned}PQ &= \sqrt{13.0030\dots} \\ &= 3.6059\dots\end{aligned}$$

$$PQ \doteq 3.6 \text{ cm}$$

$$PS = PQ \doteq 3.6 \text{ cm}$$



In right $\triangle QRT$, RT is the side opposite $\angle Q$ and QT is the side adjacent to $\angle Q$.

$$\tan Q = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Q = \frac{RT}{QT}$$

$$\tan 63.5^\circ = \frac{RT}{3.0}$$

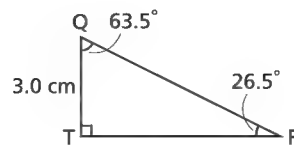
Solve for RT .

$$3.0 \times \tan 63.5^\circ = \frac{RT}{3.0} \times 3.0$$

$$3.0 \tan 63.5^\circ = RT$$

$$RT = 6.0170\dots$$

Use the Pythagorean Theorem in right $\triangle QRT$.



$$QR^2 = RT^2 + QT^2$$

$$QR^2 = (6.0170\dots)^2 + 3.0^2$$

$$= 45.2051\dots$$

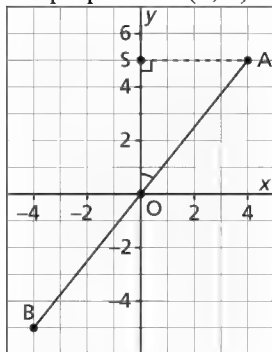
$$QR = \sqrt{45.2051\dots}$$

$$= 6.7234\dots$$

$$QR \doteq 6.7 \text{ cm}$$

$$SR = QR \doteq 6.7 \text{ cm}$$

16. a) Graph points A(4, 5) and B(-4, -5) on a coordinate grid.



Mark a point with the same y-coordinate as A on the y-axis: S(0, 5)

Then $\triangle AOS$ is a right triangle with $\angle S = 90^\circ$.

In right $\triangle AOS$, $\angle O$ is the measure of the acute angle between AB and the y-axis.

Use the tangent ratio in right $\triangle AOS$.

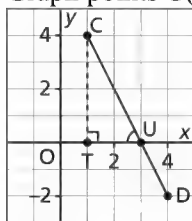
$$\tan O = \frac{AS}{OS} \quad \begin{array}{l} AS \text{ is opposite } \angle O. \\ OS \text{ is adjacent to } \angle O. \end{array}$$

$$\tan O = \frac{4}{5}$$

$$\tan O = 0.8$$

$$\angle O \doteq 38.7^\circ$$

- b) Graph points C(1, 4) and D(4, -2) on a coordinate grid.



Mark a point with the same x-coordinate as C on the x-axis: T(1, 0)

Mark the point where CD intersects the x-axis: U(3, 0)

Then $\triangle CTU$ is a right triangle with $\angle T = 90^\circ$.

In right $\triangle CTU$, $\angle U$ is the measure of the acute angle between CD and the x-axis.

Use the tangent ratio in right $\triangle CTU$.

$$\tan U = \frac{CT}{TU}$$

CT is opposite $\angle U$.
TU is adjacent to $\angle U$.

$$\tan U = \frac{4}{2}$$

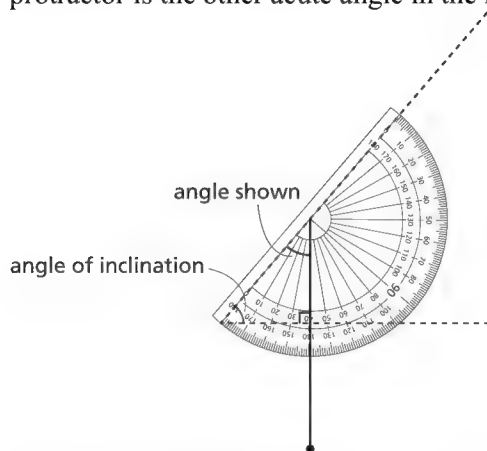
$$\tan U = 2$$

$$\angle U \doteq 63.4^\circ$$

Lesson 2.3 **Math Lab:**
Measuring an Inaccessible Height

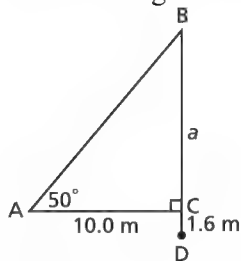
Assess Your Understanding (page 86)

1. The base of the protractor, the horizontal line that forms one arm of the angle of inclination, and the vertical piece of string form a right triangle. The angle of inclination is the acute angle formed by the base of the protractor and the horizontal line. The angle shown on the protractor is the other acute angle in the right triangle.



So, the sum of the angle of inclination and the angle shown on the protractor is 90° .

2. The angle of inclination is:
 $90^\circ - 40^\circ = 50^\circ$
Sketch a diagram to represent the situation.



In right $\triangle ABC$, BC is the side opposite $\angle A$ and AC is the side adjacent to $\angle A$.
Use the tangent ratio in right $\triangle ABC$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 50^\circ = \frac{a}{10.0}$$

Solve for a .

$$10.0 \tan 50^\circ = a$$

$$a = 11.9175\dots$$

The height of the tree is:

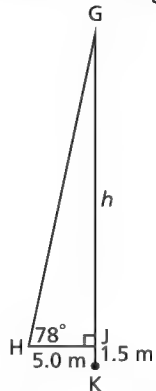
$$1.6 \text{ m} + 11.9175\dots \text{ m} = 13.5175\dots \text{ m}$$

The height of the tree is approximately 13.5 m.

3. The angle of inclination is:

$$90^\circ - 12^\circ = 78^\circ$$

Sketch a diagram to represent the situation.



In right $\triangle GHJ$, GJ is the side opposite $\angle H$ and HJ is the side adjacent to $\angle H$.
Use the tangent ratio in right $\triangle GHJ$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{GJ}{HJ}$$

$$\tan 78^\circ = \frac{h}{5.0} \quad \text{Solve for } h.$$

$$5.0 \tan 78^\circ = h$$

$$h = 23.5231\dots$$

The height of the totem pole is:

$$1.5 \text{ m} + 23.5231\dots \text{ m} = 25.0231\dots \text{ m}$$

The height of the totem pole is approximately 25 m.

Checkpoint 1

Assess Your Understanding (page 88)

2.1

1. a) In right $\triangle ABC$:

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

AB is opposite $\angle C$.

BC is adjacent to $\angle C$.

$$\tan C = \frac{2}{8}$$

$$\angle C \doteq 14^\circ$$

- b) In right $\triangle DEF$:

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan F = \frac{DE}{EF}$$

DE is opposite $\angle F$.

EF is adjacent to $\angle F$.

$$\tan F = \frac{6}{4}$$

$$\angle F \doteq 56^\circ$$

- c) First, use the Pythagorean Theorem in right $\triangle GHJ$ to determine the length of GJ.

$$HJ^2 = GH^2 + GJ^2 \quad \text{Isolate the unknown.}$$

$$GJ^2 = HJ^2 - GH^2$$

$$GJ^2 = 20^2 - 16^2$$

$$= 144$$

$$GJ = \sqrt{144}$$

$$= 12$$

Use the tangent ratio in right $\triangle GHJ$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{GH}{GJ}$$

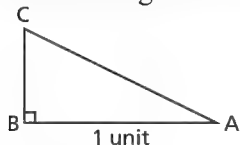
GH is opposite $\angle J$.

GJ is adjacent to $\angle J$.

$$\tan J = \frac{16}{12}$$

$$\angle J \doteq 53^\circ$$

2. Consider right $\triangle ABC$ shown below.



In right $\triangle ABC$:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AB}$$

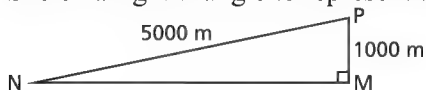
BC is opposite $\angle A$.

AB is adjacent to $\angle A$.

$$\tan A = \frac{BC}{1}$$

As $\angle A$ increases, BC increases, which means that $\tan A$ also increases.

3. Sketch a right triangle to represent the plane.



In right $\triangle MNP$, MP is the side opposite $\angle N$ and MN is the side adjacent to $\angle N$.

To use the tangent ratio to determine $\angle N$, we need to know the length of MN.

Use the Pythagorean Theorem.

$$NP^2 = MN^2 + MP^2 \quad \text{Isolate the unknown.}$$

$$MN^2 = NP^2 - MP^2$$

$$MN^2 = 5000^2 - 1000^2$$

$$= 24\,000\,000$$

$$MN = \sqrt{24\,000\,000}$$

Use the tangent ratio in right $\triangle MNP$.

$$\tan N = \frac{PM}{MN}$$

PM is opposite $\angle N$.

MN is adjacent to $\angle N$.

$$\tan N = \frac{1000}{\sqrt{24\,000\,000}}$$

$$\angle N \doteq 11.5^\circ$$

The angle between the ground and the line of sight from an observer at the beginning of the landing strip is approximately 11.5° .

2.2

4. a) In right $\triangle KMN$:

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{KM}{KN}$$

$$\tan 61^\circ = \frac{n}{6.2} \quad \text{Solve for } n.$$

$$6.2 \times \tan 61^\circ = \frac{n}{6.2} \times 6.2$$

$$6.2 \tan 61^\circ = n$$

$$n = 11.1850\dots$$

KM is approximately 11.2 cm long.

b) In right $\triangle PQR$:

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{PQ}{QR}$$

$$\tan 56^\circ = \frac{10.8}{p} \quad \text{Solve for } p.$$

$$p \times \tan 56^\circ = \frac{10.8}{p} \times p$$

$$p \times \tan 56^\circ = 10.8$$

$$\frac{p \times \tan 56^\circ}{\tan 56^\circ} = \frac{10.8}{\tan 56^\circ}$$

$$p = \frac{10.8}{\tan 56^\circ}$$

$$p = 7.2846\dots$$

QR is approximately 7.3 cm long.

- c) First, use the tangent ratio in right $\triangle STV$ to determine ST .

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{SV}{ST}$$

$$\tan 27^\circ = \frac{5.3}{ST}$$

Solve for ST .

$$ST \times \tan 27^\circ = \frac{5.3}{ST} \times ST$$

$$ST \times \tan 27^\circ = 5.3$$

$$\frac{ST \times \tan 27^\circ}{\tan 27^\circ} = \frac{5.3}{\tan 27^\circ}$$

$$ST = \frac{5.3}{\tan 27^\circ}$$

$$ST = 10.4018\dots$$

Use the Pythagorean Theorem in right $\triangle STV$ to determine s .

$$TV^2 = ST^2 + SV^2$$

$$s^2 = (10.4018\dots)^2 + 5.3^2$$

$$= 136.2881\dots$$

$$s = \sqrt{136.2881\dots}$$

$$= 11.6742\dots$$

TV is approximately 11.7 cm long.

5. Sketch a right triangle to represent the situation.

In right $\triangle HJK$, JK is the side opposite $\angle H$ and HK is the side adjacent to $\angle H$.

Use the tangent ratio in right $\triangle HJK$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{JK}{HK}$$

$$\tan 69^\circ = \frac{h}{9.1}$$

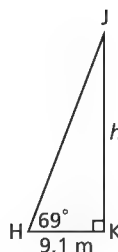
Solve for h .

$$9.1 \times \tan 69^\circ = \frac{h}{9.1} \times 9.1$$

$$9.1 \tan 69^\circ = h$$

$$h = 23.7063\dots$$

The top of the hoodoo was approximately 23.7 m above the level ground.



Lesson 2.4 The Sine and Cosine Ratios

Exercises (pages 95–96)

A

4. a) i) In right $\triangle AGH$, GH is opposite $\angle A$, AG is adjacent to $\angle A$, and AH is the hypotenuse.

- ii) In right $\triangle AKT$, KT is opposite $\angle A$, AK is adjacent to $\angle A$, and AT is the hypotenuse.

b) i) $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin A = \frac{GH}{AH}$$

$$\sin A = \frac{6}{10}$$

$$\sin A = 0.60$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AG}{AH}$$

$$\cos A = \frac{8}{10}$$

$$\cos A = 0.80$$

b) ii) $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin A = \frac{KT}{AT}$$

$$\sin A = \frac{7}{25}$$

$$\sin A = 0.28$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AK}{AT}$$

$$\cos A = \frac{24}{25}$$

$$\cos A = 0.96$$

5. Use a calculator.

a) $\sin 57^\circ \doteq 0.84$
 $\cos 57^\circ \doteq 0.54$

b) $\sin 5^\circ \doteq 0.09$
 $\cos 5^\circ \doteq 1.00$

c) $\sin 19^\circ \doteq 0.33$
 $\cos 19^\circ \doteq 0.95$

d) $\sin 81^\circ \doteq 0.99$
 $\cos 81^\circ \doteq 0.16$

6. Use a calculator.

a) $\sin X = 0.25$
 $\angle X = \sin^{-1}(0.25)$
 $\doteq 14^\circ$

b) $\cos X = 0.64$
 $\angle X = \cos^{-1}(0.64)$
 $\doteq 50^\circ$

c) $\sin X = \frac{6}{11}$
 $\angle X = \sin^{-1}\left(\frac{6}{11}\right)$
 $\doteq 33^\circ$

d) $\cos X = \frac{7}{9}$
 $\angle X = \cos^{-1}\left(\frac{7}{9}\right)$
 $\doteq 39^\circ$

B

7. a) In right $\triangle BCD$, the length of the side opposite $\angle C$ and the length of the hypotenuse are given.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{BD}{BC}$$

$$\sin C = \frac{5}{9}$$

$$\angle C \doteq 34^\circ$$

- b) In right $\triangle EFG$, the length of the side opposite $\angle E$ and the length of the hypotenuse are given.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{FG}{EG}$$

$$\sin E = \frac{4}{7}$$

$$\angle E \doteq 35^\circ$$

- c) In right $\triangle HJK$, the length of the side opposite $\angle H$ and the length of the hypotenuse are given.

$$\sin H = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin H = \frac{JK}{HJ}$$

$$\sin H = \frac{10}{16}$$

$$\angle H \doteq 39^\circ$$

- d) In right $\triangle MNP$, the length of the side opposite $\angle N$ and the length of the hypotenuse are given.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin N = \frac{MP}{NP}$$

$$\sin N = \frac{6}{11}$$

$$\angle N \doteq 33^\circ$$

8. a) In right $\triangle QRS$, the length of the side adjacent to $\angle Q$ and the length of the hypotenuse are given.

$$\cos Q = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Q = \frac{QR}{QS}$$

$$\cos Q = \frac{18}{24}$$

$$\angle Q \doteq 41^\circ$$

- b) In right $\triangle TUV$, the length of the side adjacent to $\angle U$ and the length of the hypotenuse are given.

$$\cos U = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos U = \frac{TU}{UV}$$

$$\cos U = \frac{5}{24}$$

$$\angle U \doteq 78^\circ$$

- c) In right $\triangle WYX$, the length of the side adjacent to $\angle Y$ and the length of the hypotenuse are given.

$$\cos Y = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Y = \frac{XY}{WY}$$

$$\cos Y = \frac{9}{10}$$

$$\angle Y \doteq 26^\circ$$

- d) In right $\triangle ABZ$, the length of the side adjacent to $\angle A$ and the length of the hypotenuse are given.

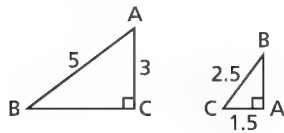
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AB}{AZ}$$

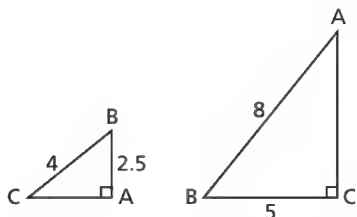
$$\cos A = \frac{2}{5}$$

$$\angle A \doteq 66^\circ$$

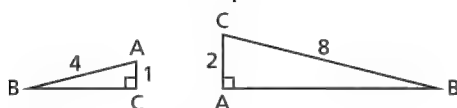
9. a) Sketch the triangles so that the ratio of the length of the side opposite $\angle B$ to the length of the hypotenuse is $\frac{3}{5}$. For example:



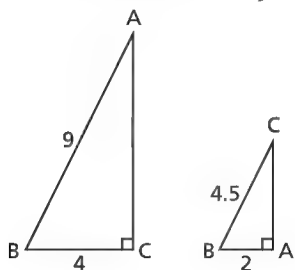
- b) Sketch the triangles so that the ratio of the length of the side adjacent to $\angle B$ to the length of the hypotenuse is $\frac{5}{8}$. For example:



- c) Sketch the triangles so that the ratio of the length of the side opposite $\angle B$ to the length of the hypotenuse is $\frac{1}{4}$. For example:



- d) Sketch the triangles so that the ratio of the length of the side adjacent to $\angle B$ to the length of the hypotenuse is $\frac{4}{9}$. For example:



10. I could use the sine and cosine ratios to determine the measures of both acute angles, or I could use the sine or cosine ratio to determine the measure of one angle and then use the fact that the sum of the angles in a triangle is 180° to determine the measure of the other angle.

- a) In right $\triangle CDE$:

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CD}{CE}$$

$$\cos C = \frac{2.4}{2.5}$$

$$\angle C \doteq 16.3^\circ$$

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{CE}$$

$$\sin E = \frac{2.4}{2.5}$$

$$\angle E \doteq 73.7^\circ$$

b) In right $\triangle FGH$:

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos F = \frac{FG}{FH}$$

$$\cos F = \frac{1.1}{2.5}$$

$$\angle F \doteq 63.9^\circ$$

$$\sin H = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin H = \frac{FG}{FH}$$

$$\sin H = \frac{1.1}{2.5}$$

$$\angle H \doteq 26.1^\circ$$

c) In right $\triangle JKM$:

$$\cos J = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos J = \frac{JM}{JK}$$

$$\cos J = \frac{2.6}{3.3}$$

$$\angle J \doteq 38.0^\circ$$

$$\angle K = 180^\circ - 90^\circ - \angle J$$

$$\angle K \doteq 180^\circ - 90^\circ - 38.0^\circ$$

$$\doteq 52.0^\circ$$

d) In right $\triangle PQR$:

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin P = \frac{QR}{PQ}$$

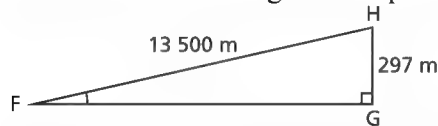
$$\sin P = \frac{2.2}{2.9}$$

$$\angle P \doteq 49.3^\circ$$

$$\angle Q = 180^\circ - 90^\circ - \angle P$$

$$\begin{aligned}\angle Q &\doteq 180^\circ - 90^\circ - 49.3^\circ \\ &\doteq 40.7^\circ\end{aligned}$$

11. Sketch and label a diagram to represent the information in the problem.



$\angle F$ is the angle of inclination of the track.

In right $\triangle FGH$:

$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$

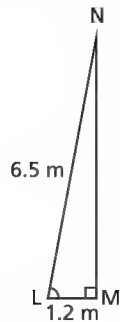
$$\sin F = \frac{GH}{FH}$$

$$\sin F = \frac{297}{13\,500}$$

$$\angle F = 1.2606\dots^\circ$$

The angle of inclination of the track is approximately 1.3° .

12. Sketch and label a diagram to represent the information in the problem.



$\angle L$ is the angle of inclination of the ladder.

LN is the length of the ladder.

LM is the distance from the base of the ladder to the wall.

In right $\triangle LMN$:

$$\cos L = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos L = \frac{LM}{LN}$$

$$\cos L = \frac{1.2}{6.5}$$

$$\angle L = 79.3612\dots^\circ$$

The angle of inclination of the ladder is approximately 79.4° .

13. Sketch and label a diagram to represent the information in the problem.

$\angle P$ is the angle of inclination of the rope.

PQ is the length of the rope.

QR is the height of the end of the rope that is attached to the tent.

In right $\triangle PQR$:

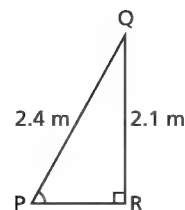
$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin P = \frac{QR}{PQ}$$

$$\sin P = \frac{2.1}{2.4}$$

$$\angle P = 61.0449\dots^\circ$$

The angle of inclination of the rope is approximately 61° .



14. Sketch and label a diagram to represent the information in the problem.

In right $\triangle ABC$:

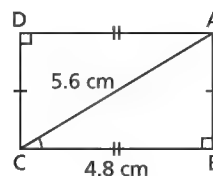
$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{BC}{AC}$$

$$\cos C = \frac{4.8}{5.6}$$

$$\angle C = 31.0027\dots^\circ$$

The angle between a diagonal and the longest side of the rectangle is approximately 31° .



15. a) i) $\sin 10^\circ = 0.1736\dots$

ii) $\sin 20^\circ = 0.3420\dots$

iii) $\sin 40^\circ = 0.6427\dots$

iv) $\sin 50^\circ = 0.7660\dots$

v) $\sin 60^\circ = 0.8660\dots$

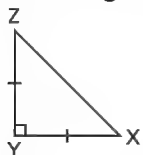
vi) $\sin 80^\circ = 0.9848\dots$

- b) Consider a right triangle with acute $\angle A$ and hypotenuse length 1 unit. Then:

$$\sin A = \frac{\text{length of side opposite } \angle A}{1 \text{ unit}}$$

As $\angle A$ increases, the side opposite $\angle A$ increases, so the ratio $\sin A$ increases.

16. Sketch right $\triangle XYZ$:



$$\cos X = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos X = \frac{XY}{XZ}$$

and

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XZ}$$

Since $XY = YZ$, $\cos X = \frac{YZ}{XZ}$ and $\sin X = \cos X$.

Similarly, $\sin Z = \cos Z$

C

17. Suppose the staircase were straightened out.

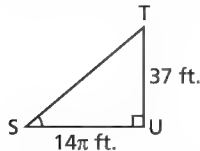
The staircase would cover a horizontal distance equal to the circumference of the silo:

$$C = \pi d$$

$$C = \pi(14)$$

The circumference of the silo is 14π ft.

The staircase would look like the diagram below.



$\angle S$ is the angle of inclination.

SU is the horizontal distance of the staircase.

TU is the height of the silo.

In right $\triangle STU$:

$$\tan S = \frac{TU}{SU}$$

$$\tan S = \frac{37}{14\pi}$$

$$\angle S = 40.0721\dots^\circ$$

The angle of inclination of the staircase is approximately 40° .

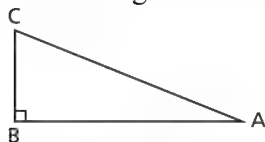
18. a) i) $\sin 90^\circ = 1$

ii) $\sin 0^\circ = 0$

iii) $\cos 90^\circ = 0$

iv) $\cos 0^\circ = 1$

b) Consider right $\triangle ABC$ with acute $\angle A$.



In right $\triangle ABC$:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

and

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sin A = \frac{BC}{AC}$$

$$\cos A = \frac{AB}{AC}$$

As vertex A moves closer to vertex B, $\angle A$ increases.

When $\angle A$ is close to 90° , the length of the hypotenuse, AC, is close to the length of BC.

Then, $\sin A$ is close to $\frac{BC}{BC}$, or 1.

When $\angle A$ is close to 90° , the length of the side adjacent to $\angle A$, AB, is close to 0.

Then, $\cos A$ is close to $\frac{0}{AC}$, or 0.

As vertex C moves closer to vertex B, $\angle A$ decreases.

When $\angle A$ is close to 0° , the length of the hypotenuse, AC, is close to the length of AB.

Then, $\cos A$ is close to $\frac{AB}{AB}$, or 1.

When $\angle A$ is close to 0° , the length of the side opposite $\angle A$, BC, is close to 0.

Then, $\sin A$ is close to $\frac{0}{AC}$, or 0.

Lesson 2.5 **Using the Sine and Cosine Ratios
to Calculate Lengths**

Exercises (pages 101–102)

A

- 3. a)** In right $\triangle XYZ$, XZ is the hypotenuse and YZ is the side opposite $\angle X$.
Use the sine ratio.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XZ}$$

$$\sin 50^\circ = \frac{x}{4.0} \quad \text{Solve for } x.$$

$$4.0 \sin 50^\circ = \frac{x(4.0)}{4.0}$$

$$4.0 \sin 50^\circ = x$$

$$x = 3.0641\dots$$

YZ is approximately 3.1 cm long.

- b)** In right $\triangle UVW$, VW is the hypotenuse and UV is the side opposite $\angle W$.
Use the sine ratio.

$$\sin W = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin W = \frac{UV}{VW}$$

$$\sin 30^\circ = \frac{w}{3.0} \quad \text{Solve for } w.$$

$$3.0 \sin 30^\circ = \frac{w(3.0)}{3.0}$$

$$3.0 \sin 30^\circ = w$$

$$w = 1.5$$

UV is 1.5 cm long.

- c) In right $\triangle RST$, RT is the hypotenuse and ST is the side opposite $\angle R$.
Use the sine ratio.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin R = \frac{ST}{RT}$$

$$\sin 25^\circ = \frac{r}{3.5} \quad \text{Solve for } r.$$

$$3.5 \sin 25^\circ = \frac{r(3.5)}{3.5}$$

$$3.5 \sin 25^\circ = r$$

$$r = 1.4791\dots$$

ST is approximately 1.5 cm long.

- d) In right $\triangle PNQ$, NQ is the hypotenuse and PQ is the side opposite $\angle N$.
Use the sine ratio.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin N = \frac{PQ}{NQ}$$

$$\sin 55^\circ = \frac{n}{4.5} \quad \text{Solve for } n.$$

$$4.5 \sin 55^\circ = \frac{n(4.5)}{4.5}$$

$$4.5 \sin 55^\circ = n$$

$$n = 3.6861\dots$$

PQ is approximately 3.7 cm long.

4. a) In right $\triangle KMN$, KM is the hypotenuse and KN is the side adjacent to $\angle K$.
Use the cosine ratio.

$$\cos K = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos K = \frac{KN}{KM}$$

$$\cos 72^\circ = \frac{m}{5.5} \quad \text{Solve for } m.$$

$$5.5 \cos 72^\circ = \frac{m(5.5)}{5.5}$$

$$5.5 \cos 72^\circ = m$$

$$m = 1.6995\dots$$

KN is approximately 1.7 cm long.

- b) In right $\triangle GHJ$, JH is the hypotenuse and GH is the side adjacent to $\angle H$.
Use the cosine ratio.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos H = \frac{GH}{JH}$$

$$\cos 41^\circ = \frac{j}{4.2} \quad \text{Solve for } j.$$

$$4.2 \cos 41^\circ = \frac{j(4.2)}{4.2}$$

$$4.2 \cos 41^\circ = j$$

$$j = 3.1697 \dots$$

GH is approximately 3.2 cm long.

- c) In right $\triangle DEF$, DF is the hypotenuse and DE is the side adjacent to $\angle D$.
Use the cosine ratio.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{DE}{DF}$$

$$\cos 62^\circ = \frac{f}{11.4} \quad \text{Solve for } f.$$

$$11.4 \cos 62^\circ = \frac{f(11.4)}{11.4}$$

$$11.4 \cos 62^\circ = f$$

$$f = 5.3519 \dots$$

DE is approximately 5.4 cm long.

- d) In right $\triangle ABC$, AB is the hypotenuse and BC is the side adjacent to $\angle B$.
Use the cosine ratio.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{BC}{AB}$$

$$\cos 23^\circ = \frac{a}{8.6} \quad \text{Solve for } a.$$

$$8.6 \cos 23^\circ = \frac{a(8.6)}{8.6}$$

$$8.6 \cos 23^\circ = a$$

$$a = 7.9163 \dots$$

BC is approximately 7.9 cm long.

B

5. a) In right $\triangle MNP$, NP is the hypotenuse and MP is the side adjacent to $\angle P$.
Use the cosine ratio.

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos P = \frac{MP}{NP}$$

$$\cos 57^\circ = \frac{13.8}{m} \quad \text{Solve for } m.$$

$$m \cos 57^\circ = \frac{13.8m}{m}$$

$$m \cos 57^\circ = 13.8$$

$$\frac{m \cos 57^\circ}{\cos 57^\circ} = \frac{13.8}{\cos 57^\circ}$$

$$m = \frac{13.8}{\cos 57^\circ}$$

$$m = 25.3378\dots$$

NP is approximately 25.3 cm long.

- b) In right $\triangle HJK$, HK is the hypotenuse and HJ is the side opposite $\angle K$.
Use the sine ratio.

$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{HJ}{HK}$$

$$\sin 51^\circ = \frac{6.2}{j} \quad \text{Solve for } j.$$

$$j \sin 51^\circ = \frac{6.2j}{j}$$

$$j \sin 51^\circ = 6.2$$

$$\frac{j \sin 51^\circ}{\sin 51^\circ} = \frac{6.2}{\sin 51^\circ}$$

$$j = \frac{6.2}{\sin 51^\circ}$$

$$j = 7.9779\dots$$

HK is approximately 8.0 cm long.

- c) In right $\triangle EFG$, EG is the hypotenuse and EF is the side adjacent to $\angle E$.
Use the cosine ratio.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{EF}{EG}$$

$$\cos 28^\circ = \frac{6.8}{f} \quad \text{Solve for } f.$$

$$f \cos 28^\circ = \frac{6.8f}{f}$$

$$f \cos 28^\circ = 6.8$$

$$\frac{f \cos 28^\circ}{\cos 28^\circ} = \frac{6.8}{\cos 28^\circ}$$

$$f = \frac{6.8}{\cos 28^\circ}$$

$$f = 7.7014\dots$$

EG is approximately 7.7 cm long.

- d) In right $\triangle BCD$, BD is the hypotenuse and BC is the side opposite $\angle D$.
Use the sine ratio.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{BC}{BD}$$

$$\sin 42^\circ = \frac{8.3}{c} \quad \text{Solve for } c.$$

$$c \sin 42^\circ = \frac{8.3c}{c}$$

$$c \sin 42^\circ = 8.3$$

$$\frac{c \sin 42^\circ}{\sin 42^\circ} = \frac{8.3}{\sin 42^\circ}$$

$$c = \frac{8.3}{\sin 42^\circ}$$

$$c = 12.4041\dots$$

BD is approximately 12.4 cm long.

6. In the diagram, DF is the height that the ladder can reach up the wall of the building.
In right $\triangle DEF$, DE is the hypotenuse and DF is the side opposite $\angle E$.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{DF}{DE}$$

$$\sin 77^\circ = \frac{DF}{30.5} \quad \text{Solve for DF.}$$

$$30.5 \sin 77^\circ = \frac{DF(30.5)}{30.5}$$

$$30.5 \sin 77^\circ = DF$$

$$DF = 29.7182\dots$$

The ladder can reach approximately 29.7 m up the wall of the apartment building.

7. a) In right $\triangle CDE$, CE is the hypotenuse and DE is the side adjacent to $\angle E$.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{DE}{CE}$$

$$\cos 58.5^\circ = \frac{25.23}{CE} \quad \text{Solve for CE.}$$

$$CE \cos 58.5^\circ = \frac{25.23(CE)}{CE}$$

$$CE \cos 58.5^\circ = 25.23$$

$$\frac{CE \cos 58.5^\circ}{\cos 58.5^\circ} = \frac{25.23}{\cos 58.5^\circ}$$

$$CE = \frac{25.23}{\cos 58.5^\circ}$$

$$CE = 48.2872\dots$$

The distance from C to E is approximately 48.3 m.

- b) If the distance from C to E is not known, the surveyor could use the tangent ratio since CD is opposite $\angle E$ and the length of the side adjacent to $\angle E$ is known.
If the distance from C to E is known, the surveyor could use the Pythagorean Theorem or the sine ratio to calculate the distance from C to D. For example, using the sine ratio:

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{CE}$$

$$\sin 58.5^\circ = \frac{CD}{48.2872\dots} \quad \text{Solve for CD.}$$

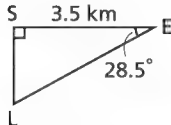
$$(48.2872\dots)\sin 58.5^\circ = \frac{CD(48.2872\dots)}{48.2872\dots}$$

$$(48.2872\dots)\sin 58.5^\circ = CD$$

$$CD = 41.1716\dots$$

The distance from C to D is approximately 41.2 m.

8. Sketch and label a diagram to represent the information in the problem.



ES is the distance travelled by the ship.

EL is the distance from the ship to the lighthouse.

$\angle E$ is the angle between the ship's path and the line of sight to the lighthouse.

In right $\triangle ELS$, EL is the hypotenuse and ES is the side adjacent to $\angle E$.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{ES}{EL}$$

$$\cos 28.5^\circ = \frac{3.5}{EL} \quad \text{Solve for EL.}$$

$$EL \cos 28.5^\circ = \frac{3.5(EL)}{EL}$$

$$EL \cos 28.5^\circ = 3.5$$

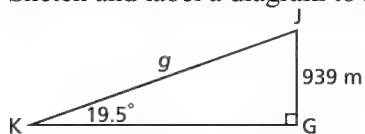
$$\frac{EL \cos 28.5^\circ}{\cos 28.5^\circ} = \frac{3.5}{\cos 28.5^\circ}$$

$$EL = \frac{3.5}{\cos 28.5^\circ}$$

$$EL = 3.9826\dots$$

The ship is approximately 4.0 km from the lighthouse.

9. Sketch and label a diagram to represent the information in the problem.



GJ is the height of the plane.

KJ is the distance from the plane to the airport.

$\angle K$ is the angle of elevation of the plane measured from the airport.

In right $\triangle GJK$, JK is the hypotenuse and GJ is the side opposite $\angle K$.

$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{GJ}{JK}$$

$$\sin 19.5^\circ = \frac{939}{g} \quad \text{Solve for } g.$$

$$g \sin 19.5^\circ = \frac{939g}{g}$$

$$g \sin 19.5^\circ = 939$$

$$\frac{g \sin 19.5^\circ}{\sin 19.5^\circ} = \frac{939}{\sin 19.5^\circ}$$

$$g = \frac{939}{\sin 19.5^\circ}$$

$$g = 2813.0039\dots$$

The plane is approximately 2813 m from the airport.

10. In right $\triangle CDE$, CE is the hypotenuse and CD is the side adjacent to $\angle C$.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CD}{CE}$$

$$\cos 76^\circ = \frac{CD}{18.9} \quad \text{Solve for } CD.$$

$$18.9 \cos 76^\circ = \frac{CD(18.9)}{18.9}$$

$$18.9 \cos 76^\circ = CD$$

$$CD = 4.5723\dots$$

In right $\triangle CDE$, CE is the hypotenuse and DE is the side opposite $\angle C$.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{DE}{CE}$$

$$\sin 76^\circ = \frac{DE}{18.9} \quad \text{Solve for } DE.$$

$$18.9 \sin 76^\circ = \frac{DE(18.9)}{18.9}$$

$$18.9 \sin 76^\circ = DE$$

$$DE = 18.3385\dots$$

So, the dimensions of the rectangle are approximately 4.6 cm by 18.3 cm.

11. a) The required length of the bookcase is AB .

In right $\triangle ABC$, AB is the hypotenuse and AC is the side adjacent to $\angle A$.

Use the cosine ratio.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos 40^\circ = \frac{3.24}{AB} \quad \text{Solve for } AB.$$

$$AB \cos 40^\circ = \frac{3.24(AB)}{AB}$$

$$AB \cos 40^\circ = 3.24$$

$$\frac{AB \cos 40^\circ}{\cos 40^\circ} = \frac{3.24}{\cos 40^\circ}$$

$$AB = \frac{3.24}{\cos 40^\circ}$$

$$AB = 4.2295\dots$$

The length of the top of the bookcase is approximately 4.23 m.

- b) The greatest height of the bookcase is side BC.
In right $\triangle ABC$, AB is the hypotenuse and BC is the side opposite $\angle A$.
Use the sine ratio.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BC}{AB}$$

$$\sin 40^\circ = \frac{BC}{4.2295\dots} \quad \text{Solve for BC.}$$

$$(4.2295\dots)\sin 40^\circ = \frac{BC(4.2295\dots)}{4.2295\dots}$$

$$(4.2295\dots)\sin 40^\circ = BC$$

$$BC = 2.7186\dots$$

The greatest height of the bookcase is approximately 2.72 m.

12. a) i) In right $\triangle CDE$, DE is the hypotenuse and CD is the side opposite $\angle E$.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{DE}$$

$$\sin 34^\circ = \frac{CD}{8.8} \quad \text{Solve for CD.}$$

$$8.8\sin 34^\circ = \frac{CD(8.8)}{8.8}$$

$$8.8\sin 34^\circ = CD$$

$$CD = 4.9208\dots$$

In right $\triangle CDE$, DE is the hypotenuse and CE is the side adjacent to $\angle E$.

$$\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos E = \frac{CE}{DE}$$

$$\cos 34^\circ = \frac{CE}{8.8} \quad \text{Solve for CE.}$$

$$8.8\cos 34^\circ = \frac{CE(8.8)}{8.8}$$

$$8.8\cos 34^\circ = CE$$

$$CE = 7.2955\dots$$

Perimeter of $\triangle CDE$:

$$4.9208\dots \text{ cm} + 7.2955\dots \text{ cm} + 8.8 \text{ cm} = 21.0164\dots \text{ cm}$$

The perimeter of $\triangle CDE$ is approximately 21.0 cm.

- ii) In right $\triangle JKM$, JM is the hypotenuse and JK is the side opposite $\angle M$.

$$\sin M = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin M = \frac{JK}{JM}$$

$$\sin 63^\circ = \frac{JK}{5.6} \quad \text{Solve for JK.}$$

$$5.6 \sin 63^\circ = \frac{JK(5.6)}{5.6}$$

$$5.6 \sin 63^\circ = JK$$

$$JK = 4.9896\dots$$

In right $\triangle JKM$, JM is the hypotenuse and KM is the side adjacent to $\angle M$.

$$\cos M = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos M = \frac{KM}{JM}$$

$$\cos 63^\circ = \frac{KM}{5.6} \quad \text{Solve for KM.}$$

$$5.6 \cos 63^\circ = \frac{KM(5.6)}{5.6}$$

$$5.6 \cos 63^\circ = KM$$

$$KM = 2.5423\dots$$

Perimeter of the rectangle is:

$$2(4.9896\dots \text{ cm}) + 2(2.5423\dots \text{ cm}) = 15.0639\dots \text{ cm}$$

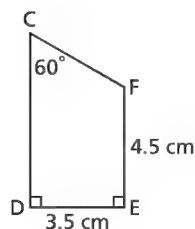
The perimeter of the rectangle is approximately 15.1 cm.

- b) In part a, I used the sine and cosine ratios to determine the side lengths. In each case, I could have used the tangent ratio or the Pythagorean Theorem to determine the second side length.

C

13. Sketch and label a diagram to represent the information in the problem.

The trapezoid is made up of a right triangle and a rectangle.



In right $\triangle BCF$, CF is the hypotenuse and BF is the side opposite $\angle C$.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{BF}{CF}$$

$$\sin 60^\circ = \frac{3.5}{y} \quad \text{Solve for } y.$$

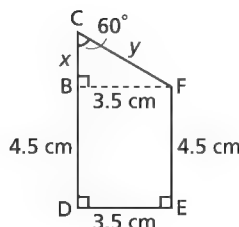
$$y \sin 60^\circ = \frac{3.5y}{y}$$

$$y \sin 60^\circ = 3.5$$

$$\frac{y \sin 60^\circ}{\sin 60^\circ} = \frac{3.5}{\sin 60^\circ}$$

$$y = \frac{3.5}{\sin 60^\circ}$$

$$y = 4.0414\dots$$



In right $\triangle BCF$, CF is the hypotenuse and BC is the side adjacent to $\angle C$.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{BC}{CF}$$

$$\cos 60^\circ = \frac{x}{4.0414\dots} \quad \text{Solve for } x.$$

$$(4.0414\dots)\cos 60^\circ = \frac{(4.0414\dots)x}{4.0414\dots}$$

$$(4.0414\dots)\cos 60^\circ = x$$

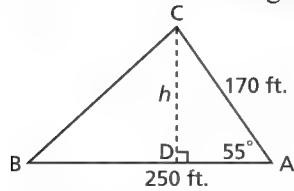
$$x = 2.0207\dots$$

The perimeter of trapezoid CDEF is:

$$2(4.5 \text{ cm}) + 3.5 \text{ cm} + 4.0414\dots \text{ cm} + 2.0207\dots \text{ cm} = 18.5621\dots \text{ cm}$$

The perimeter of trapezoid CDEF is approximately 18.6 cm.

14. a) Sketch and label a diagram to represent the information in the problem.



In right $\triangle ACD$, AC is the hypotenuse and CD is the side opposite $\angle A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{CD}{AC}$$

$$\sin 55^\circ = \frac{h}{170} \quad \text{Solve for } h.$$

$$170 \sin 55^\circ = \frac{h(170)}{170}$$

$$170 \sin 55^\circ = h$$

$$h = 139.2558\dots$$

The height of the triangle is approximately 139 ft.

- b) The area of the triangle is:

$$\frac{1}{2}(250)(139.2558\dots) = 17\,406.9809\dots$$

The area of the lot is approximately 17 407 square feet.

Checkpoint 2

Assess Your Understanding (page 104)

2.4

1. a) In right $\triangle PQR$, PR is opposite $\angle Q$ and PQ is the hypotenuse.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Q = \frac{PR}{PQ}$$

$$\sin Q = \frac{4}{8}$$

$$\angle Q = 30^\circ$$

- b) In right $\triangle STU$, SU is adjacent to $\angle S$ and ST is the hypotenuse.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{SU}{ST}$$

$$\cos S = \frac{6}{9}$$

$$\angle S \doteq 48^\circ$$

- c) In right $\triangle JKM$, JM is opposite $\angle K$ and MK is the hypotenuse.

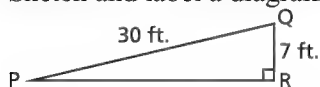
$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{JM}{MK}$$

$$\sin K = \frac{9.5}{11.5}$$

$$\angle K \doteq 56^\circ$$

2. Sketch and label a diagram to represent the information in the problem.



$\angle P$ is the angle of inclination of the conveyor.

PQ is the length of the conveyor.

QR is the height of the loading dock.

In right $\triangle PQR$:

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin P = \frac{QR}{PQ}$$

$$\sin P = \frac{7}{30}$$

$$\angle P = 13.4934\dots^\circ$$

The angle of inclination of the conveyor is approximately 13° .

3. a) i) $\cos 10^\circ = 0.9848\dots$

ii) $\cos 20^\circ = 0.9396\dots$

iii) $\cos 30^\circ = 0.8660\dots$

iv) $\cos 40^\circ = 0.7660\dots$

v) $\cos 50^\circ = 0.6427\dots$

vi) $\cos 60^\circ = 0.5$

vii) $\cos 70^\circ = 0.3420\dots$

viii) $\cos 80^\circ = 0.1736\dots$

- b) Consider a right triangle with acute $\angle A$ and hypotenuse length 1 unit. Then:

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{1 \text{ unit}}$$

As $\angle A$ increases, the side adjacent to $\angle A$ decreases, so the ratio $\cos A$ decreases.

2.5

4. a) In right $\triangle NPQ$, PQ is adjacent to $\angle Q$ and NQ is the hypotenuse.

$$\cos Q = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Q = \frac{PQ}{NQ}$$

$$\cos 65^\circ = \frac{n}{10.0} \quad \text{Solve for } n.$$

$$10.0 \cos 65^\circ = \frac{n(10.0)}{10.0}$$

$$10.0 \cos 65^\circ = n$$

$$n = 4.2261\dots$$

PQ is approximately 4.2 cm long.

- b) In right $\triangle RST$, ST is opposite $\angle R$ and RS is the hypotenuse.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin R = \frac{ST}{RS}$$

$$\sin 20^\circ = \frac{r}{8.0} \quad \text{Solve for } r.$$

$$8.0 \sin 20^\circ = \frac{r(8.0)}{8.0}$$

$$8.0 \sin 20^\circ = r$$

$$r = 2.7361\dots$$

ST is approximately 2.7 cm long.

- c) In right $\triangle UVW$, UV is adjacent to $\angle U$ and UW is the hypotenuse.

$$\cos U = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos U = \frac{UV}{UW}$$

$$\cos 50^\circ = \frac{9.0}{v} \quad \text{Solve for } v.$$

$$v \cos 50^\circ = \frac{9.0v}{v}$$

$$v \cos 50^\circ = 9.0$$

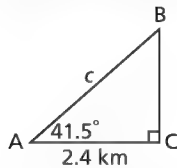
$$\frac{v \cos 50^\circ}{\cos 50^\circ} = \frac{9.0}{\cos 50^\circ}$$

$$v = \frac{9.0}{\cos 50^\circ}$$

$$v = 14.0015\dots$$

UW is approximately 14.0 cm long.

5. Sketch and label a diagram to represent the information in the problem.



AB is the distance from the ship to the beacon.

AC is the distance the ship travelled due west.

$\angle A$ is the angle between the ship's path and the line of sight to the beacon.

In right $\triangle ABC$:

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos 41.5^\circ = \frac{2.4}{c} \quad \text{Solve for } c.$$

$$c \cos 41.5^\circ = \frac{2.4c}{c}$$

$$c \cos 41.5^\circ = 2.4$$

$$\frac{c \cos 41.5^\circ}{\cos 41.5^\circ} = \frac{2.4}{\cos 41.5^\circ}$$

$$c = \frac{2.4}{\cos 41.5^\circ}$$

$$c = 3.2044\dots$$

The ship is approximately 3.2 km from the beacon.

Lesson 2.6 Applying the Trigonometric Ratios

Exercises (pages 111–112)

A

3. a) In right $\triangle ABC$, the lengths of AC and BC are given.
AC is the hypotenuse and BC is the side opposite $\angle A$.
So, I would use the sine ratio.
- b) In right $\triangle DEF$, the lengths of DE and EF are given.
DE is the side adjacent to $\angle D$ and EF is the side opposite $\angle D$.
So, I would use the tangent ratio.
- c) In right $\triangle GHJ$, the lengths of GH and GJ are given.
GJ is the hypotenuse and GH is the side adjacent to $\angle G$.
So, I would use the cosine ratio.
- d) In right $\triangle XYZ$, the lengths of XZ and YZ are given.
YZ is the side adjacent to $\angle Y$ and XZ is the side opposite $\angle Y$.
So, I would use the tangent ratio.
4. a) In right $\triangle KMN$, the length of KM is given and I need to determine the length of KN.
KM is the hypotenuse and KN is the side adjacent to $\angle K$.
So, I will use the cosine ratio.

$$\cos K = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos K = \frac{KN}{KM}$$

$$\cos 37^\circ = \frac{m}{5.8} \quad \text{Solve for } m.$$

$$5.8 \cos 37^\circ = \frac{m(5.8)}{5.8}$$

$$5.8 \cos 37^\circ = m$$

$$m = 4.6320\dots$$

KN is approximately 4.6 cm long.

- b) In right $\triangle PQR$, the length of QR is given and I need to determine the length of PQ .
 QR is the side adjacent to $\angle R$ and PQ is the side opposite $\angle R$.
 So, I will use the tangent ratio.

$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan R = \frac{PQ}{QR}$$

$$\tan 52^\circ = \frac{r}{3.7} \quad \text{Solve for } r.$$

$$3.7 \times \tan 52^\circ = \frac{r}{3.7} \times 3.7$$

$$3.7 \tan 52^\circ = r$$

$$r = 4.7357\dots$$

PQ is approximately 4.7 cm long.

- c) In right $\triangle AYZ$, the length of YZ is given and I need to determine the length of AY .
 AY is the hypotenuse and YZ is the side opposite $\angle A$.
 So, I will use the sine ratio.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{YZ}{AY}$$

$$\sin 62^\circ = \frac{10.4}{z} \quad \text{Solve for } z.$$

$$z \sin 62^\circ = \frac{10.4z}{z}$$

$$z \sin 62^\circ = 10.4$$

$$\frac{z \sin 62^\circ}{\sin 62^\circ} = \frac{10.4}{\sin 62^\circ}$$

$$z = \frac{10.4}{\sin 62^\circ}$$

$$z = 11.7787\dots$$

AY is approximately 11.8 cm long.

- d) In right $\triangle BCD$, the length of CD is given and I need to determine the length of BD .
 BD is the hypotenuse and CD is the side adjacent to $\angle D$.
 So, I will use the cosine ratio.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{CD}{BD}$$

$$\cos 55^\circ = \frac{8.3}{c} \quad \text{Solve for } c.$$

$$c \cos 55^\circ = \frac{8.3c}{c}$$

$$c \cos 55^\circ = 8.3$$

$$\frac{c \cos 55^\circ}{\cos 55^\circ} = \frac{8.3}{\cos 55^\circ}$$

$$c = \frac{8.3}{\cos 55^\circ}$$

$$c = 14.4706\dots$$

BD is approximately 14.5 cm long.

5. a) In right $\triangle EFG$, the lengths of the legs, EF and FG , are given.
 So, I would use the Pythagorean Theorem to determine the length of the hypotenuse, EG .
- b) In right $\triangle HJK$, the length of HK and the measure of $\angle J$ are given.
 HK is the side opposite $\angle J$, and HJ is the hypotenuse.
 So, I would use the sine ratio to determine the length of HJ .
- c) In right $\triangle MNP$, the lengths of one leg, MN , and the hypotenuse, MP , are given. So, I would use the Pythagorean Theorem to determine the length of the other leg, PN .
- d) In right $\triangle QRS$, the lengths of the legs, RS and QS , are given.
 So, I would use the Pythagorean Theorem to determine the length of the hypotenuse, QR .

B

6. a) Determine the measure of $\angle T$.

$$\begin{aligned}\angle T &= 90^\circ - \angle U \\ &= 90^\circ - 33^\circ \\ &= 57^\circ\end{aligned}$$

Determine the length of UV.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{UV}{TV}$$

$$\tan 57^\circ = \frac{UV}{12.5} \quad \text{Solve for UV.}$$

$$12.5 \tan 57^\circ = UV$$

$$UV = 19.2483\dots$$

Determine the length of TU. Use the Pythagorean Theorem.

$$TU^2 = TV^2 + UV^2$$

$$\begin{aligned}TU^2 &= 12.5^2 + (19.2483\dots)^2 \\ &= 526.7475\dots\end{aligned}$$

$$\begin{aligned}TU &= \sqrt{526.7475\dots} \\ &= 22.9509\dots\end{aligned}$$

$\angle T$ is 57° , TU is approximately 23.0 cm, and UV is approximately 19.2 cm.

- b) Determine the length of XY.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{XY}{WX}$$

$$\tan 47^\circ = \frac{XY}{5.9} \quad \text{Solve for XY.}$$

$$5.9 \tan 47^\circ = XY$$

$$XY = 6.3269\dots$$

Determine the length of WY. Use the Pythagorean Theorem.

$$WY^2 = WX^2 + XY^2$$

$$\begin{aligned} WY^2 &= 5.9^2 + (6.3269\dots)^2 \\ &= 74.8406\dots \end{aligned}$$

$$\begin{aligned} WY &= \sqrt{74.8406\dots} \\ &= 8.6510\dots \end{aligned}$$

Determine the measure of $\angle Y$.

$$\begin{aligned} \angle Y &= 90^\circ - \angle W \\ &= 90^\circ - 47^\circ \\ &= 43^\circ \end{aligned}$$

$\angle Y$ is 43° , XY is approximately 6.3 cm, and WY is approximately 8.7 cm.

- c) Determine the length of BZ. Use the Pythagorean Theorem.

$$BZ^2 = AB^2 + AZ^2$$

$$BZ^2 = 5.6^2 + 9.8^2$$

$$= 127.4$$

$$BZ = \sqrt{127.4}$$

$$= 11.2871\dots$$

Determine the measure of $\angle B$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AZ}{AB}$$

$$\tan B = \frac{9.8}{5.6}$$

$$\angle B = 60.2551\dots^\circ$$

Determine the measure of $\angle Z$.

$$\angle Z = 90^\circ - \angle B$$

$$= 90^\circ - 60.2551\dots^\circ$$

$$= 29.7448\dots^\circ$$

$\angle B$ is approximately 60.3° , $\angle Z$ is approximately 29.7° , and BZ is approximately 11.3 cm.

- d) Determine the measure of $\angle E$.

$$\begin{aligned}\angle E &= 90^\circ - \angle D \\ &= 90^\circ - 29^\circ \\ &= 61^\circ\end{aligned}$$

Determine the length of CD.

$$\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos D = \frac{CD}{DE}$$

$$\cos 29^\circ = \frac{CD}{13.7} \quad \text{Solve for CD.}$$

$$13.7 \cos 29^\circ = CD$$

$$CD = 11.9822\dots$$

Determine the length of CE.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{CE}{DE}$$

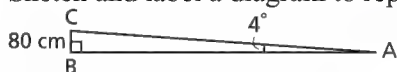
$$\sin 29^\circ = \frac{CE}{13.7} \quad \text{Solve for CE.}$$

$$13.7 \sin 29^\circ = CE$$

$$CE = 6.6418\dots$$

$\angle E$ is 61° , CD is approximately 12.0 cm, and CE is approximately 6.6 cm.

7. Sketch and label a diagram to represent the information in the problem.



AC is the length of the ramp.

AB is the horizontal distance the ramp will take up.

BC is the maximum height of the ramp.

$\angle A$ is the angle of elevation of the ramp.

- a) In right $\triangle ABC$, AC is the hypotenuse and BC is the side opposite $\angle A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BC}{AC}$$

$$\sin 4^\circ = \frac{80}{AC} \quad \text{Solve for AC.}$$

$$AC \sin 4^\circ = 80$$

$$AC = \frac{80}{\sin 4^\circ}$$

$$AC = 1146.8469\dots$$

The length of the ramp is approximately 1147 cm.

- b) In right $\triangle ABC$, AC is the hypotenuse and BC and AB are the legs.
Use the Pythagorean Theorem.

$$AC^2 = AB^2 + BC^2 \quad \text{Solve for the unknown.}$$

$$AB^2 = AC^2 - BC^2$$

$$AB^2 = (1146.8469\dots)^2 - 80^2$$

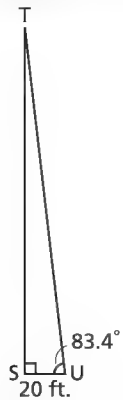
$$= 1\,308\,857.954\dots$$

$$AB = \sqrt{1\,308\,857.954\dots}$$

$$= 1144.0533\dots$$

The ramp will take up a horizontal distance of approximately 1144 cm.

8. Sketch and label a diagram to represent the information in the problem.



Assume the ground is horizontal.

ST is the height of the totem pole.

SU is the distance from the base of the totem pole.

$\angle U$ is the angle of elevation of the top of the pole.

In right $\triangle STU$, ST is the side opposite $\angle U$ and SU is side adjacent to $\angle U$.

$$\tan U = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan U = \frac{ST}{SU}$$

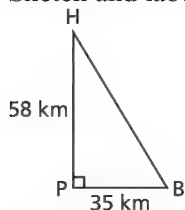
$$\tan 83.4^\circ = \frac{ST}{20} \quad \text{Solve for ST.}$$

$$20 \tan 83.4^\circ = ST$$

$$ST = 172.8549\dots$$

The totem pole is approximately 173 ft. tall.

9. Sketch and label a diagram to represent the information in the problem.



BP is the distance from the base to the sick person.

PH is the distance from the sick person to the hospital.

BH is the distance from the base to the hospital.

$\angle H$ is the angle between the path the helicopter took due north and the path it will take to return directly to its base.

- a) In right $\triangle BHP$, BH is the hypotenuse and BP and PH are the legs.

Use the Pythagorean Theorem.

$$BH^2 = BP^2 + PH^2$$

$$BH^2 = 35^2 + 58^2$$

$$= 4589$$

$$BH = \sqrt{4589}$$

$$= 67.7421\dots$$

The distance between the hospital and the base is approximately 68 km.

- b) In right $\triangle BHP$, BP is the side opposite $\angle H$ and PH is the side adjacent to $\angle H$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

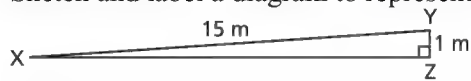
$$\tan H = \frac{BP}{PH}$$

$$\tan H = \frac{35}{58}$$

$$\angle H = 31.1088\dots^\circ$$

The angle between the path the helicopter took due north and the path it will take to return directly to its base is approximately 31° .

10. Sketch and label a diagram to represent the information in the problem.



XY is the distance travelled along the road.

YZ is the rise of the road.

XZ is the horizontal distance travelled.

$\angle X$ is the angle of inclination of the road.

a) In right $\triangle XYZ$, YZ is the side opposite $\angle X$ and XY is the hypotenuse.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin X = \frac{YZ}{XY}$$

$$\sin X = \frac{1}{15}$$

$$\angle X = 3.8225\dots^\circ$$

The angle of inclination of the road is approximately 4° .

b) In right $\triangle XYZ$, XY is the hypotenuse and XZ and YZ are the legs.
Use the Pythagorean Theorem.

$$XY^2 = XZ^2 + YZ^2 \quad \text{Isolate the unknown.}$$

$$XZ^2 = XY^2 - YZ^2$$

$$XZ^2 = 15^2 - 1^2$$

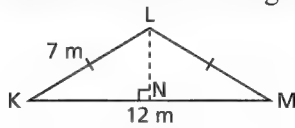
$$= 224$$

$$XZ = \sqrt{224}$$

$$= 14.9666\dots$$

The horizontal distance travelled is approximately 15.0 m.

11. Sketch and label a diagram to represent the information in the problem.



$\angle K$ is the angle of inclination of the roof.

$\angle KLM$ is the angle at the peak of the roof.

- a) In right $\triangle KLN$, KN is the side adjacent to $\angle K$ and it is $\frac{1}{2} KM = 6$ m; and KL is the hypotenuse.

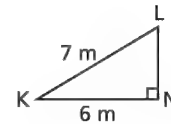
$$\cos K = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos K = \frac{KN}{KL}$$

$$\cos K = \frac{6}{7}$$

$$\angle K = 31.0027\dots^\circ$$

The angle of inclination of the roof is approximately 31° .



- b) In isosceles $\triangle KLM$, $\angle M = \angle K \doteq 31^\circ$
The sum of the angles in a triangle is 180° .
So, $\angle L = 180^\circ - 31^\circ - 31^\circ$
 $= 118^\circ$

The measure of the angle at the peak of the roof is approximately 118° .

12. a) In right $\triangle CDE$, DE is the hypotenuse and CE is the length of the side opposite $\angle D$.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{CE}{DE}$$

$$\sin 45^\circ = \frac{CE}{5.6}$$

Solve for CE .

$$5.6 \sin 45^\circ = CE$$

$$CE = 3.9597\dots$$

$$\angle E = 90^\circ - \angle D$$

$$= 90^\circ - 45^\circ$$

$$= 45^\circ$$

Since $\angle D = \angle E$, $\triangle CDE$ is an isosceles right triangle with $CD = CE$.

Perimeter of $\triangle CDE$:

$$5.6 \text{ cm} + 2(3.9597\dots \text{ cm}) = 13.5195\dots \text{ cm}$$

Area of $\triangle CDE$:

$$\frac{1}{2}(CE)(CD) = \frac{1}{2}(CE)^2$$

$$= \frac{1}{2}(3.9597\dots)^2$$

$$= 7.84$$

The perimeter of $\triangle CDE$ is approximately 13.5 cm and its area is approximately 7.8 cm^2 .

- b) In right $\triangle FGH$, FH is the hypotenuse and GH is the side opposite $\angle F$.

$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin F = \frac{GH}{FH}$$

$$\sin 28^\circ = \frac{GH}{10.7} \quad \text{Solve for GH.}$$

$$10.7 \sin 28^\circ = GH$$

$$GH = 5.0233\dots$$

In right $\triangle FGH$, FH is the hypotenuse and FG is the side adjacent to $\angle F$.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos F = \frac{FG}{FH}$$

$$\cos 28^\circ = \frac{FG}{10.7} \quad \text{Solve for FG.}$$

$$10.7 \cos 28^\circ = FG$$

$$FG = 9.4475\dots$$

Perimeter of rectangle:

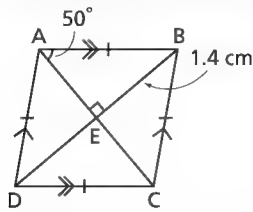
$$2(5.0233\dots \text{ cm} + 9.4475\dots \text{ cm}) = 28.9417\dots \text{ cm}$$

Area of rectangle:

$$(5.0233\dots \text{ cm})(9.4475\dots \text{ cm}) = 47.4582\dots \text{ cm}^2$$

The perimeter of rectangle FGHJ is approximately 28.9 cm and its area is approximately 47.5 cm^2 .

13. Sketch and label a copy of the rhombus.



In right $\triangle ABE$, AB is the hypotenuse and BE is the side opposite $\angle A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BE}{AB}$$

$$\sin 50^\circ = \frac{1.4}{AB} \quad \text{Solve for AB.}$$

$$AB \sin 50^\circ = 1.4$$

$$AB = \frac{1.4}{\sin 50^\circ}$$

$$AB = 1.8275\dots$$

Perimeter of rhombus:

$$4(1.8275\dots \text{ cm}) = 7.3102\dots \text{ cm}$$

The perimeter of rhombus ABCD is approximately 7.3 cm.

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Chapter 2
Trigonometry

- 14. a)** Sketch and label the base of the candle.

The base is a regular 12-sided polygon and the distance from one vertex to the opposite vertex is 2 in., so the distance from the centre of the polygon to a vertex is 1 in. The polygon can be divided into 12 congruent isosceles triangles. In each triangle, the angle at the centre of the polygon is:

$$\frac{360^\circ}{12} = 30^\circ$$

$\triangle ABO$ is an isosceles triangle.

So, draw the perpendicular bisector OD to form two congruent right triangles.

OD bisects $\angle AOB$, so $\angle AOD = 15^\circ$

In right $\triangle AOD$, AO is the hypotenuse and AD is the side opposite $\angle O$.

$$\sin O = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin O = \frac{AD}{AO}$$

$$\sin 15^\circ = \frac{AD}{1} \quad \text{Solve for AD.}$$

$$AD = \sin 15^\circ$$

So:

$$\begin{aligned} AB &= 2(AD) \\ &= 2\sin 15^\circ \end{aligned}$$

In right $\triangle AOD$, AO is the hypotenuse and DO is the side adjacent to $\angle O$.

$$\cos O = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos O = \frac{DO}{AO}$$

$$\cos 15^\circ = \frac{DO}{1} \quad \text{Solve for DO.}$$

$$DO = \cos 15^\circ$$

The area of $\triangle ABO$ is:

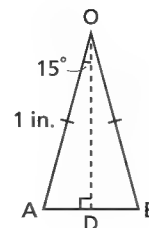
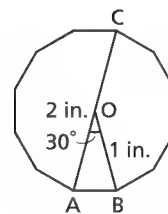
$$\begin{aligned} \frac{1}{2}(AB)(DO) &= \frac{1}{2}(2\sin 15^\circ)(\cos 15^\circ) \\ &= (\sin 15^\circ)(\cos 15^\circ) \end{aligned}$$

The 12-sided polygon is made up of 12 triangles congruent to $\triangle ABO$.

So, the area of the polygon, in square inches, is:

$$12 \times (\sin 15^\circ)(\cos 15^\circ) = 3$$

The area of the base is 3 square inches.



- b) The volume of wax in the candle, in cubic inches, is:
 $\text{volume} = (\text{base area})(\text{height})$
 $= (3)(5)$
 $= 15$

The volume of wax in the candle is 15 cubic inches.

C

15. Sketch and label a diagram to represent the information in the problem.

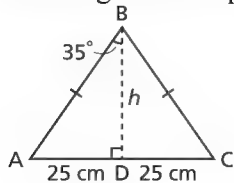
Each nozzle should spray water to at least one-half the distance between it and the next nozzle. Assume the ground is horizontal and the nozzles spray an equal distance to the left and right.

Triangle ABC is isosceles, so $AB = BC$ and BD bisects $\angle ABC$ and bisects AC .

So, AD is $\frac{1}{2}(50 \text{ cm}) = 25 \text{ cm}$

And, $\angle ABD$ is $\frac{1}{2}(70^\circ) = 35^\circ$

The height of the sprayer above the crops is the length of BD .



In right $\triangle ABD$, AD is the side opposite $\angle B$ and BD is the side adjacent to $\angle B$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{AD}{BD}$$

$$\tan 35^\circ = \frac{25}{h}$$

Solve for h .

$$h \tan 35^\circ = 25$$

$$h = \frac{25}{\tan 35^\circ}$$

$$h = 35.7037 \dots$$

The sprayer should be placed at least 36 cm above the crops.

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Chapter 2
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16. Sketch and label the trapezoid.

The trapezoid is isosceles, so $AF = DE$ and $AB = CD$.

In right $\triangle ABF$, AB is the hypotenuse and BF is the side opposite $\angle A$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

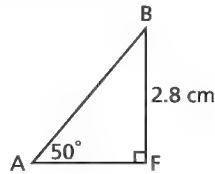
$$\sin A = \frac{BF}{AB}$$

$$\sin 50^\circ = \frac{2.8}{AB}$$

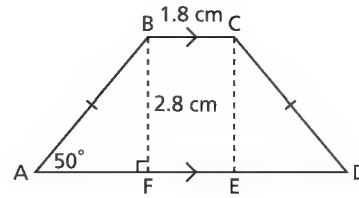
$$AB \sin 50^\circ = 2.8$$

$$AB = \frac{2.8}{\sin 50^\circ}$$

$$AB = 3.6551\dots$$



Solve for AB .



In right $\triangle ABF$, BF is the side opposite $\angle A$ and AF is the side adjacent to $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BF}{AF}$$

$$\tan 50^\circ = \frac{2.8}{AF}$$

$$AF \tan 50^\circ = 2.8$$

$$AF = \frac{2.8}{\tan 50^\circ}$$

$$AF = 2.3494\dots$$

Solve for AF .

Perimeter of the trapezoid:

$$2(1.8 \text{ cm}) + 2(3.6551\dots \text{ cm}) + 2(2.3494\dots \text{ cm}) = 15.6092\dots \text{ cm}$$

The area of the trapezoid is the sum of the areas of 2 congruent right triangles with base $2.3494\dots \text{ cm}$ and height 2.8 cm , and the area of a rectangle with width 1.8 cm and height 2.8 cm .

Area of trapezoid:

$$2 \times \frac{1}{2} (2.3494\dots \text{ cm})(2.8 \text{ cm}) + (1.8 \text{ cm})(2.8 \text{ cm}) = 11.6185\dots \text{ cm}^2$$

The perimeter of the trapezoid is approximately 15.6 cm and its area is approximately 11.6 cm^2 .

Lesson 2.7 **Solving Problems Involving More
than One Right Triangle**

Exercises (pages 118–121)

A

3. a) $JK = JM + KM$

In right $\triangle JMN$, JM is opposite $\angle N$ and MN is adjacent to $\angle N$.
Use the tangent ratio in $\triangle JMN$.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{JM}{MN}$$

$$\tan 40^\circ = \frac{JM}{5.0} \quad \text{Solve for } JM.$$

$$5.0 \tan 40^\circ = JM$$

$$JM = 4.1954\dots$$

In right $\triangle KMN$, KM is opposite $\angle N$ and MN is adjacent to $\angle N$.
Use the tangent ratio in $\triangle KMN$.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{KM}{MN}$$

$$\tan 20^\circ = \frac{KM}{5.0} \quad \text{Solve for } KM.$$

$$5.0 \tan 20^\circ = KM$$

$$KM = 1.8198\dots$$

$$JK = JM + KM$$

$$= 4.1954\dots + 1.8198\dots$$

$$= 6.0153\dots$$

JK is approximately 6.0 cm long.

b) $JK = AJ + AK$

In right $\triangle ABJ$, AJ is opposite $\angle B$ and AB is adjacent to $\angle B$.

Use the tangent ratio in right $\triangle ABJ$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AJ}{AB}$$

$$\tan 15^\circ = \frac{AJ}{3.0}$$

Solve for AJ .

$$3.0 \tan 15^\circ = AJ$$

$$AJ = 0.8038\dots$$

In right $\triangle ABK$, AK is opposite $\angle B$ and AB is adjacent to $\angle B$.

Use the tangent ratio in $\triangle ABK$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AK}{AB}$$

$$\tan 60^\circ = \frac{AK}{3.0}$$

Solve for AK .

$$3.0 \tan 60^\circ = AK$$

$$AK = 5.1961\dots$$

$$JK = AJ + AK$$

$$= 0.8038\dots + 5.1961\dots$$

$$= 6$$

JK is 6.0 cm long.

- c) In right $\triangle CDK$, CK is opposite $\angle D$ and CD is adjacent to $\angle D$.
Use the tangent ratio in right $\triangle CDK$.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{CK}{CD}$$

$$\tan 35^\circ = \frac{CK}{3.0} \quad \text{Solve for CK.}$$

$$3.0 \tan 35^\circ = CK$$

$$CK = 2.1006\dots$$

In right $\triangle CDJ$, CJ is opposite $\angle D$ and CD is adjacent to $\angle D$.
 $\angle D = 35^\circ + 30^\circ$, or 65°

Use the tangent ratio in $\triangle CDJ$.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{CJ}{CD}$$

$$\tan 65^\circ = \frac{CJ}{3.0} \quad \text{Solve for CJ.}$$

$$3.0 \tan 65^\circ = CJ$$

$$CJ = 6.4335\dots$$

$$JK = CJ - CK$$

$$= 6.4335\dots - 2.1006\dots$$

$$= 4.3328\dots$$

JK is approximately 4.3 cm.

d) $JK = EK - EJ$

In right $\triangle EFJ$, EF is opposite $\angle J$ and EJ is adjacent to $\angle J$.

Use the tangent ratio in right $\triangle EFJ$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{EF}{EJ}$$

$$\tan 60^\circ = \frac{4.2}{EJ}$$

Solve for EJ .

$$EJ \tan 60^\circ = 4.2$$

$$EJ = \frac{4.2}{\tan 60^\circ}$$

$$EJ = 2.4248\dots$$

In right $\triangle EFK$, EF is opposite $\angle K$ and EK is adjacent to $\angle K$.

Use the tangent ratio in right $\triangle EFK$.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{EF}{EK}$$

$$\tan 35^\circ = \frac{4.2}{EK}$$

Solve for EK .

$$EK \tan 35^\circ = 4.2$$

$$EK = \frac{4.2}{\tan 35^\circ}$$

$$EK = 5.9982\dots$$

$$JK = EK - EJ$$

$$= 5.9982\dots - 2.4248\dots$$

$$= 3.5733\dots$$

JK is approximately 3.6 cm.

4. a) To determine the length of GH, first determine the length of GD.
In right $\triangle ADG$, AD is opposite $\angle G$ and GD is the hypotenuse.
Use the sine ratio in right $\triangle ADG$.

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin G = \frac{AD}{GD}$$

$$\sin 46^\circ = \frac{4.5}{GD} \quad \text{Solve for GD.}$$

$$GD \sin 46^\circ = 4.5$$

$$GD = \frac{4.5}{\sin 46^\circ}$$

$$GD = 6.2557\dots$$

In right $\triangle DGH$, GH is opposite $\angle D$ and GD is the hypotenuse.
Use the sine ratio in right $\triangle DGH$.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{GH}{GD}$$

$$\sin 66^\circ = \frac{GH}{6.2557\dots} \quad \text{Solve for GH.}$$

$$(6.2557\dots) \sin 66^\circ = GH$$

$$GH = 5.7148\dots$$

GH is approximately 5.7 cm long.

- b) To determine the length of GH, first determine the length of EH.

In right $\triangle EFH$, FH is adjacent to $\angle H$
and EH is the hypotenuse.

Use the cosine ratio in right $\triangle EFH$.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos H = \frac{FH}{EH}$$

$$\cos 59^\circ = \frac{3.4}{EH} \quad \text{Solve for EH.}$$

$$EH \cos 59^\circ = 3.4$$

$$EH = \frac{3.4}{\cos 59^\circ}$$

$$EH = 6.6014\dots$$

In right $\triangle EGH$, GH is adjacent to $\angle H$ and EH is the hypotenuse.

Use the cosine ratio in right $\triangle EGH$.

$$\cos H = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos H = \frac{GH}{EH}$$

$$\cos 42^\circ = \frac{GH}{6.6014\dots} \quad \text{Solve for GH.}$$

$$(6.6014\dots) \cos 42^\circ = GH$$

$$GH = 4.9058\dots$$

GH is approximately 4.9 cm long.

- c) To determine the length of GH, first determine the length of GD.
In right $\triangle CDG$, CG is adjacent to $\angle G$ and GD is the hypotenuse.
Use the cosine ratio in right $\triangle CDG$.

$$\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos G = \frac{CG}{GD}$$

$$\cos 52^\circ = \frac{3.9}{GD} \quad \text{Solve for GD.}$$

$$GD \cos 52^\circ = 3.9$$

$$GD = \frac{3.9}{\cos 52^\circ}$$

$$GD = 6.3346\dots$$

In right $\triangle DGH$, GH is opposite $\angle D$ and GD is the hypotenuse.
Use the sine ratio in right $\triangle DGH$.

$$\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin D = \frac{GH}{GD}$$

$$\sin 64^\circ = \frac{GH}{6.3346\dots} \quad \text{Solve for GH.}$$

$$(6.3346\dots)\sin 64^\circ = GH$$

$$GH = 5.6935\dots$$

GH is approximately 5.7 cm long.

B

5. a) $\angle XYZ = \angle WYX + \angle WYZ$

In right $\triangle WXY$, WX is opposite $\angle Y$ and WY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle WXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{WX}{WY}$$

$$\tan Y = \frac{3}{4}$$

$$\angle Y = 36.8698\dots^\circ$$

In right $\triangle WYZ$, WZ is opposite $\angle Y$ and WY is adjacent to $\angle Y$.
Use the tangent ratio in right $\triangle WYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{WZ}{WY}$$

$$\tan Y = \frac{6}{4}$$

$$\angle Y = 56.3099\dots^\circ$$

$$\begin{aligned}\angle XYZ &= \angle WYX + \angle WYZ \\ &= 36.8698\dots^\circ + 56.3099\dots^\circ \\ &= 93.1798\dots^\circ \\ \angle XYZ &\text{ is approximately } 93.2^\circ.\end{aligned}$$

b) $\angle XYZ = \angle VYX + \angle VYZ$

Determine $\angle VYZ$ first because two sides of $\triangle VYZ$ are known.

In right $\triangle VYZ$, VZ is opposite $\angle Y$ and YZ is the hypotenuse.

Use the sine ratio in right $\triangle VYZ$.

$$\sin Y = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Y = \frac{VZ}{YZ}$$

$$\sin Y = \frac{8}{10}$$

$$\angle Y = 53.1301\dots^\circ$$

Use the Pythagorean Theorem in right $\triangle VYZ$.

$$YZ^2 = VY^2 + VZ^2 \quad \text{Isolate the unknown.}$$

$$VY^2 = YZ^2 - VZ^2$$

$$VY^2 = 10^2 - 8^2$$

$$= 36$$

$$VY = \sqrt{36}$$

$$= 6$$

In right $\triangle VXY$, VX is opposite $\angle Y$ and VY is adjacent to $\angle Y$.

Use the tangent ratio in right $\triangle VXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{VX}{VY}$$

$$\tan Y = \frac{17}{6}$$

$$\angle Y = 70.5599\dots^\circ$$

$$\angle XYZ = \angle VYZ + \angle VYX$$

$$= 53.1301\dots^\circ + 70.5599\dots^\circ$$

$$= 123.6900\dots^\circ$$

$\angle XYZ$ is approximately 123.7° .

c) $\angle XYZ = \angle XYU - \angle ZYU$

In right $\triangle UXY$, UX is opposite $\angle Y$ and UY is adjacent to $\angle Y$.

Use the tangent ratio in right $\triangle UXY$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{UX}{UY}$$

$$\tan Y = \frac{3+5}{12}$$

$$\angle Y = 33.6900\dots^\circ$$

In right $\triangle UYZ$, UZ is opposite $\angle Y$ and UY is adjacent to $\angle Y$.

Use the tangent ratio in right $\triangle UYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{UZ}{UY}$$

$$\tan Y = \frac{5}{12}$$

$$\angle Y = 22.6198\dots^\circ$$

$$\angle XYZ = \angle XYU - \angle ZYU$$

$$= 33.6900\dots^\circ - 22.6198\dots^\circ$$

$$= 11.0702\dots^\circ$$

$\angle XYZ$ is approximately 11.1° .

d) $\angle XYZ = \angle TYZ - \angle TYX$

Determine $\angle TYX$ first because two sides of $\triangle TXY$ are known.

In right $\triangle TXY$, TY is adjacent to $\angle Y$ and XY is the hypotenuse.

Use the cosine ratio in right $\triangle TXY$.

$$\cos Y = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Y = \frac{TY}{XY}$$

$$\cos Y = \frac{15}{17}$$

$$\angle Y = 28.0724\dots^\circ$$

Use the Pythagorean Theorem in right $\triangle TXY$.

$$XY^2 = TX^2 + TY^2 \quad \text{Isolate the unknown.}$$

$$TX^2 = XY^2 - TY^2$$

$$TX^2 = 17^2 - 15^2$$

$$= 64$$

$$TX = \sqrt{64}$$

$$= 8$$

So, the length of TZ is:

$$TZ = TX + XZ$$

$$= 8 + 6$$

$$= 14$$

In right $\triangle TYZ$, TZ is opposite $\angle Y$ and TY is adjacent to $\angle Y$.

Use the tangent ratio in right $\triangle TYZ$.

$$\tan Y = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Y = \frac{TZ}{TY}$$

$$\tan Y = \frac{14}{15}$$

$$\angle Y = 43.0250\dots^\circ$$

$$\angle XYZ = \angle TYZ - \angle TYX$$

$$= 43.0250\dots^\circ - 28.0724\dots^\circ$$

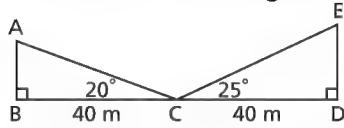
$$= 14.9525\dots^\circ$$

$\angle XYZ$ is approximately 15.0° .

6. The point from which the angles are measured is halfway between the trees, so its distance from each tree is:

$$80 \text{ m} \div 2 = 40 \text{ m}$$

Sketch and label a diagram to represent the information in the problem.



In right $\triangle ABC$, AB is opposite $\angle C$ and BC is adjacent to $\angle C$.

Use the tangent ratio in right $\triangle ABC$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

$$\tan 20^\circ = \frac{AB}{40} \quad \text{Solve for AB.}$$

$$40 \tan 20^\circ = AB$$

$$AB = 14.5588\dots$$

In right $\triangle CDE$, DE is opposite $\angle C$ and CD is adjacent to $\angle C$.

Use the tangent ratio in right $\triangle CDE$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

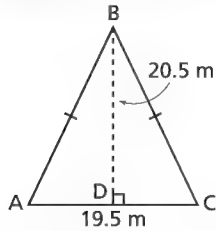
$$\tan 25^\circ = \frac{DE}{40} \quad \text{Solve for DE.}$$

$$40 \tan 25^\circ = DE$$

$$DE = 18.6523\dots$$

The heights of the trees are approximately 15 m and 19 m.

7. Sketch and label a diagram to represent one face of the pyramid.



$\triangle ABC$ is an isosceles triangle with $AB = BC$ and AC is 19.5 m, so CD is:
 $19.5 \text{ m} \div 2 = 9.75 \text{ m}$

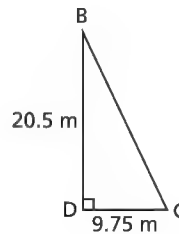
In right $\triangle BCD$, BD is opposite $\angle C$ and CD is adjacent to $\angle C$.
 Use the tangent ratio in right $\triangle BCD$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BD}{CD}$$

$$\tan C = \frac{20.5}{9.75}$$

$$\angle C = 64.5637\dots^\circ$$



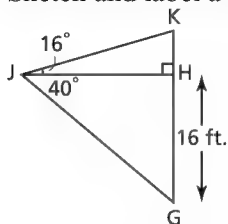
$\triangle ABC$ is an isosceles triangle with $AB = BC$, so $\angle A = \angle C$.

The sum of the angles in a triangle is 180° , so in right $\triangle ABC$:
 $\angle B = 180^\circ - 2(64.5637\dots^\circ)$
 $= 50.8724\dots^\circ$

Since the sum of the angles is 180° , round $\angle B$ down to 50° .

So, the measures of the angles in a triangular face of the pyramid are approximately 65° , 65° , and 50° .

8. Sketch and label a diagram to represent the information in the problem.



HJ represents the horizontal distance between the student and the tree.

GK represents the height of the tree.

- a) In right $\triangle GHJ$, GH is opposite $\angle J$ and HJ is adjacent to $\angle J$.

Use the tangent ratio in right $\triangle GHJ$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{GH}{HJ}$$

$$\tan 40^\circ = \frac{16}{HJ} \quad \text{Solve for HJ.}$$

$$HJ \tan 40^\circ = 16$$

$$HJ = \frac{16}{\tan 40^\circ}$$

$$HJ = 19.0680\dots$$

The horizontal distance between the student and the tree is approximately 19 ft.

- b) In right $\triangle HJK$, KH is opposite $\angle J$ and HJ is adjacent to $\angle J$.

Use the tangent ratio in right $\triangle HJK$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{HK}{HJ}$$

$$\tan 16^\circ = \frac{HK}{19.0680\dots} \quad \text{Solve for HK.}$$

$$(19.0680\dots) \tan 16^\circ = HK$$

$$HK = 5.4676\dots$$

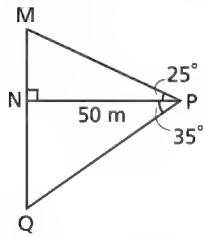
$$GK = HK + GH$$

$$= 5.4676\dots + 16$$

$$= 21.4676\dots$$

The height of the tree is approximately 21 ft.

9. Sketch and label a diagram to represent the information in the problem.



Assume the ground is horizontal.

NQ represents the height of the shorter tower.

MQ represents the height of the taller tower.

In right $\triangle NPQ$, NQ is opposite $\angle P$ and NP is adjacent to $\angle P$.

Use the tangent ratio in right $\triangle NPQ$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{NQ}{NP}$$

$$\tan 35^\circ = \frac{NQ}{50} \quad \text{Solve for NQ.}$$

$$50 \tan 35^\circ = NQ$$

$$NQ = 35.0103\dots$$

In right $\triangle MNP$, MN is opposite $\angle P$ and NP is adjacent to $\angle P$.

Use the tangent ratio in right $\triangle MNP$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{MN}{NP}$$

$$\tan 25^\circ = \frac{MN}{50} \quad \text{Solve for MN.}$$

$$50 \tan 25^\circ = MN$$

$$MN = 23.3153\dots$$

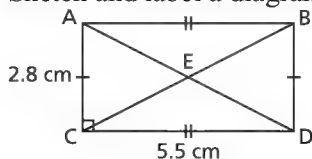
$$MQ = MN + NQ$$

$$= 23.3153\dots + 35.0103\dots$$

$$= 58.3257\dots$$

The shorter tower is approximately 35 m high and the taller tower is approximately 58 m high.

10. Sketch and label a diagram to represent the rectangle.



The angles at E cannot be determined directly because the 4 triangles that contain angles at E are not right triangles.

In right $\triangle ACD$, CD is opposite $\angle A$ and AC is adjacent to $\angle A$.

Use the tangent ratio in right $\triangle ACD$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{CD}{AC}$$

$$\tan A = \frac{5.5}{2.8}$$

$$\angle A = 63.0197\dots^\circ$$

$\triangle ACE$ is an isosceles triangle with $AE = CE$, so $\angle C = \angle A$.

The sum of the angles in a triangle is 180° , so in $\triangle ACE$:

$$\angle E = 180^\circ - 2(63.0197\dots^\circ)$$

$$= 53.9604\dots^\circ$$

$\angle AEC$ and $\angle CED$ form a straight angle, so:

$$\angle CED = 180^\circ - \angle AEC$$

$$= 180^\circ - 53.9604\dots^\circ$$

$$= 126.0395\dots^\circ$$

Opposite angles are equal, so:

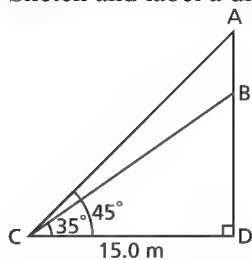
$$\angle BED = \angle AEC \doteq 54^\circ$$

$$\text{And } \angle AEB = \angle CED \doteq 126^\circ$$

So, the angles at the point where the diagonals intersect are approximately 54° , 54° , 126° , and 126° .

I could also have started by determining $\angle D$ in $\triangle ACD$, and then used a similar strategy in $\triangle CDE$ to determine the angles. I could also have divided each isosceles triangle in half to form 2 congruent right triangles and determined the angles in these triangles.

11. Sketch and label a diagram to represent the information in the problem.



AB represents the distance between the 2 carvings.

$$AB = AD - BD$$

In right $\triangle ACD$, $\angle C = 45^\circ$

$$\angle A = 90^\circ - \angle C$$

$$\angle A = 90^\circ - 45^\circ$$

$$= 45^\circ$$

So, $\triangle ACD$ is an isosceles right triangle, and $CD = AD$.

So, AD is 15.0 m.

In right $\triangle BCD$, BD is opposite $\angle C$ and CD is adjacent to $\angle C$.

Use the tangent ratio in right $\triangle BCD$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BD}{CD}$$

$$\tan 35^\circ = \frac{BD}{15.0}$$

Solve for BD.

$$15.0 \tan 35^\circ = BD$$

$$BD = 10.5031\dots$$

$$AB = AD - BD$$

$$AB = 15.0 - 10.5031\dots$$

$$= 4.4968\dots$$

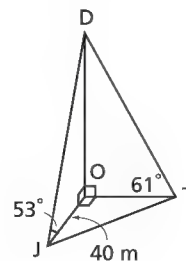
The distance between the carvings is approximately 4.5 m.

12. Sketch and label a diagram to represent the information in the problem.

DO represents the height of the top of the dome.

OT represents the distance between Troy and a point directly beneath the dome.

JT represents the distance between Janelle and Troy.



- a) In right $\triangle DJO$, JO is adjacent to $\angle J$ and DO is opposite $\angle J$.
Use the tangent ratio in right $\triangle DJO$.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan J = \frac{DO}{JO}$$

$$\tan 53^\circ = \frac{DO}{40} \quad \text{Solve for DO.}$$

$$40 \tan 53^\circ = DO$$

$$DO = 53.0817\dots$$

The dome is approximately 53 m high.

- b) In right $\triangle DOT$, OT is adjacent to $\angle T$ and DO is opposite $\angle T$.
Use the tangent ratio in right $\triangle DOT$.

$$\tan T = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan T = \frac{DO}{OT}$$

$$\tan 61^\circ = \frac{53.0817\dots}{OT} \quad \text{Solve for OT.}$$

$$OT \tan 61^\circ = 53.0817\dots$$

$$OT = \frac{53.0817\dots}{\tan 61^\circ}$$

$$OT = 29.4237\dots$$

Troy is approximately 29 m from a point directly beneath the dome.

- c) Use the Pythagorean Theorem in right $\triangle JOT$.

$$JT^2 = JO^2 + OT^2$$

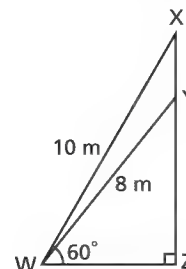
$$JT^2 = 40^2 + (29.4237\dots)^2$$

$$JT = \sqrt{40^2 + (29.4237\dots)^2}$$

$$= 49.6563\dots$$

Janelle and Troy are approximately 50 m apart.

13. Sketch and label a diagram to represent the information in the problem.
 WZ represents the distance between the base of the tower and the point where the wires are attached to the ground.
 $\angle YWZ$ represents the angle of inclination of the shorter guy wire.
 XY represents the distance between the points where the guy wires are attached to the tower.



- a) In right $\triangle WXZ$, WZ is adjacent to $\angle W$ and WX is the hypotenuse.
 Use the cosine ratio in right $\triangle WXZ$.

$$\cos W = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos W = \frac{WZ}{WX}$$

$$\cos 60^\circ = \frac{WZ}{10} \quad \text{Solve for WZ.}$$

$$10 \cos 60^\circ = WZ$$

$$WZ = 5$$

The wires are attached on the ground 5.0 m from the base of the tower.

- b) In right $\triangle WYZ$, WZ is adjacent to $\angle W$ and WY is the hypotenuse.
 Use the cosine ratio in right $\triangle WYZ$.

$$\cos W = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos W = \frac{WZ}{WY}$$

$$\cos W = \frac{5}{8}$$

$$\angle W = 51.3178\dots$$

The angle of inclination of the shorter guy wire is approximately 51.3° .

- c) Use the Pythagorean Theorem.

In right $\triangle WXZ$:

$$WX^2 = WZ^2 + XZ^2$$

$$XZ^2 = WX^2 - WZ^2$$

$$XZ^2 = 10^2 - 5^2$$

$$XZ = \sqrt{75}$$

$$XY = XZ - YZ$$

$$XY = \sqrt{75} - \sqrt{39}$$

$$= 2.4152\dots$$

The points where the guy wires are attached to the tower are about 2.4 m apart.

In right $\triangle WYZ$:

$$WY^2 = WZ^2 + YZ^2$$

$$YZ^2 = WY^2 - WZ^2$$

$$YZ^2 = 8^2 - 5^2$$

$$YZ = \sqrt{39}$$

14. a) In right $\triangle EFG$, EF is adjacent to $\angle E$
and GF is opposite $\angle E$.

Use the tangent ratio in right $\triangle EFG$.

$$\tan E = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan E = \frac{GF}{EF}$$

$$\tan 22^\circ = \frac{GF}{56}$$

Solve for GF .

$$56 \tan 22^\circ = GF$$

$$GF = 22.6254\dots$$

The width of the gorge is approximately 23 m.

- b) In right $\triangle FGH$, GF is adjacent to $\angle F$
and GH is opposite $\angle F$.

Use the tangent ratio in right $\triangle FGH$.

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan F = \frac{GH}{GF}$$

$$\tan 41^\circ = \frac{GH}{22.6254\dots}$$

Solve for GH .

$$(22.6254\dots) \tan 41^\circ = GH$$

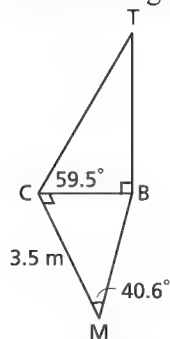
$$GH = 19.6680\dots$$

The depth of the gorge is approximately 20 m.

15. The surveyor can identify a spot directly below the cliff along the same horizontal as her eye, then use her clinometer to find a point along her side of the river that is along a path perpendicular to her line of sight to the spot. She can then measure the distance to the point, and use the clinometer to measure the angle between her path to the point and her new line of sight to the spot below the cliff. She can use the tangent ratio to determine the distance across the river.

The surveyor can then use the clinometer to measure the angle of elevation of the top of the cliff. Since she now knows the distance across the river, she can use the tangent ratio to determine the height of the cliff.

16. a) Sketch a diagram.



BT represents the height of the tower.

- b) The height cannot be calculated directly because no lengths are known in $\triangle BCT$.
So, determine the length of BC first.
In right $\triangle BCM$, CM is adjacent to $\angle M$ and BC is opposite $\angle M$.
Use the tangent ratio in right $\triangle BCM$.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{BC}{CM}$$

$$\tan 40.6^\circ = \frac{BC}{3.5} \quad \text{Solve for BC.}$$

$$3.5 \tan 40.6^\circ = BC$$

$$BC = 2.9998\dots$$

In right $\triangle BCT$, BC is adjacent to $\angle C$ and BT is opposite $\angle C$.
Use the tangent ratio in right $\triangle BCT$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{BT}{BC}$$

$$\tan 59.5^\circ = \frac{BT}{2.9998\dots} \quad \text{Solve for BT.}$$

$$(2.9998\dots) \tan 59.5^\circ = BT$$

$$BT = 5.0927\dots$$

The height of the tower is approximately 5.1 m.

C

17. Sketch and label a diagram. The longer diagonal is the perpendicular bisector of the shorter diagonal, so $BE = ED = 3.4$ cm

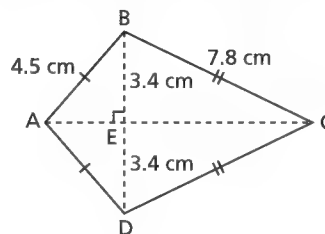
- a) In right $\triangle ABE$, BE is adjacent to $\angle B$ and AB is the hypotenuse.
Use the cosine ratio in right $\triangle ABE$.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{BE}{AB}$$

$$\cos B = \frac{3.4}{4.5}$$

$$\angle B = 40.9260\dots^\circ$$



In isosceles $\triangle ABD$, $\angle B = \angle D$

The sum of the angles in a triangle is 180° , so:

$$\angle A = 180^\circ - 2\angle B$$

$$\angle A = 180^\circ - 2(40.9260\dots^\circ)$$

$$= 98.1478\dots^\circ$$

In right $\triangle BCE$, BE is adjacent to $\angle B$ and BC is the hypotenuse.

Use the cosine ratio in right $\triangle BCE$.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{BE}{BC}$$

$$\cos B = \frac{3.4}{7.8}$$

$$\angle B = 64.1575\dots^\circ$$

In isosceles $\triangle BCD$, $\angle B = \angle D$

The sum of the angles in a triangle is 180° , so:

$$\angle C = 180^\circ - 2\angle B$$

$$\angle C = 180^\circ - 2(64.1575\dots^\circ)$$

$$= 51.6848\dots^\circ$$

$$\angle ABC = \angle ABE + \angle EBC$$

$$\angle ABC = 40.9260\dots^\circ + 64.1575\dots^\circ$$

$$= 105.0836\dots^\circ$$

In kite $ABCD$, $\angle B = \angle D$. So, $\angle D = 105.0836\dots^\circ$

So, in kite $ABCD$, $\angle A$ is approximately 98.1° , $\angle B$ is approximately 105.1° , $\angle C$ is approximately 51.7° , and $\angle D$ is approximately 105.1° .

- b) In right $\triangle ABE$, use the Pythagorean Theorem.

$$AB^2 = AE^2 + BE^2$$

$$AE^2 = AB^2 - BE^2$$

$$AE^2 = 4.5^2 - 3.4^2$$

$$AE = \sqrt{4.5^2 - 3.4^2}$$

$$= 2.9478\dots$$

In right $\triangle BCE$, use the Pythagorean Theorem.

$$BC^2 = BE^2 + CE^2$$

$$CE^2 = BC^2 - BE^2$$

$$CE^2 = 7.8^2 - 3.4^2$$

$$CE = \sqrt{7.8^2 - 3.4^2}$$

$$= 7.0199\dots$$

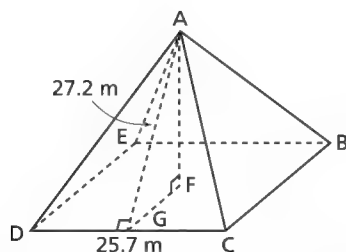
$$AC = AE + CE$$

$$AC = 2.9478\dots + 7.0199\dots$$

$$= 9.9678\dots$$

The longer diagonal is approximately 10.0 cm long.

18. a) Sketch and label the pyramid. Its height is AF.



- b) Since the 4 triangular faces are congruent, all of them have the same base length;
so, $BC = CD = 25.7$ m

$$FG = \frac{1}{2} BC$$

$$FG = \frac{1}{2} (25.7)$$

$$= 12.85$$

In right $\triangle AFG$, use the Pythagorean Theorem.

$$AG^2 = AF^2 + FG^2$$

$$AF^2 = AG^2 - FG^2$$

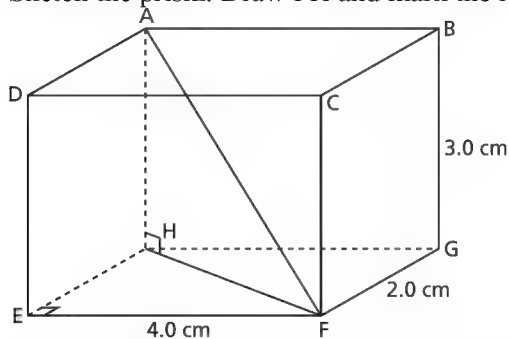
$$AF^2 = 27.2^2 - 12.85^2$$

$$AF = \sqrt{27.2^2 - 12.85^2}$$

$$= 23.9732\dots$$

The height of the pyramid is approximately 24.0 m.

19. a) Sketch the prism. Draw FH and mark the right angles.



To determine the length of AF, first determine the length of FH, then use the Pythagorean Theorem in $\triangle AFH$.

In right $\triangle EFH$, use the Pythagorean Theorem.

$$FH^2 = EF^2 + EH^2$$

$$FH^2 = 4.0^2 + 2.0^2$$

$$FH = \sqrt{4.0^2 + 2.0^2}$$

$$= 4.4721\dots$$

In right $\triangle AFH$, use the Pythagorean Theorem.

$$AF^2 = AH^2 + FH^2$$

$$AF^2 = 3.0^2 + (4.4721\dots)^2$$

$$AF = \sqrt{3.0^2 + (4.4721\dots)^2}$$

$$= 5.3851\dots$$

The length of the body diagonal is approximately 5.4 cm.

- b) In right $\triangle AFH$, FH is adjacent to $\angle F$ and AF is the hypotenuse. Use the cosine ratio in right $\triangle AFH$.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$

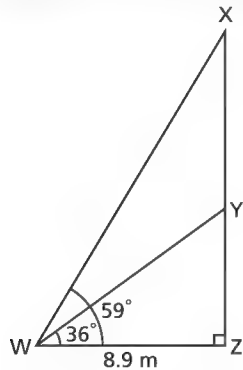
$$\cos F = \frac{FH}{AF}$$

$$\cos F = \frac{4.4721\dots}{5.3851\dots}$$

$$\angle F = 33.8545\dots^\circ$$

The measure of $\angle AFH$ is approximately 33.9° .

20. Sketch and label a diagram.



XY is the distance between the guy wire anchor and the top of the tower.

In right $\triangle WYZ$, WZ is adjacent to $\angle W$ and YZ is opposite $\angle W$.
Use the tangent ratio in right $\triangle WYZ$.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{YZ}{WZ}$$

$$\tan 36^\circ = \frac{YZ}{8.9} \quad \text{Solve for YZ.}$$

$$8.9 \tan 36^\circ = YZ$$

$$YZ = 6.4662\dots$$

In right $\triangle WXZ$, WZ is adjacent to $\angle W$ and XZ is opposite $\angle W$.
Use the tangent ratio in right $\triangle WXZ$.

$$\tan W = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan W = \frac{XZ}{WZ}$$

$$\tan 59^\circ = \frac{XZ}{8.9} \quad \text{Solve for XZ.}$$

$$8.9 \tan 59^\circ = XZ$$

$$XZ = 14.8120\dots$$

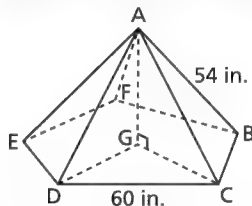
$$XY = XZ - YZ$$

$$XY = 14.8120\dots - 6.4662\dots$$

$$= 8.3458\dots$$

The guy wire is attached to the tower approximately 8.3 m below the top of the tower.

21. Sketch and label a diagram of the pyramid.



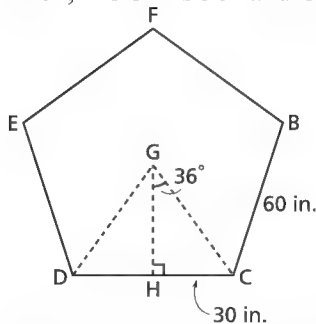
AG is the height of the pyramid.

Sketch the base of the pyramid.

In the base of the pyramid, $\angle CGD$ is $360^\circ \div 5 = 72^\circ$.

$\triangle CDG$ is isosceles, so draw the perpendicular bisector GH to divide it into two congruent right triangles: $\triangle CGH$ and $\triangle DGH$

Then, $\angle CGH$ is 36° and CH is 30 in.



To determine the height of the pyramid, use right $\triangle AGC$. First determine the length of GC using right $\triangle CGH$.

In right $\triangle CGH$, CH is opposite $\angle G$ and GC is the hypotenuse.

Use the sine ratio in right $\triangle CGH$.

$$\sin G = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin G = \frac{CH}{CG}$$

$$\sin 36^\circ = \frac{30}{CG} \quad \text{Solve for CG.}$$

$$CG \sin 36^\circ = 30$$

$$CG = \frac{30}{\sin 36^\circ}$$

$$CG = 51.0390\dots$$

In right $\triangle ACG$, use the Pythagorean Theorem.

$$AC^2 = AG^2 + GC^2$$

$$AG^2 = AC^2 - GC^2$$

$$AG^2 = 54^2 - (51.0390\dots)^2$$

$$AG = \sqrt{54^2 - (51.0390\dots)^2}$$

$$= 17.6356\dots$$

The height of the pyramid is approximately 18 in.

Review

(pages 124–126)

2.1

1. a) In right $\triangle CDE$, CE is opposite $\angle D$ and CD is adjacent to $\angle D$.
Use the tangent ratio in right $\triangle CDE$.

$$\tan D = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan D = \frac{CE}{CD}$$

$$\tan D = \frac{7}{10}$$

$$\angle D = 34.9920\dots^\circ$$

$\angle D$ is approximately 35° .

- b) In right $\triangle FGH$, FG is opposite $\angle H$ and GH is adjacent to $\angle H$.
Use the tangent ratio in right $\triangle FGH$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{FG}{GH}$$

$$\tan H = \frac{3.2}{1.5}$$

$$\angle H = 64.8851\dots^\circ$$

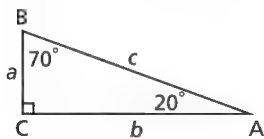
$\angle H$ is approximately 65° .

2. a) $\tan 20^\circ = 0.3639\dots$, so $\tan 20^\circ < 1$.

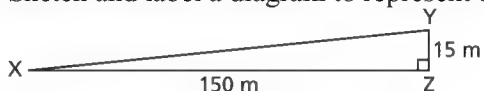
- b) $\tan 70^\circ = 2.7474\dots$, so $\tan 70^\circ > 1$.

- c) I know that as the measure of an acute angle in a right triangle increases, the length of the side opposite the angle also increases. In the triangle below, side b is opposite $\angle B = 70^\circ$, so it is longer than side a , which is opposite $\angle A = 20^\circ$. Since b is greater than a , the ratio

$\tan 20^\circ = \frac{a}{b}$ is less than 1 and the ratio $\tan 70^\circ = \frac{b}{a}$ is greater than 1.



3. Sketch and label a diagram to represent the information in the problem.



$\angle X$ is the angle of inclination of the road.

In right $\triangle XYZ$, YZ is opposite $\angle X$ and XZ is adjacent to $\angle X$.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

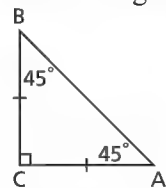
$$\tan X = \frac{YZ}{XZ}$$

$$\tan X = \frac{15}{150}$$

$$\angle X = 5.7105\dots^\circ$$

The angle of inclination of the road is approximately 6° .

4. Sketch a right triangle with $\angle A = 45^\circ$.



In right $\triangle ABC$, $\angle A = \angle B$, so the triangle is isosceles and $BC = AC$.

So:

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 45^\circ = \frac{AC}{AC}$$

$$\tan 45^\circ = 1$$

The triangle is an isosceles right triangle.

2.2

5. a) i) In right $\triangle JKM$, JM is opposite $\angle K$ and KM is adjacent to $\angle K$.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{JM}{KM}$$

$$\tan 63^\circ = \frac{k}{1.9}$$

$$1.9 \tan 63^\circ = k$$

$$k = 3.7289\dots$$

JM is approximately 3.7 cm long.

- ii) In right $\triangle NPQ$, PQ is opposite $\angle N$ and NQ is adjacent to $\angle N$.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan N = \frac{PQ}{NQ}$$

$$\tan 42^\circ = \frac{2.7}{p}$$

$$p \tan 42^\circ = 2.7$$

$$p = \frac{2.7}{\tan 42^\circ}$$

$$p = 2.9986\dots$$

NQ is approximately 3.0 cm long.

- b) i) Use the Pythagorean Theorem in right $\triangle JKM$.

$$JK^2 = KM^2 + JM^2$$

$$JK^2 = 1.9^2 + (3.7289\dots)^2$$

$$JK = \sqrt{1.9^2 + (3.7289\dots)^2}$$

$$= 4.1851\dots$$

JK is approximately 4.2 cm long.

- ii) Use the Pythagorean Theorem in right $\triangle NPQ$.

$$NP^2 = NQ^2 + PQ^2$$

$$NP^2 = (2.9986\dots)^2 + 2.7^2$$

$$NP = \sqrt{(2.9986\dots)^2 + 2.7^2}$$

$$= 4.0350\dots$$

NP is approximately 4.0 cm long.

I could also have used the sine or cosine ratios to determine the length of each hypotenuse.

Pearson
Foundations and Pre-calculus Mathematics 10

Chapter 2
Trigonometry

6. Sketch and label a diagram to represent the information in the problem.
 YZ represents the height of the Eiffel tower.

In right $\triangle XYZ$, YZ is opposite $\angle X$ and XZ is adjacent to $\angle X$.

$$\tan X = \frac{\text{opposite}}{\text{adjacent}}$$

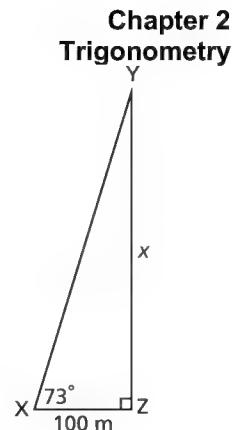
$$\tan X = \frac{YZ}{XZ}$$

$$\tan 73^\circ = \frac{x}{100}$$

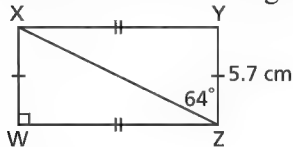
$$100 \tan 73^\circ = x$$

$$x = 327.0852 \dots$$

The tower is approximately 327 m tall.



7. Sketch and label a diagram to represent the information in the problem.



- a) In right $\triangle XYZ$, XY is opposite $\angle Z$ and YZ is adjacent to $\angle Z$.

$$\tan Z = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan Z = \frac{XY}{YZ}$$

$$\tan 64^\circ = \frac{XY}{5.7}$$

$$5.7 \tan 64^\circ = XY$$

$$XY = 11.6867 \dots$$

The rectangle is approximately 11.7 cm long.

- b) In right $\triangle XYZ$, use the Pythagorean Theorem.

$$XZ^2 = XY^2 + YZ^2$$

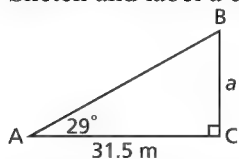
$$XZ^2 = (11.6867 \dots)^2 + 5.7^2$$

$$XZ = \sqrt{(11.6867 \dots)^2 + 5.7^2}$$

$$= 13.0026 \dots$$

The diagonal is approximately 13.0 cm long.

8. Sketch and label a diagram to represent the information in the problem.



BC represents the height of the tree.

In right $\triangle ABC$, BC is opposite $\angle A$ and AC is adjacent to $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 29^\circ = \frac{a}{31.5}$$

$$31.5 \tan 29^\circ = a$$

$$a = 17.4607\dots$$

The tree is approximately 17.5 m tall.

9. Sketch and label a diagram to represent the information in the problem.
PR represents Aidan's distance from the base of the Lookout.

In right $\triangle PQR$, QR is opposite $\angle P$
and PR is adjacent to $\angle P$.

$$\tan P = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan P = \frac{QR}{PR}$$

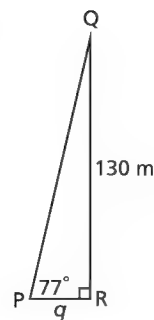
$$\tan 77^\circ = \frac{130}{q}$$

$$q \tan 77^\circ = 130$$

$$q = \frac{130}{\tan 77^\circ}$$

$$q = 30.0128\dots$$

Aidan is approximately 30 m from the base of the Lookout.



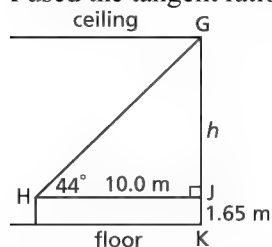
2.3

10. I measured 10 m from the wall. Then, I measured the angle of elevation of the point where the ceiling and the wall meet. The angle shown on the clinometer was 46° , so the angle of elevation was:

$$90^\circ - 46^\circ = 44^\circ$$

My eye was approximately 1.65 m above the ground.

I used the tangent ratio to determine the height of the ceiling, h .



In right $\triangle GHJ$, GJ is opposite $\angle H$ and HJ is adjacent to $\angle H$.

$$\tan H = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan H = \frac{GJ}{HJ}$$

$$\tan 44^\circ = \frac{h}{10}$$

$$10 \tan 44^\circ = h$$

$$h = 9.6568\dots$$

The height of the gym is:

$$1.65 \text{ m} + 9.6568\dots \text{ m} = 11.3068\dots \text{ m}$$

The height of the gymnasium is approximately 11.3 m.

2.4

11. a) In right $\triangle RST$, RS is adjacent to $\angle S$ and ST is the hypotenuse. So, use the cosine ratio in right $\triangle RST$.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{RS}{ST}$$

$$\cos S = \frac{3}{10}$$

$$\angle S = 72.5423\dots^\circ$$

$\angle S$ is approximately 73° .

- b) In right $\triangle UVW$, VW is opposite $\angle U$ and UV is the hypotenuse. So, use the sine ratio in right $\triangle UVW$.

$$\sin U = \frac{\text{opposite}}{\text{hypotenuse}}$$

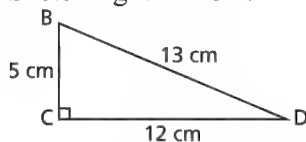
$$\sin U = \frac{VW}{UV}$$

$$\sin U = \frac{3.7}{8.0}$$

$$\angle U = 27.5485\dots^\circ$$

$\angle U$ is approximately 28° .

12. Sketch right $\triangle BCD$.



a) i) $\sin D = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin D = \frac{BC}{BD}$$

$$\sin D = \frac{5}{13}$$

ii) $\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin B = \frac{CD}{BD}$$

$$\sin B = \frac{12}{13}$$

iii) $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$

$$\cos B = \frac{BC}{BD}$$

$$\cos B = \frac{5}{13}$$

iv) $\cos D = \frac{\text{adjacent}}{\text{hypotenuse}}$

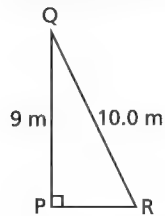
$$\cos D = \frac{CD}{BD}$$

$$\cos D = \frac{12}{13}$$

- b) $\sin D = \cos B$ and $\sin B = \cos D$

This occurs because the side opposite $\angle B$ is adjacent to $\angle D$, and the side adjacent to $\angle B$ is opposite $\angle D$.

13. Sketch and label a diagram to represent the information in the problem.



$\angle R$ represents the angle of inclination of the pole.

In right $\triangle PQR$, PQ is opposite $\angle R$ and QR is the hypotenuse.
So, use the sine ratio in right $\triangle PQR$.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin R = \frac{PQ}{QR}$$

$$\sin R = \frac{9}{10.0}$$

$$\angle R = 64.1580\dots^\circ$$

The angle of inclination of the pole is approximately 64.2° .

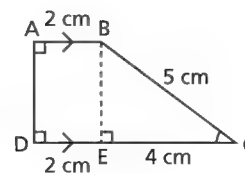
14. Sketch and label a diagram to represent the trapezoid.

$ABED$ is a rectangle, so $DE = AB = 2$ cm

$$CE = CD - DE$$

$$CE = 6 \text{ cm} - 2 \text{ cm}$$

$$= 4 \text{ cm}$$



In right $\triangle BCE$, CE is adjacent to $\angle C$ and BC is the hypotenuse.

Use the cosine ratio in right $\triangle BCE$.

$$\cos C = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos C = \frac{CE}{BC}$$

$$\cos C = \frac{4}{5}$$

$$\angle C = 36.8698\dots^\circ$$

$\angle C$ is approximately 36.9° .

2.5

15. a) In right $\triangle EFG$, FG is adjacent to $\angle G$ and EG is hypotenuse.
So, use the cosine ratio in right $\triangle EFG$.

$$\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos G = \frac{FG}{EG}$$

$$\cos 33^\circ = \frac{e}{4.7}$$

$$4.7 \cos 33^\circ = e$$

$$e = 3.9417 \dots$$

FG is approximately 3.9 cm long.

- b) In right $\triangle HJK$, HJ is opposite $\angle K$ and JK is the hypotenuse.
So, use the sine ratio in right $\triangle HJK$.

$$\sin K = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin K = \frac{HJ}{JK}$$

$$\sin 48^\circ = \frac{k}{5.9}$$

$$5.9 \sin 48^\circ = k$$

$$k = 4.3845 \dots$$

HJ is approximately 4.4 cm long.

- c) In right $\triangle MNP$, MP is opposite $\angle N$ and NP is the hypotenuse.
So, use the sine ratio in right $\triangle MNP$.

$$\sin N = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin N = \frac{MP}{NP}$$

$$\sin 52^\circ = \frac{3.7}{m}$$

$$m \sin 52^\circ = 3.7$$

$$m = \frac{3.7}{\sin 52^\circ}$$

$$m = 4.6953 \dots$$

NP is approximately 4.7 cm long.

- d) In right $\triangle QRS$, RS is adjacent to $\angle S$ and QS is the hypotenuse.
So, use the cosine ratio in right $\triangle QRS$.

$$\cos S = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos S = \frac{RS}{QS}$$

$$\cos 65^\circ = \frac{1.9}{r}$$

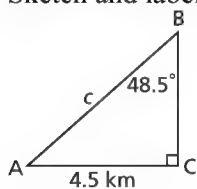
$$r \cos 65^\circ = 1.9$$

$$r = \frac{1.9}{\cos 65^\circ}$$

$$r = 4.4957\dots$$

QS is approximately 4.5 cm long.

16. Sketch and label a diagram to represent the information in the problem.



AB represents the distance between the ship and Arviat.

In right $\triangle ABC$, AC is opposite $\angle B$ and AB is the hypotenuse.
So, use the sine ratio in right $\triangle ABC$.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{AB}$$

$$\sin 48.5^\circ = \frac{4.5}{c}$$

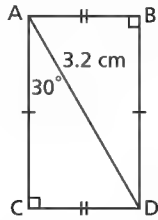
$$c \sin 48.5^\circ = 4.5$$

$$c = \frac{4.5}{\sin 48.5^\circ}$$

$$c = 6.0083\dots$$

The ship is approximately 6.0 km from Arviat.

17. Sketch and label a diagram to represent the information in the problem.



In right $\triangle ACD$, CD is opposite $\angle A$ and AD is the hypotenuse.
So, use the sine ratio in right $\triangle ACD$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{CD}{AD}$$

$$\sin 30^\circ = \frac{CD}{3.2}$$

$$3.2 \sin 30^\circ = CD$$

$$CD = 1.6$$

In right $\triangle ACD$, AC is adjacent to $\angle A$ and AD is the hypotenuse.
So, use the cosine ratio in right $\triangle ACD$.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AD}$$

$$\cos 30^\circ = \frac{AC}{3.2}$$

$$3.2 \cos 30^\circ = AC$$

$$AC = 2.7712\dots$$

The dimensions of the rectangle are 1.6 cm by approximately 2.8 cm.

2.6

18. a) In right $\triangle CDE$, DE is opposite $\angle C$ and CD is adjacent to $\angle C$.
Use the tangent ratio in right $\triangle CDE$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

$$\tan C = \frac{2.7}{4.2}$$

$$\angle C = 32.7352\dots^\circ$$

The sum of the acute angles in a right triangle is 90° , so:

$$\angle E = 90^\circ - \angle C$$

$$\angle E = 90^\circ - 32.7352\dots^\circ$$

$$= 57.2647\dots^\circ$$

Use the Pythagorean Theorem in right $\triangle CDE$.

$$CE^2 = CD^2 + DE^2$$

$$CE^2 = 4.2^2 + 2.7^2$$

$$CE = \sqrt{4.2^2 + 2.7^2}$$

$$= 4.9929\dots$$

CE is approximately 5.0 cm, $\angle C$ is approximately 32.7° , and $\angle E$ is approximately 57.3° .

- b) The sum of the acute angles in a right triangle is 90° , so:

$$\angle H = 90^\circ - \angle F$$

$$\angle H = 90^\circ - 38^\circ$$

$$= 52^\circ$$

In right $\triangle FGH$, GH is opposite $\angle F$ and FG is adjacent to $\angle F$.

Use the tangent ratio in right $\triangle FGH$.

$$\tan F = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan F = \frac{GH}{FG}$$

$$\tan 38^\circ = \frac{GH}{3.4}$$

$$3.4 \tan 38^\circ = GH$$

$$GH = 2.6563\dots$$

Use the Pythagorean Theorem in right $\triangle FGH$.

$$FH^2 = FG^2 + GH^2$$

$$FH^2 = 3.4^2 + (2.6563\dots)^2$$

$$FH = \sqrt{3.4^2 + (2.6563\dots)^2}$$

$$= 4.3146\dots$$

GH is approximately 2.7 cm, FH is approximately 4.3 cm, and $\angle H$ is 52° .

- c) The sum of the acute angles in a right triangle is 90° , so:

$$\angle K = 90^\circ - \angle J$$

$$\angle K = 90^\circ - 27^\circ$$

$$= 63^\circ$$

In right $\triangle JKM$, KM is opposite $\angle J$ and JK is the hypotenuse.

Use the sine ratio in right $\triangle JKM$.

$$\sin J = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin J = \frac{KM}{JK}$$

$$\sin 27^\circ = \frac{KM}{4.4}$$

$$4.4 \sin 27^\circ = KM$$

$$KM = 1.9975\dots$$

In right $\triangle JKM$, JM is adjacent to $\angle J$ and JK is the hypotenuse.

Use the cosine ratio in right $\triangle JKM$.

$$\cos J = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos J = \frac{JM}{JK}$$

$$\cos 27^\circ = \frac{JM}{4.4}$$

$$4.4 \cos 27^\circ = JM$$

$$JM = 3.9204\dots$$

KM is approximately 2.0 cm, JM is approximately 3.9 cm, and $\angle K$ is 63° .

19. Sketch and label a diagram to represent the information in the problem.



The angle of inclination is $\angle A$.

In right $\triangle ABC$, AC is adjacent to $\angle A$ and AB is the hypotenuse.

Use the cosine ratio in right $\triangle ABC$.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{AC}{AB}$$

$$\cos A = \frac{13}{183}$$

$$\angle A = 85.9263\dots^\circ$$

The angle of inclination is approximately 85.9° .

20. a) In right $\triangle PQR$, QR is the hypotenuse and PR is opposite $\angle Q$.
Use the sine ratio in right $\triangle PQR$.

$$\sin Q = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Q = \frac{PR}{QR}$$

$$\sin 54^\circ = \frac{PR}{14.8}$$

$$14.8 \sin 54^\circ = PR$$

$$PR = 11.9734\dots$$

In right $\triangle PQR$, QR is the hypotenuse and PQ is adjacent to $\angle Q$.
Use the cosine ratio in right $\triangle PQR$.

$$\cos Q = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos Q = \frac{PQ}{QR}$$

$$\cos 54^\circ = \frac{PQ}{14.8}$$

$$14.8 \cos 54^\circ = PQ$$

$$PQ = 8.6992\dots$$

Perimeter of $\triangle PQR$:

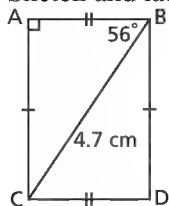
$$14.8 \text{ cm} + 11.9734\dots \text{ cm} + 8.6992\dots \text{ cm} = 35.4726\dots \text{ cm}$$

Area of $\triangle PQR$:

$$\begin{aligned} \frac{1}{2}(PQ)(PR) &= \frac{1}{2}(11.9734\dots)(8.6992\dots) \\ &= 52.0798\dots \end{aligned}$$

The perimeter of $\triangle PQR$ is approximately 35.5 cm and its area is approximately 52.1 cm^2 .

- b) Sketch and label a diagram of the rectangle.



In right $\triangle ABC$, BC is the hypotenuse and AC is opposite $\angle B$.

Use the sine ratio in right $\triangle ABC$.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{BC}$$

$$\sin 56^\circ = \frac{AC}{4.7}$$

$$4.7 \sin 56^\circ = AC$$

$$AC = 3.8964\dots$$

In right $\triangle ABC$, BC is the hypotenuse and AB is adjacent to $\angle B$.

Use the cosine ratio in right $\triangle ABC$.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{AB}{BC}$$

$$\cos 56^\circ = \frac{AB}{4.7}$$

$$4.7 \cos 56^\circ = AB$$

$$AB = 2.6282\dots$$

Perimeter of rectangle:

$$2(3.8964\dots \text{ cm} + 2.6282\dots \text{ cm}) = 13.0493\dots \text{ cm}$$

Area of rectangle:

$$(3.8964\dots \text{ cm})(2.6282\dots \text{ cm}) = 10.2407\dots \text{ cm}^2$$

The perimeter of the rectangle is approximately 13.0 cm and its area is approximately 10.2 cm^2 .

21. a) In right $\triangle ABC$, AB is the hypotenuse and AC is opposite $\angle B$.
Use the sine ratio in right $\triangle ABC$.

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{AB}$$

$$\sin 60^\circ = \frac{2.8}{AB}$$

$$AB \sin 60^\circ = 2.8$$

$$AB = \frac{2.8}{\sin 60^\circ}$$

$$AB = 3.2331\dots$$

AB is approximately 3.2 m.

- b) In right $\triangle ABC$, AC is opposite $\angle B$ and BC is adjacent to $\angle B$.
Use the tangent ratio in right $\triangle ABC$.

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan B = \frac{AC}{BC}$$

$$\tan 60^\circ = \frac{2.8}{BC}$$

$$BC \tan 60^\circ = 2.8$$

$$BC = \frac{2.8}{\tan 60^\circ}$$

$$BC = 1.6165\dots$$

$$BD = BC + DC$$

$$BD = 1.6165\dots + 6.6$$

$$= 8.2165\dots$$

BD is approximately 8.2 m.

2.7

22. a) Use the Pythagorean Theorem in right $\triangle GJK$.

$$KJ^2 = GK^2 + GJ^2$$

$$KJ^2 = 10.8^2 + (3.2 + 5.1)^2$$

$$KJ = \sqrt{10.8^2 + (3.2 + 5.1)^2}$$

$$= 13.6209\dots$$

KJ is approximately 13.6 cm long.

- b) Use the Pythagorean Theorem in right $\triangle GHK$.

$$HK^2 = GK^2 + GH^2$$

$$HK^2 = 10.8^2 + 3.2^2$$

$$HK = \sqrt{10.8^2 + 3.2^2}$$

$$= 11.2641\dots$$

HK is approximately 11.3 cm long.

- c) $\angle HKJ = \angle GKJ - \angle HKJ$

In right $\triangle GJK$, GJ is opposite $\angle K$ and GK is adjacent to $\angle K$.

Use the tangent ratio in right $\triangle GJK$ long.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{GJ}{GK}$$

$$\tan K = \frac{3.2 + 5.1}{10.8}$$

$$\tan K = \frac{8.3}{10.8}$$

$$\angle K = 37.5429\dots^\circ$$

In right $\triangle GHK$, GH is opposite $\angle K$ and GK is adjacent to $\angle K$.

Use the tangent ratio in right $\triangle GHK$.

$$\tan K = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan K = \frac{GH}{GK}$$

$$\tan K = \frac{3.2}{10.8}$$

$$\angle K = 16.5043\dots^\circ$$

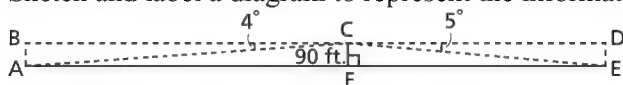
$$\angle HKJ = \angle GKJ - \angle HKJ$$

$$\angle HKJ = 37.5429\dots^\circ - 16.5043\dots^\circ$$

$$= 21.0385\dots^\circ$$

$\angle HKJ$ is approximately 21.0° .

23. Sketch and label a diagram to represent the information in the problem.



AE represents the distance between the fires.

$$AE = AF + FE$$

In right $\triangle ABC$, AB is opposite $\angle C$ and BC is adjacent to $\angle C$.

Use the tangent ratio in right $\triangle ABC$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{AB}{BC}$$

$$\tan 4^\circ = \frac{90}{BC}$$

$$BC \tan 4^\circ = 90$$

$$BC = \frac{90}{\tan 4^\circ}$$

$$BC = 1287.0599\dots$$

Since ABCF is a rectangle, $AF = BC$.

In right $\triangle CDE$, DE is opposite $\angle C$ and CD is adjacent to $\angle C$.

Use the tangent ratio in right $\triangle CDE$.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan C = \frac{DE}{CD}$$

$$\tan 5^\circ = \frac{90}{CD}$$

$$CD \tan 5^\circ = 90$$

$$CD = \frac{90}{\tan 5^\circ}$$

$$CD = 1028.7047\dots$$

Since CDEF is a rectangle, $EF = CD$.

$$AE = AF + FE$$

$$AE = 1287.0599\dots + 1028.7047\dots$$

$$= 2315.7646\dots$$

The fires are approximately 2316 ft. apart.

Practice Test

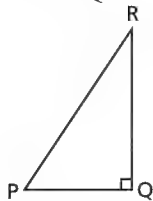
(page 127)

1. In
- $\triangle PQR$
- :

$$\begin{array}{llll} \tan Q = \frac{\text{opposite}}{\text{adjacent}} & \sin P = \frac{\text{opposite}}{\text{hypotenuse}} & \sin Q = \frac{\text{opposite}}{\text{hypotenuse}} & \tan P = \frac{\text{opposite}}{\text{adjacent}} \\ \tan Q = \frac{PR}{QR} & \sin P = \frac{QR}{PQ} & \sin Q = \frac{PR}{PQ} & \tan P = \frac{QR}{PR} \\ \tan Q = \frac{3}{4} & \sin P = \frac{4}{5} & \sin Q = \frac{3}{5} & \tan P = \frac{4}{3} \end{array}$$

3 statements are true.
Answer B is correct.

2. In
- $\triangle PQR$
- :



$\angle P$ increases when QR increases and PQ remains the same length.
Then PR also increases. So:

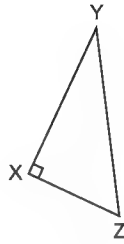
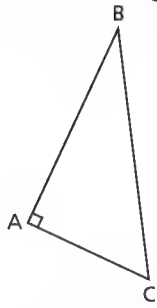
$$\tan P = \frac{QR}{PQ} \text{ increases and } \cos P = \frac{PQ}{PR} \text{ decreases}$$

$\angle P$ also increases when QR remains the same length and PQ decreases.
Then PR also decreases. So:

$$\sin P = \frac{QR}{PR} \text{ increases}$$

Answer C is correct.

3. Sketch a diagram.



In $\triangle ABC$:

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin B = \frac{AC}{BC}$$

In $\triangle XYZ$:

$$\sin Y = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin Y = \frac{XZ}{YZ}$$

Since the triangles are similar, corresponding sides are in the same ratio:

$$\frac{AC}{XZ} = \frac{BC}{YZ}$$

Multiply each side by XZ.

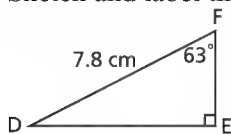
$$\frac{AC}{XZ} \cdot XZ = \frac{BC}{YZ} \cdot XZ$$

Divide each side by BC.

$$\frac{AC}{BC} = \frac{XZ}{YZ}$$

So, $\sin B = \sin Y$

4. Sketch and label the triangle.



The sum of the acute angles in a right triangle is 90° , so:

$$\angle D = 90^\circ - \angle F$$

$$\angle D = 90^\circ - 63^\circ$$

$$= 27^\circ$$

In right $\triangle DEF$, DE is opposite $\angle F$
and DF is the length of the hypotenuse.

Use the sine ratio in right $\triangle DEF$.

$$\sin F = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin F = \frac{DE}{DF}$$

$$\sin 63^\circ = \frac{DE}{7.8}$$

$$7.8 \sin 63^\circ = DE$$

$$DE = 6.9498\dots$$

In right $\triangle DEF$, EF is adjacent to $\angle F$ and DF is the hypotenuse.

Use the cosine ratio in right $\triangle DEF$.

$$\cos F = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos F = \frac{EF}{DF}$$

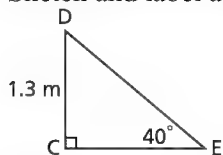
$$\cos 63^\circ = \frac{EF}{7.8}$$

$$7.8 \cos 63^\circ = EF$$

$$EF = 3.5411\dots$$

DE is approximately 6.9 cm, EF is approximately 3.5 cm, and $\angle D$ is 27° .

5. Sketch and label a diagram to represent the information in the problem.



The shortest possible length of the ramp is represented by DE.

In right $\triangle CDE$, CD is opposite $\angle E$ and DE is the hypotenuse.

Use the sine ratio in right $\triangle CDE$.

$$\sin E = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin E = \frac{CD}{DE}$$

$$\sin 40^\circ = \frac{1.3}{DE}$$

$$DE \sin 40^\circ = 1.3$$

$$DE = \frac{1.3}{\sin 40^\circ}$$

$$DE = 2.0224 \dots$$

Round up since the angle of inclination of the ramp should be less than 40° .

To the nearest centimetre, the shortest possible length of the ramp is 2.03 m.

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Chapter 2
Trigonometry

6. Sketch and label a diagram to represent the information in the problem.
 BD represents the height of the tower.

Assume the ground is horizontal.

Then quadrilateral AEDF is a rectangle and $DE = AF = 1.5$ m.

$$CE = CD - DE$$

$$CE = 50 - 1.5$$

$$= 48.5$$

In right $\triangle ACE$, CE is opposite $\angle A$ and AE is adjacent to $\angle A$.

Use the tangent ratio in right $\triangle ACE$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{CE}{AE}$$

$$\tan 37^\circ = \frac{48.5}{AE}$$

$$AE \tan 37^\circ = 48.5$$

$$AE = \frac{48.5}{\tan 37^\circ}$$

$$AE = 64.3616\dots$$

In right $\triangle ABE$, BE is opposite $\angle A$ and AE is adjacent to $\angle A$.

Use the tangent ratio in right $\triangle ABE$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{BE}{AE}$$

$$\tan 49^\circ = \frac{BE}{64.3616\dots}$$

$$(64.3616\dots) \tan 49^\circ = BE$$

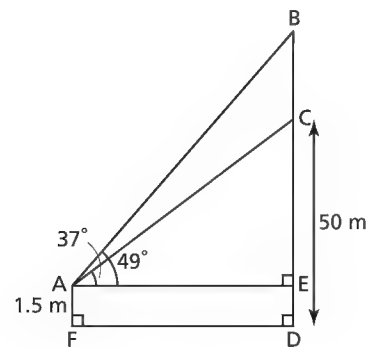
$$BE = 74.0396\dots$$

$$BD = BE + DE$$

$$BD = 74.0396\dots + 1.5$$

$$= 75.5396\dots$$

To the nearest tenth of a metre, the tower is 75.5 m high.



Lesson 3.1 Factors and Multiples of Whole Numbers Exercises (pages 140–141)**A**

3. Multiply each number by 1, 2, 3, 4, 5, and 6.

a) $6 \times 1 = 6$

$$6 \times 2 = 12$$

$$6 \times 3 = 18$$

$$6 \times 4 = 24$$

$$6 \times 5 = 30$$

$$6 \times 6 = 36$$

So, the first 6 multiples of 6 are: 6, 12, 18, 24, 30, 36

b) $13 \times 1 = 13$

$$13 \times 2 = 26$$

$$13 \times 3 = 39$$

$$13 \times 4 = 52$$

$$13 \times 5 = 65$$

$$13 \times 6 = 78$$

So, the first 6 multiples of 13 are: 13, 26, 39, 52, 65, 78

c) $22 \times 1 = 22$

$$22 \times 2 = 44$$

$$22 \times 3 = 66$$

$$22 \times 4 = 88$$

$$22 \times 5 = 110$$

$$22 \times 6 = 132$$

So, the first 6 multiples of 22 are: 22, 44, 66, 88, 110, 132

d) $31 \times 1 = 31$

$$31 \times 2 = 62$$

$$31 \times 3 = 93$$

$$31 \times 4 = 124$$

$$31 \times 5 = 155$$

$$31 \times 6 = 186$$

So, the first 6 multiples of 31 are: 31, 62, 93, 124, 155, 186

e) $45 \times 1 = 45$

$$45 \times 2 = 90$$

$$45 \times 3 = 135$$

$$45 \times 4 = 180$$

$$45 \times 5 = 225$$

$$45 \times 6 = 270$$

So, the first 6 multiples of 45 are: 45, 90, 135, 180, 225, 270

f) $27 \times 1 = 27$

$27 \times 2 = 54$

$27 \times 3 = 81$

$27 \times 4 = 108$

$27 \times 5 = 135$

$27 \times 6 = 162$

So, the first 6 multiples of 27 are: 27, 54, 81, 108, 135, 162

4. The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
Divide each number by these primes.

a) $40 \div 2 = 20$

$40 \div 3 = 13.3333\dots$

$40 \div 5 = 8$

$40 \div 7 = 5.7142\dots$

There are no more primes between 5.7142... and 7.

40 is divisible by the prime numbers 2 and 5.

So, the prime factors of 40 are 2 and 5.

b) $75 \div 2 = 37.5$

$75 \div 3 = 25$

$75 \div 5 = 15$

$75 \div 7 = 10.7142\dots$

There are no more primes between 7 and 10.7142....

75 is divisible by the prime numbers 3 and 5.

So, the prime factors of 75 are 3 and 5.

c) $81 \div 2 = 40.5$

$81 \div 3 = 27$

$81 \div 5 = 16.2$

$81 \div 7 = 11.5714\dots$

$81 \div 11 = 7.3636\dots$

There are no more primes between 7.3636... and 11.

81 is divisible by the prime number 3.

So, the prime factor of 81 is 3.

d) $120 \div 2 = 60$

$120 \div 3 = 40$

$120 \div 5 = 24$

$120 \div 7 = 17.1428\dots$

$120 \div 11 = 10.9090\dots$

There are no more primes between 10.9090... and 11.

120 is divisible by the prime numbers 2, 3, and 5.

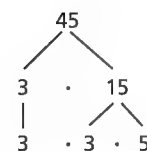
So, the prime factors of 120 are 2, 3, and 5.

- e) $140 \div 2 = 70$
 $140 \div 3 = 46.6666\dots$
 $140 \div 5 = 28$
 $140 \div 7 = 20$
 $140 \div 11 = 12.7272\dots$
 There are no more primes between 11 and 12.7272....
 140 is divisible by the prime numbers 2, 5, and 7.
 So, the prime factors of 140 are 2, 5, and 7.

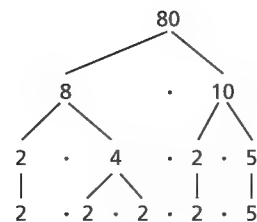
- f) $192 \div 2 = 96$
 $192 \div 3 = 64$
 $192 \div 5 = 38.4$
 $192 \div 7 = 27.4285\dots$
 $192 \div 11 = 17.4545\dots$
 $192 \div 13 = 14.7692\dots$
 There are no more primes between 13 and 14.7692....
 192 is divisible by the prime numbers 2 and 3.
 So, the prime factors of 192 are 2 and 3.

5. Draw a factor tree for each number.

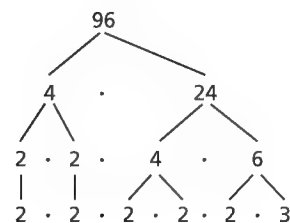
- a) Write 45 as a product of 2 factors: $45 = 3 \cdot 15$
 3 is a prime factor, but 15 can be factored further.
 $15 = 3 \cdot 5$
 3 and 5 are prime factors, so there are no more factors.
 $45 = 3 \cdot 3 \cdot 5$, or $3^2 \cdot 5$



- b) Write 80 as a product of 2 factors: $80 = 8 \cdot 10$
 Both 8 and 10 are composite numbers, so factor again.
 $8 = 2 \cdot 4$ and $10 = 2 \cdot 5$
 2 and 5 are prime factors, but 4 can be factored further.
 $4 = 2 \cdot 2$
 2 is a prime factor, so there are no more factors.
 $80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$



- c) Write 96 as a product of 2 factors: $96 = 4 \cdot 24$
 Both 4 and 24 are composite numbers, so factor again.
 $4 = 2 \cdot 2$ and $24 = 4 \cdot 6$
 2 is a prime factor, but both 4 and 6 can be factored further.
 $4 = 2 \cdot 2$ and $6 = 2 \cdot 3$
 2 and 3 are prime factors, so there are no more factors.
 $96 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^5 \cdot 3$



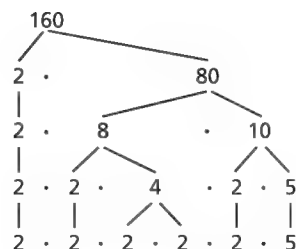
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Chapter 3
Factors and Products

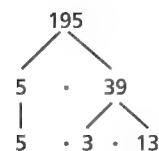
- d) Write 122 as a product of 2 factors: $122 = 2 \cdot 61$
 2 and 61 are prime factors, so there are no more factors.
 $122 = 2 \cdot 61$



- e) Write 160 as a product of 2 factors:
 $160 = 2 \cdot 80$
 From part b, $80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$
 So, $160 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^5 \cdot 5$



- f) Write 195 as a product of 2 factors: $195 = 5 \cdot 39$
 5 is a prime factor, but 39 can be factored further.
 $39 = 3 \cdot 13$
 3 and 13 are prime factors, so there are no more factors.
 $195 = 3 \cdot 5 \cdot 13$



B

6. Use a calculator and repeated division by prime factors.

- a) $600 \div 2 = 300$
 $300 \div 2 = 150$
 $150 \div 2 = 75$
 $75 \div 3 = 25$
 $25 \div 5 = 5$
 $5 \div 5 = 1$
 So, $600 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$
 Using powers: $600 = 2^3 \cdot 3 \cdot 5^2$

- b) $1150 \div 2 = 575$
 $575 \div 5 = 115$
 $115 \div 5 = 23$
 $23 \div 23 = 1$
 So, $1150 = 2 \cdot 5 \cdot 5 \cdot 23$
 Using powers: $1150 = 2 \cdot 5^2 \cdot 23$

- c) $1022 \div 2 = 511$
 $511 \div 7 = 73$
 $73 \div 73 = 1$
 So, $1150 = 2 \cdot 7 \cdot 73$

d) $2250 \div 2 = 1125$
 $1125 \div 3 = 375$
 $375 \div 3 = 125$
 $125 \div 5 = 25$
 $25 \div 5 = 5$
 $5 \div 5 = 1$
 So, $2250 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$
 Using powers: $2250 = 2 \cdot 3^2 \cdot 5^3$

e) $4500 \div 2 = 2250$
 From part b, $2250 = 2 \cdot 3^2 \cdot 5^3$
 So, $4500 = 2 \cdot 2 \cdot 3^2 \cdot 5^3$
 Using powers: $4500 = 2^2 \cdot 3^2 \cdot 5^3$

f) $6125 \div 5 = 1225$
 $1225 \div 5 = 245$
 $245 \div 5 = 49$
 $49 \div 7 = 7$
 $7 \div 7 = 1$
 So, $6125 = 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7$
 Using powers: $6125 = 5^3 \cdot 7^2$

7. I cannot write 0 as a product of prime numbers, so 0 has no prime factors.
 1 is the only whole number that divides into 1, so 1 has no prime factors.

8. Write the prime factorization of each number.
 Highlight the factors that appear in each prime factorization.

a) $46 = 2 \cdot 23$
 $84 = 2 \cdot 2 \cdot 3 \cdot 7$
 The greatest common factor is 2.

b) $64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
 $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$
 The greatest common factor is $2 \cdot 2 \cdot 2$, which is 8.

c) $81 = 3 \cdot 3 \cdot 3 \cdot 3$
 $216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
 The greatest common factor is $3 \cdot 3 \cdot 3$, which is 27.

d) $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$
 $224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$
 The greatest common factor is $2 \cdot 2$, which is 4.

e) $160 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$
 $672 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$
 The greatest common factor is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, which is 32.

f) $220 = 2 \cdot 2 \cdot 5 \cdot 11$
 $860 = 2 \cdot 2 \cdot 5 \cdot 43$
 The greatest common factor is $2 \cdot 2 \cdot 5$, which is 20.

9. Write the prime factorization of each number.

Highlight the factors that appear in each prime factorization.

a) $150 = 2 \cdot 3 \cdot 5 \cdot 5$

$$275 = 5 \cdot 5 \cdot 11$$

$$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$$

The greatest common factor is 5.

b) $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$$960 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

$$1400 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7$$

The greatest common factor is $2 \cdot 2 \cdot 2 \cdot 5$, which is 40.

c) $126 = 2 \cdot 3 \cdot 3 \cdot 7$

$$210 = 2 \cdot 3 \cdot 5 \cdot 7$$

$$546 = 2 \cdot 3 \cdot 7 \cdot 13$$

$$714 = 2 \cdot 3 \cdot 7 \cdot 17$$

The greatest common factor is $2 \cdot 3 \cdot 7$, which is 42.

d) $220 = 2 \cdot 2 \cdot 5 \cdot 11$

$$308 = 2 \cdot 2 \cdot 7 \cdot 11$$

$$484 = 2 \cdot 2 \cdot 11 \cdot 11$$

$$988 = 2 \cdot 2 \cdot 13 \cdot 19$$

The greatest common factor is $2 \cdot 2$, which is 4.

10. Write the prime factorization of each number. Highlight the greater power of each prime factor in any list. The least common multiple is the product of the greater power of each prime factor.

a) $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

$$14 = 2 \cdot 7$$

The greater power of 2 in either list is 2^2 .

The greater power of 3 in either list is 3.

The greater power of 7 in either list is 7.

The least common multiple is:

$$2^2 \cdot 3 \cdot 7 = 4 \cdot 3 \cdot 7$$

$$= 84$$

So, the least common multiple of 12 and 14 is 84.

b) $21 = 3 \cdot 7$

$$45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$$

The greater power of 3 in either list is 3^2 .

The greater power of 5 in either list is 5.

The greater power of 7 in either list is 7.

The least common multiple is:

$$3^2 \cdot 5 \cdot 7 = 9 \cdot 5 \cdot 7$$

$$= 315$$

So, the least common multiple of 21 and 45 is 315.

c) $45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

The greater power of 2 in either list is 2^2 .

The greater power of 3 in either list is 3^2 .

The greater power of 5 in either list is 5.

The least common multiple is:

$$2^2 \cdot 3^2 \cdot 5 = 4 \cdot 9 \cdot 5$$

$$= 180$$

So, the least common multiple of 45 and 60 is 180.

d) $38 = 2 \cdot 19$

$$42 = 2 \cdot 3 \cdot 7$$

The greater power of 2 in either list is 2.

The greater power of 3 in either list is 3.

The greater power of 7 in either list is 7.

The greater power of 19 in either list is 19.

The least common multiple is:

$$2 \cdot 3 \cdot 7 \cdot 19 = 798$$

So, the least common multiple of 38 and 42 is 798.

e) $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

$$45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$$

The greater power of 2 in either list is 2^5 .

The greater power of 3 in either list is 3^2 .

The greater power of 5 in either list is 5.

The least common multiple is:

$$2^5 \cdot 3^2 \cdot 5 = 32 \cdot 9 \cdot 5$$

$$= 1440$$

So, the least common multiple of 32 and 45 is 1440.

f) $28 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$

$$52 = 2 \cdot 2 \cdot 13 = 2^2 \cdot 13$$

The greater power of 2 in either list is 2^2 .

The greater power of 7 in either list is 7.

The greater power of 13 in either list is 13.

The least common multiple is:

$$2^2 \cdot 7 \cdot 13 = 4 \cdot 7 \cdot 13$$

$$= 364$$

So, the least common multiple of 28 and 52 is 364.

11. Write the prime factorization of each number. Highlight the greatest power of each prime factor in any list. The least common multiple is the product of the greatest power of each prime factor.

a) $20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$

$$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$$

$$38 = 2 \cdot 19$$

The greatest power of 2 in any list is 2^2 .

The greatest power of 3 in any list is 3^2 .

The greatest power of 5 in any list is 5.

The greatest power of 19 in any list is 19.

The least common multiple is:

$$2^2 \cdot 3^2 \cdot 5 \cdot 19 = 4 \cdot 9 \cdot 5 \cdot 19$$

$$= 3420$$

So, the least common multiple of 20, 36, and 38 is 3420.

b) $15 = 3 \cdot 5$

$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

$$44 = 2 \cdot 2 \cdot 11 = 2^2 \cdot 11$$

The greatest power of 2 in any list is 2^5 .

The greatest power of 3 in any list is 3.

The greatest power of 5 in any list is 5.

The greatest power of 11 in any list is 11.

The least common multiple is:

$$2^5 \cdot 3 \cdot 5 \cdot 11 = 32 \cdot 3 \cdot 5 \cdot 11$$

$$= 5280$$

So, the least common multiple of 15, 32, and 44 is 5280.

c) $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

$$18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$$

$$25 = 5 \cdot 5 = 5^2$$

$$30 = 2 \cdot 3 \cdot 5$$

The greatest power of 2 in any list is 2^2 .

The greatest power of 3 in any list is 3^2 .

The greatest power of 5 in any list is 5^2 .

The least common multiple is:

$$2^2 \cdot 3^2 \cdot 5^2 = 4 \cdot 9 \cdot 25$$

$$= 900$$

So, the least common multiple of 12, 18, 25, and 30 is 900.

d) $15 = 3 \cdot 5$

$$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$$

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$$

$$27 = 3 \cdot 3 \cdot 3 = 3^3$$

The greatest power of 2 in any list is 2^3 .

The greatest power of 3 in any list is 3^3 .

The greatest power of 5 in any list is 5.

The least common multiple is:

$$2^3 \cdot 3^3 \cdot 5 = 8 \cdot 27 \cdot 5$$

$$= 1080$$

So, the least common multiple of 15, 20, 24, and 27 is 1080.

12. The greatest common factor of 12 and 14 is the greatest number that divides into 12 and 14 with no remainder. This number is less than both 12 and 14.

Since $12 = 2 \cdot 6$, and $14 = 2 \cdot 7$, then the greatest common factor is 2.

The least common multiple of 12 and 14 is the least number that both 12 and 14 divide into with no remainder. This number is greater than both 12 and 14.

Since $12 = 2 \cdot 6$ and $14 = 2 \cdot 7$, then the least common multiple is $2 \cdot 6 \cdot 7 = 84$.

13. The first band has 42 members. So, the number of columns in the array is a factor of 42.
The second band has 36 members. So, the number of columns in the array is a factor of 36.
The arrays have the same number of columns.

So, the number of columns in each array is a common factor of 42 and 36.

The greatest number of columns is the greatest common factor of 42 and 36.

Write the prime factorization of each number.

Highlight the prime factors that appear in both lists.

$$42 = 2 \cdot 3 \cdot 7$$

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

The greatest common factor is: $2 \cdot 3 = 6$

The array has no more than 6 columns.

14. The product of two numbers is equal to their least common multiple when both numbers have no common factors, except 1. For example, two prime numbers such as 5 and 11 have a product of 55 and this is also their least common multiple. And, two numbers such as 6 and 35 have a product of 210 and this is also their least common multiple.

15. A fraction is simplified when its numerator and denominator have no common factors.
So, divide the numerator and denominator of each fraction by their greatest common factor.

a) $\frac{185}{325}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$185 = 5 \cdot 37$$

$$325 = 5 \cdot 5 \cdot 13$$

The greatest common factor is 5.

$$\frac{185}{325} = \frac{185 \div 5}{325 \div 5}$$

$$= \frac{37}{65}$$

b) $\frac{340}{380}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$340 = 2 \cdot 2 \cdot 5 \cdot 17$$

$$380 = 2 \cdot 2 \cdot 5 \cdot 19$$

The greatest common factor is $2 \cdot 2 \cdot 5$, or 20.

$$\frac{340}{380} = \frac{340 \div 20}{380 \div 20}$$

$$= \frac{17}{19}$$

c) $\frac{650}{900}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$650 = 2 \cdot 5 \cdot 5 \cdot 13$$

$$900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$$

The greatest common factor is $2 \cdot 5 \cdot 5$, or 50.

$$\frac{650}{900} = \frac{650 \div 50}{900 \div 50}$$

$$= \frac{13}{18}$$

d) $\frac{840}{1220}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$840 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$$

$$1220 = 2 \cdot 2 \cdot 5 \cdot 61$$

The greatest common factor is $2 \cdot 2 \cdot 5$, or 20.

$$\frac{840}{1220} = \frac{840 \div 20}{1220 \div 20}$$

$$= \frac{42}{61}$$

e) $\frac{1225}{2750}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$1225 = 5 \cdot 5 \cdot 7 \cdot 7$$

$$2750 = 2 \cdot 5 \cdot 5 \cdot 5 \cdot 11$$

The greatest common factor is $5 \cdot 5$, or 25.

$$\begin{aligned}\frac{1225}{2750} &= \frac{1225 \div 25}{2750 \div 25} \\ &= \frac{49}{110}\end{aligned}$$

f) $\frac{2145}{1105}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$2145 = 3 \cdot 5 \cdot 11 \cdot 13$$

$$1105 = 5 \cdot 13 \cdot 17$$

The greatest common factor is $5 \cdot 13$, or 65.

$$\begin{aligned}\frac{2145}{1105} &= \frac{2145 \div 65}{1105 \div 65} \\ &= \frac{33}{17}\end{aligned}$$

- 16.** Add, subtract, or divide fractions with a common denominator by adding, subtracting, or dividing their numerators.
So, write the fractions in each part with denominator equal to the least common multiple of the original denominators.

a) $\frac{9}{14} + \frac{11}{16}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$14 = 2 \cdot 7$$

$$16 = 2^4$$

The least common multiple is $2^4 \cdot 7$, or 112.

$$\begin{aligned}\frac{9}{14} + \frac{11}{16} &= \frac{72}{112} + \frac{77}{112} \\ &= \frac{72 + 77}{112} \\ &= \frac{149}{112}\end{aligned}$$

b) $\frac{8}{15} + \frac{11}{20}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$15 = 3 \cdot 5$$

$$20 = 2^2 \cdot 5$$

The least common multiple is $2^2 \cdot 3 \cdot 5$, or 60.

$$\begin{aligned}\frac{8}{15} + \frac{11}{20} &= \frac{32}{60} + \frac{33}{60} \\ &= \frac{32 + 33}{60} \\ &= \frac{65}{60}, \text{ or } \frac{13}{12}\end{aligned}$$

c) $\frac{5}{24} - \frac{1}{22}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$24 = 2^3 \cdot 3$$

$$22 = 2 \cdot 11$$

The least common multiple is $2^3 \cdot 3 \cdot 11$, or 264.

$$\begin{aligned}\frac{5}{24} - \frac{1}{22} &= \frac{55}{264} - \frac{12}{264} \\ &= \frac{55 - 12}{264} \\ &= \frac{43}{264}\end{aligned}$$

d) $\frac{9}{10} + \frac{5}{14} + \frac{4}{21}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in any list.

$$10 = 2 \cdot 5$$

$$14 = 2 \cdot 7$$

$$21 = 3 \cdot 7$$

The least common multiple is $2 \cdot 3 \cdot 5 \cdot 7$, or 210.

$$\begin{aligned}\frac{9}{10} + \frac{5}{14} + \frac{4}{21} &= \frac{189}{210} + \frac{75}{210} + \frac{40}{210} \\ &= \frac{189 + 75 + 40}{210} \\ &= \frac{304}{210}, \text{ or } \frac{152}{105}\end{aligned}$$

e) $\frac{9}{25} + \frac{7}{15} - \frac{5}{8}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in any list.

$$25 = 5^2$$

$$15 = 3 \cdot 5$$

$$8 = 2^3$$

The least common multiple is $2^3 \cdot 3 \cdot 5^2$, or 600.

$$\begin{aligned}\frac{9}{25} + \frac{7}{15} - \frac{5}{8} &= \frac{216}{600} + \frac{280}{600} - \frac{375}{600} \\ &= \frac{216 + 280 - 375}{600} \\ &= \frac{121}{600}\end{aligned}$$

f) $\frac{3}{5} - \frac{5}{18} + \frac{7}{3}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in any list.

$$5 = 5$$

$$18 = 2 \cdot 3^2$$

$$3 = 3$$

The least common multiple is $2 \cdot 3^2 \cdot 5$, or 90.

$$\begin{aligned}\frac{3}{5} - \frac{5}{18} + \frac{7}{3} &= \frac{54}{90} - \frac{25}{90} + \frac{210}{90} \\ &= \frac{54 - 25 + 210}{90} \\ &= \frac{239}{90}\end{aligned}$$

g) $\frac{3}{5} \div \frac{4}{9}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$5 = 5$$

$$9 = 3^2$$

The least common multiple is $3^2 \cdot 5$, or 45.

$$\begin{aligned}\frac{3}{5} \div \frac{4}{9} &= \frac{27}{45} \div \frac{20}{45} \\ &= 27 \div 20 \\ &= \frac{27}{20}\end{aligned}$$

h) $\frac{11}{6} \div \frac{2}{7}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$6 = 2 \cdot 3$$

$$7 = 7$$

The least common multiple is $2 \cdot 3 \cdot 7$, or 42.

$$\begin{aligned}\frac{11}{6} \div \frac{2}{7} &= \frac{77}{42} \div \frac{12}{42} \\ &= 77 \div 12 \\ &= \frac{77}{12}\end{aligned}$$

17. The shorter side of the rectangular plot of land measures 2400 m. So, the side length of the square plot must be a factor of 2400.

The longer side of the rectangular plot of land measures 3200 m. So, the side length of the square plot must be a factor of 3200.

So, the side length of the square must be a common factor of 2400 and 3200.

Write the prime factorization of each number.

Highlight the prime factors that appear in both lists.

$$2400 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$$

$$3200 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$$

The greatest common factor is:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 800$$

The largest square has side length 800 m.

18. No. The number 1 is a whole number, and its only factor is 1.

A prime number has exactly 2 factors: 1 and itself

So, 1 is a whole number, but not a prime number, and it has no prime factors.

19. a) The side length of the square is a common multiple of 18 and 24. The least common multiple will produce the smallest square.

The multiples of 18 are: 18, 36, 54, 72, ...

The multiples of 24 are: 24, 48, 72, ...

The least common multiple of 18 and 24 is 72.

So, the dimensions of the square are 72 cm by 72 cm.

- b) The dimensions of the floor in centimetres are 648 cm by 1512 cm.

If 18 or 24 is a factor of each dimension, then the tiles will cover the floor.

$$648 \div 18 = 36$$

$$648 \div 24 = 27$$

$$1512 \div 18 = 84$$

$$1512 \div 24 = 63$$

Since both dimensions of the tile are factors of the dimensions of the floor, the tiles could be used to cover the floor.

20. a) A square with side length 1 mi. measures 5280 ft. by 5280 ft.
Determine whether 660 is a factor of 5280.
 $5280 \div 660 = 8$
Since 66 is a factor of 660, then 66 is a factor of 5280, and $5280 \div 66 = 80$
So, the rectangles for 1 acre fit exactly into a section.
- b) From part a, 8 rectangles fit along the one side of a section and 80 rectangles fit along the side at right angles.
For a quarter section, which is a square with side length $\frac{1}{2}$ mi., 4 rectangles will fit along one side and 40 rectangles will fit along the side at right angles. So, the rectangles for 1 acre do fit exactly into a quarter section.
- c) The side length of the square is a common multiple of 660 and 66. The least common multiple will produce the smallest square.
Since $660 = 66 \cdot 10$, then the least common multiple of 660 and 66 is 660.
So, the side length of the smallest square is 660 ft.

C

21. Yes, $61 \div 7 = 8.7142\dots$
And, 61 is not divisible by 8 because 61 is an odd number.
There are no more natural numbers between 7 and $8.7142\dots$
So, 61 is not divisible by any natural number.
22. The edge length, in centimetres, of the smallest cube that could be filled with these bars is the least common multiple of 10, 6, and 3.
The multiples of 10 are: 10, 20, 30, ...
The multiples of 6 are: 6, 12, 18, 24, 30, ...
The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...
The least common multiple is 30.
So, the edge length of the cube is 30 cm.

Lesson 3.2 **Perfect Squares, Perfect Cubes, and Their Roots** **Exercises (pages 146–147)**

A

4. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 2 equal groups. The product of the factors in one group is the square root.

$$\begin{aligned}\text{a) } 196 &= 2 \cdot 2 \cdot 7 \cdot 7 \\ &= (2 \cdot 7)(2 \cdot 7) \\ &= 14 \cdot 14 \\ \sqrt{196} &= 14\end{aligned}$$

$$\begin{aligned}\text{b) } 256 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) \\ &= 16 \cdot 16 \\ \sqrt{256} &= 16\end{aligned}$$

$$\begin{aligned}\text{c) } 361 &= 19 \cdot 19 \\ \sqrt{361} &= 19\end{aligned}$$

$$\begin{aligned}\text{d) } 289 &= 17 \cdot 17 \\ \sqrt{289} &= 17\end{aligned}$$

$$\begin{aligned}\text{e) } 441 &= 3 \cdot 3 \cdot 7 \cdot 7 \\ &= (3 \cdot 7)(3 \cdot 7) \\ &= 21 \cdot 21 \\ \sqrt{441} &= 21\end{aligned}$$

5. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 3 equal groups. The product of the factors in one group is the cube root.

$$\begin{aligned}\text{a) } 343 &= 7 \cdot 7 \cdot 7 \\ \sqrt[3]{343} &= 7\end{aligned}$$

$$\begin{aligned}\text{b) } 512 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2) \\ &= 8 \cdot 8 \cdot 8 \\ \sqrt[3]{512} &= 8\end{aligned}$$

$$\begin{aligned}\text{c) } 1000 &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \\ &= (2 \cdot 5)(2 \cdot 5)(2 \cdot 5) \\ &= 10 \cdot 10 \cdot 10 \\ \sqrt[3]{1000} &= 10\end{aligned}$$

$$\begin{aligned}\text{d) } 1331 &= 11 \cdot 11 \cdot 11 \\ \sqrt[3]{1331} &= 11\end{aligned}$$

$$\begin{aligned}\text{e) } 3375 &= 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \\ &= (3 \cdot 5)(3 \cdot 5)(3 \cdot 5) \\ &= 15 \cdot 15 \cdot 15 \\ \sqrt[3]{3375} &= 15\end{aligned}$$

B

6. Use a calculator to write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

$$\begin{aligned}\text{a) } 225 &= 3 \cdot 3 \cdot 5 \cdot 5 \\ &= (3 \cdot 5)(3 \cdot 5)\end{aligned}$$

The factors can be arranged in 2 equal groups, so 225 is a perfect square.

$$\begin{aligned}\text{b) } 729 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) \\ &= (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)\end{aligned}$$

The factors can be arranged in 2 equal groups and in 3 equal groups, so 729 is both a perfect square and a perfect cube.

$$\text{c) } 1944 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

The factors cannot be arranged in 2 or 3 equal groups, so 1944 is neither a perfect square nor a perfect cube.

$$\begin{aligned}\text{d) } 1444 &= 2 \cdot 2 \cdot 19 \cdot 19 \\ &= (2 \cdot 19)(2 \cdot 19)\end{aligned}$$

The factors can be arranged in 2 equal groups, so 1444 is a perfect square.

$$\begin{aligned}\text{e) } 4096 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \\ &= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)\end{aligned}$$

The factors can be arranged in 2 equal groups and 3 equal groups, so 4096 is both a perfect square and a perfect cube.

$$\begin{aligned}\text{f) } 13\,824 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)\end{aligned}$$

The factors can be arranged 3 equal groups, so 13 824 is a perfect cube.

7. The side length of each square is the square root of its area. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 2 equal groups. The product of the factors in one group is the square root.

$$\begin{aligned}\text{a) } 484 &= 2 \cdot 2 \cdot 11 \cdot 11 \\ &= (2 \cdot 11)(2 \cdot 11) \\ &= 22 \cdot 22\end{aligned}$$

$$\sqrt{484} = 22$$

The side length of the square is 22 mm.

$$\begin{aligned}\text{b) } 1764 &= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \\ &= (2 \cdot 3 \cdot 7)(2 \cdot 3 \cdot 7) \\ &= 42 \cdot 42\end{aligned}$$

$$\sqrt{1764} = 42$$

The side length of the square is 42 yd.

8. The edge length of each cube is the cube root of its volume. Use a calculator to write each number as a product of its prime factors, then arrange the factors in 3 equal groups. The product of the factors in one group is the cube root.

$$\begin{aligned}\text{a) } 5832 &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3) \\ &= 18 \cdot 18 \cdot 18\end{aligned}$$

$$\sqrt[3]{5832} = 18$$

The edge length of the cube is 18 in.

$$\begin{aligned}\text{b) } 15\,625 &= 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \\ &= (5 \cdot 5)(5 \cdot 5)(5 \cdot 5) \\ &= 25 \cdot 25 \cdot 25\end{aligned}$$

$$\sqrt[3]{15\,625} = 25$$

The edge length of the cube is 25 ft.

9. The volume of the cube is 64 cubic feet.

The edge length of the cube, in feet, is: $\sqrt[3]{64} = 4$

The surface area of the cube, in square feet, is 6 times the area of one face: $6(4^2) = 96$

The surface area of the cube was 96 square feet.

10. The surface area of the cube is 6534 square feet.

The area of one face, in square feet, is: $\frac{6534}{6} = 1089$

The side length of a face, in feet, is $\sqrt{1089}$.

$$\begin{aligned}1089 &= 3 \cdot 3 \cdot 11 \cdot 11 \\ &= (3 \cdot 11)(3 \cdot 11) \\ &= 33 \cdot 33\end{aligned}$$

$$\sqrt{1089} = 33$$

The volume of the cube, in cubic feet, is: $33^3 = 35\,937$

The volume of the cube is 35 937 cubic feet.

11. If 2000 is a perfect cube, then a cube could be constructed with 2000 interlocking cubes.

Determine the factors of 2000.

$$2000 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$$

Since these factors cannot be arranged in 3 equal groups, 2000 is not a perfect cube, and a cube could not be constructed with 2000 interlocking cubes.

12. Use estimation or guess and check to determine the perfect square and perfect cube closest to the first number in each given pair, then calculate the squares and cubes of all whole numbers until the second number in each pair is reached or exceeded.

$$\text{a) } 315 - 390$$

$$17^2 = 289 \quad 18^2 = 324 \quad 19^2 = 361 \quad 20^2 = 400$$

$$6^3 = 216 \quad 7^3 = 343 \quad 8^3 = 512$$

Between 315 and 390, the perfect squares are 324 and 361; and the perfect cube is 343.

b) $650 - 750$

$$\begin{array}{llll} 25^2 = 625 & 26^2 = 676 & 27^2 = 729 & 28^2 = 784 \\ 8^3 = 512 & 9^3 = 729 & 10^3 = 1000 & \end{array}$$

Between 650 and 750, the perfect squares are 676 and 729; and the perfect cube is 729.

c) $800 - 925$

$$\begin{array}{llll} 28^2 = 784 & 29^2 = 841 & 30^2 = 900 & 31^2 = 961 \\ 9^3 = 729 & 10^3 = 1000 & & \end{array}$$

Between 800 and 925, the perfect squares are 841 and 900; there are no perfect cubes.

d) $1200 - 1350$

$$\begin{array}{llll} 34^2 = 1156 & 35^2 = 1225 & 36^2 = 1296 & 37^2 = 1369 \\ 10^3 = 1000 & 11^3 = 1331 & 12^3 = 1728 & \end{array}$$

Between 1200 and 1350, the perfect squares are 1225 and 1296; and the perfect cube is 1331.

13. For a number to be a perfect square and a perfect cube, its prime factors must be arranged in 2 equal groups and 3 equal groups; that is, each factor occurs 2(3), or 6 times, or a multiple of 6 times.

The first number, after 0 and 1, that is a perfect square and perfect cube is:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$$

Another number that is a perfect square and a perfect cube is: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 729$

Another number that is a perfect square and a perfect cube is: $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 15\,625$

14. Since the rectangular prism has a square cross-section, the prism is a square prism. Its volume is 1440 cubic feet.

Its height is 10 ft., so its base area, in square feet, is: $\frac{1440}{10} = 144$

The side length of the square base, in feet, is: $\sqrt{144} = 12$

So, the length and width of the base are 12 ft.

C

15. a) The tent has 4 congruent square faces; 2 congruent rectangular faces, and 2 congruent triangular faces.

The area, in square feet, of each square face is: $(x)(x) = x^2$

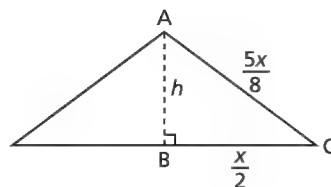
The area, in square feet, of each rectangular face is: $(x)\left(\frac{5x}{8}\right) = \frac{5x^2}{8}$

To determine the area of each triangle, first determine its height, h .

Use the Pythagorean Theorem in $\triangle ABC$.

$$h^2 = \left(\frac{5x}{8}\right)^2 - \left(\frac{x}{2}\right)^2$$

$$h^2 = \frac{25x^2}{64} - \frac{x^2}{4}$$



$$h^2 = \frac{25x^2}{64} - \frac{16x^2}{64}$$

$$h^2 = \frac{9x^2}{64}$$

$$h = \sqrt{\frac{9x^2}{64}}$$

$$h = \frac{3x}{8}$$

The area, in square feet, of each triangular face is: $\frac{1}{2}(x)\left(\frac{3x}{8}\right) = \frac{3x^2}{16}$

So, the surface area of the tent is:

$$\begin{aligned} 4(x^2) + 2\left(\frac{5x^2}{8}\right) + 2\left(\frac{3x^2}{16}\right) &= 4x^2 + \frac{10x^2}{8} + \frac{3x^2}{8} \\ &= \frac{32x^2}{8} + \frac{13x^2}{8} \\ &= \frac{45x^2}{8} \end{aligned}$$

The surface area of the tent is $\frac{45x^2}{8}$ square feet.

b) Write an equation.

$$\frac{45x^2}{8} = 90 \quad \text{Multiply each side by 8.}$$

$$45x^2 = 720 \quad \text{Divide each side by 45.}$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = 4$$

16. Let the edge length of the cube be x .

Then the area of one face is x^2 .

And its surface area is $6x^2$.

The volume of the cube is x^3 .

The volume and surface area are equal, so:

$$x^3 = 6x^2$$

Since x is not equal to 0, divide each side by x^2 .

$$x = 6$$

The dimensions of a cube that has its surface area numerically the same as its volume are 6 units by 6 units by 6 units.

17. a) The side length of a square with area $121x^4y^2$ is: $\sqrt{121x^4y^2}$

Factor $121x^4y^2$: $11 \cdot 11 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$

Rearrange the factors in 2 equal groups: $(11 \cdot x \cdot x \cdot y)(11 \cdot x \cdot x \cdot y)$

$$\text{So, } \sqrt{121x^4y^2} = 11x^2y$$

The side length of the square is $11x^2y$.

b) The edge length of a cube with volume $64x^6y^3$ is: $\sqrt[3]{64x^6y^3}$

Factor $64x^6y^3$: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

Rearrange the factors in 3 equal groups:

$(2 \cdot 2 \cdot x \cdot x \cdot y)(2 \cdot 2 \cdot x \cdot x \cdot y)(2 \cdot 2 \cdot x \cdot x \cdot y)$

So, $\sqrt[3]{64x^6y^3} = 4x^2y$

The edge length of the cube is $4x^2y$.

18. List all the perfect cubes up to a number close to 1729:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728

Use guess and check.

The 1st and 12th perfect cubes have the sum: $1 + 1728 = 1729$

The 9th and 10th perfect cubes have the sum: $729 + 1000 = 1729$

No other perfect cubes have the sum 1729.

Checkpoint 1

Assess Your Understanding (page 149)

3.1

1. Use a calculator to divide each number by its prime factors.

a) $1260 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$
 $= 2^2 \cdot 3^2 \cdot 5 \cdot 7$

b) $4224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 11$
 $= 2^7 \cdot 3 \cdot 11$

c) $6120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 17$
 $= 2^3 \cdot 3^2 \cdot 5 \cdot 17$

d) $1045 = 5 \cdot 11 \cdot 19$

e) $3024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7$
 $= 2^4 \cdot 3^3 \cdot 7$

f) $3675 = 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7$
 $= 3 \cdot 5^2 \cdot 7^2$

2. List the factors of each number, then highlight the common factors.

a) $40 = 2 \cdot 2 \cdot 2 \cdot 5$
 $48 = 2 \cdot 2 \cdot 2 \cdot 6$
 $56 = 2 \cdot 2 \cdot 2 \cdot 7$
 The greatest common factor is: $2 \cdot 2 \cdot 2 = 8$

b) $84 = 2 \cdot 2 \cdot 3 \cdot 7$
 $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$
 $144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
 The greatest common factor is: $2 \cdot 2 \cdot 3 = 12$

c) $145 = 5 \cdot 29$
 $205 = 5 \cdot 41$
 $320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$
 The greatest common factor is: 5

d) $208 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 13$
 $368 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 23$
 $528 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 11$
 The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

e) $856 = 2 \cdot 2 \cdot 2 \cdot 107$
 $1200 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5$
 $1368 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 19$
 The greatest common factor is: $2 \cdot 2 \cdot 2 = 8$

f) $950 = 2 \cdot 5 \cdot 5 \cdot 19$
 $1225 = 5 \cdot 5 \cdot 7 \cdot 7$
 $1550 = 2 \cdot 5 \cdot 5 \cdot 31$
 The greatest common factor is: $5 \cdot 5 = 25$

3. Factor each number.

Highlight the greatest power of each prime factor in any list.

The least common multiple is the product of the greatest power of each prime factor.

a) $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

$15 = 3 \cdot 5$

$21 = 3 \cdot 7$

The least common multiple is: $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$

b) $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

$20 = 2 \cdot 2 \cdot 5 = 2^2 \cdot 5$

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

The least common multiple is: $2^5 \cdot 3 \cdot 5 = 480$

c) $18 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$

$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$

$30 = 2 \cdot 3 \cdot 5$

The least common multiple is: $2^3 \cdot 3^2 \cdot 5 = 360$

d) $30 = 2 \cdot 3 \cdot 5$

$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

$40 = 2 \cdot 2 \cdot 2 \cdot 5 = 2^3 \cdot 5$

The least common multiple is: $2^5 \cdot 3 \cdot 5 = 480$

e) $49 = 7 \cdot 7 = 7^2$

$56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

The least common multiple is: $2^6 \cdot 7^2 = 3136$

f) $50 = 2 \cdot 5 \cdot 5 = 2 \cdot 5^2$

$55 = 5 \cdot 11$

$66 = 2 \cdot 3 \cdot 11$

The least common multiple is: $2 \cdot 3 \cdot 5^2 \cdot 11 = 1650$

4. a) $\frac{8}{3} + \frac{5}{11} = \frac{88}{33} + \frac{15}{33}$
 $= \frac{103}{33}$

b) $\frac{13}{5} - \frac{4}{7} = \frac{91}{35} - \frac{20}{35}$
 $= \frac{71}{35}$

c) $\frac{9}{10} \div \frac{7}{3} = \frac{27}{30} \div \frac{70}{30}$
 $= \frac{27}{70}$

5. Find the least common multiple of 365 and 260.

$$365 = 5 \cdot 73$$

$$260 = 2 \cdot 2 \cdot 5 \cdot 13$$

The least common multiple is: $2 \cdot 2 \cdot 5 \cdot 13 \cdot 73 = 18\,980$

The first day on both calendars would occur again after 18 980 days, which is $\frac{18\,980}{365}$ years, or 52 years.

3.2

6. I could list the prime factors, then arrange them in two equal groups, or I could estimate then use guess and check. I will use prime factors.

a) $400 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$$= (2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5)$$

$$= 20 \cdot 20$$

$$\sqrt{400} = 20$$

b) $784 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$

$$= (2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 7)$$

$$= 28 \cdot 28$$

$$\sqrt{784} = 28$$

c) $576 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$$= (2 \cdot 2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 2 \cdot 3)$$

$$= 24 \cdot 24$$

$$\sqrt{576} = 24$$

d) $1089 = 3 \cdot 3 \cdot 11 \cdot 11$

$$= (3 \cdot 11)(3 \cdot 11)$$

$$= 33 \cdot 33$$

$$\sqrt{1089} = 33$$

e) $1521 = 3 \cdot 3 \cdot 13 \cdot 13$

$$= (3 \cdot 13)(3 \cdot 13)$$

$$= 39 \cdot 39$$

$$\sqrt{1521} = 39$$

f) $3025 = 5 \cdot 5 \cdot 11 \cdot 11$

$$= (5 \cdot 11)(5 \cdot 11)$$

$$= 55 \cdot 55$$

$$\sqrt{3025} = 55$$

7. I could list the prime factors, then arrange them in three equal groups, or I could estimate then use guess and check. I will use prime factors.

a) $1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$$= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$$

$$= 12 \cdot 12 \cdot 12$$

$$\sqrt[3]{1728} = 12$$

$$\begin{aligned}\text{b) } 3375 &= 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \\ &= (3 \cdot 5)(3 \cdot 5)(3 \cdot 5) \\ &= 15 \cdot 15 \cdot 15 \\ \sqrt[3]{3375} &= 15\end{aligned}$$

$$\begin{aligned}\text{c) } 8000 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \\ &= (2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5)(2 \cdot 2 \cdot 5) \\ &= 20 \cdot 20 \cdot 20 \\ \sqrt[3]{8000} &= 20\end{aligned}$$

$$\begin{aligned}\text{d) } 5832 &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3) \\ &= 18 \cdot 18 \cdot 18 \\ \sqrt[3]{5832} &= 18\end{aligned}$$

$$\begin{aligned}\text{e) } 10\,648 &= 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \cdot 11 \\ &= (2 \cdot 11)(2 \cdot 11)(2 \cdot 11) \\ &= 22 \cdot 22 \cdot 22 \\ \sqrt[3]{10\,648} &= 22\end{aligned}$$

$$\begin{aligned}\text{f) } 9261 &= 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7 \\ &= (3 \cdot 7)(3 \cdot 7)(3 \cdot 7) \\ &= 21 \cdot 21 \cdot 21 \\ \sqrt[3]{9261} &= 21\end{aligned}$$

8. Use a calculator to write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

a) $2808 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 13$

The factors cannot be arranged in 2 or 3 equal groups, so 2808 is neither a perfect square nor a perfect cube.

b) $3136 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
 $= (2 \cdot 2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 2 \cdot 7)$

The factors can be arranged in 2 equal groups, so 3136 is a perfect square.

c) $4096 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
 $= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
 $= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$

The factors can be arranged in 2 equal groups and 3 equal groups, so 4096 is both a perfect square and a perfect cube.

d) $4624 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 17 \cdot 17$
 $= (2 \cdot 2 \cdot 17)(2 \cdot 2 \cdot 17)$

The factors can be arranged in 2 equal groups, so 4624 is a perfect square.

e) $5832 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
 $= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3)$

The factors can be arranged in 3 equal groups, so 5832 is a perfect cube.

f) $9270 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 103$

The factors cannot be arranged in 2 or 3 equal groups, so 9270 is neither a perfect square nor a perfect cube.

9. Use estimation or guess and check to determine the perfect square and perfect cube closest to the first number in each given pair, then calculate the squares and cubes of all whole numbers until the second number in each pair is reached or exceeded.

a) $400 - 500$

$$20^2 = 400$$

$$21^2 = 441$$

$$22^2 = 484$$

$$23^2 = 529$$

$$7^3 = 343$$

$$8^3 = 512$$

Between 400 and 500, the perfect squares are 441 and 484; there are no perfect cubes.

b) $900 - 1000$

$$30^2 = 900$$

$$31^2 = 961$$

$$32^2 = 1024$$

$$9^3 = 729$$

$$10^3 = 1000$$

Between 900 and 1000, the perfect square is 961; there are no perfect cubes.

c) $1100 - 1175$

$$33^2 = 1089$$

$$34^2 = 1156$$

$$35^2 = 1225$$

$$10^3 = 1000$$

$$11^3 = 1331$$

Between 1100 and 1175, the perfect square is 1156; there are no perfect cubes.

10. The edge length of the cube, in metres, is $\sqrt[3]{2197}$.

$$2197 = 13 \cdot 13 \cdot 13$$

So, $\sqrt[3]{2197} = 13$

The surface area of one face, in square metres, is: $13^2 = 169$

The surface area of the cube, in square metres, is: $6(169) = 1014$

One can of paint covers about 40 m^2 .

So, the number of cans of paint needed is: $\frac{1014}{40} = 25.35$

Twenty-six cans of paint are needed.

Lesson 3.3 Common Factors of a Polynomial

Exercises (pages 155–156)

A

4. a) There are 3 x -tiles and twelve 1-tiles.
So, the polynomial is: $3x + 12$
The length of the rectangle is $x + 4$ and the width is 3; these are the factors.
So, $3x + 12 = 3(x + 4)$
- b) There are 4 x^2 -tiles and 10 x -tiles.
So, the polynomial is: $4x^2 + 10x$
The length of the rectangle is $2x + 5$ and the width is $2x$; these are the factors.
So, $4x^2 + 10x = 2x(2x + 5)$
- c) There are 12 x^2 -tiles, 8 negative x -tiles, and sixteen 1-tiles.
So, the polynomial is: $12x^2 - 8x + 16$
There are 4 equal groups of tiles; 4 is one factor.
Each group of tiles contains 3 x^2 -tiles, 2 negative x -tiles, and four 1-tiles; $3x^2 - 2x + 4$ is the other factor.
So, $12x^2 - 8x + 16 = 4(3x^2 - 2x + 4)$
5. a) $6 = 2 \cdot 3$
 $15n = 3 \cdot 5 \cdot n$
The greatest common factor of 6 and $15n$ is 3.
- b) $4m = 2 \cdot 2 \cdot m$
 $m^2 = m \cdot m$
The greatest common factor of $4m$ and m^2 is m .
6. For each polynomial, write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.
- a) The greatest common factor of 6 and $15n$ is 3.
- i) $6 + 15n = 3(2) + 3(5n)$
 $= 3(2 + 5n)$
- ii) $6 - 15n = 3(2) - 3(5n)$
 $= 3(2 - 5n)$
- iii) $15n - 6 = 3(5n) - 3(2)$
 $= 3(5n - 2)$
- iv) $-15n + 6 = 3(-5n) + 3(2)$
 $= 3(-5n + 2)$
- b) The greatest common factor of $4m$ and m^2 is m .
- i) $4m + m^2 = m(4) + m(m)$
 $= m(4 + m)$
- ii) $m^2 + 4m = m(m) + m(4)$
 $= m(m + 4)$
- iii) $4m - m^2 = m(4) - m(m)$
 $= m(4 - m)$
- iv) $m^2 - 4m = m(m) - m(4)$
 $= m(m - 4)$

B

7. Arrange the algebra tiles in a rectangle; if more than one rectangle is possible, make the rectangle that is closest to a square. Or, make as many equal groups of tiles as possible.

- a) Arrange 5 y -tiles and ten 1-tiles to form a rectangle.

The length of the rectangle is $y + 2$ and the width is 5.

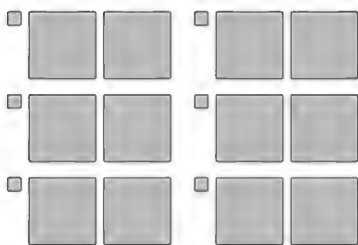
$$5y + 10 = 5(y + 2)$$



- b) Arrange six 1-tiles and 12 x^2 -tiles into equal groups.

There are 6 groups; each contains one 1-tile and 2 x^2 -tiles.

$$6 + 12x^2 = 6(1 + 2x^2)$$



- c) Arrange 9 k -tiles and six 1-tiles in a rectangle.

The length of the rectangle is $3k + 2$ and the width is 3.

$$9k + 6 = 3(3k + 2)$$



- d) Arrange 4 s^2 -tiles and 14 s -tiles in a rectangle.

The length of the rectangle is $2s + 7$ and the width is $2s$.

$$4s^2 + 14s = 2s(2s + 7)$$



- e) Arrange 1 y -tile and 1 y^2 -tile in a rectangle.

The length of the rectangle is $1 + y$ and the width is y .

$$y + y^2 = y(1 + y)$$



- f) Arrange 3 h -tiles and 7 h^2 -tiles in a rectangle.

The length of the rectangle is $3 + 7h$ and the width is h .

$$3h + 7h^2 = h(3 + 7h)$$



8. For each binomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

I cannot use algebra tiles because I do not have x^3 -tiles, or tiles for greater powers of x .

a) $9b^2 - 12b^3$

$$9b^2 = 3 \cdot 3 \cdot b \cdot b$$

$$12b^3 = 2 \cdot 2 \cdot 3 \cdot b \cdot b \cdot b$$

The greatest common factor is: $3 \cdot b \cdot b = 3b^2$

$$9b^2 - 12b^3 = 3b^2(3) - 3b^2(4b)$$

$$= 3b^2(3 - 4b)$$

$$\text{Check: } 3b^2(3 - 4b) = 3b^2(3) - 3b^2(4b)$$

$$= 9b^2 - 12b^3$$

b) $48s^3 - 12$

$$48s^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot b \cdot b$$

$$12 = 2 \cdot 2 \cdot 3$$

The greatest common factor is: $2 \cdot 2 \cdot 3 = 12$

$$48s^3 - 12 = 12(4s^3) - 12(1)$$

$$= 12(4s^3 - 1)$$

$$\text{Check: } 12(4s^3 - 1) = 12(4s^3) - 12(1)$$

$$= 48s^3 - 12$$

c) $-a^2 - a^3$

$$a^2 = a \cdot a$$

$$a^3 = a \cdot a \cdot a$$

The greatest common factor is: $a \cdot a = a^2$

$$-a^2 - a^3 = a^2(-1) - a^2(a)$$

$$= a^2(-1 - a)$$

$$= a^2(-1)(1 + a)$$

$$= -a^2(1 + a)$$

$$\text{Check: } -a^2(1 + a) = -a^2(1) + (-a^2)(a)$$

$$= -a^2 - a^3$$

d) $3x^2 + 6x^4$

$$3x^2 = 3 \cdot x \cdot x$$

$$6x^4 = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x$$

The greatest common factor is: $3 \cdot x \cdot x = 3x^2$

$$3x^2 + 6x^4 = 3x^2(1) + 3x^2(2x^2)$$

$$= 3x^2(1 + 2x^2)$$

$$\text{Check: } 3x^2(1 + 2x^2) = 3x^2(1) + 3x^2(2x^2)$$

$$= 3x^2 + 6x^4$$

e) $8y^3 - 12y$

$$8y^3 = 2 \cdot 2 \cdot 2 \cdot y \cdot y \cdot y$$

$$12y = 2 \cdot 2 \cdot 3 \cdot y$$

The greatest common factor is: $2 \cdot 2 \cdot y = 4y$

$$8y^3 - 12y = 4y(2y^2) - 4y(3)$$

$$= 4y(2y^2 - 3)$$

$$\text{Check: } 4y(2y^2 - 3) = 4y(2y^2) - 4y(3)$$

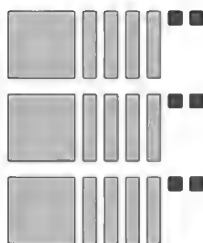
$$= 8y^3 - 12y$$

f) $-7d - 14d^4$
 $7d = 7 \cdot d$
 $14d^4 = 2 \cdot 7 \cdot d \cdot d \cdot d \cdot d$
 The greatest common factor is: $7 \cdot d = 7d$
 $-7d - 14d^4 = 7d(-1) - 7d(2d^3)$
 $= 7d(-1 - 2d^3)$
 $= 7d(-1)(1 + 2d^3)$
 $= -7d(1 + 2d^3)$
 Check: $-7d(1 + 2d^3) = -7d(1) + (-7d)(2d^3)$
 $= -7d - 14d^4$

9. Arrange the algebra tiles in as many equal groups as possible.

a) Arrange 3 x^2 -tiles, 12 x -tiles, and 6 negative 1-tiles into equal groups. There are 3 groups; each contains 1 x^2 -tile, 4 x -tiles, and 2 negative 1-tiles.

$$3x^2 + 12x - 6 = 3(x^2 + 4x - 2)$$



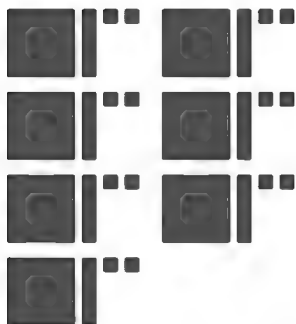
b) Arrange four 1-tiles, 6 negative y -tiles, and 8 negative y^2 -tiles into equal groups. There are 2 groups; each contains two 1-tiles, 3 negative y -tiles, and 4 negative y^2 -tiles.

$$4 - 6y - 8y^2 = 2(2 - 3y - 4y^2)$$



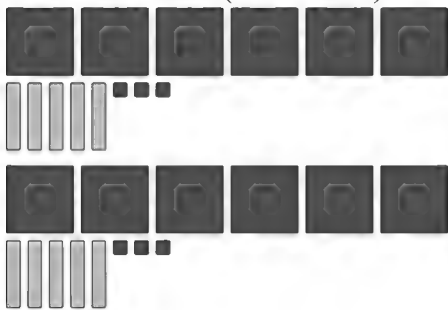
c) Arrange 7 negative m -tiles, 7 negative m^2 -tiles, and 14 negative 1-tiles into equal groups. There are 7 groups; each contains 1 negative m -tile, 1 negative m^2 -tile, and 2 negative 1-tiles.

$$\begin{aligned} -7m - 7m^2 - 14 &= 7(-m - m^2 - 2) \\ &= 7(-1)(m + m^2 + 2) \\ &= -7(m + m^2 + 2) \end{aligned}$$



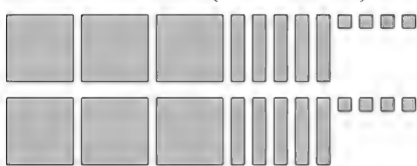
d) Arrange 10 n -tiles, 6 negative 1-tiles, and 12 negative n^2 -tiles into equal groups. There are 2 groups; each contains 5 n -tiles, 3 negative 1-tiles, and 6 negative n^2 -tiles.

$$10n - 6 - 12n^2 = 2(5n - 3 - 6n^2)$$



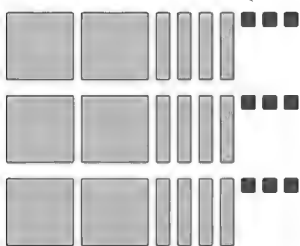
- e) Arrange eight 1-tiles, 10 x -tiles, and 6 x^2 -tiles into equal groups. There are 2 groups; each contains four 1-tiles, 5 x -tiles, and 3 x^2 -tiles.

$$8 + 10x + 6x^2 = 2(4 + 5x + 3x^2)$$



- f) Arrange 9 negative 1-tiles, 12 b -tiles, and 6 b^2 -tiles into equal groups. There are 3 groups; each contains 3 negative 1-tiles, 4 b -tiles, and 2 b^2 -tiles.

$$\begin{aligned} -9 + 12b + 6b^2 &= 3(-3 + 4b + 2b^2) \\ &= 3(-1)(3 - 4b - 2b^2) \\ &= -3(3 - 4b - 2b^2) \end{aligned}$$



10. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

I cannot use algebra tiles because I do not have x^3 -tiles, or tiles for greater powers of x .

a) $5 + 15m^2 - 10m^3$

$$5 = 5$$

$$15m^2 = 3 \cdot 5 \cdot m \cdot m$$

$$10m^3 = 2 \cdot 5 \cdot m \cdot m \cdot m$$

The greatest common factor is: 5

$$5 + 15m^2 - 10m^3 = 5(1) + 5(3m^2) - 5(2m^3)$$

$$= 5(1 + 3m^2 - 2m^3)$$

$$\begin{aligned} \text{Check: } 5(1 + 3m^2 - 2m^3) &= 5(1) + 5(3m^2) - 5(2m^3) \\ &= 5 + 15m^2 - 10m^3 \end{aligned}$$

b) $27n + 36 - 18n^3$

$$27n = 3 \cdot 3 \cdot 3 \cdot n$$

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

$$18n^3 = 2 \cdot 3 \cdot 3 \cdot n \cdot n \cdot n$$

The greatest common factor is: $3 \cdot 3 = 9$
 $27n + 36 - 18n^3 = 9(3n) + 9(4) - 9(2n^3)$
 $= 9(3n + 4 - 2n^3)$
 Check: $9(3n + 4 - 2n^3) = 9(3n) + 9(4) - 9(2n^3)$
 $= 27n + 36 - 18n^3$

c) $6v^4 + 7v - 8v^3$
 $6v^4 = 2 \cdot 3 \cdot v \cdot v \cdot v \cdot v$
 $7v = 7 \cdot v$
 $8v^3 = 2 \cdot 2 \cdot 2 \cdot v \cdot v \cdot v$
 The greatest common factor is: v
 $6v^4 + 7v - 8v^3 = v(6v^3) + v(7) - v(8v^2)$
 $= v(6v^3 + 7 - 8v^2)$
 Check: $v(6v^3 + 7 - 8v^2) = v(6v^3) + v(7) - v(8v^2)$
 $= 6v^4 + 7v - 8v^3$

d) $-3c^2 - 13c^4 - 12c^3$
 $3c^2 = 3 \cdot c \cdot c$
 $13c^4 = 13 \cdot c \cdot c \cdot c \cdot c$
 $12c^3 = 2 \cdot 2 \cdot 3 \cdot c \cdot c \cdot c$
 The greatest common factor is: $c \cdot c = c^2$
 $-3c^2 - 13c^4 - 12c^3 = c^2(-3) - c^2(13c^2) - c^2(12c)$
 $= c^2(-3 - 13c^2 - 12c)$
 $= c^2(-1)(3 + 13c^2 + 12c)$
 $= -c^2(3 + 13c^2 + 12c)$
 Check: $-c^2(3 + 13c^2 + 12c) = -c^2(3) + (-c^2)(13c^2) + (-c^2)(12c)$
 $= -3c^2 - 13c^4 - 12c^3$

e) $24x + 30x^2 - 12x^4$
 $24x = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x$
 $30x^2 = 2 \cdot 3 \cdot 5 \cdot x \cdot x$
 $12x^4 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x$
 The greatest common factor is: $2 \cdot 3 \cdot x = 6x$
 $24x + 30x^2 - 12x^4 = 6x(4) + 6x(5x) - 6x(2x^3)$
 $= 6x(4 + 5x - 2x^3)$
 Check: $6x(4 + 5x - 2x^3) = 6x(4) + 6x(5x) - 6x(2x^3)$
 $= 24x + 30x^2 - 12x^4$

f) $s^4 + s^2 - 4s$
 $s^4 = s \cdot s \cdot s \cdot s$
 $s^2 = s \cdot s$
 $4s = 2 \cdot 2 \cdot s$
 The greatest common factor is: s
 $s^4 + s^2 - 4s = s(s^3) + s(s) - s(4)$
 $= s(s^3 + s - 4)$
 Check: $s(s^3 + s - 4) = s(s^3) + s(s) - s(4)$
 $= s^4 + s^2 - 4s$

11. a) There are 12 negative x^2 -tiles and 20 x -tiles.
 So, the polynomial is: $-12x^2 + 20x$

- b) Factor each term, then identify the greatest common factor.

$$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

$$20x = 2 \cdot 2 \cdot 5 \cdot x$$

The greatest common factor is: $2 \cdot 2 \cdot x = 4x$

Write each term as the product of the greatest common factor and another monomial.

$$-12x^2 + 20x = 4x(-3x) + 4x(5)$$

Use the distributive property to write the expression as a product.

$$\begin{aligned} -12x^2 + 20x &= 4x(-3x + 5) \\ &= 4x(-1)(3x - 5) \\ &= -4x(3x - 5) \end{aligned}$$

- c) The factors can be written in two ways: $4x$ and $-3x + 5$; or $-4x$ and $3x - 5$

The width of the rectangle is $-3x + 5$ and its length is $-4x$.

So, for each way the factors are written, only one of the factors matches the dimensions of the rectangle.

12. a) i) When the student wrote each term as a product of the common factor and another monomial, he probably wrote the 3rd term, $3m$, as $3m(0)$, so when the student used the distributive property, this term disappeared.

The correct solution is:

$$\begin{aligned} 3m^2 + 9m^3 - 3m &= 3m(m) + 3m(3m^2) - 3m(1) \\ &= 3m(m + 3m^2 - 1) \end{aligned}$$

- ii) When the student removed -4 as a common factor, he or she forgot to consider the negative sign when writing the 2nd term as the product of the common factor and another monomial. Also, the student wrote the other monomial for the 3rd term incorrectly.

The correct solution is:

$$\begin{aligned} -16 + 8n - 4n^3 &= -4(4) + (-4)(-2n) + (-4)(n^3) \\ &= -4(4 - 2n + n^3) \end{aligned}$$

- b) The student should have expanded to check that the factored expression is equal to the given expression.

13. The monomial is 1 when the factor is equal to the term; for example, if the factor is $2x$ and the term is $2x$, I write: $2x = 2x(1)$

The monomial is -1 when the factor and the term are opposites; for example, if the factor is $2x$ and the term is $-2x$, I write: $-2x = 2x(-1)$

14. a)
$$\begin{aligned} x^2 + 6x - 7 - x^2 - 2x + 3 &= x^2 - x^2 + 6x - 2x - 7 + 3 \\ &= 4x - 4 \end{aligned}$$

Factor. The common factor is 4.

$$\begin{aligned} 4x - 4 &= 4(x) - 4(1) \\ &= 4(x - 1) \end{aligned}$$

- b)
$$\begin{aligned} 12m^2 - 24m - 3 + 4m^2 - 13 &= 12m^2 + 4m^2 - 24m - 3 - 13 \\ &= 16m^2 - 24m - 16 \end{aligned}$$

Factor. The common factor is 8.

$$\begin{aligned} 16m^2 - 24m - 16 &= 8(2m^2) - 8(3m) - 8(2) \\ &= 8(2m^2 - 3m - 2) \end{aligned}$$

$$\begin{aligned} \text{c) } -7n^3 - 5n^2 + 2n - n^2 - n^3 - 12n &= -7n^3 - n^3 - 5n^2 - n^2 + 2n - 12n \\ &= -8n^3 - 6n^2 - 10n \end{aligned}$$

Factor. The common factor is $2n$.

$$\begin{aligned} -8n^3 - 6n^2 - 10n &= 2n(-4n^2) - 2n(3n) - 2n(5) \\ &= 2n(-4n^2 - 3n - 5) \\ &= 2n(-1)(4n^2 + 3n + 5) \\ &= -2n(4n^2 + 3n + 5) \end{aligned}$$

15. a) i) $4s^2t^2 = 2 \cdot 2 \cdot s \cdot s \cdot t \cdot t$
 $12s^2t^3 = 2 \cdot 2 \cdot 3 \cdot s \cdot s \cdot t \cdot t \cdot t$
 $36st^2 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot s \cdot t \cdot t$
 The greatest common factor is: $2 \cdot 2 \cdot s \cdot t \cdot t = 4st^2$
- ii) $3a^3b = 3 \cdot a \cdot a \cdot a \cdot b$
 $8a^2b = 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot b$
 $9a^4b = 3 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot b$
 The greatest common factor is: $a \cdot a \cdot b = a^2b$
- iii) $12x^3y^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$
 $12x^4y^3 = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$
 $36x^2y^4 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
 The greatest common factor is: $2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$
- b) i) $4s^2t^2 + 12s^2t^3 + 36st^2 = 4st^2(s) + 4st^2(3st) + 4st^2(9)$
 $= 4st^2(s + 3st + 9)$
- ii) $12s^2t^3 - 4s^2t^2 - 36st^2 = 4st^2(3st) - 4st^2(s) - 4st^2(9)$
 $= 4st^2(3st - s - 9)$
- iii) $-3a^3b - 9a^4b + 8a^2b = a^2b(-3a) - a^2b(9a^2) + a^2b(8)$
 $= a^2b(-3a - 9a^2 + 8)$
 $= a^2b(-1)(3a + 9a^2 - 8)$
 $= -a^2b(3a + 9a^2 - 8)$
- iv) $9a^4b + 3a^3b - 8a^2b = a^2b(9a^2) + a^2b(3a) - a^2b(8)$
 $= a^2b(9a^2 + 3a - 8)$
- v) $36x^2y^4 + 12x^3y^2 + 12x^4y^3 = 12x^2y^2(3y^2) + 12x^2y^2(x) + 12x^2y^2(x^2y)$
 $= 12x^2y^2(3y^2 + x + x^2y)$
- vi) $-36x^2y^4 - 12x^4y^3 - 12x^3y^2 = 12x^2y^2(-3y^2) + 12x^2y^2(-x^2y) + 12x^2y^2(-x)$
 $= 12x^2y^2(-3y^2 - x^2y - x)$
 $= 12x^2y^2(-1)(3y^2 + x^2y + x)$
 $= -12x^2y^2(3y^2 + x^2y + x)$

16. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

a) $25xy + 15x^2 - 30x^2y^2$
 $25xy = 5 \cdot 5 \cdot x \cdot y$
 $15x^2 = 3 \cdot 5 \cdot x \cdot x$

$$30x^2y^2 = 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y$$

The greatest common factor is: $5 \cdot x = 5x$

$$\begin{aligned} 25xy + 15x^2 - 30x^2y^2 &= 5x(5y) + 5x(3x) - 5x(6xy^2) \\ &= 5x(5y + 3x - 6xy^2) \end{aligned}$$

$$\begin{aligned} \text{Check: } 5x(5y + 3x - 6xy^2) &= 5x(5y) + 5x(3x) - 5x(6xy^2) \\ &= 25xy + 15x^2 - 30x^2y^2 \end{aligned}$$

b) $51m^2n + 39mn^2 - 72mn$

$$51m^2n = 3 \cdot 17 \cdot m \cdot m \cdot n$$

$$39mn^2 = 3 \cdot 13 \cdot m \cdot n \cdot n$$

$$72mn = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot m \cdot n$$

The greatest common factor is: $3 \cdot m \cdot n = 3mn$

$$\begin{aligned} 51m^2n + 39mn^2 - 72mn &= 3mn(17m) + 3mn(13n) - 3mn(24) \\ &= 3mn(17m + 13n - 24) \end{aligned}$$

$$\begin{aligned} \text{Check: } 3mn(17m + 13n - 24) &= 3mn(17m) + 3mn(13n) - 3mn(24) \\ &= 51m^2n + 39mn^2 - 72mn \end{aligned}$$

c) $9p^4q^2 - 6p^3q^3 + 12p^2q^4$

$$9p^4q^2 = 3 \cdot 3 \cdot p \cdot p \cdot p \cdot p \cdot q \cdot q$$

$$6p^3q^3 = 2 \cdot 3 \cdot p \cdot p \cdot p \cdot q \cdot q \cdot q$$

$$12p^2q^4 = 2 \cdot 2 \cdot 3 \cdot p \cdot p \cdot q \cdot q \cdot q \cdot q$$

The greatest common factor is: $3 \cdot p \cdot p \cdot q \cdot q = 3p^2q^2$

$$\begin{aligned} 9p^4q^2 - 6p^3q^3 + 12p^2q^4 &= 3p^2q^2(3p^2) - 3p^2q^2(2pq) + 3p^2q^2(4q^2) \\ &= 3p^2q^2(3p^2 - 2pq + 4q^2) \end{aligned}$$

$$\begin{aligned} \text{Check: } 3p^2q^2(3p^2 - 2pq + 4q^2) &= 3p^2q^2(3p^2) - 3p^2q^2(2pq) + 3p^2q^2(4q^2) \\ &= 9p^4q^2 - 6p^3q^3 + 12p^2q^4 \end{aligned}$$

d) $10a^3b^2 + 12a^2b^4 - 5a^2b^2$

$$10a^3b^2 = 2 \cdot 5 \cdot a \cdot a \cdot a \cdot b \cdot b$$

$$12a^2b^4 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$$

$$5a^2b^2 = 5 \cdot a \cdot a \cdot b \cdot b$$

The greatest common factor is: $a \cdot a \cdot b \cdot b = a^2b^2$

$$\begin{aligned} 10a^3b^2 + 12a^2b^4 - 5a^2b^2 &= a^2b^2(10a) + a^2b^2(12b^2) - a^2b^2(5) \\ &= a^2b^2(10a + 12b^2 - 5) \end{aligned}$$

$$\begin{aligned} \text{Check: } a^2b^2(10a + 12b^2 - 5) &= a^2b^2(10a) + a^2b^2(12b^2) - a^2b^2(5) \\ &= 10a^3b^2 + 12a^2b^4 - 5a^2b^2 \end{aligned}$$

e) $12cd^2 - 8cd - 20c^2d$

$$12cd^2 = 2 \cdot 2 \cdot 3 \cdot c \cdot d \cdot d$$

$$8cd = 2 \cdot 2 \cdot 2 \cdot c \cdot d$$

$$20c^2d = 2 \cdot 2 \cdot 5 \cdot c \cdot c \cdot d$$

The greatest common factor is: $2 \cdot 2 \cdot c \cdot d = 4cd$

$$\begin{aligned} 12cd^2 - 8cd - 20c^2d &= 4cd(3d) - 4cd(2) - 4cd(5c) \\ &= 4cd(3d - 2 - 5c) \end{aligned}$$

$$\begin{aligned} \text{Check: } 4cd(3d - 2 - 5c) &= 4cd(3d) - 4cd(2) - 4cd(5c) \\ &= 12cd^2 - 8cd - 20c^2d \end{aligned}$$

f) $7r^3s^3 + 14r^2s^2 - 21rs^2$

$$7r^3s^3 = 7 \cdot r \cdot r \cdot r \cdot s \cdot s \cdot s$$

$$14r^2s^2 = 2 \cdot 7 \cdot r \cdot r \cdot s \cdot s$$

$$21rs^2 = 3 \cdot 7 \cdot r \cdot s \cdot s$$

$$\begin{aligned}\text{The greatest common factor is: } 7 \cdot r \cdot s \cdot s &= 7rs^2 \\ 7r^3s^3 + 14r^2s^2 - 21rs^2 &= 7rs^2(r^2s) + 7rs^2(2r) - 7rs^2(3) \\ &= 7rs^2(r^2s + 2r - 3) \\ \text{Check: } 7rs^2(r^2s + 2r - 3) &= 7rs^2(r^2s) + 7rs^2(2r) - 7rs^2(3) \\ &= 7r^3s^3 + 14r^2s^2 - 21rs^2\end{aligned}$$

17. a) Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$SA = 2\pi r^2 + 2\pi rh$$

$$2\pi r^2 = 2 \cdot \pi \cdot r \cdot r$$

$$2\pi rh = 2 \cdot \pi \cdot r \cdot h$$

$$\text{The greatest common factor is: } 2 \cdot \pi \cdot r = 2\pi r$$

$$2\pi r^2 + 2\pi rh = 2\pi r(r) + 2\pi r(h)$$

$$= 2\pi r(r + h)$$

$$\text{So, in factored form, } SA = 2\pi r(r + h)$$

- b) Use: $SA = 2\pi r^2 + 2\pi rh$ Substitute: $r = 12$ and $h = 23$

$$SA = 2\pi(12)^2 + 2\pi(12)(23)$$

$$= 288\pi + 552\pi$$

$$= 840\pi$$

$$= 2638.9378\dots$$

$$\text{Use: } SA = 2\pi r(r + h)$$

$$\text{Substitute: } r = 12 \text{ and } h = 23$$

$$SA = 2\pi(12)(12 + 23)$$

$$= 24\pi(35)$$

$$= 840\pi$$

$$= 2638.9378\dots$$

The surface area of the cylinder is approximately 2639 cm^2 .

The factored form of the formula is more efficient to use because fewer calculations are required, and I can use mental math for some calculations.

18. a) Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$SA = \pi r^2 + \pi rs$$

$$\pi r^2 = \pi \cdot r \cdot r$$

$$\pi rs = \pi \cdot r \cdot s$$

$$\text{The greatest common factor is: } \pi \cdot r = \pi r$$

$$\pi r^2 + \pi rs = \pi r(r) + \pi r(s)$$

$$= \pi r(r + s)$$

$$\text{So, in factored form, } SA = \pi r(r + s)$$

- b) Use: $SA = \pi r^2 + \pi rs$ Substitute: $r = 9$ and $s = 15$

$$SA = \pi(9)^2 + \pi(9)(15)$$

$$= 81\pi + 135\pi$$

$$= 216\pi$$

$$= 678.5840\dots$$

$$\text{Use: } SA = \pi r(r + s)$$

$$\text{Substitute: } r = 9 \text{ and } s = 15$$

$$SA = \pi(9)(9 + 15)$$

$$\begin{aligned}
 &= 9\pi(24) \\
 &= 216\pi \\
 &= 678.5840\dots
 \end{aligned}$$

The surface area of the cone is approximately 679 cm^2 .

The factored form of the formula is more efficient to use because fewer calculations are required, and I can use mental math for some calculations.

19. a) Assume the base of the silo is not included in the surface area.

The surface area of the silo is:

the curved surface area of a cylinder + the curved surface area of a hemisphere

The curved surface area of the cylinder is: $2\pi rh$

The curved surface area of the hemisphere is: $2\pi r^2$

The surface area of the silo is:

$$SA = 2\pi rh + 2\pi r^2$$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$2\pi rh = 2 \cdot \pi \cdot r \cdot h$$

$$2\pi r^2 = 2 \cdot \pi \cdot r \cdot r$$

The greatest common factor is: $2 \cdot \pi \cdot r = 2\pi r$

$$2\pi rh + 2\pi r^2 = 2\pi r(h) + 2\pi r(r)$$

$$= 2\pi r(h + r)$$

I will use the factored form because I have fewer calculations and I can use mental math for some of them.

$$SA = 2\pi r(h + r) \quad \text{Substitute: } r = 6 \text{ and } h = 10$$

$$= 2\pi(6)(10 + 6)$$

$$= 12\pi(16)$$

$$= 192\pi$$

$$= 603.1857$$

The surface area of the silo is approximately 603 m^2 .

- b) The volume of the silo is:

the volume of a cylinder + the volume of a hemisphere

The volume of the cylinder is: $\pi r^2 h$

The volume of the hemisphere is: $\frac{2}{3}\pi r^3$

The volume of the silo is:

$$V = \pi r^2 h + \frac{2}{3}\pi r^3$$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$\pi r^2 h = \pi \cdot r \cdot r \cdot h$$

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \cdot \pi \cdot r \cdot r \cdot r$$

The greatest common factor is: $\pi \cdot r \cdot r = \pi r^2$

$$\pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2(h) + \pi r^2\left(\frac{2}{3}r\right)$$

$$= \pi r^2 \left(h + \frac{2}{3}r \right)$$

So, in factored form, $V = \pi r^2 \left(h + \frac{2}{3}r \right)$

I will use the factored form because I have fewer calculations and I can use mental math for some of them.

$$V = \pi r^2 \left(h + \frac{2}{3}r \right) \quad \text{Substitute: } r = 6 \text{ and } h = 10$$

$$= \pi(6)^2 \left(10 + \frac{2}{3}(6) \right)$$

$$= 36\pi(14)$$

$$= 504\pi$$

$$= 1583.3626\dots$$

The volume of the silo is approximately 1583 m^3 .

20. Factor: $n^2 - n = n(n - 1)$

When n is an integer, $n - 1$ is also an integer, and the product of two integers is an integer.

So, when n is an integer, then $n^2 - n$ is always an integer.

C

21. a) The surface area of a cylinder with base radius r and height h is: $2\pi r^2 + 2\pi rh$

From question 17a, this expression factors as: $2\pi r(r + h)$

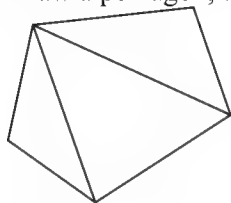
The curved surface area of the cylinder is: $2\pi rh$

The fraction of the surface area that will be painted is: $\frac{2\pi rh}{2\pi r(r + h)}$

b) To simplify the fraction $\frac{2\pi rh}{2\pi r(r + h)}$, divide numerator and denominator by their common factor, $2\pi r$.

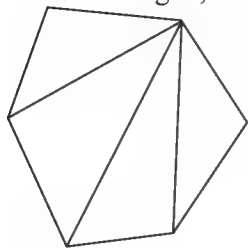
The fraction simplifies to: $\frac{h}{r + h}$

22. a) Draw a pentagon, then draw the diagonals from one vertex.



From the diagram, 2 diagonals can be drawn from one vertex of a pentagon.

Draw a hexagon, then draw the diagonals from one vertex.



From the diagram, 3 diagonals can be drawn from one vertex of a hexagon.

- b)** For a polygon with n sides and n vertices, $(n - 1)$ line segments can be drawn from one vertex to all the other vertices. But two of these line segments are sides of the polygon, so $(n - 3)$ of the line segments are diagonals.
For a polygon with n sides, $(n - 3)$ diagonals can be drawn from one vertex.

c) Factor: $\frac{n^2}{2} - \frac{3n}{2}$

The common factor of the two terms is $\frac{n}{2}$.

$$\begin{aligned}\text{So, } \frac{n^2}{2} - \frac{3n}{2} &= \frac{n}{2}(n) - \frac{n}{2}(3) \\ &= \frac{n}{2}(n - 3)\end{aligned}$$

The total number of diagonals in a polygon can be written as $\frac{n}{2}(n - 3)$.

To explain why this formula is reasonable:

There are n vertices; so from all the vertices, $n(n - 3)$ diagonals can be drawn. But each diagonal has been counted twice, so divide by 2 to get the total number of diagonals:

$$\frac{n}{2}(n - 3)$$

Lesson 3.4 **Math Lab:** **Assess Your Understanding (page 158)**
Modelling Trinomials as Binomial Products

1. a) Try to arrange 1 y^2 -tile, 4 y -tiles, and three 1-tiles in a rectangle. A rectangle is possible.



- b) Try to arrange 1 d^2 -tile, 7 d -tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



- c) Try to arrange 1 m^2 -tile, 7 m -tiles, and seven 1-tiles in a rectangle. A rectangle is not possible.
- d) Try to arrange 1 r^2 -tile, 14 r -tiles, and fourteen 1-tiles in a rectangle. A rectangle is not possible.
- e) Try to arrange 1 t^2 -tile, 6 t -tiles, and six 1-tiles in a rectangle. A rectangle is not possible.
- f) Try to arrange 1 p^2 -tile, 9 p -tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

2. a) Try to arrange 2 s^2 -tiles, 7 s -tiles, and three 1-tiles in a rectangle. A rectangle is possible.



- b) Try to arrange 3 w^2 -tiles, 5 w -tiles, and two 1-tiles in a rectangle. A rectangle is possible.



- c) Try to arrange 2 f^2 -tiles, 3 f -tiles, and two 1-tiles in a rectangle. A rectangle is not possible.
- d) Try to arrange 2 h^2 -tiles, 10 h -tiles, and six 1-tiles in a rectangle. A rectangle is not possible.
- e) Try to arrange 4 n^2 -tiles, 2 n -tiles, and one 1-tile in a rectangle. A rectangle is not possible.

Lesson 3.4 **Math Lab:** **Assess Your Understanding (page 158)**
Modelling Trinomials as Binomial Products

1. a) Try to arrange 1 y^2 -tile, 4 y -tiles, and three 1-tiles in a rectangle. A rectangle is possible.



- b) Try to arrange 1 d^2 -tile, 7 d -tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



- c) Try to arrange 1 m^2 -tile, 7 m -tiles, and seven 1-tiles in a rectangle. A rectangle is not possible.
- d) Try to arrange 1 r^2 -tile, 14 r -tiles, and fourteen 1-tiles in a rectangle. A rectangle is not possible.
- e) Try to arrange 1 t^2 -tile, 6 t -tiles, and six 1-tiles in a rectangle. A rectangle is not possible.
- f) Try to arrange 1 p^2 -tile, 9 p -tiles, and two 1-tiles in a rectangle. A rectangle is not possible.

2. a) Try to arrange 2 s^2 -tiles, 7 s -tiles, and three 1-tiles in a rectangle. A rectangle is possible.

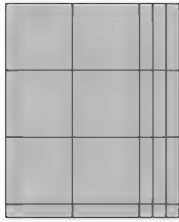


- b) Try to arrange 3 w^2 -tiles, 5 w -tiles, and two 1-tiles in a rectangle. A rectangle is possible.



- c) Try to arrange 2 f^2 -tiles, 3 f -tiles, and two 1-tiles in a rectangle. A rectangle is not possible.
- d) Try to arrange 2 h^2 -tiles, 10 h -tiles, and six 1-tiles in a rectangle. A rectangle is not possible.
- e) Try to arrange 4 n^2 -tiles, 2 n -tiles, and one 1-tile in a rectangle. A rectangle is not possible.

- f) Try to arrange 6 k^2 -tiles, 11 k -tiles, and three 1-tiles in a rectangle. A rectangle is possible.



3. Use the patterns you found in *Construct Understanding*. There are twelve 1-tiles. Write all the pairs of numbers that have a product of 12. The numbers of x -tiles that you could use to make rectangles are equal to the sums of the numbers in these pairs.

$$12 = 1 \cdot 12 \quad 1 + 12 = 13$$

$$12 = 2 \cdot 6 \quad 2 + 6 = 8$$

$$12 = 3 \cdot 4 \quad 3 + 4 = 7$$

These sets of tiles make rectangles:

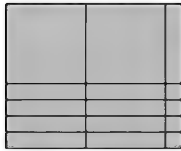
1 x^2 -tile, 13 x -tiles, and twelve 1-tiles

1 x^2 -tile, 8 x -tiles, and twelve 1-tiles

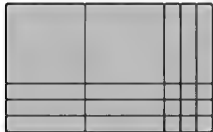
1 x^2 -tile, 7 x -tiles, and twelve 1-tiles

4. Use algebra tiles. Arrange 9 x -tiles on two sides of 2 x^2 -tiles in as many different ways as possible, so that rectangles can be completed by adding 1-tiles. For each arrangement, add 1-tiles to complete a rectangle. Here are the possible arrangements.

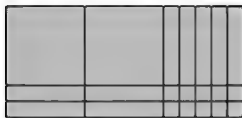
With four 1-tiles:



With nine 1-tiles:



With ten 1-tiles:



With seven 1-tiles:



Lesson 3.5 Polynomials of the Form $x^2 + bx + c$

Exercises (pages 166–167)

A

4. a) There are 1 x^2 -tile, 4 x -tiles, and three 1-tiles.
So, the trinomial is: $x^2 + 4x + 3$
The length of the rectangle is $x + 3$ and the width is $x + 1$.
So, the multiplication sentence is: $(x + 3)(x + 1) = x^2 + 4x + 3$
- b) There are 1 x^2 -tile, 6 x -tiles, and eight 1-tiles.
So, the trinomial is: $x^2 + 6x + 8$
The length of the rectangle is $x + 4$ and the width is $x + 2$.
So, the multiplication sentence is: $(x + 4)(x + 2) = x^2 + 6x + 8$
- c) There are 1 x^2 -tile, 10 x -tiles, and twenty-five 1-tiles.
So, the trinomial is: $x^2 + 10x + 25$
The length of the rectangle is $x + 5$ and the width is also $x + 5$.
So, the multiplication sentence is: $(x + 5)(x + 5) = x^2 + 10x + 25$
- d) There are 1 x^2 -tile, 9 x -tiles, and eighteen 1-tiles.
So, the trinomial is: $x^2 + 9x + 18$
The length of the rectangle is $x + 6$ and the width is $x + 3$.
So, the multiplication sentence is: $(x + 6)(x + 3) = x^2 + 9x + 18$

5. a) Make a rectangle with length $b + 5$ and width $b + 2$.



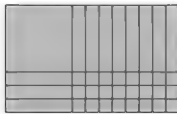
The tiles that form the product are: 1 b^2 -tile, 7 b -tiles, and ten 1-tiles.
So, $(b + 2)(b + 5) = b^2 + 7b + 10$

- b) Make a rectangle with length $n + 7$ and width $n + 4$.



The tiles that form the product are: 1 n^2 -tile, 11 n -tiles, and twenty-eight 1-tiles.
So, $(n + 4)(n + 7) = n^2 + 11n + 28$

- c) Make a rectangle with length $h + 8$ and width $h + 3$.



The tiles that form the product are: 1 h^2 -tile, 11 h -tiles, and twenty-four 1-tiles.
So, $(h + 8)(h + 3) = h^2 + 11h + 24$

- d) Make a rectangle with length $k + 6$ and width $k + 1$.



The tiles that form the product are: 1 k^2 -tile, 7 k -tiles, and six 1-tiles.
So, $(k + 1)(k + 6) = k^2 + 7k + 6$

6. a) i) There are 1 x^2 -tile, 4 x -tiles, and four 1-tiles.
So, the trinomial is: $x^2 + 4x + 4$

ii)



- iii) The length of the rectangle is $x + 2$ and the width is also $x + 2$.
So, the factors of $x^2 + 4x + 4$ are: $(x + 2)(x + 2)$

- b) i) There are 1 x^2 -tile, 5 x -tiles, and four 1-tiles.
So, the trinomial is: $x^2 + 5x + 4$

ii)



- iii) The length of the rectangle is $x + 4$ and the width is $x + 1$.
So, the factors of $x^2 + 5x + 4$ are: $(x + 4)(x + 1)$

- c) i) There are 1 x^2 -tile, 6 x -tiles, and eight 1-tiles.
So, the trinomial is: $x^2 + 6x + 8$

ii)



- iii) The length of the rectangle is $x + 4$ and the width is $x + 2$.
So, the factors of $x^2 + 6x + 8$ are: $(x + 4)(x + 2)$

- d) i) There are 1 x^2 -tile, 7 x -tiles, and twelve 1-tiles.
So, the trinomial is: $x^2 + 7x + 12$

ii)



- iii) The length of the rectangle is $x + 4$ and the width is $x + 3$.
So, the factors of $x^2 + 7x + 12$ are: $(x + 4)(x + 3)$

7. a) i) Only two numbers have a product of 2; the numbers are 1 and 2.
The sum is: $1 + 2 = 3$
So, $a = 1$ and $b = 2$

- ii) The numbers with a product of 6 are: 1 and 6; 2 and 3
The numbers with a sum of 5 are 2 and 3.
So, $a = 2$ and $b = 3$
- iii) The numbers with a product of 9 are: 1 and 9; 3 and 3
The numbers with a sum of 10 are 1 and 9.
So, $a = 1$ and $b = 9$
- iv) The numbers with a product of 10 are: 1 and 10; 2 and 5
The numbers with a sum of 7 are 2 and 5.
So, $a = 2$ and $b = 5$
- v) The numbers with a product of 12 are: 1 and 12; 2 and 6; 3 and 4
The numbers with a sum of 7 are 3 and 4.
So, $a = 3$ and $b = 4$
- vi) The numbers with a product of 15 are: 1 and 15; 3 and 5
The numbers with a sum of 8 are 3 and 5.
So, $a = 3$ and $b = 5$
- b) i) To factor $v^2 + 3v + 2$, find two numbers with a sum of 3 and a product of 2.
From part a, i, these numbers are 1 and 2.
So, $v^2 + 3v + 2 = (v + 1)(v + 2)$
- ii) To factor $w^2 + 5w + 6$, find two numbers with a sum of 5 and a product of 6.
From part a, ii, these numbers are 2 and 3.
So, $w^2 + 5w + 6 = (w + 2)(w + 3)$
- iii) To factor $s^2 + 10s + 9$, find two numbers with a sum of 10 and a product of 9.
From part a, iii, these numbers are 1 and 9.
So, $s^2 + 10s + 9 = (s + 1)(s + 9)$
- iv) To factor $t^2 + 7t + 10$, find two numbers with a sum of 7 and a product of 10.
From part a, iv, these numbers are 2 and 5.
So, $t^2 + 7t + 10 = (t + 2)(t + 5)$
- v) To factor $y^2 + 7y + 12$, find two numbers with a sum of 7 and a product of 12.
From part a, v, these numbers are 3 and 4.
So, $y^2 + 7y + 12 = (y + 3)(y + 4)$
- vi) To factor $h^2 + 8h + 15$, find two numbers with a sum of 8 and a product of 15.
From part a, vi, these numbers are 3 and 5.
So, $h^2 + 8h + 15 = (h + 3)(h + 5)$

B

8. a) i) Make a rectangle with 1 v^2 -tile, 2 v -tiles, and one 1-tile.



The length of the rectangle is $v + 1$ and the width is also $v + 1$.
So, $v^2 + 2v + 1 = (v + 1)(v + 1)$

- ii) Make a rectangle with 1 v^2 -tile, 4 v -tiles, and four 1-tiles.



The length of the rectangle is $v + 2$ and the width is also $v + 2$.
So, $v^2 + 4v + 4 = (v + 2)(v + 2)$

- iii) Make a rectangle with 1 v^2 -tile, 6 v -tiles, and nine 1-tiles.



The length of the rectangle is $v + 3$ and the width is also $v + 3$.
So, $v^2 + 6v + 9 = (v + 3)(v + 3)$

- iv) Make a rectangle with 1 v^2 -tile, 8 v -tiles, and sixteen 1-tiles.



The length of the rectangle is $v + 4$ and the width is also $v + 4$.
So, $v^2 + 8v + 16 = (v + 4)(v + 4)$

- b) All the rectangles are squares. For each trinomial, its binomial factors are equal because the sides of a square are equal.

- c) In the trinomials in part a, the coefficients of the v -terms are consecutive even numbers and the constant terms are the squares of one-half of these coefficients.

So, the next 3 trinomials and their factors are:

$$v^2 + 10v + 25 = (v + 5)(v + 5)$$

$$v^2 + 12v + 36 = (v + 6)(v + 6)$$

$$v^2 + 14v + 49 = (v + 7)(v + 7)$$

9. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(m + 5)(m + 8) = m(m + 8) + 5(m + 8)$
 $= m(m) + m(8) + 5(m) + 5(8)$
 $= m^2 + 8m + 5m + 40$

$$= m^2 + 13m + 40$$

	m	8
m	$(m)(m) = m^2$	$(m)(8) = 8m$
5	$(5)(m) = 5m$	$(5)(8) = 40$

b) $(y + 9)(y + 3) = y(y + 3) + 9(y + 3)$
 $= y(y) + y(3) + 9(y) + 9(3)$
 $= y^2 + 3y + 9y + 27$
 $= y^2 + 12y + 27$

	y	3
y	$(y)(y) = y^2$	$(y)(3) = 3y$
9	$(9)(y) = 9y$	$(9)(3) = 27$

c) $(w + 2)(w + 16) = w(w + 16) + 2(w + 16)$
 $= w(w) + w(16) + 2(w) + 2(16)$
 $= w^2 + 16w + 2w + 32$
 $= w^2 + 18w + 32$

	w	16
w	$(w)(w) = w^2$	$(w)(16) = 16w$
2	$(2)(w) = 2w$	$(2)(16) = 32$

d) $(k + 13)(k + 1) = k(k + 1) + 13(k + 1)$
 $= k(k) + k(1) + 13(k) + 13(1)$
 $= k^2 + 1k + 13k + 13$

$$= k^2 + 14k + 13$$

	k	1
k	$(k)(k) = k^2$	$(k)(1) = k$
13	$(13)(k) = 13k$	$(13)(1) = 13$

10. a) $(w + 3)(w + 2) = w^2 + \square w + 6$
 The coefficient of w in the trinomial is the sum of the constant terms in the binomial factors.
 So, $\square = 3 + 2$, or 5
 Then, $(w + 3)(w + 2) = w^2 + 5w + 6$

- b) $(x + 5)(x + \square) = x^2 + \bigcirc x + 10$
 The product of the constant terms in the binomial factors is equal to the constant term in the trinomial.
 This means that 5 times the $\square$ in the binomial factor is 10.
 So, $\square = 2$
 The coefficient of x in the trinomial is the sum of the constant terms in the binomial factors.
 So, $\bigcirc = 5 + 2$, or 7
 Then, $(x + 5)(x + 2) = x^2 + 7x + 10$

- c) $(y + \bigcirc)(y + \square) = y^2 + 12y + 20$
 The constant terms in the binomial factors have a sum of 12 and a product of 20.
 Two numbers with a product of 20 are: 1 and 20; 2 and 10; 4 and 5
 The two numbers with a sum of 12 are 2 and 10.
 So, $\bigcirc = 2$ and $\square = 10$
 Then, $(y + 2)(y + 10) = y^2 + 12y + 20$

11. For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

- a) $x^2 + 10x + 24$
 Since all the terms are positive, consider only positive factors of 24.
 The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6
 The two factors with a sum of 10 are 4 and 6.
 So, $x^2 + 10x + 24 = (x + 4)(x + 6)$
 Check by expanding.

$$\begin{aligned} (x + 4)(x + 6) &= x(x + 6) + 4(x + 6) \\ &= x(x) + x(6) + 4(x) + 4(6) \\ &= x^2 + 6x + 4x + 24 \\ &= x^2 + 10x + 24 \end{aligned}$$

b) $m^2 + 10m + 16$

Since all the terms are positive, consider only positive factors of 16.

The factors of 16 are: 1 and 16; 2 and 8; 4 and 4

The two factors with a sum of 10 are 2 and 8.

So, $m^2 + 10m + 16 = (m + 2)(m + 8)$

Check by expanding.

$$\begin{aligned}(m + 2)(m + 8) &= m(m + 8) + 2(m + 8) \\ &= m(m) + m(8) + 2(m) + 2(8) \\ &= m^2 + 8m + 2m + 16 \\ &= m^2 + 10m + 16\end{aligned}$$

c) $p^2 + 13p + 12$

Since all the terms are positive, consider only positive factors of 12.

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

The two factors with a sum of 13 are 1 and 12.

So, $p^2 + 13p + 12 = (p + 1)(p + 12)$

Check by expanding.

$$\begin{aligned}(p + 1)(p + 12) &= p(p + 12) + 1(p + 12) \\ &= p(p) + p(12) + 1(p) + 1(12) \\ &= p^2 + 12p + 1p + 12 \\ &= p^2 + 13p + 12\end{aligned}$$

d) $s^2 + 12s + 20$

Since all the terms are positive, consider only positive factors of 20.

The factors of 20 are: 1 and 20; 2 and 10; 4 and 5

The two factors with a sum of 12 are 2 and 10.

So, $s^2 + 12s + 20 = (s + 2)(s + 10)$

Check by expanding.

$$\begin{aligned}(s + 2)(s + 10) &= s(s + 10) + 2(s + 10) \\ &= s(s) + s(10) + 2(s) + 2(10) \\ &= s^2 + 10s + 2s + 20 \\ &= s^2 + 12s + 20\end{aligned}$$

e) $n^2 + 12n + 11$

Since all the terms are positive, consider only positive factors of 11.

The factors of 11 are: 1 and 11; these numbers have a sum of 12.

So, $n^2 + 12n + 11 = (n + 1)(n + 11)$

Check by expanding.

$$\begin{aligned}(n + 1)(n + 11) &= n(n + 11) + 1(n + 11) \\ &= n(n) + n(11) + 1(n) + 1(11) \\ &= n^2 + 11n + 1n + 11 \\ &= n^2 + 12n + 11\end{aligned}$$

f) $h^2 + 8h + 12$

Since all the terms are positive, consider only positive factors of 12.

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

The two factors with a sum of 8 are 2 and 6.

So, $h^2 + 8h + 12 = (h + 2)(h + 6)$

Check by expanding.

$$\begin{aligned}(h + 2)(h + 6) &= h(h + 6) + 2(h + 6) \\ &= h(h) + h(6) + 2(h) + 2(6)\end{aligned}$$

$$= h^2 + 6h + 2h + 12$$

$$= h^2 + 8h + 12$$

g) $q^2 + 7q + 6$

Since all the terms are positive, consider only positive factors of 6.

The factors of 6 are: 1 and 6; 2 and 3

The two factors with a sum of 7 are 1 and 6.

So, $q^2 + 7q + 6 = (q + 1)(q + 6)$

Check by expanding.

$$(q + 1)(q + 6) = q(q + 6) + 1(q + 6)$$

$$= q(q) + q(6) + 1(q) + 1(6)$$

$$= q^2 + 6q + 1q + 6$$

$$= q^2 + 7q + 6$$

h) $b^2 + 11b + 18$

Since all the terms are positive, consider only positive factors of 18.

The factors of 18 are: 1 and 18; 2 and 9; 3 and 6

The two factors with a sum of 11 are 2 and 9.

So, $b^2 + 11b + 18 = (b + 2)(b + 9)$

Check by expanding.

$$(b + 2)(b + 9) = b(b + 9) + 2(b + 9)$$

$$= b(b) + b(9) + 2(b) + 2(9)$$

$$= b^2 + 9b + 2b + 18$$

$$= b^2 + 11b + 18$$

12. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(g - 3)(g + 7) = g(g + 7) - 3(g + 7)$

$$= g(g) + g(7) - 3(g) - 3(7)$$

$$= g^2 + 7g - 3g - 21$$

$$= g^2 + 4g - 21$$

	g	7
g	$(g)(g) = g^2$	$(g)(7) = 7g$
-3	$(-3)(g) = -3g$	$(-3)(7) = -21$

b) $(h + 2)(h - 7) = h(h - 7) + 2(h - 7)$

$$= h(h) + h(-7) + 2(h) + 2(-7)$$

$$= h^2 - 7h + 2h - 14$$

$$= h^2 - 5h - 14$$

	h	-7
h	$(h)(h) = h^2$	$(h)(-7) = -7h$
2	$(2)(h) = 2h$	$(2)(-7) = -14$

c) $(11 - j)(2 - j) = 11(2 - j) - j(2 - j)$

$$= 11(2) + 11(-j) - j(2) - j(-j)$$

$$= 22 - 11j - 2j + j^2$$

$$= 22 - 13j + j^2$$

	2		-j
11	$(11)(2) = 22$	$(11)(-j) = -11j$	
-j	$(-j)(2) = -2j$	$(-j)(-j) = j^2$	

d) $(k-3)(k+11) = k(k+11) - 3(k+11)$

$$= k(k) + k(11) - 3(k) - 3(11)$$

$$= k^2 + 11k - 3k - 33$$

$$= k^2 + 8k - 33$$

	k		11
k	$(k)(k) = k^2$	$(k)(11) = 11k$	
-3	$(-3)(k) = -3k$	$(-3)(11) = -33$	

e) $(12+h)(7-h) = 12(7-h) + h(7-h)$

$$= 12(7) + 12(-h) + h(7) + h(-h)$$

$$= 84 - 12h + 7h - h^2$$

$$= 84 - 5h - h^2$$

	7		-h
12	$(12)(7) = 84$	$(12)(-h) = -12h$	
h	$(h)(7) = 7h$	$(h)(-h) = -h^2$	

f) $(m-9)(m+9) = m(m+9) - 9(m+9)$

$$= m(m) + m(9) - 9(m) - 9(9)$$

$$= m^2 + 9m - 9m - 81$$

$$= m^2 - 81$$

	m		9
m	$(m)(m) = m^2$	$(m)(9) = 9m$	
-9	$(-9)(m) = -9m$	$(-9)(9) = -81$	

g) $(n-14)(n-4) = n(n-4) - 14(n-4)$

$$= n(n) + n(-4) - 14(n) - 14(-4)$$

$$= n^2 - 4n - 14n + 56$$

$$= n^2 - 18n + 56$$

	n		-4
n	$(n)(n) = n^2$	$(n)(-4) = -4n$	
-14	$(-14)(n) = -14n$	$(-14)(-4) = 56$	

$$\begin{aligned}
 \text{h) } (p+6)(p-17) &= p(p-17) + 6(p-17) \\
 &= p(p) + p(-17) + 6(p) + 6(-17) \\
 &= p^2 - 17p + 6p - 102 \\
 &= p^2 - 11p - 102
 \end{aligned}$$

	p	
p	$(p)(p) = p^2$	$(p)(-17) = -17p$
6	$(6)(p) = 6p$	$(6)(-17) = -102$

13. a) In the second line of the solution, the product of -13 and 4 is -52 ; and the sum $+4r - 13r$ is $-9r$. The correct solution is:

$$\begin{aligned}
 (r-13)(r+4) &= r(r+4) - 13(r+4) \\
 &= r^2 + 4r - 13r - 52 \\
 &= r^2 - 9r - 52
 \end{aligned}$$

- b) In the first line of the solution, on the right side of the equals sign, both binomial factors should be $(s-5)$; and the sign between the products of a monomial and a binomial should be negative. The correct solution is:

$$\begin{aligned}
 (s-15)(s-5) &= s(s-5) - 15(s-5) \\
 &= s^2 - 5s - 15s + 75 \\
 &= s^2 - 20s + 75
 \end{aligned}$$

14. For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) $b^2 + 19b - 20$

The factors of -20 are: 1 and -20 ; -1 and 20 ; 2 and -10 ; -2 and 10 ; 4 and -5 ; -4 and 5
Use mental math to calculate each sum.

The two factors with a sum of 19 are -1 and 20 .

So, $b^2 + 19b - 20 = (b-1)(b+20)$

Check by expanding.

$$\begin{aligned}
 (b-1)(b+20) &= b(b+20) - 1(b+20) \\
 &= b(b) + b(20) - 1(b) - 1(20) \\
 &= b^2 + 20b - 1b - 20 \\
 &= b^2 + 19b - 20
 \end{aligned}$$

b) $t^2 + 15t - 54$

The factors of -54 are: 1 and -54 ; -1 and 54 ; 2 and -27 ; -2 and 27 ; 3 and -18 ; -3 and 18 ; 6 and -9 ; -6 and 9

Use mental math to calculate each sum.

The two factors with a sum of 15 are -3 and 18 .

So, $t^2 + 15t - 54 = (t-3)(t+18)$

Check by expanding.

$$\begin{aligned}
 (t-3)(t+18) &= t(t+18) - 3(t+18) \\
 &= t(t) + t(18) - 3(t) - 3(18) \\
 &= t^2 + 18t - 3t - 54 \\
 &= t^2 + 15t - 54
 \end{aligned}$$

c) $x^2 + 12x - 28$

The factors of -28 are: 1 and -28 ; -1 and 28 ; 2 and -14 ; -2 and 14 ; 4 and -7 ; -4 and 7

Use mental math to calculate each sum.

The two factors with a sum of 12 are -2 and 14 .

So, $x^2 + 12x - 28 = (x - 2)(x + 14)$

Check by expanding.

$$\begin{aligned}(x - 2)(x + 14) &= x(x + 14) - 2(x + 14) \\ &= x(x) + x(14) - 2(x) - 2(14) \\ &= x^2 + 14x - 2x - 28 \\ &= x^2 + 12x - 28\end{aligned}$$

d) $n^2 - 5n - 24$

The factors of -24 are: 1 and -24 ; -1 and 24 ; 2 and -12 ; -2 and 12 ; 3 and -8 ; -3 and 8 ;

4 and -6 ; -4 and 6

Use mental math to calculate each sum.

The two factors with a sum of -5 are 3 and -8 .

So, $n^2 - 5n - 24 = (n + 3)(n - 8)$

Check by expanding.

$$\begin{aligned}(n + 3)(n - 8) &= n(n - 8) + 3(n - 8) \\ &= n(n) + n(-8) + 3(n) + 3(-8) \\ &= n^2 - 8n + 3n - 24 \\ &= n^2 - 5n - 24\end{aligned}$$

e) $a^2 - a - 20$

The factors of -20 are: 1 and -20 ; -1 and 20 ; 2 and -10 ; -2 and 10 ; 4 and -5 ; -4 and 5

Use mental math to calculate each sum.

The two factors with a sum of -1 are 4 and -5 .

So, $a^2 - a - 20 = (a + 4)(a - 5)$

Check by expanding.

$$\begin{aligned}(a + 4)(a - 5) &= a(a - 5) + 4(a - 5) \\ &= a(a) + a(-5) + 4(a) + 4(-5) \\ &= a^2 - 5a + 4a - 20 \\ &= a^2 - a - 20\end{aligned}$$

f) $y^2 - 2y - 48$

The factors of -48 are: 1 and -48 ; -1 and 48 ; 2 and -24 ; -2 and 24 ; 3 and -16 ; -3 and 16 ;

4 and -12 ; -4 and 12 ; 6 and -8 ; -6 and 8

Use mental math to calculate each sum.

The two factors with a sum of -2 are 6 and -8 .

So, $y^2 - 2y - 48 = (y + 6)(y - 8)$

Check by expanding.

$$\begin{aligned}(y + 6)(y - 8) &= y(y - 8) + 6(y - 8) \\ &= y(y) + y(-8) + 6(y) + 6(-8) \\ &= y^2 - 8y + 6y - 48 \\ &= y^2 - 2y - 48\end{aligned}$$

g) $m^2 - 15m + 50$

Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of m is negative, both numbers are negative. So, list only the negative factors of 50.

The negative factors of 50 are: -1 and -50 ; -2 and -25 ; -5 and -10

Use mental math to calculate each sum.

The two factors with a sum of -15 are -5 and -10 .

So, $m^2 - 15m + 50 = (m - 5)(m - 10)$

Check by expanding.

$$\begin{aligned}(m - 5)(m - 10) &= m(m - 10) - 5(m - 10) \\ &= m(m) + m(-10) - 5(m) - 5(-10) \\ &= m^2 - 10m - 5m + 50 \\ &= m^2 - 15m + 50\end{aligned}$$

h) $a^2 - 12a + 36$

Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of a is negative, both numbers are negative. So, list only the negative factors of 36.

The negative factors of 36 are: -1 and -36 ; -2 and -18 ; -3 and -12 ; -4 and -9 ; -6 and -6

Use mental math to calculate each sum.

The two factors with a sum of -12 are -6 and -6 .

So, $a^2 - 12a + 36 = (a - 6)(a - 6)$

Check by expanding.

$$\begin{aligned}(a - 6)(a - 6) &= a(a - 6) - 6(a - 6) \\ &= a(a) + a(-6) - 6(a) - 6(-6) \\ &= a^2 - 6a - 6a + 36 \\ &= a^2 - 12a + 36\end{aligned}$$

15. For each trinomial, find two numbers whose product is equal to the constant term and whose sum is equal to the coefficient of the middle term. These numbers are the constants in the binomial factors.

a) $12 + 13k + k^2$

Since all the terms are positive, consider only positive factors of 12.

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

The two factors with a sum of 13 are 1 and 12.

So, $12 + 13k + k^2 = (1 + k)(12 + k)$

Check by expanding.

$$\begin{aligned}(1 + k)(12 + k) &= 1(12 + k) + k(12 + k) \\ &= 1(12) + 1(k) + k(12) + k(k) \\ &= 12 + 1k + 12k + k^2 \\ &= 12 + 13k + k^2\end{aligned}$$

b) $-16 - 6g + g^2$

The factors of -16 are: 1 and -16 ; -1 and 16 ; 2 and -8 ; -2 and 8 ; 4 and -4

The two factors with a sum of -6 are 2 and -8 .

So, $-16 - 6g + g^2 = (2 + g)(-8 + g)$

Check by expanding.

$$\begin{aligned}(2 + g)(-8 + g) &= 2(-8 + g) + g(-8 + g) \\ &= 2(-8) + 2(g) + g(-8) + g(g) \\ &= -16 + 2g - 8g + g^2 \\ &= -16 - 6g + g^2\end{aligned}$$

c) $60 + 17y + y^2$

Since all the terms are positive, consider only positive factors of 60.

The factors of 60 are: 1 and 60; 2 and 30; 3 and 20; 4 and 15; 5 and 12; 6 and 10

The two factors with a sum of 17 are 5 and 12.

So, $60 + 17y + y^2 = (5 + y)(12 + y)$

Check by expanding.

$$\begin{aligned}(5 + y)(12 + y) &= 5(12 + y) + y(12 + y) \\ &= 5(12) + 5(y) + y(12) + y(y) \\ &= 60 + 5y + 12y + y^2 \\ &= 60 + 17y + y^2\end{aligned}$$

d) $72 - z - z^2$

Remove -1 as a common factor to make the z^2 -term positive.

$$72 - z - z^2 = -1(-72 + z + z^2)$$

The factors of -72 are: 1 and -72 ; -1 and 72 ; 2 and -36 ; -2 and 36 ; 3 and -24 ; -3 and 24 ; 4 and -18 ; -4 and 18 ; 6 and -12 ; -6 and 12 ; 8 and -9 ; -8 and 9

The two factors with a sum of 1 are -8 and 9 .

$$\begin{aligned}\text{So, } 72 - z - z^2 &= -(-8 + z)(9 + z) \\ &= (8 - z)(9 + z)\end{aligned}$$

Check by expanding.

$$\begin{aligned}(8 - z)(9 + z) &= 8(9 + z) - z(9 + z) \\ &= 8(9) + 8(z) - z(9) - z(z) \\ &= 72 + 8z - 9z - z^2 \\ &= 72 - z - z^2\end{aligned}$$

16. a) i) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$
 $= x(x) + x(2) + 1(x) + 1(2)$
 $= x^2 + 2x + 1x + 2$
 $= x^2 + 3x + 2$
 $11 \cdot 12 = (10 + 1)(10 + 2)$
 $= 10(10 + 2) + 1(10 + 2)$
 $= 10(10) + 10(2) + 1(10) + 1(2)$
 $= 100 + 20 + 10 + 2$
 $= 132$

ii) $(x + 1)(x + 3) = x(x + 3) + 1(x + 3)$
 $= x(x) + x(3) + 1(x) + 1(3)$
 $= x^2 + 3x + 1x + 3$
 $= x^2 + 4x + 3$
 $11 \cdot 13 = (10 + 1)(10 + 3)$
 $= 10(10 + 3) + 1(10 + 3)$
 $= 10(10) + 10(3) + 1(10) + 1(3)$
 $= 100 + 30 + 10 + 3$
 $= 143$

- b) For each pair of products, if x is substituted for 10, then the products are equal. The coefficients in the terms of the trinomial are the same as the digits in the product of the whole numbers.

17. a) The constant terms in the binomials have a sum of -17 and a product of 60. Their sum should be -7 and their product should be -60 . So, these constant terms should be 5 and -12 . The correct solution is:
 $m^2 - 7m - 60 = (m + 5)(m - 12)$

- b) The constant terms in the binomials have a sum of -12 and a product of -45 . Their sum should be -14 and their product should be 45. So, these constant terms should be -5 and -9 . The correct solution is:
 $w^2 - 14w + 45 = (w - 5)(w - 9)$

- c) The constant terms in the binomials have a sum of -9 and a product of -36 . Their sum should be 9. So, these constant terms should be -3 and 12. The correct solution is:
 $b^2 + 9b - 36 = (b - 3)(b + 12)$

18. a) i) $(t + 4)(t + 7) = t(t + 7) + 4(t + 7)$
 $= t^2 + 7t + 4t + 28$
 $= t^2 + 11t + 28$

ii) $(t - 4)(t - 7) = t(t - 7) - 4(t - 7)$
 $= t^2 - 7t - 4t + 28$
 $= t^2 - 11t + 28$

iii) $(t - 4)(t + 7) = t(t + 7) - 4(t + 7)$
 $= t^2 + 7t - 4t - 28$
 $= t^2 + 3t - 28$

$$\begin{aligned}\text{iv)} \quad (t+4)(t-7) &= t(t-7) + 4(t-7) \\ &= t^2 - 7t + 4t - 28 \\ &= t^2 - 3t - 28\end{aligned}$$

- b) i) The constant terms in the trinomials in parts i and ii above are positive because the constant terms in the binomials have the same sign.
- ii) The constant terms in the trinomials in parts iii and iv above are negative because the constant terms in the binomials have opposite signs.
- iii) The coefficient of the t -term in the trinomial is the sum of the constant terms in the binomials.

19. To be able to factor each trinomial, the coefficient of the middle term must be the sum of the factors of the constant term.

a) $x^2 + \square x + 10$

The factors of 10 are: 1 and 10; -1 and -10; 2 and 5; -2 and -5

The sums of the factors are: $1 + 10 = 11$; $-1 - 10 = -11$; $2 + 5 = 7$; $-2 - 5 = -7$

So, $\square = 11, -11, 7, \text{ or } -7$

b) $a^2 + \square a - 9$

The factors of -9 are: 1 and -9; -1 and 9; 3 and -3

The sums of the factors are: $1 - 9 = -8$; $-1 + 9 = 8$; $3 - 3 = 0$

So, $\square = -8, 8, \text{ or } 0$

c) $t^2 + \square t + 8$

The factors of 8 are: 1 and 8; -1 and -8; 2 and 4; -2 and -4

The sums of the factors are: $1 + 8 = 9$; $-1 - 8 = -9$; $2 + 4 = 6$; $-2 - 4 = -6$

So, $\square = 9, -9, 6, \text{ or } -6$

d) $y^2 + \square y - 12$

The factors of -12 are: 1 and -12; -1 and 12; 2 and -6; -2 and 6; 3 and -4; -3 and 4

The sums of the factors are: $1 - 12 = -11$; $-1 + 12 = 11$; $2 - 6 = -4$; $-2 + 6 = 4$;

$3 - 4 = -1$; $-3 + 4 = 1$

So, $\square = -11, 11, -4, 4, -1, \text{ or } 1$

e) $h^2 + \square h + 18$

The factors of 18 are: 1 and 18; -1 and -18; 2 and 9; -2 and -9; 3 and 6; -3 and -6

The sums of the factors are: $1 + 18 = 19$; $-1 - 18 = -19$; $2 + 9 = 11$; $-2 - 9 = -11$;

$3 + 6 = 9$; $-3 - 6 = -9$

So, $\square = 19, -19, 11, -11, 9, \text{ or } -9$

f) $p^2 + \square p - 16$

The factors of -16 are: 1 and -16; -1 and 16; 2 and -8; -2 and 8; 4 and -4

The sums of the factors are: $1 - 16 = -15$; $-1 + 16 = 15$; $2 - 8 = -6$; $-2 + 8 = 6$;

$4 - 4 = 0$

So, $\square = -15, 15, -6, 6, \text{ or } 0$

20. To be able to factor each trinomial, the constant term must be the product of two numbers whose sum is the coefficient of the middle term.

a) $r^2 + r + \square$

Two numbers whose sum is 1 are: $0 + 1$; $2 - 1$; $3 - 2$; $4 - 3$; $5 - 4$; and so on

The products of these numbers are: $0(1) = 0$; $2(-1) = -2$; $3(-2) = -6$; $4(-3) = -12$; $5(-4) = -20$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, -2, -6, -12, and -20

b) $h^2 - h + \square$

Two numbers whose sum is -1 are: $0 - 1$; $1 - 2$; $2 - 3$; $3 - 4$; $4 - 5$; and so on

The products of these numbers are: $0(-1) = 0$; $1(-2) = -2$; $2(-3) = -6$; $3(-4) = -12$; $4(-5) = -20$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, -2, -6, -12, and -20

c) $b^2 + 2b + \square$

Two numbers whose sum is 2 are: $0 + 2$; $1 + 1$; $3 - 1$; $4 - 2$; $5 - 3$; and so on

The products of these numbers are: $0(2) = 0$; $1(1) = 1$; $3(-1) = -3$; $4(-2) = -8$; $5(-3) = -15$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, 1, -3, -8, and -15

d) $z^2 - 2z + \square$

Two numbers whose sum is -2 are: $0 - 2$; $-1 - 1$; $1 - 3$; $2 - 4$; $3 - 5$; and so on

The products of these numbers are: $0(-2) = 0$; $(-1)(-1) = 1$; $1(-3) = -3$; $2(-4) = -8$; $3(-5) = -15$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, 1, -3, -8, and -15

e) $q^2 + 3q + \square$

Two numbers whose sum is 3 are: $0 + 3$; $1 + 2$; $4 - 1$; $5 - 2$; $6 - 3$; and so on

The products of these numbers are: $0(3) = 0$; $1(2) = 2$; $4(-1) = -4$; $5(-2) = -10$; $6(-3) = -18$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, 2, -4, -10, and -18

f) $g^2 - 3g + \square$

Two numbers whose sum is -3 are: $0 - 3$; $-1 - 2$; $1 - 4$; $2 - 5$; $3 - 6$; and so on

The products of these numbers are: $0(-3) = 0$; $(-1)(-2) = 2$; $1(-4) = -4$; $2(-5) = -10$; $3(-6) = -18$; and so on

So, there are infinitely many values of $\square$.

Some of these values are: 0, 2, -4, -10, and -18

21. Remove common factors first.

a) $4y^2 - 20y - 56$

The common factor is 4.

$$\begin{aligned} 4y^2 - 20y - 56 &= 4(y^2) - 4(5y) - 4(14) \\ &= 4(y^2 - 5y - 14) \end{aligned}$$

Find two numbers with a product of -14 and a sum of -5.

The numbers are -7 and 2.

So, $4y^2 - 20y - 56 = 4(y - 7)(y + 2)$

b) $-3m^2 - 18m - 24$

The common factor is -3 .

$$\begin{aligned} -3m^2 - 18m - 24 &= -3(m^2) + (-3)(6m) + (-3)(8) \\ &= -3(m^2 + 6m + 8) \end{aligned}$$

Find two numbers with a product of 8 and a sum of 6.

The numbers are 2 and 4.

So, $-3m^2 - 18m - 24 = -3(m + 2)(m + 4)$

c) $4x^2 + 4x - 48$

The common factor is 4.

$$\begin{aligned} 4x^2 + 4x - 48 &= 4(x^2) + 4(x) - 4(12) \\ &= 4(x^2 + x - 12) \end{aligned}$$

Find two numbers with a product of -12 and a sum of 1.

The numbers are 4 and -3 .

So, $4x^2 + 4x - 48 = 4(x + 4)(x - 3)$

d) $10x^2 + 80x + 120$

The common factor is 10.

$$\begin{aligned} 10x^2 + 80x + 120 &= 10(x^2) + 10(8x) + 10(12) \\ &= 10(x^2 + 8x + 12) \end{aligned}$$

Find two numbers with a product of 12 and a sum of 8.

The numbers are 2 and 6.

So, $10x^2 + 80x + 120 = 10(x + 2)(x + 6)$

e) $-5n^2 + 40n - 35$

The common factor is -5 .

$$\begin{aligned} -5n^2 + 40n - 35 &= -5(n^2) + (-5)(-8n) + (-5)(7) \\ &= -5(n^2 - 8n + 7) \end{aligned}$$

Find two numbers with a product of 7 and a sum of -8 .

The numbers are -1 and -7 .

So, $-5n^2 + 40n - 35 = -5(n - 1)(n - 7)$

f) $7c^2 - 35c + 42$

The common factor is 7.

$$\begin{aligned} 7c^2 - 35c + 42 &= 7(c^2) - 7(5c) + 7(6) \\ &= 7(c^2 - 5c + 6) \end{aligned}$$

Find two numbers with a product of 6 and a sum of -5 .

The numbers are -2 and -3 .

So, $7c^2 - 35c + 42 = 7(c - 2)(c - 3)$

C

22. a) To expand $(r - 4)(r + 1)$ with algebra tiles:

Make a rectangle with length $r - 4$ and width $r + 1$; that is, use 1 r^2 -tile, then 4 negative r -tiles for the length and 1 r -tile for the width. Complete the rectangle with 4 negative 1-tiles.



From the diagram, the polynomial is: $r^2 - 4r + 1r - 4 = r^2 - 3r - 4$
So, $(r - 4)(r + 1) = r^2 - 3r - 4$

- b) To factor $t^2 + t - 6$ with algebra tiles:

Use 1 t^2 -tile. Arrange positive and negative t -tiles on two sides of the t^2 -tile, so there is 1 more positive t -tile than negative t -tile, and the rectangle is completed with 6 negative 1-tiles.



From the diagram, the length of the rectangle is $t + 3$, and the width is $t - 2$.
So, $t^2 + t - 6 = (t + 3)(t - 2)$

23. a) For each trinomial, find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

i) $h^2 - 10h - 24$

The factors of -24 are: 1 and -24 ; -1 and 24 ; 2 and -12 ; -2 and 12 ; 3 and -8 ; -3 and 8 ; 4 and -6 ; -4 and 6

Use mental math to calculate each sum.

The two factors with a sum of -10 are 2 and -12 .

So, $h^2 - 10h - 24 = (h + 2)(h - 12)$

ii) $h^2 + 10h - 24$

Use the factors from part i.

The two factors with a sum of 10 are -2 and 12 .

So, $h^2 + 10h - 24 = (h - 2)(h + 12)$

iii) $h^2 - 10h + 24$

The factors of 24 are: 1 and 24 ; -1 and -24 ; 2 and 12 ; -2 and -12 ; 3 and 8 ; -3 and -8 ; 4 and 6 ; -4 and -6

Use mental math to calculate each sum.

The two factors with a sum of -10 are -4 and -6 .

So, $h^2 - 10h + 24 = (h - 4)(h - 6)$

iv) $h^2 + 10h + 24$

Use the factors from part iii.

The two factors with a sum of 10 are 4 and 6 .

So, $h^2 + 10h + 24 = (h + 4)(h + 6)$

- b) In part a, there are 4 pairs of factors. The two numbers in two pairs are opposites, and the 4 sums are two opposite numbers. All these numbers must be factors of the same number.

$$2 - 12 = -10$$

$$4 + 6 = 10$$

$$-4 - 6 = -10$$

$$-2 + 12 = 10$$

Use guess and check to find other numbers.

The factors of 6 are: 1, 2, 3, 6

$$2 + 3 = 5 \text{ and } 6 - 1 = 5$$

So, four trinomials and their factors are:

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

$$(x - 2)(x - 3) = x^2 - 5x + 6$$

$$(x - 6)(x + 1) = x^2 - 5x - 6$$

$$(x + 6)(x - 1) = x^2 + 5x - 6$$

The factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30

$$3 + 10 = 13 \text{ and } 15 - 2 = 13$$

So, four trinomials and their factors are:

$$(x + 3)(x + 10) = x^2 + 13x + 30$$

$$(x - 3)(x - 10) = x^2 - 13x + 30$$

$$(x - 15)(x + 2) = x^2 - 13x - 30$$

$$(x + 15)(x - 2) = x^2 + 13x - 30$$

The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54

$$6 + 9 = 15 \text{ and } 18 - 3 = 15$$

So, four trinomials and their factors are:

$$(x + 6)(x + 9) = x^2 + 15x + 54$$

$$(x - 6)(x - 9) = x^2 - 15x + 54$$

$$(x - 18)(x + 3) = x^2 - 15x - 54$$

$$(x + 18)(x - 3) = x^2 + 15x - 54$$

The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

$$5 + 12 = 17 \text{ and } 20 - 3 = 17$$

So, four trinomials and their factors are:

$$(x + 5)(x + 12) = x^2 + 17x + 60$$

$$(x - 5)(x - 12) = x^2 - 17x + 60$$

$$(x - 20)(x + 3) = x^2 - 17x - 60$$

$$(x + 20)(x - 3) = x^2 + 17x - 60$$

There are many other possible trinomials.

Lesson 3.6 Polynomials of the Form $ax^2 + bx + c$

Exercises (pages 177–178)

A

5. a) There are 2 x^2 -tiles, 7 x -tiles, and three 1-tiles.

So, the trinomial is: $2x^2 + 7x + 3$

The length of the rectangle is $2x + 1$ and the width is $x + 3$.

So, the multiplication sentence is: $(2x + 1)(x + 3) = 2x^2 + 7x + 3$

- b) There are 3 x^2 -tiles, 14 x -tiles, and eight 1-tiles.

So, the trinomial is: $3x^2 + 14x + 8$

The length of the rectangle is $3x + 2$ and the width is $x + 4$.

So, the multiplication sentence is: $(3x + 2)(x + 4) = 3x^2 + 14x + 8$

- c) There are 6 x^2 -tiles, 5 x -tiles, and one 1-tile.

So, the trinomial is: $6x^2 + 5x + 1$

The length of the rectangle is $3x + 1$ and the width is $2x + 1$.

So, the multiplication sentence is: $(3x + 1)(2x + 1) = 6x^2 + 5x + 1$

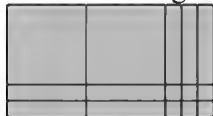
- d) There are 12 x^2 -tiles, 17 x -tiles, and six 1-tiles.

So, the trinomial is: $12x^2 + 17x + 6$

The length of the rectangle is $4x + 3$ and the width is $3x + 2$.

So, the multiplication sentence is: $(4x + 3)(3x + 2) = 12x^2 + 17x + 6$

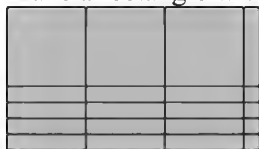
6. a) Make a rectangle with length $2v + 3$ and width $v + 2$.



The tiles that form the product are: 2 v^2 -tiles, 7 v -tiles, and six 1-tiles

So, $(2v + 3)(v + 2) = 2v^2 + 7v + 6$

- b) Make a rectangle with length $3r + 1$ and width $r + 4$.



The tiles that form the product are: 3 r^2 -tiles, 13 r -tiles, and four 1-tiles

So, $(3r + 1)(r + 4) = 3r^2 + 13r + 4$

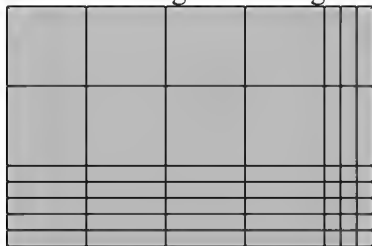
- c) Make a rectangle with length $2g + 3$ and width $3g + 2$.



The tiles that form the product are: 6 g^2 -tiles, 13 g -tiles, and six 1-tiles

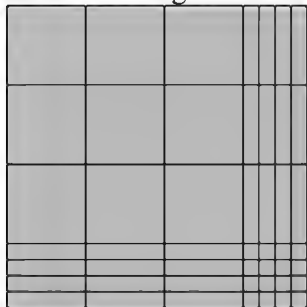
So, $(2g + 3)(3g + 2) = 6g^2 + 13g + 6$

- d) Make a rectangle with length $4z + 3$ and width $2z + 5$.



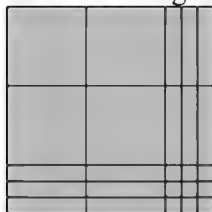
The tiles that form the product are: 8 z^2 -tiles, 26 z -tiles, and fifteen 1-tiles
So, $(4z + 3)(2z + 5) = 8z^2 + 26z + 15$

- e) Make a rectangle with length $3t + 4$ and width $3t + 4$.



The tiles that form the product are: 9 t^2 -tiles, 24 t -tiles, and sixteen 1-tiles
So, $(3t + 4)(3t + 4) = 9t^2 + 24t + 16$

- f) Make a rectangle with length $2r + 3$ and width $2r + 3$.



The tiles that form the product are: 4 r^2 -tiles, 12 r -tiles, and nine 1-tiles
So, $(2r + 3)(2r + 3) = 4r^2 + 12r + 9$

7. a) i) There are 2 x^2 -tiles, 5 x -tiles, and two 1-tiles.
So, the trinomial is: $2x^2 + 5x + 2$

ii)



- iii) The length of the rectangle is $2x + 1$ and the width is $x + 2$.
So, the factors of $2x^2 + 5x + 2$ are: $(2x + 1)(x + 2)$

- b) i) There are 3 x^2 -tiles, 11 x -tiles, and six 1-tiles.
So, the trinomial is: $3x^2 + 11x + 6$

ii)



iii) The length of the rectangle is $3x + 2$ and the width is $x + 3$.
So, the factors of $3x^2 + 11x + 6$ are: $(3x + 2)(x + 3)$

c) i) There are 3 x^2 -tiles, 8 x -tiles, and four 1-tiles.
So, the trinomial is: $3x^2 + 8x + 4$

ii)



iii) The length of the rectangle is $3x + 2$ and the width is $x + 2$.
So, the factors of $3x^2 + 8x + 4$ are: $(3x + 2)(x + 2)$

d) i) There are 4 x^2 -tiles, 9 x -tiles, and two 1-tiles.
So, the trinomial is: $4x^2 + 9x + 2$

ii)



iii) The length of the rectangle is $4x + 1$ and the width is $x + 2$.
So, the factors of $4x^2 + 9x + 2$ are: $(4x + 1)(x + 2)$

B

8. a) $(2w + 1)(w + 6) = 2w^2 + \square w + 6$
The coefficient of w in the trinomial is: (the coefficient of the 1st term in the 1st binomial)(the constant term in the 2nd binomial) + (the constant term in the 1st binomial)(the coefficient of the 1st term in the 2nd binomial)
That is: $2(6) + 1(1) = 13$
So, $\square = 13$
Then, $(2w + 1)(w + 6) = 2w^2 + 13w + 6$
- b) $(2g - 5)(3g - 3) = 6g^2 + \square + \bigcirc$
 $\square$ is: (the 1st term in the 1st binomial)(the constant term in the 2nd binomial) + (the constant term in the 1st binomial)(the 1st term in the 2nd binomial)
That is: $2g(-3) + (-5)(3g) = -6g - 15g = -21g$
 $\bigcirc$ is the product of the constant terms in the binomials.
That is: $(-5)(-3) = 15$
Then, $(2g - 5)(3g - 3) = 6g^2 - 21g + 15$

c) $(-4v - 3)(-2v - 7) = \square + \bigcirc + 21$

$\square$ is the product of the 1st terms in the binomials.

That is: $(-4v)(-2v) = 8v^2$

$\bigcirc$ is: (the 1st term in the 1st binomial)(the constant term in the 2nd binomial) +
(the constant term in the 1st binomial)(the 1st term in the 2nd binomial)

That is: $(-4v)(-7) + (-3)(-2v) = 28v + 6v$
 $= 34v$

Then, $(-4v - 3)(-2v - 7) = 8v^2 + 34v + 21$

9. Use the distributive property, then collect and combine like terms.

a) $(5 + f)(3 + 4f) = 5(3 + 4f) + f(3 + 4f)$
 $= 5(3) + 5(4f) + f(3) + f(4f)$
 $= 15 + 20f + 3f + 4f^2$
 $= 15 + 23f + 4f^2$

b) $(3 - 4t)(5 - 3t) = 3(5 - 3t) - 4t(5 - 3t)$
 $= 3(5) + 3(-3t) - 4t(5) - 4t(-3t)$
 $= 15 - 9t - 20t + 12t^2$
 $= 15 - 29t + 12t^2$

c) $(10 - r)(9 + 2r) = 10(9 + 2r) - r(9 + 2r)$
 $= 10(9) + 10(2r) - r(9) - r(2r)$
 $= 90 + 20r - 9r - 2r^2$
 $= 90 + 11r - 2r^2$

d) $(-6 + 2m)(-6 + 2m) = (-6)(-6 + 2m) + 2m(-6 + 2m)$
 $= (-6)(-6) + (-6)(2m) + 2m(-6) + 2m(2m)$
 $= 36 - 12m - 12m + 4m^2$
 $= 36 - 24m + 4m^2$

e) $(-8 - 2x)(3 - 7x) = (-8)(3 - 7x) - 2x(3 - 7x)$
 $= (-8)(3) - 8(-7x) - 2x(3) - 2x(-7x)$
 $= -24 + 56x - 6x + 14x^2$
 $= -24 + 50x + 14x^2$

f) $(6 - 5n)(-6 + 5n) = 6(-6 + 5n) - 5n(-6 + 5n)$
 $= -36 + 30n + 30n - 25n^2$
 $= -36 + 60n - 25n^2$

10. Use the distributive property, then collect and combine like terms.

a) $(3c + 4)(5 + 2c) = 3c(5 + 2c) + 4(5 + 2c)$
 $= 3c(5) + 3c(2c) + 4(5) + 4(2c)$
 $= 15c + 6c^2 + 20 + 8c$ Arrange the terms in descending order.
 $= 6c^2 + 15c + 8c + 20$
 $= 6c^2 + 23c + 20$

b) $(1 - 7t)(3t + 5) = 1(3t + 5) - 7t(3t + 5)$
 $= 3t + 5 - 21t^2 - 35t$ Arrange the terms in descending order.
 $= -21t^2 - 35t + 3t + 5$
 $= -21t^2 - 32t + 5$

$$\begin{aligned}
 \text{c) } (-4r - 7)(2 - 8r) &= (-4r)(2 - 8r) - 7(2 - 8r) \\
 &= (-4r)(2) - 4r(-8r) - 7(2) - 7(-8r) \\
 &= -8r + 32r^2 - 14 + 56r && \text{Arrange the terms in descending order.} \\
 &= 32r^2 - 8r + 56r - 14 \\
 &= 32r^2 + 48r - 14
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } (-9 - t)(-5t - 1) &= (-9)(-5t - 1) - t(-5t - 1) \\
 &= (-9)(-5t) - 9(-1) - t(-5t) - t(-1) \\
 &= 45t + 9 + 5t^2 + t && \text{Arrange the terms in descending order.} \\
 &= 5t^2 + 45t + t + 9 \\
 &= 5t^2 + 46t + 9
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } (7h + 10)(-3 + 5h) &= 7h(-3 + 5h) + 10(-3 + 5h) \\
 &= 7h(-3) + 7h(5h) + 10(-3) + 10(5h) \\
 &= -21h + 35h^2 - 30 + 50h && \text{Arrange the terms in descending order.} \\
 &= 35h^2 - 21h + 50h - 30 \\
 &= 35h^2 + 29h - 30
 \end{aligned}$$

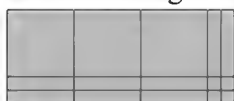
$$\begin{aligned}
 \text{f) } (7 - 6y)(6y - 7) &= 7(6y - 7) - 6y(6y - 7) \\
 &= 7(6y) + 7(-7) - 6y(6y) - 6y(-7) \\
 &= 42y - 49 - 36y^2 + 42y && \text{Arrange the terms in descending order.} \\
 &= -36y^2 + 42y + 42y - 49 \\
 &= -36y^2 + 84y - 49
 \end{aligned}$$

11. a) i) Make a rectangle with 3 t^2 -tiles, 4 t -tiles, and one 1-tile.



The length of the rectangle is $3t + 1$ and the width is $t + 1$.
So, $3t^2 + 4t + 1 = (3t + 1)(t + 1)$

- ii) Make a rectangle with 3 t^2 -tiles, 8 t -tiles, and four 1-tiles.



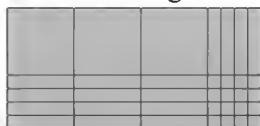
The length of the rectangle is $3t + 2$ and the width is $t + 2$.
So, $3t^2 + 8t + 4 = (3t + 2)(t + 2)$

- iii) Make a rectangle with 3 t^2 -tiles, 12 t -tiles, and nine 1-tiles.



The length of the rectangle is $3t + 3$ and the width is $t + 3$.
So, $3t^2 + 12t + 9 = (3t + 3)(t + 3)$

- iv) Make a rectangle with 3 t^2 -tiles, 16 t -tiles, and sixteen 1-tiles.



The length of the rectangle is $3t + 4$ and the width is $t + 4$.

$$\text{So, } 3t^2 + 16t + 16 = (3t + 4)(t + 4)$$

- b) In the rectangles, each side length increases by 1 each time. In the binomial factors, the constant terms increase by 1 each time.
- c) In the trinomials in part a:
- The coefficients of the t^2 -terms are 3, so these do not change.
 - The coefficients of the t -terms are consecutive multiples of 4.
 - The constant terms are consecutive squares.

So, the next 3 trinomials and their factors are:

$$3t^2 + 20t + 25 = (3t + 5)(t + 5)$$

$$3t^2 + 24t + 36 = (3t + 6)(t + 6)$$

$$3t^2 + 28t + 49 = (3t + 7)(t + 7)$$

12. To factor each trinomial, use number sense and reasoning, with mental math.

a) i) $2n^2 + 13n + 6$

The factors of $2n^2$ are: $2n$ and n

The factors of 6 are: 1 and 6; and 2 and 3

Do not consider negative factors of 6 because there are no negative terms in the trinomial.

Arrange the factor combinations vertically. Then form the products and add.

$$\begin{array}{r} 2n \quad 1 \\ \times \\ n \quad 6 \end{array}$$

$$12n + 1n = 13n$$

$$\begin{array}{r} 2n \quad 6 \\ \times \\ n \quad 1 \end{array}$$

$$2n + 6n = 8n$$

$$\begin{array}{r} 2n \quad 2 \\ \times \\ n \quad 3 \end{array}$$

$$6n + 2n = 8n$$

$$\begin{array}{r} 2n \quad 3 \\ \times \\ n \quad 2 \end{array}$$

$$4n + 3n = 7n$$

The first set of factors is correct because the coefficient of n is 13.

$$\text{So, } 2n^2 + 13n + 6 = (2n + 1)(n + 6)$$

ii) $2n^2 - 13n + 6$

The factors of 6 in the binomials are numerically the same as those in part i; but both factors are negative because the coefficient of n in the trinomial is negative, and the constant term is positive.

$$\text{So, } 2n^2 - 13n + 6 = (2n - 1)(n - 6)$$

b) i) $2n^2 + 11n - 6$

The factors of 6 in the binomials have opposite signs because the constant term in the trinomial is negative.

Use the factors from part a, but amend the signs so they are opposites.

Stop when the sum of the n -terms is $11n$.

$$\begin{array}{r} 2n \quad 1 \\ \times \\ n \quad -6 \end{array}$$

$$-12n + 1n = -11n$$

$$\begin{array}{r} 2n \quad -1 \\ \times \\ n \quad 6 \end{array}$$

$$12n - 1n = 11n$$

$$\text{So, } 2n^2 + 11n - 6 = (2n - 1)(n + 6)$$

ii) $2n^2 - 11n - 6$

The factors of 6 in the binomials have opposite signs because the constant term in the trinomial is negative.

The factors that produce the sum of n -terms as $-11n$ are shown in part i.

$$\text{So, } 2n^2 - 11n - 6 = (2n + 1)(n - 6)$$

c) i) $2n^2 + 7n + 6$

The factors of 6 in the binomials have the same sign because the constant term in the trinomial is positive. Both factors are positive because the n -term in the trinomial is positive.

The factors that produce the sum of n -terms as $7n$ are shown in part a, i.

So, $2n^2 + 7n + 6 = (2n + 3)(n + 2)$

ii) $2n^2 - 7n + 6$

The factors of 6 in the binomials have the same sign because the constant term in the trinomial is positive. Both factors are negative because the n -term in the trinomial is negative.

The factors that produce the sum of n -terms as $-7n$ are the opposites of those in part i.

So, $2n^2 - 7n + 6 = (2n - 3)(n - 2)$

All 6 trinomials have the same coefficient of n^2 , and either 6 or -6 as their constant term. All 6 pairs of binomial factors have $2n$ as the first term in each binomial, and a factor of 6 as the constant term in each binomial.

For each pair of trinomials, their constant terms are equal and the coefficients of their n -terms are opposites. For each related pair of binomial factors, the constant terms in one pair of factors are the opposites of the constant terms in the other pair.

13. To factor each trinomial, use number sense and reasoning, with mental math.

a) $2y^2 + 5y + 2$

The factors of $2y^2$ are: $2y$ and y

The factors of 2 are: 1 and 2

The possible binomial factors are:

$(2y + 2)(y + 1)$ The middle term is: $2y(1) + 2(y) = 4y$

$(2y + 1)(y + 2)$ The middle term is: $2y(2) + 1(y) = 5y$

The second set of factors is correct because the middle term in the trinomial is $5y$.

$2y^2 + 5y + 2 = (2y + 1)(y + 2)$

Check: $(2y + 1)(y + 2) = 2y(y + 2) + 1(y + 2)$
 $= 2y^2 + 4y + y + 2$
 $= 2y^2 + 5y + 2$

b) $2a^2 + 11a + 12$

The factors of $2a^2$ are: $2a$ and a

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

Do not consider negative factors of 6 because there are no negative terms in the trinomial.

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the a -terms is $11a$.

$$\begin{array}{r} 2a \quad 1 \\ \diagdown \quad \diagup \\ a \quad 12 \\ 24a + 1a = 25a \end{array}$$

$$\begin{array}{r} 2a \quad 12 \\ \diagdown \quad \diagup \\ a \quad 1 \\ 2a + 12a = 14a \end{array}$$

$$\begin{array}{r} 2a \quad 2 \\ \diagdown \quad \diagup \\ a \quad 6 \\ 12a + 2a = 14a \end{array}$$

$$\begin{array}{r} 2a \quad 6 \\ \diagdown \quad \diagup \\ a \quad 2 \\ 4a + 6a = 10a \end{array}$$

$$\begin{array}{r} 2a \quad 3 \\ \diagdown \quad \diagup \\ a \quad 4 \\ 8a + 3a = 11a \end{array}$$

So, $2a^2 + 11a + 12 = (2a + 3)(a + 4)$

Check: $(2a + 3)(a + 4) = 2a(a + 4) + 3(a + 4)$
 $= 2a^2 + 8a + 3a + 12$
 $= 2a^2 + 11a + 12$

c) $2k^2 + 13k + 15$

The factors of $2k^2$ are: $2k$ and k

The factors of 15 are: 1 and 15; 3 and 5

Do not consider negative factors of 15 because there are no negative terms in the trinomial.

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the k -terms is $13k$.

$$\begin{array}{r} 2k \quad 1 \\ k \quad 15 \\ 30k + 1k = 31k \end{array}$$

$$\begin{array}{cc} 2k & 15 \\ k & 1 \end{array}$$

$$2k + 15k = 17k$$

$$\begin{array}{cc} 2k & 3 \\ k & 5 \end{array}$$

$$10k + 3k = 13k$$

So, $2k^2 + 13k + 15 = (2k + 3)(k + 5)$

$$\begin{aligned}\text{Check: } (2k+3)(k+5) &= 2k(k+5) + 3(k+5) \\ &= 2k^2 + 10k + 3k + 15 \\ &= 2k^2 + 13k + 15\end{aligned}$$

d) $2m^2 - 11m + 12$

The factors of $2m^2$ are: $2m$ and m

The factors of 12 are: -1 and -12 ; -2 and -6 ; -3 and -4

Consider only negative factors of 12 because the constant term in the trinomial is positive, and the m -term is negative.

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the m -terms is $-11m$.

$$\begin{array}{r} 2m \quad -1 \\ m \quad -12 \\ -24m - 1m = -25m \end{array}$$

$$\begin{array}{r} 2m \quad -12 \\ m \quad -1 \\ \hline -2m - 12m = -14m \end{array}$$

$$\begin{array}{cc} 2m & -2 \\ m & -6 \end{array}$$

$$-12m - 2m = -14m$$

$$\begin{array}{cc} 2m & -6 \\ m & -2 \end{array}$$

$$-4m - 6m = -10m$$

$$\begin{array}{r} 2m \quad -3 \\ m \quad -4 \\ -8m - 3m = -11m \end{array}$$

$$\text{So, } 2m^2 - 11m + 12 = (2m - 3)(m - 4)$$

$$\begin{aligned}\text{Check: } (2m-3)(m-4) &= 2m(m-4) - 3(m-4) \\ &= 2m^2 - 8m - 3m + 12 \\ &= 2m^2 - 11m + 12\end{aligned}$$

e) $2k^2 - 11k + 15$

The factors of $2k^2$ are: $2k$ and k

The factors of 15 are: -1 and -15 ; -3 and -5

Consider only negative factors of 15 because the constant term in the trinomial is positive, and the k -term is negative.

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the k -terms is $-11k$.

$$\begin{array}{r} 2k \quad -1 \\ k \quad -15 \\ -30k - 1k = -31k \end{array}$$

$$\begin{array}{r} 2k \quad -15 \\ k \quad -1 \\ \hline -2k - 15k = -17k \end{array}$$

$$\begin{array}{r} 2k \quad -3 \\ k \quad -5 \\ -10k - 3k = -13k \end{array}$$

$$\begin{array}{r} 2k \quad -5 \\ k \quad -3 \\ \hline -6k - 5k = -11k \end{array}$$

So, $2k^2 - 11k + 15 = (2k - 5)(k - 3)$

$$\begin{aligned}\text{Check: } (2k-5)(k-3) &= 2k(k-3) - 5(k-3) \\ &= 2k^2 - 6k - 5k + 15 \\ &= 2k^2 - 11k + 15\end{aligned}$$

f) $2m^2 + 15m + 7$

The factors of $2m^2$ are: $2m$ and m

The factors of 7 are: 1 and 7

The possible binomial factors are:

$(2m + 7)(m + 1)$ The middle term is: $2m(1) + 7(m) = 9m$

The middle term is: $2m(7) + 1(m) = 15m$

The second set of factors is correct because the middle term in the trinomial is $15m$.

$$2m^2 + 15m + 7 = (2m + 1)(m + 7)$$

Check: $(2m+1)(m+7) = 2m(m+7) + 1(m+7)$
 $= 2m^2 + 14m + m + 7$
 $= 2m^2 + 15m + 7$

g) $2g^2 + 15g + 18$

The factors of $2g^2$ are: $2g$ and g

The factors of 18 are: 1 and 18; 2 and 9; 3 and 6

Do not consider negative factors of 18 because there are no negative terms in the trinomial.

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the g -terms is $15g$.

$$\begin{array}{r} 2g \quad 1 \\ g \quad 18 \\ 36g + 1g = 37g \end{array}$$

$$\begin{array}{r} 2g \quad 18 \\ g \quad 1 \\ \hline 2g + 18g = 20g \end{array}$$

$$\begin{array}{cc} 2g & 2 \\ g & 9 \end{array}$$

$$18g + 2g = 20g$$

$$\begin{array}{cc} 2g & 9 \\ g & 2 \end{array}$$

$$4g + 9g = 13g$$

$$\begin{array}{cc} 2g & 3 \\ g & 6 \end{array}$$

$$12g + 3g = 15g$$

$$\text{So, } 2g^2 + 15g + 18 = (2g + 3)(g + 6)$$

$$\begin{aligned}\text{Check: } (2g+3)(g+6) &= 2g(g+6) + 3(g+6) \\ &= 2g^2 + 12g + 3g + 18 \\ &= 2g^2 + 15g + 18\end{aligned}$$

h) $2n^2 + 9n - 18$

The factors of $2n^2$ are: $2n$ and n

The factors of -18 are: 1 and -18 ; -1 and 18; 2 and -9 ; -2 and 9; 3 and -6 ; -3 and 6

Arrange the factor combinations vertically. Then form the products and add.

Stop when the sum of the n -terms is $9n$.

$$\begin{array}{r} 2n \quad 1 \\ \times \quad \times \\ n \quad -18 \\ \hline -36n + 1n = -35n \end{array}$$

$$\begin{array}{r} 2n \quad -18 \\ \times \quad \times \\ n \quad 1 \\ \hline 2n - 18n = -16n \end{array}$$

$$\begin{array}{r} 2n \quad -1 \\ \times \quad \times \\ n \quad 18 \\ \hline 36n - 1n = 35n \end{array}$$

$$\begin{array}{r} 2n \quad 18 \\ \times \quad \times \\ n \quad -1 \\ \hline -2n + 18n = 16n \end{array}$$

$$\begin{array}{r} 2n \quad 2 \\ \times \quad \times \\ n \quad -9 \\ \hline -18n + 2n = -16n \end{array}$$

$$\begin{array}{r} 2n \quad -9 \\ \times \quad \times \\ n \quad 2 \\ \hline 4n - 9n = -5n \end{array}$$

$$\begin{array}{r} 2n \quad -2 \\ \times \quad \times \\ n \quad 9 \\ \hline 18n - 2n = 16n \end{array}$$

$$\begin{array}{r} 2n \quad 9 \\ \times \quad \times \\ n \quad -2 \\ \hline -4n + 9n = 5n \end{array}$$

$$\begin{array}{r} 2n \quad 3 \\ \times \quad \times \\ n \quad -6 \\ \hline -12n + 3n = -9n \end{array}$$

$$\begin{array}{r} 2n \quad -6 \\ \times \quad \times \\ n \quad 3 \\ \hline 6n - 6n = 0 \end{array}$$

$$\begin{array}{r} 2n \quad -3 \\ \times \quad \times \\ n \quad 6 \\ \hline 12n - 3n = 9n \end{array}$$

So, $2n^2 + 9n - 18 = (2n - 3)(n + 6)$

Check: $(2n - 3)(n + 6) = 2n(n + 6) - 3(n + 6)$
 $= 2n^2 + 12n - 3n - 18$
 $= 2n^2 + 9n - 18$

14. a) i) The factors of 15 are: 1 and 15; 3 and 5
 The sums of the factors are: 16 and 8
 So, the two integers with a sum of 16 are 1 and 15.
- ii) The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6
 The sums of the factors are: 25; 14; 11; and 10
 So, the two integers with a sum of 14 are 2 and 12.
- iii) The factors of 15 are: 1 and 15; 3 and 5
 The sums of the factors are: 16 and 8
 So, the two integers with a sum of 8 are 3 and 5.
- iv) The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
 The sums of the factors are: 13; 8; and 7
 So, the two integers with a sum of 7 are 3 and 4.
- v) The factors of 12 are: 1 and 12; 2 and 6; 3 and 4
 The sums of the factors are: 13; 8; and 7
 So, the two integers with a sum of 13 are 1 and 12.
- vi) The factors of 24 are: 1 and 24; 2 and 12; 3 and 8; 4 and 6

The sums of the factors are: 25; 14; 11; and 10
So, the two integers with a sum of 11 are 3 and 8.

- b) For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

i) $3v^2 + 16v + 5$

The product is: $3(5) = 15$

From part a) i), the factors of 15 with a sum of 16 are: 1 and 15

$$\begin{aligned}\text{So, } 3v^2 + 16v + 5 &= 3v^2 + 1v + 15v + 5 \\ &= v(3v + 1) + 5(3v + 1) \\ &= (3v + 1)(v + 5)\end{aligned}$$

ii) $3m^2 + 14m + 8$

The product is: $3(8) = 24$

From part a) ii), the factors of 24 with a sum of 14 are: 2 and 12

$$\begin{aligned}\text{So, } 3m^2 + 14m + 8 &= 3m^2 + 2m + 12m + 8 \\ &= m(3m + 2) + 4(3m + 2) \\ &= (3m + 2)(m + 4)\end{aligned}$$

iii) $3b^2 + 8b + 5$

The product is: $3(5) = 15$

From part a) iii), the factors of 15 with a sum of 8 are: 3 and 5

$$\begin{aligned}\text{So, } 3b^2 + 8b + 5 &= 3b^2 + 3b + 5b + 5 \\ &= 3b(b + 1) + 5(b + 1) \\ &= (b + 1)(3b + 5)\end{aligned}$$

iv) $4a^2 + 7a + 3$

The product is: $4(3) = 12$

From part a) iv), the factors of 12 with a sum of 7 are: 3 and 4

$$\begin{aligned}\text{So, } 4a^2 + 7a + 3 &= 4a^2 + 3a + 4a + 3 \\ &= a(4a + 3) + 1(4a + 3) \\ &= (4a + 3)(a + 1)\end{aligned}$$

v) $4d^2 + 13d + 3$

The product is: $4(3) = 12$

From part a) v), the factors of 12 with a sum of 13 are: 1 and 12

$$\begin{aligned}\text{So, } 4d^2 + 13d + 3 &= 4d^2 + 1d + 12d + 3 \\ &= d(4d + 1) + 3(4d + 1) \\ &= (4d + 1)(d + 3)\end{aligned}$$

vi) $4v^2 + 11v + 6$

The product is: $4(6) = 24$

From part a) vi), the factors of 24 with a sum of 11 are: 3 and 8

$$\begin{aligned}\text{So, } 4v^2 + 11v + 6 &= 4v^2 + 3v + 8v + 6 \\ &= v(4v + 3) + 2(4v + 3) \\ &= (4v + 3)(v + 2)\end{aligned}$$

15. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and

the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $5a^2 - 7a - 6$

The product is: $5(-6) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30 ; 2 and -15 ; -2 and 15 ; 3 and -10 ; -3 and 10 ; 5 and -6 ; -5 and 6

The two factors with a sum of -7 are: 3 and -10

So, $5a^2 - 7a - 6 = 5a^2 + 3a - 10a - 6$

$$= a(5a + 3) - 2(5a + 3)$$

$$= (5a + 3)(a - 2)$$

Check: $(5a + 3)(a - 2) = 5a(a - 2) + 3(a - 2)$

$$= 5a^2 - 10a + 3a - 6$$

$$= 5a^2 - 7a - 6$$

b) $3y^2 - 13y - 10$

The product is: $3(-10) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30 ; 2 and -15 ; -2 and 15 ; 3 and -10 ; -3 and 10 ; 5 and -6 ; -5 and 6

The two factors with a sum of -13 are: 2 and -15

So, $3y^2 - 13y - 10 = 3y^2 + 2y - 15y - 10$

$$= y(3y + 2) - 5(3y + 2)$$

$$= (3y + 2)(y - 5)$$

Check: $(3y + 2)(y - 5) = 3y(y - 5) + 2(y - 5)$

$$= 3y^2 - 15y + 2y - 10$$

$$= 3y^2 - 13y - 10$$

c) $5s^2 + 19s - 4$

The product is: $5(-4) = -20$

The factors of -20 are: 1 and -20 ; -1 and 20 ; 2 and -10 ; -2 and 10 ; 4 and -5 ; -4 and 5

The two factors with a sum of 19 are: -1 and 20

So, $5s^2 + 19s - 4 = 5s^2 - 1s + 20s - 4$

$$= s(5s - 1) + 4(5s - 1)$$

$$= (5s - 1)(s + 4)$$

Check: $(5s - 1)(s + 4) = 5s(s + 4) - 1(s + 4)$

$$= 5s^2 + 20s - 1s - 4$$

$$= 5s^2 + 19s - 4$$

d) $14c^2 - 19c - 3$

The product is: $14(-3) = -42$

The factors of -42 are: 1 and -42 ; -1 and 42 ; 2 and -21 ; -2 and 21 ; 3 and -14 ; -3 and 14 ; 6 and -7 ; -6 and 7

The two factors with a sum of -19 are: 2 and -21

So, $14c^2 - 19c - 3 = 14c^2 + 2c - 21c - 3$

$$= 2c(7c + 1) - 3(7c + 1)$$

$$= (7c + 1)(2c - 3)$$

Check: $(7c + 1)(2c - 3) = 7c(2c - 3) + 1(2c - 3)$

$$= 14c^2 - 21c + 2c - 3$$

$$= 14c^2 - 19c - 3$$

e) $8a^2 + 18a - 5$

The product is: $8(-5) = -40$

The factors of -40 are: 1 and -40 ; -1 and 40 ; 2 and -20 ; -2 and 20 ; 4 and -10 ; -4 and 10 ; 8 and -5 ; -8 and 5

The two factors with a sum of 18 are: -2 and 20

$$\begin{aligned}\text{So, } 8a^2 + 18a - 5 &= 8a^2 - 2a + 20a - 5 \\ &= 2a(4a - 1) + 5(4a - 1) \\ &= (4a - 1)(2a + 5)\end{aligned}$$

$$\begin{aligned}\text{Check: } (4a - 1)(2a + 5) &= 4a(2a + 5) - 1(2a + 5) \\ &= 8a^2 + 20a - 2a - 5 \\ &= 8a^2 + 18a - 5\end{aligned}$$

f) $8r^2 - 14r + 3$

The product is: $8(3) = 24$

Consider only negative factors of 24 because the constant term in the trinomial is positive, and the r -term is negative.

The factors of 24 are: -1 and -24 ; -2 and -12 ; -3 and -8 ; -4 and -6

The two factors with a sum of -14 are: -2 and -12

$$\begin{aligned}\text{So, } 8r^2 - 14r + 3 &= 8r^2 - 2r - 12r + 3 \\ &= 2r(4r - 1) - 3(4r - 1) \\ &= (4r - 1)(2r - 3)\end{aligned}$$

$$\begin{aligned}\text{Check: } (4r - 1)(2r - 3) &= 4r(2r - 3) - 1(2r - 3) \\ &= 8r^2 - 12r - 2r + 3 \\ &= 8r^2 - 14r + 3\end{aligned}$$

g) $6d^2 + d - 5$

The product is: $6(-5) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30 ; 2 and -15 ; -2 and 15 ; 3 and -10 ; -3 and 10 ; 5 and -6 ; -5 and 6

The two factors with a sum of 1 are: -5 and 6

$$\begin{aligned}\text{So, } 6d^2 + d - 5 &= 6d^2 - 5d + 6d - 5 \\ &= d(6d - 5) + 1(6d - 5) \\ &= (6d - 5)(d + 1)\end{aligned}$$

$$\begin{aligned}\text{Check: } (6d - 5)(d + 1) &= 6d(d + 1) - 5(d + 1) \\ &= 6d^2 + 6d - 5d - 5 \\ &= 6d^2 + d - 5\end{aligned}$$

h) $15e^2 - 7e - 2$

The product is: $15(-2) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30 ; 2 and -15 ; -2 and 15 ; 3 and -10 ; -3 and 10 ; 5 and -6 ; -5 and 6

The two factors with a sum of -7 are: 3 and -10

$$\begin{aligned}\text{So, } 15e^2 - 7e - 2 &= 15e^2 + 3e - 10e - 2 \\ &= 3e(5e + 1) - 2(5e + 1) \\ &= (5e + 1)(3e - 2)\end{aligned}$$

$$\begin{aligned}\text{Check: } (5e + 1)(3e - 2) &= 5e(3e - 2) + 1(3e - 2) \\ &= 15e^2 - 10e + 3e - 2 \\ &= 15e^2 - 7e - 2\end{aligned}$$

16. a) $6u^2 + 17u - 14 = (2u - 7)(3u + 2)$

Consider how the mistake might have been made.

The incorrect signs could have been used in the binomial factors.

Check to see if the factors are: $(2u + 7)(3u - 2)$

$$\begin{aligned}\text{Expand: } (2u + 7)(3u - 2) &= 2u(3u - 2) + 7(3u - 2) \\ &= 6u^2 - 4u + 21u - 14 \\ &= 6u^2 + 17u - 14\end{aligned}$$

This trinomial is the same as the given trinomial.

So, the correct factorization is: $6u^2 + 17u - 14 = (2u + 7)(3u - 2)$

Alternative solution:

Expand the binomial factors.

$$\begin{aligned}(2u - 7)(3u + 2) &= 2u(3u + 2) - 7(3u + 2) \\ &= 6u^2 + 4u - 21u - 14 \\ &= 6u^2 - 17u - 14\end{aligned}$$

Compare this trinomial with the given trinomial.

The sign of $17u$ should be positive.

So, to correct the error, transpose the signs in the binomial factors.

The correct factorization is:

$$6u^2 + 17u - 14 = (2u + 7)(3u - 2)$$

b) $3k^2 - k - 30 = (3k - 3)(k + 10)$

Consider how the mistake might have been made.

The incorrect signs could have been used in the binomial factors.

Check to see if the factors are: $(3k + 3)(k - 10)$

$$\begin{aligned}\text{Expand: } (3k + 3)(k - 10) &= 3k(k - 10) + 3(k - 10) \\ &= 3k^2 - 30k + 3k - 30 \\ &= 3k^2 - 27k - 30\end{aligned}$$

This is not the correct trinomial.

The constant terms in the binomial factors could have been transposed.

Check to see if the factors are: $(3k - 10)(k + 3)$

$$\begin{aligned}\text{Expand: } (3k - 10)(k + 3) &= 3k(k + 3) - 10(k + 3) \\ &= 3k^2 + 9k - 10k - 30 \\ &= 3k^2 - k - 30\end{aligned}$$

This is the correct trinomial.

The correct factorization is:

$$3k^2 - k - 30 = (3k - 10)(k + 3)$$

Alternative solution:

Expand the binomial factors.

$$\begin{aligned}(3k - 3)(k + 10) &= 3k(k + 10) - 3(k + 10) \\ &= 3k^2 + 30k - 3k - 30 \\ &= 3k^2 + 27k - 30\end{aligned}$$

Compare this trinomial with the given trinomial.

The middle term, $27k$, should be $-k$.

Factor $3k^2 - k - 30$ by decomposition.

The product is: $3(-30) = -90$

Since the difference of the factors is -1 , find factors that are close in numerical value: -10 and 9

$$\begin{aligned}\text{So, } 3k^2 - k - 30 &= 3k^2 - 10k + 9k - 30 \\ &= k(3k - 10) + 3(3k - 10) \\ &= (3k - 10)(k + 3)\end{aligned}$$

The correct factorization is:

$$3k^2 - k - 30 = (3k - 10)(k + 3)$$

c) $4v^2 - 21v + 20 = (4v - 4)(v + 5)$

Consider how the mistake might have been made.

The constant term in the trinomial is positive and the v -term is negative, so both the constant terms in the binomials must be negative.

Check to see if the factors are: $(4v - 4)(v - 5)$

$$\begin{aligned}\text{Expand: } (4v - 4)(v - 5) &= 4v(v - 5) - 4(v - 5) \\ &= 4v^2 - 20v - 4v + 20 \\ &= 4v^2 - 24v + 20\end{aligned}$$

This is not the correct trinomial.

The constant terms in the binomial factors could have been transposed.

Check to see if the factors are: $(4v - 5)(v - 4)$

$$\begin{aligned}\text{Expand: } (4v - 5)(v - 4) &= 4v(v - 4) - 5(v - 4) \\ &= 4v^2 - 16v - 5v + 20 \\ &= 4v^2 - 21v + 20\end{aligned}$$

This is the correct trinomial.

The correct factorization is:

$$4v^2 - 21v + 20 = (4v - 5)(v - 4)$$

Alternative solution:

$$4v^2 - 21v + 20 = (4v - 4)(v + 5)$$

Expand the binomial factors.

$$\begin{aligned}(4v - 4)(v + 5) &= 4v(v + 5) - 4(v + 5) \\ &= 4v^2 + 20v - 4v - 20 \\ &= 4v^2 + 16v - 20\end{aligned}$$

Compare this trinomial with the given trinomial.

The middle term, $16v$, should be $-21v$ and the constant term, -20 , should be 20 .

Factor $4v^2 - 21v + 20$ by decomposition.

The product is: $4(20) = 80$

The factors of 80 are: -1 and -80 ; -2 and -40 ; -4 and -20 ; -5 and -16 , -8 and -10

Consider only negative factors of 80 because the constant term in the trinomial is positive, and the v -term is negative.

The two factors with a sum of -21 are: -5 and -16

$$\begin{aligned}\text{So, } 4v^2 - 21v + 20 &= 4v^2 - 5v - 16v + 20 \\ &= v(4v - 5) - 4(4v - 5) \\ &= (4v - 5)(v - 4)\end{aligned}$$

The correct factorization is:

$$4v^2 - 21v + 20 = (4v - 5)(v - 4)$$

17. In the second line of the solution, when 7 is removed as the common factor of the last 2 terms, the sign in the binomial should be negative because $(-42) \div 7 = -6$

The correct solution is:

$$\begin{aligned}15g^2 + 17g - 42 &= 15g^2 - 18g + 35g - 42 \\ &= 3g(5g - 6) + 7(5g - 6) \\ &= (5g - 6)(3g + 7)\end{aligned}$$

18. Remove any common factors before factoring each trinomial by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $20r^2 + 70r + 60 = 10(2r^2 + 7r + 6)$

Factor: $2r^2 + 7r + 6$

The product is: $2(6) = 12$

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

The factors with a sum of 7 are 3 and 4.

$$\begin{aligned} 2r^2 + 7r + 6 &= 2r^2 + 3r + 4r + 6 \\ &= r(2r + 3) + 2(2r + 3) \\ &= (2r + 3)(r + 2) \end{aligned}$$

$$\text{So, } 20r^2 + 70r + 60 = 10(2r + 3)(r + 2)$$

b) $15a^2 - 65a + 20 = 5(3a^2 - 13a + 4)$

Factor: $3a^2 - 13a + 4$

The product is: $3(4) = 12$

Consider only the negative factors of 12 because the a -term is negative and the constant term is positive.

The negative factors of 12 are: -1 and -12 ; -2 and -6 ; -3 and -4

The factors with a sum of -13 are -1 and -12 .

$$\begin{aligned} 3a^2 - 13a + 4 &= 3a^2 - 1a - 12a + 4 \\ &= a(3a - 1) - 4(3a - 1) \\ &= (3a - 1)(a - 4) \end{aligned}$$

$$\text{So, } 15a^2 - 65a + 20 = 5(3a - 1)(a - 4)$$

c) $18h^2 + 15h - 18 = 3(6h^2 + 5h - 6)$

Factor: $6h^2 + 5h - 6$

The product is: $6(-6) = -36$

The factors of -36 are: 1 and -36 ; -1 and 36; 2 and -18 ; -2 and 18; 3 and -12 ; -3 and 12; 4 and -9 ; -4 and 9; 6 and -6

The factors with a sum of 5 are -4 and 9.

$$\begin{aligned} 6h^2 + 5h - 6 &= 6h^2 - 4h + 9h - 6 \\ &= 2h(3h - 2) + 3(3h - 2) \\ &= (3h - 2)(2h + 3) \end{aligned}$$

$$\text{So, } 18h^2 + 15h - 18 = 3(3h - 2)(2h + 3)$$

d) $24u^2 - 72u + 54 = 6(4u^2 - 12u + 9)$

Factor: $4u^2 - 12u + 9$

The product is: $4(9) = 36$

Consider only the negative factors of 36 because the u -term is negative and the constant term is positive.

The negative factors of 36 are: -1 and -36 ; -2 and -18 ; -3 and -12 ; -4 and -9 ; -6 and -6

The factors with a sum of -12 are -6 and -6 .

$$\begin{aligned} 4u^2 - 12u + 9 &= 4u^2 - 6u - 6u + 9 \\ &= 2u(2u - 3) - 3(2u - 3) \\ &= (2u - 3)(2u - 3) \end{aligned}$$

$$\text{So, } 24u^2 - 72u + 54 = 6(2u - 3)(2u - 3)$$

e) $12m^2 - 52m - 40 = 4(3m^2 - 13m - 10)$

Factor: $3m^2 - 13m - 10$

The product is: $3(-10) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30; 2 and -15 ; -2 and 15; 3 and -10 ; -3 and 10; 5 and -6 ; -5 and 6

The factors with a sum of -13 are 2 and -15 .

$$3m^2 - 13m - 10 = 3m^2 + 2m - 15m - 10$$

$$= m(3m + 2) - 5(3m + 2)$$

$$= (3m + 2)(m - 5)$$

$$\text{So, } 12m^2 - 52m - 40 = 4(3m + 2)(m - 5)$$

f) $24g^2 - 2g - 70 = 2(12g^2 - g - 35)$

Factor: $12g^2 - g - 35$

The product is: $12(-35) = -420$

Since the coefficient of g is -1 , start with factors of -420 that are close in numerical value: 20 and -21

$$12g^2 - g - 35 = 12g^2 + 20g - 21g - 35$$

$$= 4g(3g + 5) - 7(3g + 5)$$

$$= (3g + 5)(4g - 7)$$

$$\text{So, } 24g^2 - 2g - 70 = 2(3g + 5)(4g - 7)$$

19. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $14y^2 - 13y + 3$

The product is: $14(3) = 42$

Consider only the negative factors of 42 because the constant term in the trinomial is positive, and the y -term is negative.

The negative factors of 42 are: -1 and -42 ; -2 and -21 ; -3 and -14 ; -6 and -7

The two factors with a sum of -13 are: -6 and -7

$$\text{So, } 14y^2 - 13y + 3 = 14y^2 - 6y - 7y + 3$$

$$= 2y(7y - 3) - 1(7y - 3)$$

$$= (7y - 3)(2y - 1)$$

b) $10p^2 - 17p - 6$

The product is: $10(-6) = -60$

The factors of -60 are: 1 and -60 ; -1 and 60; 2 and -30 ; -2 and 30; 3 and -20 ; -3 and 20; 4 and -15 ; -4 and 15; 5 and -12 ; -5 and 12; 6 and -10 ; -6 and 10

The factors with a sum of -17 are 3 and -20 .

$$10p^2 - 17p - 6 = 10p^2 - 20p + 3p - 6$$

$$= 10p(p - 2) + 3(p - 2)$$

$$= (p - 2)(10p + 3)$$

c) $10r^2 - 33r - 7$

The product is: $10(-7) = -70$

The factors of -70 are: 1 and -70 ; -1 and 70; 2 and -35 ; -2 and 35; 5 and -14 ; -5 and 14; 7 and -10 ; -7 and 10

The factors with a sum of -33 are 2 and -35 .

$$10r^2 - 33r - 7 = 10r^2 + 2r - 35r - 7$$

$$= 2r(5r + 1) - 7(5r + 1)$$

$$= (5r + 1)(2r - 7)$$

d) $15g^2 - g - 2$

The product is: $15(-2) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30; 2 and -15 ; -2 and 15; 3 and -10 ; -3 and 10; 5 and -6 ; -5 and 6

The factors with a sum of -1 are 5 and -6 .

$$\begin{aligned} 15g^2 - g - 2 &= 15g^2 + 5g - 6g - 2 \\ &= 5g(3g + 1) - 2(3g + 1) \\ &= (3g + 1)(5g - 2) \end{aligned}$$

e) $4x^2 + 4x - 15$

The product is: $4(-15) = -60$

The factors of -60 are: 1 and -60 ; -1 and 60 ; 2 and -30 ; -2 and 30 ; 3 and -20 ; -3 and 20 ; 4 and -15 ; -4 and 15 ; 5 and -12 ; -5 and 12 ; 6 and -10 ; -6 and 10

The factors with a sum of 4 are -6 and 10 .

$$\begin{aligned} 4x^2 + 4x - 15 &= 4x^2 - 6x + 10x - 15 \\ &= 2x(2x - 3) + 5(2x - 3) \\ &= (2x - 3)(2x + 5) \end{aligned}$$

f) $9d^2 - 24d + 16$

The product is: $9(16) = 144$

Consider only the negative factors of 144 because the constant term in the trinomial is positive, and the d -term is negative.

The negative factors of 144 are: -1 and -144 ; -2 and -72 ; -3 and -48 ; -4 and -36 ; -6 and -24 ; -8 and -18 ; -9 and -16 ; -12 and -12

The two factors with a sum of -24 are -12 and -12 .

$$\begin{aligned} \text{So, } 9d^2 - 24d + 16 &= 9d^2 - 12d - 12d + 16 \\ &= 3d(3d - 4) - 4(3d - 4) \\ &= (3d - 4)(3d - 4) \end{aligned}$$

g) $9t^2 + 12t + 4$

The product is: $9(4) = 36$

Consider only the positive factors of 36 because both the constant term and the t -term in the trinomial are positive.

The positive factors of 36 are: 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6

The two factors with a sum of 12 are: 6 and 6

$$\begin{aligned} \text{So, } 9t^2 + 12t + 4 &= 9t^2 + 6t + 6t + 4 \\ &= 3t(3t + 2) + 2(3t + 2) \\ &= (3t + 2)(3t + 2) \end{aligned}$$

h) $40y^2 + y - 6$

The product is: $40(-6) = -240$

Since the coefficient of y is 1, start with factors of -240 that are close in numerical value: -15 and 16

$$\begin{aligned} 40y^2 + y - 6 &= 40y^2 - 15y + 16y - 6 \\ &= 5y(8y - 3) + 2(8y - 3) \\ &= (8y - 3)(5y + 2) \end{aligned}$$

i) $24c^2 + 26c - 15$

The product is: $24(-15) = -360$

Since the coefficient of c is 26, start with factors of -360 that have a difference close to 26: -10 and 36

$$\begin{aligned} 24c^2 + 26c - 15 &= 24c^2 - 10c + 36c - 15 \\ &= 2c(12c - 5) + 3(12c - 5) \\ &= (12c - 5)(2c + 3) \end{aligned}$$

j) $8x^2 + 14x - 15$

The product is: $8(-15) = -120$

Since the coefficient of x is 14, start with factors of -120 that have a difference close to 14: -6 and 20

$$\begin{aligned} 8x^2 + 14x - 15 &= 8x^2 - 6x + 20x - 15 \\ &= 2x(4x - 3) + 5(4x - 3) \\ &= (4x - 3)(2x + 5) \end{aligned}$$

20. a) $4s^2 + \square s + 3$

The possible values of $\square$ are the sums of the factors of $4(3) = 12$

The factors of 12 are: 1 and 12; -1 and -12 ; 2 and 6; -2 and -6 ; 3 and 4; -3 and -4

The sums of these factors are: 13; -13 ; 8; -8 ; 7; -7

So, $\square = 13; -13; 8; -8; 7; -7$

b) $4h^2 + \square h + 25$

The possible values of $\square$ are the sums of the factors of $4(25) = 100$

The factors of 100 are: 1 and 100; -1 and -100 ; 2 and 50; -2 and -50 ; 4 and 25; -4 and -25 ; 5 and 20; -5 and -20 ; 10 and 10; -10 and -10

The sums of these factors are: 101; -101 ; 52; -52 ; 29; -29 ; 25; -25 ; 20; -20

So, $\square = 101; -101; 52; -52; 29; -29; 25; -25; 20; -20$

c) $6y^2 + \square y - 9$

The possible values of $\square$ are the sums of the factors of $6(-9) = -54$

The factors of -54 are: 1 and -54 ; -1 and 54; 2 and -27 ; -2 and 27; 3 and -18 ; -3 and 18; 6 and -9 ; -6 and 9

The sums of these factors are: -53 ; 53; -25 ; 25; -15 ; 15; -3 ; 3

So, $\square = -53; 53; -25; 25; -15; 15; -3; 3$

d) $12t^2 + \square t + 10$

The possible values of $\square$ are the sums of the factors of $12(10) = 120$

The factors of 120 are: 1 and 120; -1 and -120 ; 2 and 60; -2 and -60 ; 3 and 40; -3 and -40 ; 4 and 30; -4 and -30 ; 5 and 24; -5 and -24 ; 6 and 20; -6 and -20 ; 8 and 15; -8 and -15 ; 10 and 12; -10 and -12

The sums of these factors are: 121; -121 ; 62; -62 ; 43; -43 ; 34; -34 ; 29; -29 ; 26; -26 ; 23; -23 ; 22; -22

So, $\square = 121; -121; 62; -62; 43; -43; 34; -34; 29; -29; 26; -26; 23; -23; 22; -22$

e) $9z^2 + \square z + 1$

The possible values of $\square$ are the sums of the factors of $9(1) = 9$

The factors of 9 are: 1 and 9; -1 and -9 ; 3 and 3; -3 and -3

The sums of these factors are: 10; -10 ; 6; -6

So, $\square = 10; -10; 6; -6$

f) $\square f^2 + 2f + \square$

The possible values of the two $\square$ are numbers whose products have factors with a sum or difference of 2.

Factors with a sum of 2 are: $1 + 1$; $3 - 1$; $4 - 2$; $5 - 3$; $6 - 4$; and so on

If both $\square$ have the same value; then, $\square = 1$

If both $\square$ have different values; then these values are: 3 and -1 ; or 4 and -2 ; or 5 and -3 ; or 6 and -4 ; and so on; there are infinite possible values.

C

21. a) i) $4r^2 - r - 5$

Attempt to factor by decomposition.

The product is: $4(-5) = -20$

Find two factors of -20 that have a sum of -1 ; these factors are 4 and -5 .

$$\begin{aligned}\text{So, } 4r^2 - r - 5 &= 4r^2 + 4r - 5r - 5 \\ &= 4r(r + 1) - 5(r + 1) \\ &= (r + 1)(4r - 5)\end{aligned}$$

ii) $2t^2 + 10t + 3$

Attempt to factor by decomposition.

The product is: $2(3) = 6$

There are no two factors of 6 that have a sum of 10 .

So, $2t^2 + 10t + 3$ does not factor.

iii) $5y^2 + 4y - 2$

Attempt to factor by decomposition.

The product is: $5(-2) = -10$

The factors of -10 are: 1 and -10 ; -1 and 10 ; 2 and -5 ; -2 and 5

There are no two factors of -10 that have a sum of 4 .

So, $5y^2 + 4y - 2$ does not factor.

iv) $2w^2 - 5w + 2$

Attempt to factor by decomposition.

The product is: $2(2) = 4$

Find two factors of 4 that have a sum of -5 ; these factors are -1 and -4 .

$$\begin{aligned}\text{So, } 2w^2 - 5w + 2 &= 2w^2 - 1w - 4w + 2 \\ &= w(2w - 1) - 2(2w - 1) \\ &= (2w - 1)(w - 2)\end{aligned}$$

v) $3h^2 - 8h - 3$

Attempt to factor by decomposition.

The product is: $3(-3) = -9$

Find two factors of -9 that have a sum of -8 ; these factors are 1 and -9 .

$$\begin{aligned}\text{So, } 3h^2 - 8h - 3 &= 3h^2 + 1h - 9h - 3 \\ &= h(3h + 1) - 3(3h + 1) \\ &= (3h + 1)(h - 3)\end{aligned}$$

vi) $2f^2 - f + 1$

Attempt to factor by decomposition.

The product is: $2(1) = 2$

There are no two factors of 2 that have a sum of -1 .

So, $2f^2 - f + 1$ does not factor.

- b) From part a) v), $3h^2 - 8h - 3$ can be factored because there are two numbers with a product of -9 and a sum of -8 . From part a) vi), $2f^2 - f + 1$ cannot be factored because there are no two numbers with a product of 2 and a sum of -1 .

22. Factor using decomposition.

a) i) $3n^2 + 11n + 10$

The product is: $3(10) = 30$

The positive factors of 30 are: 1 and 30; 2 and 15; 3 and 10; 5 and 6

The factors of 30 with a sum of 11 are 5 and 6.

$$\begin{aligned}\text{So, } 3n^2 + 11n + 10 &= 3n^2 + 5n + 6n + 10 \\ &= n(3n + 5) + 2(3n + 5) \\ &= (3n + 5)(n + 2)\end{aligned}$$

ii) $3n^2 - 11n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of -11 are -5 and -6 .

$$\begin{aligned}\text{So, } 3n^2 - 11n + 10 &= 3n^2 - 5n - 6n + 10 \\ &= n(3n - 5) - 2(3n - 5) \\ &= (3n - 5)(n - 2)\end{aligned}$$

iii) $3n^2 + 13n + 10$

The product is: $3(10) = 30$

From part i, two factors of 30 with a sum of 13 are 3 and 10.

$$\begin{aligned}\text{So, } 3n^2 + 13n + 10 &= 3n^2 + 3n + 10n + 10 \\ &= 3n(n + 1) + 10(n + 1) \\ &= (n + 1)(3n + 10)\end{aligned}$$

iv) $3n^2 - 13n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of -13 are -3 and -10 .

$$\begin{aligned}\text{So, } 3n^2 - 13n + 10 &= 3n^2 - 3n - 10n + 10 \\ &= 3n(n - 1) - 10(n - 1) \\ &= (n - 1)(3n - 10)\end{aligned}$$

v) $3n^2 + 17n + 10$

The product is: $3(10) = 30$

From part i, two factors of 30 with a sum of 17 are 2 and 15.

$$\begin{aligned}\text{So, } 3n^2 + 17n + 10 &= 3n^2 + 2n + 15n + 10 \\ &= n(3n + 2) + 5(3n + 2) \\ &= (3n + 2)(n + 5)\end{aligned}$$

vi) $3n^2 - 17n + 10$

The product is: $3(10) = 30$

Two factors of 30 with a sum of -17 are -2 and -15 .

$$\begin{aligned}\text{So, } 3n^2 - 17n + 10 &= 3n^2 - 2n - 15n + 10 \\ &= n(3n - 2) - 5(3n - 2) \\ &= (3n - 2)(n - 5)\end{aligned}$$

- b) There are 2 more trinomials that begin with $3n^2$ and end in $+10$. From the list of factors of 30 in part a) i), these are the products of the binomial factors with constant terms: 1 and 30; and the negative constant terms -1 and -30 .
The sums of these factors are: 31 and -31 .
So, the trinomials are: $3n^2 + 31n + 10$ and $3n^2 - 31n + 10$

23. For all the trinomials that begin with $9m^2$ and end with $+16$, find the sums of the pairs of factors of $9(16) = 144$. These sums are the coefficients of the m -terms in the trinomials.
 The factors of 144 are: 1 and 144; -1 and -144 ; 2 and 72; -2 and -72 ; 3 and 48; -3 and -48 ; 4 and 36; -4 and -36 ; 6 and 24; -6 and -24 ; 8 and 18; -8 and -18 ; 9 and 16; -9 and -16 ; 12 and 12; -12 and -12
 The sums of these factors are: 145; -145 ; 74; -74 ; 51; -51 ; 40; -40 ; 30; -30 ; 26; -26 ; 25; 25; 24; -24
 So, the trinomials are: $9m^2 + 145m + 16$; $9m^2 - 145m + 16$; $9m^2 + 74m + 16$; $9m^2 - 74m + 16$; $9m^2 + 51m + 16$; $9m^2 - 51m + 16$; $9m^2 + 40m + 16$; $9m^2 - 40m + 16$; $9m^2 + 30m + 16$; $9m^2 - 30m + 16$; $9m^2 + 26m + 16$; $9m^2 - 26m + 16$; $9m^2 + 25m + 16$; $9m^2 - 25m + 16$; $9m^2 + 24m + 16$; $9m^2 - 24m + 16$

Checkpoint 2

Assess Your Understanding (pages 180–181)

3.3

1. Arrange each set of tiles to form a rectangle.

- a) There are 6 x -tiles and fifteen 1-tiles.

So, the polynomial is: $6x + 15$

The length of the rectangle is $2x + 5$ and the width is 3; these are the factors.

So, $6x + 15 = 3(2x + 5)$



- b) There are 4 x -tiles and twelve 1-tiles.

So, the polynomial is: $4x + 12$

The length of the rectangle is $x + 3$ and the width is 4; these are the factors.

So, $4x + 12 = 4(x + 3)$

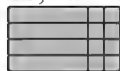


2. a) i) $4a + 8$

Arrange 4 a -tiles and eight 1-tiles to form a rectangle.

The length of the rectangle is $a + 2$ and the width is 4.

So, $4a + 8 = 4(a + 2)$



- ii) $3c - 6$

Arrange 3 c -tiles and 6 negative 1-tiles to form a rectangle.

The length of the rectangle is $c - 2$ and the width is 3.

So, $3c - 6 = 3(c - 2)$



- iii) $-2v^2 - 5v$

The greatest common factor is v .

Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$-2v^2 - 5v = v(-2v) - v(5)$$

$$= v(-2v - 5)$$

$$= -v(2v + 5)$$

Remove -1 as a common factor.

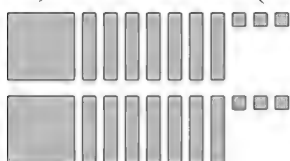
- iv) $2x^2 + 14x + 6$

Arrange 2 x^2 -tiles, 14 x -tiles, and six 1-tiles in equal groups.

There are 2 equal groups.

Each group has 1 x^2 -tile, 7 x -tiles, and three 1-tiles.

So, $2x^2 + 14x + 6 = 2(x^2 + 7x + 3)$



v) $-3r^2 + 15r - 3$

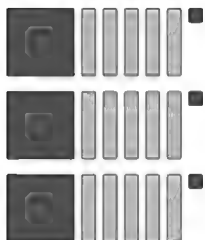
Arrange 3 negative r^2 -tiles, 15 r -tiles, and 3 negative 1-tiles in equal groups.

There are 3 equal groups.

Each group has 1 negative r^2 -tile, 5 r -tiles, and 1 negative 1-tile.

So, $-3r^2 + 15r - 3 = 3(-r^2 + 5r - 1)$ Remove -1 as a common factor.

$$= -3(r^2 - 5r + 1)$$



vi) $15a^3 - 3a^2b - 6ab^2$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$15a^3 = 3 \cdot 5 \cdot a \cdot a \cdot a$$

$$3a^2b = 3 \cdot a \cdot a \cdot b$$

$$6ab^2 = 2 \cdot 3 \cdot a \cdot b \cdot b$$

The greatest common factor is: $3 \cdot a = 3a$

$$15a^3 - 3a^2b - 6ab^2 = 3a(5a^2) - 3a(ab) - 3a(2b^2)$$

$$= 3a(5a^2 - ab - 2b^2)$$

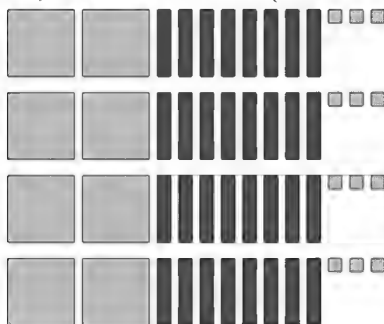
vii) $12 - 32x + 8x^2$

Arrange twelve 1-tiles, 32 negative x -tiles, and 8 x^2 -tiles in equal groups.

There are 4 equal groups.

Each group has three 1-tiles, 8 negative x -tiles, and 2 x^2 -tiles.

So, $12 - 32x + 8x^2 = 4(3 - 8x + 2x^2)$



viii) $12x^2y - 8xy - 16y$

Factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

$$12x^2y = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y$$

$$8xy = 2 \cdot 2 \cdot 2 \cdot x \cdot y$$

$$16y = 2 \cdot 2 \cdot 2 \cdot 2 \cdot y$$

The greatest common factor is: $2 \cdot 2 \cdot y = 4y$

$$12x^2y - 8xy - 16y = 4y(3x^2) + 4y(-2x) + 4y(-4)$$

$$= 4y(3x^2 - 2x - 4)$$

- b) iii) For $-2v^2 - 5v$, I could have used algebra tiles to make a rectangle, but both sides of the rectangle appear to be negative, which would suggest both factors are negative. But, from part a) iii), the factors have opposite signs. So, the algebra tiles show the terms in the factors, but I have to check the signs of the factors.



vi), viii) I cannot use algebra tiles when terms have more than one variable.

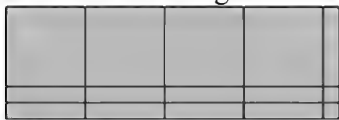
3.4

3. I used 1 x^2 -tile, 9 x -tiles, and twenty 1-tiles.
I made this rectangle.



This rectangle represents the multiplication sentence:
 $(x + 4)(x + 5) = x^2 + 9x + 20$

4. I used 4 x^2 -tiles, 9 x -tiles, and two 1-tiles.
I made this rectangle.



This rectangle represents the multiplication sentence:
 $(4x + 1)(x + 2) = 4x^2 + 9x + 2$

3.5

5. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(x + 1)(x + 4) = x(x + 4) + 1(x + 4)$
 $= x^2 + 4x + x + 4$
 $= x^2 + 5x + 4$

	x	4
x	$(x)(x) = x^2$	$(x)(4) = 4x$
1	$(1)(x) = x$	$(1)(4) = 4$

b) $(d - 2)(d + 3) = d(d + 3) - 2(d + 3)$
 $= d^2 + 3d - 2d - 6$

$$= d^2 + d - 6$$

d	$(d)(d) = d^2$	$(d)(3) = 3d$
-2	$(-2)(d) = -2d$	$(-2)(3) = -6$

c) $(x-4)(x-2) = x(x-2) - 4(x-2)$
 $= x^2 - 2x - 4x + 8$
 $= x^2 - 6x + 8$

x	$(x)(x) = x^2$	$(x)(-2) = -2x$
-4	$(-4)(x) = -4x$	$(-4)(-2) = 8$

d) $(5-r)(6+r) = 5(6+r) - r(6+r)$
 $= 30 + 5r - 6r - r^2$
 $= 30 - r - r^2$

5	$(5)(6) = 30$	$(5)(r) = 5r$
$-r$	$(-r)(6) = -6r$	$(-r)(r) = -r^2$

e) $(g+5)(g-1) = g(g-1) + 5(g-1)$
 $= g^2 - g + 5g - 5$
 $= g^2 + 4g - 5$

g	$(g)(g) = g^2$	$(g)(-1) = -g$
5	$(5)(g) = 5g$	$(5)(-1) = -5$

f) $(2-t)(10-t) = 2(10-t) - t(10-t)$
 $= 20 - 2t - 10t + t^2$
 $= 20 - 12t + t^2$

2	$(2)(10) = 20$	$(2)(-t) = -2t$
$-t$	$(-t)(10) = -10t$	$(-t)(-t) = t^2$

6. For each trinomial, find two integers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term.

a) $s^2 + 11s + 30$

Two numbers whose sum is 11 and whose product is 30 are 5 and 6.

So, $s^2 + 11s + 30 = (s+5)(s+6)$

$$\begin{aligned}\text{Check: } (s+5)(s+6) &= s(s+6) + 5(s+6) \\ &= s^2 + 6s + 5s + 30 \\ &= s^2 + 11s + 30\end{aligned}$$

b) $n^2 - n - 30$

Two numbers whose sum is -1 and whose product is -30 are 5 and -6 .

$$\text{So, } n^2 - n - 30 = (n+5)(n-6)$$

$$\begin{aligned}\text{Check: } (n+5)(n-6) &= n(n-6) + 5(n-6) \\ &= n^2 - 6n + 5n - 30 \\ &= n^2 - n - 30\end{aligned}$$

c) $20 - 9b + b^2$

Two numbers whose sum is -9 and whose product is 20 are -4 and -5 .

$$\text{So, } 20 - 9b + b^2 = (-4+b)(-5+b)$$

This can be written as $(4-b)(5-b)$, or $(b-4)(b-5)$

$$\begin{aligned}\text{Check: } (b-4)(b-5) &= b(b-5) - 4(b-5) \\ &= b^2 - 5b - 4b + 20 \\ &= b^2 - 9b + 20\end{aligned}$$

d) $-11 - 10t + t^2$

Rearrange the trinomial and write it as: $t^2 - 10t - 11$

Two numbers whose sum is -10 and whose product is -11 are 1 and -11 .

$$\text{So, } t^2 - 10t - 11 = (t+1)(t-11)$$

$$\begin{aligned}\text{Check: } (t+1)(t-11) &= t(t-11) + 1(t-11) \\ &= t^2 - 11t + 1t - 11 \\ &= t^2 - 10t - 11\end{aligned}$$

e) $z^2 + 13z + 30$

Two numbers whose sum is 13 and whose product is 30 are 3 and 10 .

$$\text{So, } z^2 + 13z + 30 = (z+3)(z+10)$$

$$\begin{aligned}\text{Check: } (z+3)(z+10) &= z(z+10) + 3(z+10) \\ &= z^2 + 10z + 3z + 30 \\ &= z^2 + 13z + 30\end{aligned}$$

f) $-k^2 + 9k - 18$

Remove -1 as a common factor.

$$-k^2 + 9k - 18 = -(k^2 - 9k + 18)$$

Two numbers whose sum is -9 and whose product is 18 are -3 and -6 .

$$\text{So, } -k^2 + 9k - 18 = -(k-3)(k-6)$$

$$\begin{aligned}\text{Check: } -(k-3)(k-6) &= -[k(k-6) - 3(k-6)] \\ &= -[k^2 - 6k - 3k + 18] \\ &= -[k^2 - 9k + 18] \\ &= -k^2 + 9k - 18\end{aligned}$$

7. Remove a common factor first.

a) $3x^2 + 15x - 42 = 3(x^2 + 5x - 14)$

Two numbers whose sum is 5 and whose product is -14 are -2 and 7 .

So, $3x^2 + 15x - 42 = 3(x - 2)(x + 7)$

b) $-2y^2 + 22y - 48 = -2(y^2 - 11y + 24)$

Two numbers whose sum is -11 and whose product is 24 are -3 and -8 .

So, $-2y^2 + 22y - 48 = -2(y - 3)(y - 8)$

c) $-24 - 11m - m^2 = -(24 + 11m + m^2)$

Two numbers whose sum is 11 and whose product is 24 are 3 and 8 .

So, $-24 - 11m - m^2 = -(3 + m)(8 + m)$

d) $50 - 23y - y^2 = -(-50 + 23y + y^2)$

Two numbers whose sum is 23 and whose product is -50 are -2 and 25 .

So, $50 - 23y - y^2 = -(-2 + y)(25 + y)$

This can be written as: $(2 - y)(25 + y)$

3.6

8. Use the distributive property.

a) $(2c + 1)(c + 3) = 2c(c + 3) + 1(c + 3)$
 $= 2c^2 + 6c + c + 3$
 $= 2c^2 + 7c + 3$

b) $(-m + 5)(4m - 1) = (-m)(4m - 1) + 5(4m - 1)$
 $= -4m^2 + m + 20m - 5$
 $= -4m^2 + 21m - 5$

c) $(3f - 4)(3f + 1) = 3f(3f + 1) - 4(3f + 1)$
 $= 9f^2 + 3f - 12f - 4$
 $= 9f^2 - 9f - 4$

d) $(6z - 1)(2z - 3) = 6z(2z - 3) - 1(2z - 3)$
 $= 12z^2 - 18z - 2z + 3$
 $= 12z^2 - 20z + 3$

e) $(5 - 3r)(6 + 2r) = 5(6 + 2r) - 3r(6 + 2r)$
 $= 30 + 10r - 18r - 6r^2$
 $= 30 - 8r - 6r^2$

f) $(-4 - 2h)(-2 - 4h) = -4(-2 - 4h) - 2h(-2 - 4h)$
 $= 8 + 16h + 4h + 8h^2$
 $= 8 + 20h + 8h^2$

9. Factor using decomposition.

a) $2j^2 + 13j + 20$

The product is: $2(20) = 40$

Consider only positive factors of 40 because all the terms in the trinomial are positive.

The factors of 40 are: 1 and 40; 2 and 20; 4 and 10; 5 and 8

The factors of 40 with a sum of 13 are 5 and 8.

So, $2j^2 + 13j + 20 = 2j^2 + 5j + 8j + 20$

$$= j(2j + 5) + 4(2j + 5)$$

$$= (2j + 5)(j + 4)$$

Check: $(2j + 5)(j + 4) = 2j(j + 4) + 5(j + 4)$

$$= 2j^2 + 8j + 5j + 20$$

$$= 2j^2 + 13j + 20$$

b) $3v^2 + v - 10$

The product is: $3(-10) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30; 2 and -15 ; -2 and 15; 3 and -10 ; -3 and 10;

5 and -6 ; -5 and 6

The factors of -30 with a sum of 1 are -5 and 6.

So, $3v^2 + v - 10 = 3v^2 - 5v + 6v - 10$

$$= v(3v - 5) + 2(3v - 5)$$

$$= (3v - 5)(v + 2)$$

Check: $(3v - 5)(v + 2) = 3v(v + 2) - 5(v + 2)$

$$= 3v^2 + 6v - 5v - 10$$

$$= 3v^2 + v - 10$$

c) $5k^2 - 23k + 12$

The product is: $5(12) = 60$

Consider only negative factors of 60 because the constant term in the trinomial is positive, and the k -term is negative.

The factors of 60 are: -1 and -60 ; -2 and -30 ; -3 and -20 ; -4 and -15 ; -5 and -12 ; -6 and -10

The factors of 60 with a sum of -23 are -3 and -20 .

So, $5k^2 - 23k + 12 = 5k^2 - 3k - 20k + 12$

$$= k(5k - 3) - 4(5k - 3)$$

$$= (5k - 3)(k - 4)$$

Check: $(5k - 3)(k - 4) = 5k(k - 4) - 3(k - 4)$

$$= 5k^2 - 20k - 3k + 12$$

$$= 5k^2 - 23k + 12$$

d) $9h^2 + 18h + 8$

The product is: $9(8) = 72$

Consider only positive factors of 72 because all the terms in the trinomial are positive.

The factors of 72 are: 1 and 72; 2 and 36; 3 and 24; 4 and 18; 6 and 12; 8 and 9

The factors of 72 with a sum of 18 are 6 and 12.

So, $9h^2 + 18h + 8 = 9h^2 + 6h + 12h + 8$

$$= 3h(3h + 2) + 4(3h + 2)$$

$$= (3h + 2)(3h + 4)$$

Check: $(3h + 2)(3h + 4) = 3h(3h + 4) + 2(3h + 4)$

$$= 9h^2 + 12h + 6h + 8$$

$$= 9h^2 + 18h + 8$$

e) $8y^2 - 2y - 1$

The product is: $8(-1) = -8$

The factors of -8 are: 1 and -8 ; -1 and 8; 2 and -4 ; -2 and 4

The factors of -8 with a sum of -2 are 2 and -4 .

So, $8y^2 - 2y - 1 = 8y^2 + 2y - 4y - 1$

$$= 2y(4y + 1) - 1(4y + 1)$$

$$= (4y + 1)(2y - 1)$$

Check: $(4y + 1)(2y - 1) = 4y(2y - 1) + 1(2y - 1)$

$$= 8y^2 - 4y + 2y - 1$$

$$= 8y^2 - 2y - 1$$

f) $6 - 23u + 20u^2$

The product is: $6(20) = 120$

Consider only negative factors of 120 because the constant term in the trinomial is positive, and the u -term is negative.

The negative factors of 120 are: -1 and -120 ; -2 and -60 ; -3 and -40 ; -4 and -30 ; -5 and -24 ; -6 and -20 ; -8 and -15 ; -10 and -12

The factors of 120 with a sum of -23 are -8 and -15 .

So, $6 - 23u + 20u^2 = 6 - 8u - 15u + 20u^2$

$$= 2(3 - 4u) - 5u(3 - 4u)$$

$$= (3 - 4u)(2 - 5u)$$

Check: $(3 - 4u)(2 - 5u) = 3(2 - 5u) - 4u(2 - 5u)$

$$= 6 - 15u - 8u + 20u^2$$

$$= 6 - 23u + 20u^2$$

Lesson 3.7 Multiplying Polynomials

Exercises (pages 186–187)

A

4. a) $(g + 1)(g^2 + 2g + 3)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$$\begin{aligned} & (g + 1)(g^2 + 2g + 3) \\ &= g(g^2 + 2g + 3) + 1(g^2 + 2g + 3) \\ &= g(g^2) + g(2g) + g(3) + 1(g^2) + 1(2g) + 1(3) \\ &= g^3 + 2g^2 + 3g + g^2 + 2g + 3 && \text{Collect like terms.} \\ &= g^3 + 2g^2 + g^2 + 3g + 2g + 3 && \text{Combine like terms.} \\ &= g^3 + 3g^2 + 5g + 3 \end{aligned}$$

b) $(2 + t + t^2)(1 + 3t + t^2)$

Use the distributive property. Multiply each term in the 1st trinomial by each term in the 2nd trinomial. Write the terms in a list.

$$\begin{aligned} & (2 + t + t^2)(1 + 3t + t^2) \\ &= 2(1 + 3t + t^2) + t(1 + 3t + t^2) + t^2(1 + 3t + t^2) \\ &= 2(1) + 2(3t) + 2(t^2) + t(1) + t(3t) + t(t^2) + t^2(1) + t^2(3t) + t^2(t^2) \\ &= 2 + 6t + 2t^2 + t + 3t^2 + t^3 + t^2 + 3t^3 + t^4 && \text{Collect like terms.} \\ &= 2 + 6t + t + 2t^2 + 3t^2 + t^2 + t^3 + 3t^3 + t^4 && \text{Combine like terms.} \\ &= 2 + 7t + 6t^2 + 4t^3 + t^4 \end{aligned}$$

c) $(2w + 3)(w^2 + 4w + 7)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$$\begin{aligned} & (2w + 3)(w^2 + 4w + 7) \\ &= 2w(w^2 + 4w + 7) + 3(w^2 + 4w + 7) \\ &= 2w(w^2) + 2w(4w) + 2w(7) + 3(w^2) + 3(4w) + 3(7) \\ &= 2w^3 + 8w^2 + 14w + 3w^2 + 12w + 21 && \text{Collect like terms.} \\ &= 2w^3 + 8w^2 + 3w^2 + 14w + 12w + 21 && \text{Combine like terms.} \\ &= 2w^3 + 11w^2 + 26w + 21 \end{aligned}$$

d) $(4 + 3n + n^2)(3 + 5n + n^2)$

Use the distributive property. Multiply each term in the 1st trinomial by each term in the 2nd trinomial. Write the terms in a list.

$$\begin{aligned} & (4 + 3n + n^2)(3 + 5n + n^2) \\ &= 4(3 + 5n + n^2) + 3n(3 + 5n + n^2) + n^2(3 + 5n + n^2) \\ &= 4(3) + 4(5n) + 4(n^2) + 3n(3) + 3n(5n) + 3n(n^2) + n^2(3) + n^2(5n) + n^2(n^2) \\ &= 12 + 20n + 4n^2 + 9n + 15n^2 + 3n^3 + 3n^2 + 5n^3 + n^4 \quad \text{Collect like terms.} \\ &= 12 + 20n + 9n + 4n^2 + 15n^2 + 3n^2 + 3n^3 + 5n^3 + n^4 \quad \text{Combine like terms.} \\ &= 12 + 29n + 22n^2 + 8n^3 + n^4 \end{aligned}$$

5. a) $(2z + y)(3z + y)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$$\begin{aligned} & (2z + y)(3z + y) \\ &= 2z(3z + y) + y(3z + y) \\ &= 2z(3z) + 2z(y) + y(3z) + y(y) \\ &= 6z^2 + 2yz + 3yz + y^2 \quad \text{Combine like terms.} \\ &= 6z^2 + 5yz + y^2 \end{aligned}$$

b) $(4f - 3g)(3f - 4g + 1)$

Use the distributive property. Multiply each term in the trinomial by each term in the binomial. Write the terms in a list.

$$\begin{aligned} & (4f - 3g)(3f - 4g + 1) \\ &= 4f(3f - 4g + 1) - 3g(3f - 4g + 1) \\ &= 4f(3f) + 4f(-4g) + 4f(1) - 3g(3f) - 3g(-4g) - 3g(1) \\ &= 12f^2 - 16fg + 4f - 9fg + 12g^2 - 3g \quad \text{Collect like terms.} \\ &= 12f^2 + 4f - 16fg - 9fg - 3g + 12g^2 \quad \text{Combine like terms.} \\ &= 12f^2 + 4f - 25fg - 3g + 12g^2 \end{aligned}$$

c) $(2a + 3b)(4a + 5b)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$$\begin{aligned} & (2a + 3b)(4a + 5b) \\ &= 2a(4a + 5b) + 3b(4a + 5b) \\ &= 2a(4a) + 2a(5b) + 3b(4a) + 3b(5b) \\ &= 8a^2 + 10ab + 12ab + 15b^2 \quad \text{Combine like terms.} \\ &= 8a^2 + 22ab + 15b^2 \end{aligned}$$

d) $(3a - 4b + 1)(4a - 5b)$

Use the distributive property. Multiply each term in the binomial by each term in the trinomial. Write the terms in a list.

$$\begin{aligned} & (3a - 4b + 1)(4a - 5b) \\ &= 3a(4a - 5b) - 4b(4a - 5b) + 1(4a - 5b) \\ &= 3a(4a) + 3a(-5b) - 4b(4a) - 4b(-5b) + 1(4a) + 1(-5b) \\ &= 12a^2 - 15ab - 16ab + 20b^2 + 4a - 5b && \text{Collect like terms.} \\ &= 12a^2 + 4a - 15ab - 16ab - 5b + 20b^2 && \text{Combine like terms.} \\ &= 12a^2 + 4a - 31ab - 5b + 20b^2 \end{aligned}$$

e) $(2r + s)^2 = (2r + s)(2r + s)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$$\begin{aligned} & (2r + s)(2r + s) \\ &= 2r(2r + s) + s(2r + s) \\ &= 2r(2r) + 2r(s) + s(2r) + s(s) \\ &= 4r^2 + 2rs + 2rs + s^2 && \text{Combine like terms.} \\ &= 4r^2 + 4rs + s^2 \end{aligned}$$

f) $(3t - 2u)^2 = (3t - 2u)(3t - 2u)$

Use the distributive property. Multiply each term in the 1st binomial by each term in the 2nd binomial. Write the terms in a list.

$$\begin{aligned} & (3t - 2u)(3t - 2u) \\ &= 3t(3t - 2u) - 2u(3t - 2u) \\ &= 3t(3t) + 3t(-2u) - 2u(3t) - 2u(-2u) \\ &= 9t^2 - 6tu - 6tu + 4u^2 && \text{Combine like terms.} \\ &= 9t^2 - 12tu + 4u^2 \end{aligned}$$

B

6. a) i) $(2x + y)(2x + y)$

$$\begin{aligned} &= 2x(2x + y) + y(2x + y) \\ &= 2x(2x) + 2x(y) + y(2x) + y(y) \\ &= 4x^2 + 2xy + 2xy + y^2 && \text{Combine like terms.} \\ &= 4x^2 + 4xy + y^2 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } (5r + 2s)(5r + 2s) &= 5r(5r + 2s) + 2s(5r + 2s) \\
 &= 5r(5r) + 5r(2s) + 2s(5r) + 2s(2s) \\
 &= 25r^2 + 10rs + 10rs + 4s^2 && \text{Combine like terms.} \\
 &= 25r^2 + 20rs + 4s^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } (6c + 5d)(6c + 5d) &= 6c(6c + 5d) + 5d(6c + 5d) \\
 &= 6c(6c) + 6c(5d) + 5d(6c) + 5d(5d) \\
 &= 36c^2 + 30cd + 30cd + 25d^2 && \text{Combine like terms.} \\
 &= 36c^2 + 60cd + 25d^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } (5v + 7w)(5v + 7w) &= 5v(5v + 7w) + 7w(5v + 7w) \\
 &= 5v(5v) + 5v(7w) + 7w(5v) + 7w(7w) \\
 &= 25v^2 + 35vw + 35vw + 49w^2 && \text{Combine like terms.} \\
 &= 25v^2 + 70vw + 49w^2
 \end{aligned}$$

$$\begin{aligned}
 \text{v) } (2x - y)(2x - y) &= 2x(2x - y) - y(2x - y) \\
 &= 2x(2x) + 2x(-y) - y(2x) - y(-y) \\
 &= 4x^2 - 2xy - 2xy + y^2 && \text{Combine like terms.} \\
 &= 4x^2 - 4xy + y^2
 \end{aligned}$$

$$\begin{aligned}
 \text{vi) } (5r - 2s)(5r - 2s) &= 5r(5r - 2s) - 2s(5r - 2s) \\
 &= 5r(5r) + 5r(-2s) - 2s(5r) - 2s(-2s) \\
 &= 25r^2 - 10rs - 10rs + 4s^2 && \text{Combine like terms.} \\
 &= 25r^2 - 20rs + 4s^2
 \end{aligned}$$

$$\begin{aligned}
 \text{vii) } (6c - 5d)(6c - 5d) &= 6c(6c - 5d) - 5d(6c - 5d) \\
 &= 6c(6c) + 6c(-5d) - 5d(6c) - 5d(-5d) \\
 &= 36c^2 - 30cd - 30cd + 25d^2 && \text{Combine like terms.} \\
 &= 36c^2 - 60cd + 25d^2
 \end{aligned}$$

$$\begin{aligned}
 \text{viii) } (5v - 7w)(5v - 7w) &= 5v(5v - 7w) - 7w(5v - 7w) \\
 &= 5v(5v) + 5v(-7w) - 7w(5v) - 7w(-7w) \\
 &= 25v^2 - 35vw - 35vw + 49w^2 && \text{Combine like terms.} \\
 &= 25v^2 - 70vw + 49w^2
 \end{aligned}$$

- b)** The factors being multiplied are the same, so each time a binomial is being squared.
 The first term in the trinomial is the square of the first term in the binomial.
 The second term in the trinomial is twice the product of the terms in the binomial.
 The third term in the trinomial is the square of the second term in the binomial.

$$\begin{aligned}
 \text{i) } (p + 3q)(p + 3q) &= p(p) + 2(p)(3q) + 3q(3q) \\
 &= p^2 + 6pq + 9q^2
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } (2s - 7t)(2s - 7t) &= 2s(2s) + 2(2s)(-7t) + (-7t)(-7t) \\
 &= 4s^2 - 28st + 49t^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } (5g + 4h)(5g + 4h) &= 5g(5g) + 2(5g)(4h) + 4h(4h) \\
 &= 25g^2 + 40gh + 16h^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } (10h - 7k)(10h - 7k) &= 10h(10h) + 2(10h)(-7k) + (-7k)(-7k) \\
 &= 100h^2 - 140hk + 49k^2
 \end{aligned}$$

$$\begin{aligned}
 7. \text{ a) i) } (x + 2y)(x - 2y) &= x(x - 2y) + 2y(x - 2y) \\
 &= x(x) + x(-2y) + 2y(x) + 2y(-2y) \\
 &= x^2 - 2xy + 2xy - 4y^2 && \text{Combine like terms.} \\
 &= x^2 - 4y^2
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } (3r - 4s)(3r + 4s) &= 3r(3r + 4s) - 4s(3r + 4s) \\
 &= 3r(3r) + 3r(4s) - 4s(3r) - 4s(4s) \\
 &= 9r^2 + 12rs - 12rs - 16s^2 && \text{Combine like terms.} \\
 &= 9r^2 - 16s^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } (5c + 3d)(5c - 3d) &= 5c(5c - 3d) + 3d(5c - 3d) \\
 &= 5c(5c) + 5c(-3d) + 3d(5c) + 3d(-3d) \\
 &= 25c^2 - 15cd + 15cd - 9d^2 && \text{Combine like terms.} \\
 &= 25c^2 - 9d^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } (2v - 7w)(2v + 7w) &= 2v(2v + 7w) - 7w(2v + 7w) \\
 &= 2v(2v) + 2v(7w) - 7w(2v) - 7w(7w) \\
 &= 4v^2 + 14vw - 14vw - 49w^2 && \text{Combine like terms.} \\
 &= 4v^2 - 49w^2
 \end{aligned}$$

- b)** The factors in each pair are the same except the second terms have opposite signs.
The product is the square of the first term in the binomial minus the square of the second term in the binomial.

$$\begin{aligned}
 \text{i) } (11g + 5h)(11g - 5h) &= 11g(11g) - 5h(5h) \\
 &= 121g^2 - 25h^2
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } (25m - 7n)(25m + 7n) &= 25m(25m) - 7n(7n) \\
 &= 625m^2 - 49n^2
 \end{aligned}$$

$$\begin{aligned}
 \text{8. a) } (3y - 2)(y^2 + y - 8) &= 3y(y^2 + y - 8) - 2(y^2 + y - 8) \\
 &= 3y(y^2) + 3y(y) + 3y(-8) - 2(y^2) - 2(y) - 2(-8) \\
 &= 3y^3 + 3y^2 - 24y - 2y^2 - 2y + 16 && \text{Collect like terms.} \\
 &= 3y^3 + 3y^2 - 2y^2 - 24y - 2y + 16 && \text{Combine like terms.} \\
 &= 3y^3 + y^2 - 26y + 16
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (4r + 1)(r^2 - 2r - 3) \\
 &= 4r(r^2 - 2r - 3) + 1(r^2 - 2r - 3) \\
 &= 4r(r^2) + 4r(-2r) + 4r(-3) + 1(r^2) + 1(-2r) + 1(-3) \\
 &= 4r^3 - 8r^2 - 12r + r^2 - 2r - 3 && \text{Collect like terms.} \\
 &= 4r^3 - 8r^2 + r^2 - 12r - 2r - 3 && \text{Combine like terms.} \\
 &= 4r^3 - 7r^2 - 14r - 3
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & (b^2 + 9b - 2)(2b - 1) \\
 &= b^2(2b - 1) + 9b(2b - 1) - 2(2b - 1) \\
 &= b^2(2b) + b^2(-1) + 9b(2b) + 9b(-1) - 2(2b) - 2(-1) \\
 &= 2b^3 - b^2 + 18b^2 - 9b - 4b + 2 && \text{Combine like terms.} \\
 &= 2b^3 + 17b^2 - 13b + 2
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & (x^2 + 6x + 1)(3x - 7) \\
 &= x^2(3x - 7) + 6x(3x - 7) + 1(3x - 7) \\
 &= x^2(3x) + x^2(-7) + 6x(3x) + 6x(-7) + 1(3x) + 1(-7) \\
 &= 3x^3 - 7x^2 + 18x^2 - 42x + 3x - 7 && \text{Combine like terms.} \\
 &= 3x^3 + 11x^2 - 39x - 7
 \end{aligned}$$

$$\begin{aligned}
 \text{9. a) } & (x + y)(x + y + 3) \\
 &= x(x + y + 3) + y(x + y + 3) \\
 &= x(x) + x(y) + x(3) + y(x) + y(y) + y(3) \\
 &= x^2 + xy + 3x + xy + y^2 + 3y && \text{Collect like terms.} \\
 &= x^2 + 3x + xy + xy + 3y + y^2 && \text{Combine like terms.} \\
 &= x^2 + 3x + 2xy + 3y + y^2
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (x + 2)(x + y + 1) \\
 &= x(x + y + 1) + 2(x + y + 1) \\
 &= x(x) + x(y) + x(1) + 2(x) + 2(y) + 2(1) \\
 &= x^2 + xy + x + 2x + 2y + 2 && \text{Collect like terms.} \\
 &= x^2 + 2x + x + xy + 2y + 2 && \text{Combine like terms.} \\
 &= x^2 + 3x + xy + 2y + 2
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & (a+b)(a+b+c) \\
 &= a(a+b+c) + b(a+b+c) \\
 &= a(a) + a(b) + a(c) + b(a) + b(b) + b(c) \\
 &= a^2 + ab + ac + ab + b^2 + bc && \text{Collect like terms.} \\
 &= a^2 + ab + ab + b^2 + ac + bc && \text{Combine like terms.} \\
 &= a^2 + 2ab + b^2 + ac + bc
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & (3+t)(2+t+s) \\
 &= 3(2+t+s) + t(2+t+s) \\
 &= 3(2) + 3(t) + 3(s) + t(2) + t(t) + t(s) \\
 &= 6 + 3t + 3s + 2t + t^2 + st && \text{Collect like terms.} \\
 &= 3s + st + 3t + 2t + t^2 + 6 && \text{Combine like terms.} \\
 &= 3s + st + 5t + t^2 + 6
 \end{aligned}$$

$$\begin{aligned}
 \text{10. a) } & (x+2y)(x-2y-1) \\
 &= x(x-2y-1) + 2y(x-2y-1) \\
 &= x(x) + x(-2y) + x(-1) + 2y(x) + 2y(-2y) + 2y(-1) \\
 &= x^2 - 2xy - x + 2xy - 4y^2 - 2y && \text{Collect like terms.} \\
 &= x^2 - x - 2y - 2xy + 2xy - 4y^2 && \text{Combine like terms.} \\
 &= x^2 - x - 2y - 4y^2
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (2c-3d)(c+d+1) \\
 &= 2c(c+d+1) - 3d(c+d+1) \\
 &= 2c(c) + 2c(d) + 2c(1) - 3d(c) - 3d(d) - 3d(1) \\
 &= 2c^2 + 2cd + 2c - 3cd - 3d^2 - 3d && \text{Collect like terms.} \\
 &= 2c^2 + 2c + 2cd - 3cd - 3d - 3d^2 && \text{Combine like terms.} \\
 &= 2c^2 + 2c - cd - 3d - 3d^2
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & (a-5b)(a+2b-4) \\
 &= a(a+2b-4) - 5b(a+2b-4) \\
 &= a(a) + a(2b) + a(-4) - 5b(a) - 5b(2b) - 5b(-4) \\
 &= a^2 + 2ab - 4a - 5ab - 10b^2 + 20b && \text{Collect like terms.} \\
 &= a^2 - 4a + 2ab - 5ab + 20b - 10b^2 && \text{Combine like terms.} \\
 &= a^2 - 4a - 3ab + 20b - 10b^2
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & (p - 2q)(p + 4q - r) \\
 &= p(p + 4q - r) - 2q(p + 4q - r) \\
 &= p(p) + p(4q) + p(-r) - 2q(p) - 2q(4q) - 2q(-r) \\
 &= p^2 + 4pq - pr - 2pq - 8q^2 + 2qr \quad \text{Collect like terms.} \\
 &= p^2 + 4pq - 2pq - 8q^2 - pr + 2qr \quad \text{Combine like terms.} \\
 &= p^2 + 2pq - 8q^2 - pr + 2qr
 \end{aligned}$$

11. There are 2 errors in the third line of the solution:

- the product of $2r$ and $-5s$ should be $-10rs$, not $-5rs$.
- the product of $-3s$ and $-5s$ should be $+15s^2$, not $-15s^2$.

There is an error in the fourth line of the solution:

- $-15s^2$ and $-18s$ were added to get $-33s^2$. These two terms cannot be added because they are not like terms.

A correct solution is:

$$\begin{aligned}
 & (2r - 3s)(r - 5s + 6) \\
 &= 2r(r - 5s + 6) - 3s(r - 5s + 6) \\
 &= 2r(r) + 2r(-5s) + 2r(6) - 3s(r) - 3s(-5s) - 3s(6) \\
 &= 2r^2 - 10rs + 12r - 3rs + 15s^2 - 18s \quad \text{Collect like terms.} \\
 &= 2r^2 - 10rs - 3rs + 12r + 15s^2 - 18s \quad \text{Combine like terms.} \\
 &= 2r^2 - 13rs + 12r + 15s^2 - 18s
 \end{aligned}$$

12. The formula for the volume of a right rectangular prism is:

$V = Ah$, where A represents the area of the base and h represents the height.

Substitute: $A = x^2 + 3x + 2$ and $h = x + 7$ into the formula, then simplify.

$$\begin{aligned}
 V &= Ah \\
 &= (x^2 + 3x + 2)(x + 7) \\
 &= x^2(x + 7) + 3x(x + 7) + 2(x + 7) \\
 &= x^2(x) + x^2(7) + 3x(x) + 3x(7) + 2(x) + 2(7) \\
 &= x^3 + 7x^2 + 3x^2 + 21x + 2x + 14 \quad \text{Combine like terms.} \\
 &= x^3 + 10x^2 + 23x + 14
 \end{aligned}$$

An expression for the volume of the prism is: $x^3 + 10x^2 + 23x + 14$

$$\begin{aligned}
 \text{13. a) } & (r^2 + 3r + 2)(4r^2 + r + 1) \\
 &= r^2(4r^2 + r + 1) + 3r(4r^2 + r + 1) + 2(4r^2 + r + 1) \\
 &= r^2(4r^2) + r^2(r) + r^2(1) + 3r(4r^2) + 3r(r) + 3r(1) + 2(4r^2) + 2(r) + 2(1) \\
 &= 4r^4 + r^3 + r^2 + 12r^3 + 3r^2 + 3r + 8r^2 + 2r + 2 \quad \text{Collect like terms.} \\
 &= 4r^4 + r^3 + 12r^3 + r^2 + 3r^2 + 8r^2 + 3r + 2r + 2 \quad \text{Combine like terms.} \\
 &= 4r^4 + 13r^3 + 12r^2 + 5r + 2
 \end{aligned}$$

To check, substitute $r = 2$ into the trinomial product and its simplification.

$$(r^2 + 3r + 2)(4r^2 + r + 1) = 4r^4 + 13r^3 + 12r^2 + 5r + 2$$

$$\begin{aligned}\text{Left side: } (r^2 + 3r + 2)(4r^2 + r + 1) &= [(2)^2 + 3(2) + 2][4(2)^2 + 2 + 1] \\ &= (4 + 6 + 2)(16 + 3) \\ &= (12)(19) \\ &= 228\end{aligned}$$

$$\begin{aligned}\text{Right side: } 4r^4 + 13r^3 + 12r^2 + 5r + 2 &= 4(2)^4 + 13(2)^3 + 12(2)^2 + 5(2) + 2 \\ &= 64 + 104 + 48 + 10 + 2 \\ &= 228\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

$$\begin{aligned}\text{b) } (2d^2 + 2d + 1)(d^2 + 6d + 3) &= 2d^2(d^2 + 6d + 3) + 2d(d^2 + 6d + 3) + 1(d^2 + 6d + 3) \\ &= 2d^2(d^2) + 2d^2(6d) + 2d^2(3) + 2d(d^2) + 2d(6d) + 2d(3) + 1(d^2) + 1(6d) + 1(3) \\ &= 2d^4 + 12d^3 + 6d^2 + 2d^3 + 12d^2 + 6d + d^2 + 6d + 3 \quad \text{Collect like terms.} \\ &= 2d^4 + 12d^3 + 2d^3 + 6d^2 + 12d^2 + d^2 + 6d + 6d + 3 \quad \text{Combine like terms.} \\ &= 2d^4 + 14d^3 + 19d^2 + 12d + 3\end{aligned}$$

To check, substitute $d = 2$ into the trinomial product and its simplification.

$$(2d^2 + 2d + 1)(d^2 + 6d + 3) = 2d^4 + 14d^3 + 19d^2 + 12d + 3$$

$$\begin{aligned}\text{Left side: } (2d^2 + 2d + 1)(d^2 + 6d + 3) &= [2(2)^2 + 2(2) + 1][(2)^2 + 6(2) + 3] \\ &= (8 + 4 + 1)(4 + 12 + 3) \\ &= (13)(19) \\ &= 247\end{aligned}$$

$$\begin{aligned}\text{Right side: } 2d^4 + 14d^3 + 19d^2 + 12d + 3 &= 2(2)^4 + 14(2)^3 + 19(2)^2 + 12(2) + 3 \\ &= 32 + 112 + 76 + 24 + 3 \\ &= 247\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

$$\begin{aligned}\text{c) } (4c^2 - 2c - 3)(-c^2 + 6c + 2) &= 4c^2(-c^2 + 6c + 2) - 2c(-c^2 + 6c + 2) - 3(-c^2 + 6c + 2) \\ &= 4c^2(-c^2) + 4c^2(6c) + 4c^2(2) - 2c(-c^2) - 2c(6c) - 2c(2) - 3(-c^2) - 3(6c) - 3(2) \\ &= -4c^4 + 24c^3 + 8c^2 + 2c^3 - 12c^2 - 4c + 3c^2 - 18c - 6 \quad \text{Collect like terms.} \\ &= -4c^4 + 24c^3 + 2c^3 + 8c^2 - 12c^2 + 3c^2 - 4c - 18c - 6 \quad \text{Combine like terms.} \\ &= -4c^4 + 26c^3 - c^2 - 22c - 6\end{aligned}$$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$$(4c^2 - 2c - 3)(-c^2 + 6c + 2) = -4c^4 + 26c^3 - c^2 - 22c - 6$$

$$\begin{aligned}\text{Left side: } (4c^2 - 2c - 3)(-c^2 + 6c + 2) &= [4(2)^2 - 2(2) - 3][-(2)^2 + 6(2) + 2] \\ &= (16 - 4 - 3)(-4 + 12 + 2) \\ &= (9)(10) \\ &= 90\end{aligned}$$

$$\begin{aligned}\text{Right side: } -4c^4 + 26c^3 - c^2 - 22c - 6 &= -4(2)^4 + 26(2)^3 - (2)^2 - 22(2) - 6 \\ &= -64 + 208 - 4 - 44 - 6 \\ &= 90\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

d)

$$\begin{aligned} &(-4n^2 - n + 3)(-2n^2 + 5n - 1) \\ &= -4n^2(-2n^2 + 5n - 1) - n(-2n^2 + 5n - 1) + 3(-2n^2 + 5n - 1) \\ &= -4n^2(-2n^2) - 4n^2(5n) - 4n^2(-1) - n(-2n^2) - n(5n) - n(-1) + 3(-2n^2) + 3(5n) + 3(-1) \\ &= 8n^4 - 20n^3 + 4n^2 + 2n^3 - 5n^2 + n - 6n^2 + 15n - 3 && \text{Collect like terms.} \\ &= 8n^4 - 20n^3 + 2n^3 + 4n^2 - 5n^2 - 6n^2 + n + 15n - 3 && \text{Combine like terms.} \\ &= 8n^4 - 18n^3 - 7n^2 + 16n - 3\end{aligned}$$

To check, substitute $n = 2$ into the trinomial product and its simplification.

$$(-4n^2 - n + 3)(-2n^2 + 5n - 1) = 8n^4 - 18n^3 - 7n^2 + 16n - 3$$

$$\begin{aligned}\text{Left side: } (-4n^2 - n + 3)(-2n^2 + 5n - 1) &= [-4(2)^2 - 2 + 3][-(2)^2 + 5(2) - 1] \\ &= (-16 + 1)(-8 + 10 - 1) \\ &= (-15)(1) \\ &= -15\end{aligned}$$

$$\begin{aligned}\text{Right side: } 8n^4 - 18n^3 - 7n^2 + 16n - 3 &= 8(2)^4 - 18(2)^3 - 7(2)^2 + 16(2) - 3 \\ &= 128 - 144 - 28 + 32 - 3 \\ &= -15\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

14. There are 3 errors in the second line of the solution:

- the product of $4g$ and $-g$ should be $-4g^2$, not $+4g^2$.
- the product of $4g$ and 4 should be $16g$, not $8g$.
- the product of -2 and 4 should be -8 , not $+8$.

There is an error in the third line of the solution:

- the sum of $-3g^3$ and $-4g^3$ should be $-7g^3$, not $5g^3$.
- the sum of $12g^2$, $4g^2$, and $2g^2$ should be $18g^2$, not $6g^2$.

A correct solution is:

$$\begin{aligned}
 & (3g^2 + 4g - 2)(-g^2 - g + 4) \\
 &= 3g^2(-g^2 - g + 4) + 4g(-g^2 - g + 4) - 2(-g^2 - g + 4) \\
 &= 3g^2(-g^2) + 3g^2(-g) + 3g^2(4) + 4g(-g^2) + 4g(-g) + 4g(4) - 2(-g^2) - 2(-g) - 2(4) \\
 &= -3g^4 - 3g^3 + 12g^2 - 4g^3 - 4g^2 + 16g + 2g^2 + 2g - 8 && \text{Collect like terms.} \\
 &= -3g^4 - 3g^3 - 4g^3 + 12g^2 - 4g^2 + 2g^2 + 16g + 2g - 8 && \text{Combine like terms.} \\
 &= -3g^4 - 7g^3 + 10g^2 + 18g - 8
 \end{aligned}$$

15. a) $(3s + 5)(2s + 2) + (3s + 7)(s + 6)$

$$\begin{aligned}
 &= 3s(2s + 2) + 5(2s + 2) + 3s(s + 6) + 7(s + 6) \\
 &= 3s(2s) + 3s(2) + 5(2s) + 5(2) + 3s(s) + 3s(6) + 7(s) + 7(6) \\
 &= 6s^2 + 6s + 10s + 10 + 3s^2 + 18s + 7s + 42 && \text{Collect like terms.} \\
 &= 6s^2 + 3s^2 + 6s + 10s + 18s + 7s + 10 + 42 && \text{Combine like terms.} \\
 &= 9s^2 + 41s + 52
 \end{aligned}$$

b) $(2x + 3)(5x + 4) + (x - 4)(3x - 7)$

$$\begin{aligned}
 &= 2x(5x + 4) + 3(5x + 4) + x(3x - 7) - 4(3x - 7) \\
 &= 2x(5x) + 2x(4) + 3(5x) + 3(4) + x(3x) + x(-7) - 4(3x) - 4(-7) \\
 &= 10x^2 + 8x + 15x + 12 + 3x^2 - 7x - 12x + 28 && \text{Collect like terms.} \\
 &= 10x^2 + 3x^2 + 8x + 15x - 7x - 12x + 12 + 28 && \text{Combine like terms.} \\
 &= 13x^2 + 4x + 40
 \end{aligned}$$

c)

$$\begin{aligned}
 & (3m + 4)(m - 4n) + (5m - 2)(3m - 6n) \\
 &= 3m(m - 4n) + 4(m - 4n) + 5m(3m - 6n) - 2(3m - 6n) \\
 &= 3m(m) + 3m(-4n) + 4(m) + 4(-4n) + 5m(3m) + 5m(-6n) - 2(3m) - 2(-6n) \\
 &= 3m^2 - 12mn + 4m - 16n + 15m^2 - 30mn - 6m + 12n && \text{Collect like terms.} \\
 &= 3m^2 + 15m^2 + 4m - 6m - 12mn - 30mn - 16n + 12n && \text{Combine like terms.} \\
 &= 18m^2 - 2m - 42mn - 4n
 \end{aligned}$$

d)

$$\begin{aligned}
 & (4y - 5)(3y + 2) - (3y + 2)(4y - 5) \\
 &= 4y(3y + 2) - 5(3y + 2) - 3y(4y - 5) - 2(4y - 5) \\
 &= 4y(3y) + 4y(2) - 5(3y) - 5(2) - 3y(4y) - 3y(-5) - 2(4y) - 2(-5) \\
 &= 12y^2 + 8y - 15y - 10 - 12y^2 + 15y - 8y + 10 && \text{Collect like terms.} \\
 &= 12y^2 - 12y^2 + 8y - 15y + 15y - 8y - 10 + 10 && \text{Combine like terms.} \\
 &= 0
 \end{aligned}$$

e)

$$\begin{aligned}
 & (3x - 2)^2 - (2x + 6)(3x - 1) \\
 &= (3x - 2)(3x - 2) - (2x + 6)(3x - 1) \\
 &= 3x(3x - 2) - 2(3x - 2) - 2x(3x - 1) - 6(3x - 1) \\
 &= 3x(3x) + 3x(-2) - 2(3x) - 2(-2) - 2x(3x) - 2x(-1) - 6(3x) - 6(-1) \\
 &= 9x^2 - 6x - 6x + 4 - 6x^2 + 2x - 18x + 6 && \text{Collect like terms.} \\
 &= 9x^2 - 6x^2 - 6x - 6x + 2x - 18x + 4 + 6 && \text{Combine like terms.} \\
 &= 3x^2 - 28x + 10
 \end{aligned}$$

f) $(2a + 1)(4a - 3) - (a - 2)^2$

$$\begin{aligned}
 &= (2a + 1)(4a - 3) - (a - 2)(a - 2) \\
 &= 2a(4a - 3) + 1(4a - 3) - a(a - 2) - (-2)(a - 2) \\
 &= 2a(4a) + 2a(-3) + 4a - 3 - a(a) - a(-2) - (-2)(a) - (-2)(-2) \\
 &= 8a^2 - 6a + 4a - 3 - a^2 + 2a + 2a - 4 && \text{Collect like terms.} \\
 &= 8a^2 - a^2 - 6a + 4a + 2a + 2a - 3 - 4 && \text{Combine like terms.} \\
 &= 7a^2 + 2a - 7
 \end{aligned}$$

16. a) The cardboard has length 20 cm.

A length of x cm is cut from each of its ends.

So, a polynomial that represents the length of the box in centimetres is:

$20 - x - x$, or $20 - 2x$

b) The cardboard has width 10 cm.

A length of x cm is cut from each of its ends.

So, a polynomial that represents the width of the box in centimetres is:

$10 - x - x$, or $10 - 2x$

- c) The base of the box is a rectangle.

The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.

Use the formula $A = lw$. Substitute: $l = 20 - 2x$ and $w = 10 - 2x$

A polynomial that represents the area of the base of the box is:

$$A = (20 - 2x)(10 - 2x)$$

Expand and simplify.

$$A = (20 - 2x)(10 - 2x)$$

$$= 20(10 - 2x) - 2x(10 - 2x)$$

$$= 20(10) + 20(-2x) - 2x(10) - 2x(-2x)$$

$$= 200 - 40x - 20x + 4x^2 \quad \text{Combine like terms.}$$

$$= 200 - 60x + 4x^2 \quad \text{Arrange in descending order.}$$

$$= 4x^2 - 60x + 200$$

So, the area of the base of the box in square centimetres is: $4x^2 - 60x + 200$

- d) The formula for the volume of a rectangular prism is: $V = Ah$, where A is the area of the base and h is its height.

Each side that is folded up has height x .

Use the formula $V = Ah$. Substitute: $A = 4x^2 - 60x + 200$ and $h = x$

A polynomial that represents the volume of the box is:

$$V = (4x^2 - 60x + 200)x$$

Expand and simplify.

$$V = (4x^2 - 60x + 200)x$$

$$= 4x^2(x) - 60x(x) + 200(x)$$

$$= 4x^3 - 60x^2 + 200x$$

So, the volume of the box in cubic centimetres is: $4x^3 - 60x^2 + 200x$

17. a) The area of the shaded region is:

area of large rectangle – area of small rectangle

The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.

Area of large rectangle:

Use the formula $A = lw$. Substitute: $l = 5x + 8$ and $w = 6x + 2$

$$A_L = (5x + 8)(6x + 2)$$

Area of small rectangle:

Use the formula $A = lw$. Substitute: $l = x + 5$ and $w = 3x$

$$A_S = (x + 5)(3x)$$

So, area of the shaded region is:

$$\begin{aligned}
 A &= A_L - A_S \\
 &= (5x + 8)(6x + 2) - (x + 5)(3x) \\
 &= 5x(6x + 2) + 8(6x + 2) - x(3x) - 5(3x) \\
 &= 5x(6x) + 5x(2) + 8(6x) + 8(2) - 3x^2 - 15x \\
 &= 30x^2 + 10x + 48x + 16 - 3x^2 - 15x \quad \text{Collect like terms.} \\
 &= 30x^2 - 3x^2 + 10x + 48x - 15x + 16 \quad \text{Combine like terms.} \\
 &= 27x^2 + 43x + 16
 \end{aligned}$$

A polynomial that represents the area of the shaded region is:

$$27x^2 + 43x + 16$$

- b) The area of the shaded region is:
 area of large rectangle – area of small rectangle
 The formula for the area of a rectangle is: $A = lw$, where l is the length of the rectangle and w is its width.

Area of large rectangle:

Use the formula $A = lw$. Substitute: $l = 2x - 2$ and $w = x + 1$

$$A_L = (2x - 2)(x + 1)$$

Area of small rectangle:

Use the formula $A = lw$. Substitute: $l = x$ and $w = x - 2$

$$A_S = x(x - 2)$$

So, area of the shaded region is:

$$\begin{aligned}
 A &= A_L - A_S \\
 &= (2x - 2)(x + 1) - x(x - 2) \\
 &= 2x(x + 1) - 2(x + 1) - x(x) - x(-2) \\
 &= 2x(x) + 2x(1) - 2(x) - 2(1) - x^2 + 2x \\
 &= 2x^2 + 2x - 2x - 2 - x^2 + 2x \quad \text{Collect like terms.} \\
 &= 2x^2 - x^2 + 2x - 2x + 2x - 2 \quad \text{Combine like terms.} \\
 &= x^2 + 2x - 2
 \end{aligned}$$

A polynomial that represents the area of the shaded region is:

$$x^2 + 2x - 2$$

C

18. a)

$$\begin{aligned}
 (x-2)^3 &= (x-2)(x-2)(x-2) \\
 &= [(x-2)(x-2)](x-2) \\
 &= [x(x-2) - 2(x-2)](x-2) \\
 &= [x(x) + x(-2) - 2(x) - 2(-2)](x-2) \\
 &= (x^2 - 2x - 2x + 4)(x-2) \\
 &= (x^2 - 4x + 4)(x-2) \\
 &= x^2(x-2) - 4x(x-2) + 4(x-2) \\
 &= x^2(x) + x^2(-2) - 4x(x) - 4x(-2) + 4(x) + 4(-2) \\
 &= x^3 - 2x^2 - 4x^2 + 8x + 4x - 8 \\
 &= x^3 - 6x^2 + 12x - 8
 \end{aligned}$$

b)

$$\begin{aligned}
 (2y+5)^3 &= (2y+5)(2y+5)(2y+5) \\
 &= [(2y+5)(2y+5)](2y+5) \\
 &= [2y(2y+5) + 5(2y+5)](2y+5) \\
 &= [2y(2y) + 2y(5) + 5(2y) + 5(5)](2y+5) \\
 &= (4y^2 + 10y + 10y + 25)(2y+5) \\
 &= (4y^2 + 20y + 25)(2y+5) \\
 &= 4y^2(2y+5) + 20y(2y+5) + 25(2y+5) \\
 &= 4y^2(2y) + 4y^2(5) + 20y(2y) + 20y(5) + 25(2y) + 25(5) \\
 &= 8y^3 + 20y^2 + 40y^2 + 100y + 50y + 125 \\
 &= 8y^3 + 60y^2 + 150y + 125
 \end{aligned}$$

c)

$$\begin{aligned}
 (4a - 3b)^3 &= (4a - 3b)(4a - 3b)(4a - 3b) \\
 &= [(4a - 3b)(4a - 3b)](4a - 3b) \\
 &= [4a(4a - 3b) - 3b(4a - 3b)](4a - 3b) \\
 &= [4a(4a) + 4a(-3b) - 3b(4a) - 3b(-3b)](4a - 3b) \\
 &= (16a^2 - 12ab - 12ab + 9b^2)(4a - 3b) \\
 &= (16a^2 - 24ab + 9b^2)(4a - 3b) \\
 &= 16a^2(4a - 3b) - 24ab(4a - 3b) + 9b^2(4a - 3b) \\
 &= 16a^2(4a) + 16a^2(-3b) - 24ab(4a) - 24ab(-3b) + 9b^2(4a) + 9b^2(-3b) \\
 &= 64a^3 - 48a^2b - 96a^2b + 72ab^2 + 36ab^2 - 27b^3 \\
 &= 64a^3 - 144a^2b + 108ab^2 - 27b^3
 \end{aligned}$$

d)

$$\begin{aligned}
 (c + d)^3 &= (c + d)(c + d)(c + d) \\
 &= [(c + d)(c + d)](c + d) \\
 &= [c(c + d) + d(c + d)](c + d) \\
 &= [c(c) + c(d) + d(c) + d(d)](c + d) \\
 &= (c^2 + cd + cd + d^2)(c + d) \\
 &= (c^2 + 2cd + d^2)(c + d) \\
 &= c^2(c + d) + 2cd(c + d) + d^2(c + d) \\
 &= c^2(c) + c^2(d) + 2cd(c) + 2cd(d) + d^2(c) + d^2(d) \\
 &= c^3 + c^2d + 2c^2d + 2cd^2 + cd^2 + d^3 \\
 &= c^3 + 3c^2d + 3cd^2 + d^3
 \end{aligned}$$

19. a)

$$\begin{aligned}
 &2a(2a - 1)(3a + 2) \\
 &= 2a[2a(3a + 2) - 1(3a + 2)] \\
 &= 2a[2a(3a) + 2a(2) - 1(3a) - 1(2)] \\
 &= 2a(6a^2 + 4a - 3a - 2) \\
 &= 2a(6a^2 + a - 2) \\
 &= 2a(6a^2) + 2a(a) + 2a(-2) \\
 &= 12a^3 + 2a^2 - 4a
 \end{aligned}$$

b)

$$\begin{aligned}
 & -3r(r-1)(2r+1) \\
 &= -3r[r(2r+1) - 1(2r+1)] \\
 &= -3r[r(2r) + r(1) - 1(2r) - 1(1)] \\
 &= -3r(2r^2 + r - 2r - 1) \\
 &= -3r(2r^2 - r - 1) \\
 &= -3r(2r^2) - 3r(-r) - 3r(-1) \\
 &= -6r^3 + 3r^2 + 3r
 \end{aligned}$$

c)

$$\begin{aligned}
 & 5x^2(2x-1)(4x-3) \\
 &= 5x^2[2x(4x-3) - 1(4x-3)] \\
 &= 5x^2[2x(4x) + 2x(-3) - 1(4x) - 1(-3)] \\
 &= 5x^2(8x^2 - 6x - 4x + 3) \\
 &= 5x^2(8x^2 - 10x + 3) \\
 &= 5x^2(8x^2) + 5x^2(-10x) + 5x^2(3) \\
 &= 40x^4 - 50x^3 + 15x^2
 \end{aligned}$$

d)

$$\begin{aligned}
 & -xy(2x+5)(4x-5) \\
 &= -xy[2x(4x-5) + 5(4x-5)] \\
 &= -xy[2x(4x) + 2x(-5) + 5(4x) + 5(-5)] \\
 &= -xy(8x^2 - 10x + 20x - 25) \\
 &= -xy(8x^2 + 10x - 25) \\
 &= -xy(8x^2) - xy(10x) - xy(-25) \\
 &= -8x^3y - 10x^2y + 25xy
 \end{aligned}$$

e)

$$\begin{aligned}
 & 2b(2b - c)(b + c) \\
 &= 2b[2b(b + c) - c(b + c)] \\
 &= 2b[2b(b) + 2b(c) - c(b) - c(c)] \\
 &= 2b(2b^2 + 2bc - bc - c^2) \\
 &= 2b(2b^2 + bc - c^2) \\
 &= 2b(2b^2) + 2b(bc) + 2b(-c^2) \\
 &= 4b^3 + 2b^2c - 2bc^2
 \end{aligned}$$

f)

$$\begin{aligned}
 & y^2(y^2 + 1)(y^2 - 1) \\
 &= y^2[y^2(y^2 - 1) + 1(y^2 - 1)] \\
 &= y^2[y^2(y^2) + y^2(-1) + y^2 - 1] \\
 &= y^2(y^4 - y^2 + y^2 - 1) \\
 &= y^2(y^4 - 1) \\
 &= y^2(y^4) + y^2(-1) \\
 &= y^6 - y^2
 \end{aligned}$$

20. a) A cube with edge length e has volume: $V = e^3$
Substitute $e = 2x + 3$, then simplify.

$$\begin{aligned}
 V &= (2x + 3)^3 \\
 &= (2x + 3)(2x + 3)(2x + 3) \\
 &= [(2x + 3)(2x + 3)](2x + 3) \\
 &= [2x(2x + 3) + 3(2x + 3)](2x + 3) \\
 &= [2x(2x) + 2x(3) + 3(2x) + 3(3)](2x + 3) \\
 &= (4x^2 + 6x + 6x + 9)(2x + 3) \\
 &= (4x^2 + 12x + 9)(2x + 3) \\
 &= 4x^2(2x + 3) + 12x(2x + 3) + 9(2x + 3) \\
 &= 4x^2(2x) + 4x^2(3) + 12x(2x) + 12x(3) + 9(2x) + 9(3) \\
 &= 8x^3 + 12x^2 + 24x^2 + 36x + 18x + 27 \\
 &= 8x^3 + 36x^2 + 54x + 27
 \end{aligned}$$

- b) A cube has 6 congruent square faces.

Each face has area: $(2x + 3)(2x + 3)$

So, the surface area of the cube is: $6(2x + 3)(2x + 3)$, or $6(2x + 3)^2$

Simplify the expression.

$$\begin{aligned}
 & 6(2x + 3)^2 \\
 &= 6(2x + 3)(2x + 3) \\
 &= 6[2x(2x + 3) + 3(2x + 3)] \\
 &= 6[2x(2x) + 2x(3) + 3(2x) + 3(3)] \\
 &= 6(4x^2 + 6x + 6x + 9) \\
 &= 6(4x^2 + 12x + 9) \\
 &= 6(4x^2) + 6(12x) + 6(9) \\
 &= 24x^2 + 72x + 54
 \end{aligned}$$

21. a)

$$\begin{aligned}
 & (3x + 4)(x - 5)(2x + 8) \\
 &= [3x(x - 5) + 4(x - 5)](2x + 8) \\
 &= [3x(x) + 3x(-5) + 4(x) + 4(-5)](2x + 8) \\
 &= (3x^2 - 15x + 4x - 20)(2x + 8) \\
 &= (3x^2 - 11x - 20)(2x + 8) \\
 &= 3x^2(2x + 8) - 11x(2x + 8) - 20(2x + 8) \\
 &= 3x^2(2x) + 3x^2(8) - 11x(2x) - 11x(8) - 20(2x) - 20(8) \\
 &= 6x^3 + 24x^2 - 22x^2 - 88x - 40x - 160 \\
 &= 6x^3 + 2x^2 - 128x - 160
 \end{aligned}$$

- b)

$$\begin{aligned}
 & (b - 7)(b + 8)(3b - 4) \\
 &= [b(b + 8) - 7(b + 8)](3b - 4) \\
 &= [b(b) + b(8) - 7(b) - 7(8)](3b - 4) \\
 &= (b^2 + 8b - 7b - 56)(3b - 4) \\
 &= (b^2 + b - 56)(3b - 4) \\
 &= b^2(3b - 4) + b(3b - 4) - 56(3b - 4) \\
 &= b^2(3b) + b^2(-4) + b(3b) + b(-4) - 56(3b) - 56(-4) \\
 &= 3b^3 - 4b^2 + 3b^2 - 4b - 168b + 224 \\
 &= 3b^3 - b^2 - 172b + 224
 \end{aligned}$$

c)

$$\begin{aligned}
 & (2x - 5)(3x + 4)^2 \\
 &= (2x - 5)(3x + 4)(3x + 4) \\
 &= [2x(3x + 4) - 5(3x + 4)](3x + 4) \\
 &= [2x(3x) + 2x(4) - 5(3x) - 5(4)](3x + 4) \\
 &= (6x^2 + 8x - 15x - 20)(3x + 4) \\
 &= (6x^2 - 7x - 20)(3x + 4) \\
 &= 6x^2(3x + 4) - 7x(3x + 4) - 20(3x + 4) \\
 &= 6x^2(3x) + 6x^2(4) - 7x(3x) - 7x(4) - 20(3x) - 20(4) \\
 &= 18x^3 + 24x^2 - 21x^2 - 28x - 60x - 80 \\
 &= 18x^3 + 3x^2 - 88x - 80
 \end{aligned}$$

d)

$$\begin{aligned}
 & (5a - 3)^2(2a - 7) \\
 &= (5a - 3)(5a - 3)(2a - 7) \\
 &= [5a(5a - 3) - 3(5a - 3)](2a - 7) \\
 &= [5a(5a) + 5a(-3) - 3(5a) - 3(-3)](2a - 7) \\
 &= (25a^2 - 15a - 15a + 9)(2a - 7) \\
 &= (25a^2 - 30a + 9)(2a - 7) \\
 &= 25a^2(2a - 7) - 30a(2a - 7) + 9(2a - 7) \\
 &= 25a^2(2a) + 25a^2(-7) - 30a(2a) - 30a(-7) + 9(2a) + 9(-7) \\
 &= 50a^3 - 175a^2 - 60a^2 + 210a + 18a - 63 \\
 &= 50a^3 - 235a^2 + 228a - 63
 \end{aligned}$$

e)

$$\begin{aligned}
 & (2k - 3)(2k + 3)^2 \\
 &= (2k - 3)(2k + 3)(2k + 3) \\
 &= [2k(2k + 3) - 3(2k + 3)](2k + 3) \\
 &= [2k(2k) + 2k(3) - 3(2k) - 3(3)](2k + 3) \\
 &= (4k^2 + 6k - 6k - 9)(2k + 3) \\
 &= (4k^2 - 9)(2k + 3) \\
 &= 4k^2(2k + 3) - 9(2k + 3) \\
 &= 4k^2(2k) + 4k^2(3) - 9(2k) - 9(3) \\
 &= 8k^3 + 12k^2 - 18k - 27
 \end{aligned}$$

22. a)

$$\begin{aligned}
 & (x + y + 1)^3 \\
 &= (x + y + 1)(x + y + 1)(x + y + 1) \\
 &= [x(x + y + 1) + y(x + y + 1) + 1(x + y + 1)](x + y + 1) \\
 &= [x(x) + x(y) + x(1) + y(x) + y(y) + y(1) + x + y + 1](x + y + 1) \\
 &= (x^2 + xy + x + xy + y^2 + y + x + y + 1)(x + y + 1) \\
 &= (x^2 + 2x + 2xy + 2y + y^2 + 1)(x + y + 1) \\
 &= x^2(x + y + 1) + 2x(x + y + 1) + 2xy(x + y + 1) + 2y(x + y + 1) + y^2(x + y + 1) + x + y + 1 \\
 &= x^2(x) + x^2(y) + x^2(1) + 2x(x) + 2x(y) + 2x(1) + 2xy(x) + 2xy(y) + 2xy(1) + 2y(x) + 2y(y) + \\
 &\quad 2y(1) + y^2(x) + y^2(y) + y^2(1) + x + y + 1 \\
 &= x^3 + x^2y + x^2 + 2x^2 + 2xy + 2x + 2x^2y + 2xy^2 + 2xy + 2xy + 2y^2 + 2y + xy^2 + y^3 + y^2 + x + y + 1 \\
 &= x^3 + 3x^2y + 3xy^2 + y^3 + 3x^2 + 6xy + 3y^2 + 3x + 3y + 1
 \end{aligned}$$

b)

$$\begin{aligned}
 & (x - y - 1)^3 \\
 &= (x - y - 1)(x - y - 1)(x - y - 1) \\
 &= [x(x - y - 1) - y(x - y - 1) - 1(x - y - 1)](x - y - 1) \\
 &= [x(x) + x(-y) + x(-1) - y(x) - y(-y) - y(-1) - x + y + 1](x - y - 1) \\
 &= (x^2 - xy - x - xy + y^2 + y - x + y + 1)(x - y - 1) \\
 &= (x^2 - 2x - 2xy + 2y + y^2 + 1)(x - y - 1) \\
 &= x^2(x - y - 1) - 2x(x - y - 1) - 2xy(x - y - 1) + 2y(x - y - 1) + y^2(x - y - 1) + x - y - 1 \\
 &= x^2(x) + x^2(-y) + x^2(-1) - 2x(x) - 2x(-y) - 2x(-1) - 2xy(x) - 2xy(-y) - 2xy(-1) + 2y(x) + \\
 &\quad 2y(-y) + 2y(-1) + y^2(x) + y^2(-y) + y^2(-1) + x - y - 1 \\
 &= x^3 - x^2y - x^2 - 2x^2 + 2xy + 2x - 2x^2y + 2xy^2 + 2xy + 2xy - 2y^2 - 2y + xy^2 - y^3 - y^2 + x - y - 1 \\
 &= x^3 - 3x^2y + 3xy^2 - y^3 - 3x^2 + 6xy - 3y^2 + 3x - 3y - 1
 \end{aligned}$$

c)

$$\begin{aligned}
 & (x + y + z)^3 \\
 &= (x + y + z)(x + y + z)(x + y + z) \\
 &= [x(x + y + z) + y(x + y + z) + z(x + y + z)](x + y + z) \\
 &= [x(x) + x(y) + x(z) + y(x) + y(y) + y(z) + z(x) + z(y) + z(z)](x + y + z) \\
 &= (x^2 + xy + xz + xy + y^2 + yz + xz + yz + z^2)(x + y + z) \\
 &= (x^2 + 2xy + y^2 + 2yz + z^2 + 2xz)(x + y + z) \\
 &= x^2(x + y + z) + 2xy(x + y + z) + y^2(x + y + z) + 2yz(x + y + z) + z^2(x + y + z) + 2xz(x + y + z) \\
 &= x^2(x) + x^2(y) + x^2(z) + 2xy(x) + 2xy(y) + 2xy(z) + y^2(x) + y^2(y) + y^2(z) + 2yz(x) + \\
 &\quad 2yz(y) + 2yz(z) + z^2(x) + z^2(y) + z^2(z) + 2xz(x) + 2xz(y) + 2xz(z) \\
 &= x^3 + x^2y + x^2z + 2x^2y + 2xy^2 + 2xyz + xy^2 + y^3 + y^2z + 2xyz + 2y^2z + 2yz^2 + xz^2 + \\
 &\quad yz^2 + z^3 + 2x^2z + 2xyz + 2xz^2 \\
 &= x^3 + 3x^2y + 3xy^2 + y^3 + 3x^2z + 6xyz + 3y^2z + 3xz^2 + 3yz^2 + z^3
 \end{aligned}$$

d)

$$\begin{aligned}
 & (x - y - z)^3 \\
 &= (x - y - z)(x - y - z)(x - y - z) \\
 &= [x(x - y - z) - y(x - y - z) - z(x - y - z)](x - y - z) \\
 &= [x(x) + x(-y) + x(-z) - y(x) - y(-y) - y(-z) - z(x) - z(-y) - z(-z)](x - y - z) \\
 &= (x^2 - xy - xz - xy + y^2 + yz - xz + yz + z^2)(x - y - z) \\
 &= (x^2 - 2xy + y^2 + 2yz + z^2 - 2xz)(x - y - z) \\
 &= x^2(x - y - z) - 2xy(x - y - z) + y^2(x - y - z) + 2yz(x - y - z) + z^2(x - y - z) - 2xz(x - y - z) \\
 &= x^2(x) + x^2(-y) + x^2(-z) - 2xy(x) - 2xy(-y) - 2xy(-z) + y^2(x) + y^2(-y) + y^2(-z) + 2yz(x) + \\
 &\quad 2yz(-y) + 2yz(-z) + z^2(x) + z^2(-y) + z^2(-z) - 2xz(x) - 2xz(-y) - 2xz(-z) \\
 &= x^3 - x^2y - x^2z - 2x^2y + 2xy^2 + 2xyz + xy^2 - y^3 - y^2z + 2xyz - 2y^2z - 2yz^2 + xz^2 - \\
 &\quad yz^2 - z^3 - 2x^2z + 2xyz + 2xz^2 \\
 &= x^3 - 3x^2y + 3xy^2 - y^3 - 3x^2z + 6xyz - 3y^2z + 3xz^2 - 3yz^2 - z^3
 \end{aligned}$$

Lesson 3.8

Factoring Special Polynomials

Exercises (pages 194–195)

A

4. a) $(x + 2)^2 = (x + 2)(x + 2)$
 $= x(x + 2) + 2(x + 2)$
 $= x(x) + x(2) + 2(x) + 2(2)$
 $= x^2 + 2x + 2x + 4$
 $= x^2 + 4x + 4$
- b) $(3 - y)^2 = (3 - y)(3 - y)$
 $= 3(3 - y) - y(3 - y)$
 $= 3(3) + 3(-y) - y(3) - y(-y)$
 $= 9 - 3y - 3y + y^2$
 $= 9 - 6y + y^2$
- c) $(5 + d)^2 = (5 + d)(5 + d)$
 $= 5(5 + d) + d(5 + d)$
 $= 5(5) + 5(d) + d(5) + d(d)$
 $= 25 + 5d + 5d + d^2$
 $= 25 + 10d + d^2$
- d) $(7 - f)^2 = (7 - f)(7 - f)$
 $= 7(7 - f) - f(7 - f)$
 $= 7(7) + 7(-f) - f(7) - f(-f)$
 $= 49 - 7f - 7f + f^2$
 $= 49 - 14f + f^2$
- e) $(x + 2)(x - 2) = x(x - 2) + 2(x - 2)$
 $= x(x) + x(-2) + 2(x) + 2(-2)$
 $= x^2 - 2x + 2x - 4$
 $= x^2 - 4$
- f) $(3 - y)(3 + y) = 3(3 + y) - y(3 + y)$
 $= 3(3) + 3(y) - y(3) - y(y)$
 $= 9 + 3y - 3y - y^2$
 $= 9 - y^2$

$$\begin{aligned}
 \text{g) } (5 + d)(5 - d) &= 5(5 - d) + d(5 - d) \\
 &= 5(5) + 5(-d) + d(5) + d(-d) \\
 &= 25 - 5d + 5d - d^2 \\
 &= 25 - d^2
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } (7 - f)(7 + f) &= 7(7 + f) - f(7 + f) \\
 &= 7(7) + 7(f) - f(7) - f(f) \\
 &= 49 + 7f - 7f - f^2 \\
 &= 49 - f^2
 \end{aligned}$$

5. a) $25 - t^2$

This polynomial is a difference of squares because it is of the form $a^2 - b^2$, and $25 = (5)(5)$ and $t^2 = (t)(t)$.

b) $16m^2 + 49n^2$

This polynomial is not a perfect square trinomial because it has only 2 terms.

It is not a difference of squares because it is not of the form $a^2 - b^2$.

So, this polynomial is neither a perfect square trinomial nor a difference of squares.

c) $4x^2 - 24xy + 9y^2$

This polynomial is not a difference of squares because it is not of the form $a^2 - b^2$.

It has 3 terms, so it may be a perfect square trinomial.

Check: $4x^2 = (2x)(2x)$ and $9y^2 = (3y)(3y)$

Since $2(2x)(3y) \neq 24xy$, the polynomial is not a perfect square trinomial.

So, this polynomial is neither a perfect square trinomial nor a difference of squares.

d) $9m^2 - 24mn + 16n^2$

This polynomial is not a difference of squares because it is not of the form $a^2 - b^2$.

It has 3 terms, so it may be a perfect square trinomial.

Check: $9m^2 = (3m)(3m)$ and $16n^2 = (4n)(4n)$

Since $2(3m)(4n) = 24mn$, the polynomial is a perfect square trinomial.

6. a) $x^2 - 49$

Write each term as a perfect square.

$x^2 - 49 = (x)^2 - (7)^2$ Write these terms in binomial factors.

$$= (x + 7)(x - 7)$$

b) $b^2 - 121$

Write each term as a perfect square.

$$\begin{aligned} b^2 - 121 &= (b)^2 - (11)^2 && \text{Write these terms in binomial factors.} \\ &= (b + 11)(b - 11) \end{aligned}$$

c) $1 - q^2$

Write each term as a perfect square.

$$\begin{aligned} 1 - q^2 &= (1)^2 - (q)^2 && \text{Write these terms in binomial factors.} \\ &= (1 + q)(1 - q) \end{aligned}$$

d) $36 - c^2$

Write each term as a perfect square.

$$\begin{aligned} 36 - c^2 &= (6)^2 - (c)^2 && \text{Write these terms in binomial factors.} \\ &= (6 + c)(6 - c) \end{aligned}$$

B

7. a) i) $a^2 + 10a + 25$

The first term is a perfect square since $a^2 = (a)(a)$.

The third term is a perfect square since $25 = (5)(5)$.

The second term is twice the product of a and 5:

$$10a = 2(a)(5)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(a + 5)(a + 5), \text{ or } (a + 5)^2$$

ii) $b^2 - 12b + 36$

The first term is a perfect square since $b^2 = (b)(b)$.

The third term is a perfect square since $36 = (6)(6)$.

The second term is twice the product of b and 6:

$$12b = 2(b)(6)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(b - 6)(b - 6), \text{ or } (b - 6)^2$$

iii) $c^2 + 14c + 49$

The first term is a perfect square since $c^2 = (c)(c)$.

The third term is a perfect square since $49 = (7)(7)$.

The second term is twice the product of c and 7:

$$14c = 2(c)(7)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(c + 7)(c + 7), \text{ or } (c + 7)^2$$

iv) $d^2 - 16d + 64$

The first term is a perfect square since $d^2 = (d)(d)$.

The third term is a perfect square since $64 = (8)(8)$.

The second term is twice the product of d and 8:

$$16d = 2(d)(8)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(d - 8)(d - 8), \text{ or } (d - 8)^2$$

v) $e^2 + 18e + 81$

The first term is a perfect square since $e^2 = (e)(e)$.

The third term is a perfect square since $81 = (9)(9)$.

The second term is twice the product of e and 9:

$$18e = 2(e)(9)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(e + 9)(e + 9), \text{ or } (e + 9)^2$$

vi) $f^2 - 20f + 100$

The first term is a perfect square since $f^2 = (f)(f)$.

The third term is a perfect square since $100 = (10)(10)$.

The second term is twice the product of f and 10:

$$20f = 2(f)(10)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(f - 10)(f - 10), \text{ or } (f - 10)^2$$

- b) The variables in the trinomials and binomial factors are consecutive letters of the alphabet, starting with a .

Trinomials:

The first term in the trinomials is the square of the variable.

The coefficient of the second term starts at 10 and increases by two each time. The variable has degree one. The signs alternate.

The third terms in the trinomials are consecutive perfect squares, starting at 25.

Binomial factors:

The first term is the variable.

The second terms are consecutive whole numbers, starting at 5. The sign alternates.

Patterns that relate the trinomial to its factors:

The first term in the trinomial is the square of the first term in the binomial.

The third term in the trinomial is the square of the second term in the binomial.

The second term in the trinomial is twice the product of the first and second terms in the binomial.

The operation in the binomial factors is the same as the sign of the second term in the trinomial.

The next 4 trinomials in the pattern and their factors are:

$$g^2 + 22g + 121 = (g + 11)^2$$

$$h^2 - 24h + 144 = (h - 12)^2$$

$$i^2 + 26i + 169 = (i + 13)^2$$

$$j^2 - 28j + 196 = (j - 14)^2$$

8. a) $4x^2 - 12x + 9$

The first term is a perfect square since $4x^2 = (2x)(2x)$.

The third term is a perfect square since $9 = (3)(3)$.

The second term is twice the product of $2x$ and 3 :

$$12x = 2(2x)(3)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(2x - 3)(2x - 3), \text{ or } (2x - 3)^2$$

To verify, multiply:

$$\begin{aligned} (2x - 3)(2x - 3) &= 2x(2x - 3) - 3(2x - 3) \\ &= 4x^2 - 6x - 6x + 9 \\ &= 4x^2 - 12x + 9 \end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

b) $9 + 30n + 25n^2$

The first term is a perfect square since $9 = (3)(3)$.

The third term is a perfect square since $25n^2 = (5n)(5n)$.

The second term is twice the product of 3 and $5n$:

$$30n = 2(3)(5n)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(3 + 5n)(3 + 5n), \text{ or } (3 + 5n)^2$$

To verify, multiply:

$$\begin{aligned}(3 + 5n)(3 + 5n) &= 3(3 + 5n) + 5n(3 + 5n) \\ &= 9 + 15n + 15n + 25n^2 \\ &= 9 + 30n + 25n^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

c) $81 - 36v + 4v^2$

The first term is a perfect square since $81 = (9)(9)$.

The third term is a perfect square since $4v^2 = (2v)(2v)$.

The second term is twice the product of 9 and $2v$:

$$36v = 2(9)(2v)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(9 - 2v)(9 - 2v), \text{ or } (9 - 2v)^2$$

To verify, multiply:

$$\begin{aligned}(9 - 2v)(9 - 2v) &= 9(9 - 2v) - 2v(9 - 2v) \\ &= 81 - 18v - 18v + 4v^2 \\ &= 81 - 36v + 4v^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

d) $25 + 40h + 16h^2$

The first term is a perfect square since $25 = (5)(5)$.

The third term is a perfect square since $16h^2 = (4h)(4h)$.

The second term is twice the product of 5 and $4h$:

$$40h = 2(5)(4h)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(5 + 4h)(5 + 4h), \text{ or } (5 + 4h)^2$$

To verify, multiply:

$$\begin{aligned}(5 + 4h)(5 + 4h) &= 5(5 + 4h) + 4h(5 + 4h) \\ &= 25 + 20h + 20h + 16h^2 \\ &= 25 + 40h + 16h^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

e) $9g^2 + 48g + 64$

The first term is a perfect square since $9g^2 = (3g)(3g)$.

The third term is a perfect square since $64 = (8)(8)$.

The second term is twice the product of $3g$ and 8 :

$$48g = 2(3g)(8)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(3g + 8)(3g + 8), \text{ or } (3g + 8)^2$$

To verify, multiply:

$$\begin{aligned}(3g + 8)(3g + 8) &= 3g(3g + 8) + 8(3g + 8) \\ &= 9g^2 + 24g + 24g + 64 \\ &= 9g^2 + 48g + 64\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

f) $49r^2 - 28r + 4$

The first term is a perfect square since $49r^2 = (7r)(7r)$.

The third term is a perfect square since $4 = (2)(2)$.

The second term is twice the product of $7r$ and 2 :

$$28r = 2(7r)(2)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(7r - 2)(7r - 2), \text{ or } (7r - 2)^2$$

To verify, multiply:

$$\begin{aligned}(7r - 2)(7r - 2) &= 7r(7r - 2) - 2(7r - 2) \\ &= 49r^2 - 14r - 14r + 4 \\ &= 49r^2 - 28r + 4\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

9. a) The formula for the area of a square, A , is: $A = s^2$, where s is the side length of the square.

The larger square has side length x .

So, an expression that represents its area is: x^2

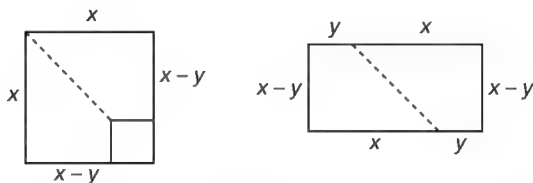
The smaller square has side length y .

So, an expression that represents its area is: y^2

The area of the piece that remains is the area of the larger square minus the area of the smaller square.

So, an expression for the area of the piece that remains is: $x^2 - y^2$

b)



From the diagrams above, the rectangle has dimensions $x - y$ and $x + y$.

The formula for the area, A , of a rectangle is:

$A = lw$, where l is the length of the rectangle and w is its width.

Substitute: $l = x + y$ and $w = x - y$

$$\begin{aligned} A &= (x + y)(x - y) \\ &= x(x - y) + y(x - y) \\ &= x^2 - xy + xy - y^2 \\ &= x^2 - y^2 \end{aligned}$$

The area of the rectangle is represented by the expression: $x^2 - y^2$

c) The two expressions represent the area of the same region, so the expressions are equal:

$$x^2 - y^2 = (x + y)(x - y)$$

This is the difference of squares.

10. a) $9d^2 - 16f^2$

Write each term as a perfect square.

$$\begin{aligned} 9d^2 - 16f^2 &= (3d)^2 - (4f)^2 \\ &= (3d + 4f)(3d - 4f) \end{aligned}$$

Write these terms in binomial factors.

To verify, multiply:

$$\begin{aligned} (3d + 4f)(3d - 4f) &= 3d(3d - 4f) + 4f(3d - 4f) \\ &= 9d^2 - 12df + 12df - 16f^2 \\ &= 9d^2 - 16f^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

b) $25s^2 - 64t^2$

Write each term as a perfect square.

$$\begin{aligned} 25s^2 - 64t^2 &= (5s)^2 - (8t)^2 \\ &= (5s + 8t)(5s - 8t) \end{aligned}$$

Write these terms in binomial factors.

To verify, multiply:

$$\begin{aligned} (5s + 8t)(5s - 8t) &= 5s(5s - 8t) + 8t(5s - 8t) \\ &= 25s^2 - 40st + 40st - 64t^2 \\ &= 25s^2 - 64t^2 \end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

c) $144a^2 - 9b^2$

The terms have a common factor 9. Remove this common factor.

$$144a^2 - 9b^2 = 9(16a^2 - b^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= 9[(4a)^2 - (b)^2] \quad \text{Write these terms in binomial factors.}$$

$$= 9(4a + b)(4a - b)$$

To verify, multiply:

$$9(4a + b)(4a - b) = 9[4a(4a - b) + b(4a - b)]$$

$$= 9(16a^2 - 4ab + 4ab - b^2)$$

$$= 9(16a^2 - b^2)$$

$$= 144a^2 - 9b^2$$

Since the binomial is the same as the original binomial, the factors are correct.

d) $121m^2 - n^2$

Write each term as a perfect square.

$$121m^2 - n^2 = (11m)^2 - (n)^2 \quad \text{Write these terms in binomial factors.}$$

$$= (11m + n)(11m - n)$$

To verify, multiply:

$$(11m + n)(11m - n) = 11m(11m - n) + n(11m - n)$$

$$= 121m^2 - 11mn + 11mn - n^2$$

$$= 121m^2 - n^2$$

Since the binomial is the same as the original binomial, the factors are correct.

e) $81k^2 - 49m^2$

Write each term as a perfect square.

$$81k^2 - 49m^2 = (9k)^2 - (7m)^2 \quad \text{Write these terms in binomial factors.}$$

$$= (9k + 7m)(9k - 7m)$$

To verify, multiply:

$$(9k + 7m)(9k - 7m) = 9k(9k - 7m) + 7m(9k - 7m)$$

$$= 81k^2 - 63km + 63km - 49m^2$$

$$= 81k^2 - 49m^2$$

Since the binomial is the same as the original binomial, the factors are correct.

f) $100y^2 - 81z^2$

Write each term as a perfect square.

$$100y^2 - 81z^2 = (10y)^2 - (9z)^2 \quad \text{Write these terms in binomial factors.}$$

$$= (10y + 9z)(10y - 9z)$$

To verify, multiply:

$$\begin{aligned}(10y + 9z)(10y - 9z) &= 10y(10y - 9z) + 9z(10y - 9z) \\ &= 100y^2 - 90yz + 90yz - 81z^2 \\ &= 100y^2 - 81z^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

g) $v^2 - 36t^2$

Write each term as a perfect square.

$$\begin{aligned}v^2 - 36t^2 &= (v)^2 - (6t)^2 && \text{Write these terms in binomial factors.} \\ &= (v + 6t)(v - 6t)\end{aligned}$$

To verify, multiply:

$$\begin{aligned}(v + 6t)(v - 6t) &= v(v - 6t) + 6t(v - 6t) \\ &= v^2 - 6tv + 6tv - 36t^2 \\ &= v^2 - 36t^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

h) $4j^2 - 225h^2$

Write each term as a perfect square.

$$\begin{aligned}4j^2 - 225h^2 &= (2j)^2 - (15h)^2 && \text{Write these terms in binomial factors.} \\ &= (2j + 15h)(2j - 15h)\end{aligned}$$

To verify, multiply:

$$\begin{aligned}(2j + 15h)(2j - 15h) &= 2j(2j - 15h) + 15h(2j - 15h) \\ &= 4j^2 - 30hj + 30hj - 225h^2 \\ &= 4j^2 - 225h^2\end{aligned}$$

Since the binomial is the same as the original binomial, the factors are correct.

11. a) $y^2 + 7yz + 10z^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is positive, these operations will be addition.

The two binomials will have the form: $(?y + ?z)(?y + ?z)$

The product of the coefficients of y^2 and z^2 is: $(1)(10) = 10$

Write $7yz$ as a sum of two terms whose coefficients have a product of 10.

Factors of 10	Sum of Factors
1, 10	$1 + 10 = 11$
2, 5	$2 + 5 = 7$

The two coefficients are 2 and 5, so write the trinomial $y^2 + 7yz + 10z^2$ as $y^2 + 2yz + 5yz + 10z^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} & y^2 + 2yz + 5yz + 10z^2 \\ &= y(y + 2z) + 5z(y + 2z) \quad \text{Remove the common binomial factor.} \\ &= (y + 2z)(y + 5z) \end{aligned}$$

$$\text{So, } y^2 + 7yz + 10z^2 = (y + 2z)(y + 5z)$$

To verify, multiply:

$$\begin{aligned} (y + 2z)(y + 5z) &= y(y + 5z) + 2z(y + 5z) \\ &= y^2 + 5yz + 2yz + 10z^2 \\ &= y^2 + 7yz + 10z^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

b) $4w^2 - 8wx - 21x^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?w + ?x)(?w - ?x)$

The product of the coefficients of w^2 and x^2 is: $(4)(-21) = -84$

Write $-8wx$ as a sum of two terms whose coefficients have a product of -84 .

Factors of -84	Sum of Factors
4, -21	$4 - 21 = -17$
-4 , 21	$-4 + 21 = 17$
6, -14	$6 - 14 = -8$
-6 , 14	$-6 + 14 = 8$

The two coefficients are 6 and -14 , so write the trinomial $4w^2 - 8wx - 21x^2$ as

$$4w^2 + 6wx - 14wx - 21x^2.$$

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} & 4w^2 + 6wx - 14wx - 21x^2 \\ &= 2w(2w + 3x) - 7x(2w + 3x) \quad \text{Remove the common binomial factor.} \\ &= (2w + 3x)(2w - 7x) \end{aligned}$$

$$\text{So, } 4w^2 - 8wx - 21x^2 = (2w + 3x)(2w - 7x)$$

To verify, multiply:

$$\begin{aligned} (2w + 3x)(2w - 7x) &= 2w(2w - 7x) + 3x(2w - 7x) \\ &= 4w^2 - 14wx + 6wx - 21x^2 \\ &= 4w^2 - 8wx - 21x^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

c) $12s^2 - 7su + u^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?s - ?u)(?s - ?u)$

The product of the coefficients of s^2 and u^2 is: $(12)(1) = 12$

Write $-7su$ as a sum of two terms whose coefficients have a product of 12.

Factors of 12	Sum of Factors
-1, -12	$-1 - 12 = -13$
-2, -6	$-2 - 6 = -8$
-3, -4	$-3 - 4 = -7$

The two coefficients are -3 and -4, so write the trinomial $12s^2 - 7su + u^2$ as $12s^2 - 3su - 4su + u^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$12s^2 - 3su - 4su + u^2$$

$$= 3s(4s - u) - u(4s - u) \quad \text{Remove the common binomial factor.}$$

$$= (4s - u)(3s - u)$$

$$\text{So, } 12s^2 - 7su + u^2 = (4s - u)(3s - u)$$

To verify, multiply:

$$\begin{aligned} (4s - u)(3s - u) &= 4s(3s - u) - u(3s - u) \\ &= 12s^2 - 4su - 3su + u^2 \\ &= 12s^2 - 7su + u^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

d) $3t^2 - 7tv + 4v^2$

Use decomposition. Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?t - ?v)(?t - ?v)$

The product of the coefficients of t^2 and v^2 is: $(3)(4) = 12$

Write $-7tv$ as a sum of two terms whose coefficients have a product of 12.

Factors of 12	Sum of Factors
-1, -12	$-1 - 12 = -13$
-2, -6	$-2 - 6 = -8$
-3, -4	$-3 - 4 = -7$

The two coefficients are -3 and -4, so write the trinomial $3t^2 - 7tv + 4v^2$ as $3t^2 - 3tv - 4tv + 4v^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} & 3t^2 - 3tv - 4tv + 4v^2 \\ &= 3t(t - v) - 4v(t - v) \quad \text{Remove the common binomial factor.} \\ &= (t - v)(3t - 4v) \end{aligned}$$

$$\text{So, } 3t^2 - 7tv + 4v^2 = (t - v)(3t - 4v)$$

To verify, multiply:

$$\begin{aligned} (t - v)(3t - 4v) &= t(3t - 4v) - v(3t - 4v) \\ &= 3t^2 - 4tv - 3tv + 4v^2 \\ &= 3t^2 - 7tv + 4v^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

e) $10r^2 + 9rs - 9s^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?r + ?s)(?r - ?s)$

The product of the coefficients of r^2 and s^2 is: $(10)(-9) = -90$

Write $9rs$ as a sum of two terms whose coefficients have a product of -90 .

Factors of -90	Sum of Factors
5, -18	$5 - 18 = -13$
-5 , 18	$-5 + 18 = 13$
6, -15	$6 - 15 = -9$
-6 , 15	$-6 + 15 = 9$

The two coefficients are -6 and 15 , so write the trinomial $10r^2 + 9rs - 9s^2$ as

$$10r^2 - 6rs + 15rs - 9s^2.$$

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} & 10r^2 - 6rs + 15rs - 9s^2 \\ &= 2r(5r - 3s) + 3s(5r - 3s) \quad \text{Remove the common binomial factor.} \\ &= (5r - 3s)(2r + 3s) \end{aligned}$$

$$\text{So, } 10r^2 + 9rs - 9s^2 = (5r - 3s)(2r + 3s)$$

To verify, multiply:

$$\begin{aligned} (5r - 3s)(2r + 3s) &= 5r(2r + 3s) - 3s(2r + 3s) \\ &= 10r^2 + 15rs - 6rs - 9s^2 \\ &= 10r^2 + 9rs - 9s^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

f) $8p^2 + 18pq - 35q^2$

Use decomposition. Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?r + ?s)(?r - ?s)$

The product of the coefficients of p^2 and q^2 is: $(8)(-35) = -280$

Write $18pq$ as a sum of two terms whose coefficients have a product of -280 .

Factors of -280	Sum of Factors
8, -35	$8 - 35 = -27$
-8 , 35	$-8 + 35 = 27$
10, -28	$10 - 28 = -18$
-10 , 28	$-10 + 28 = 18$

The two coefficients are -10 and 28 , so write the trinomial $8p^2 + 18pq - 35q^2$ as $8p^2 - 10pq + 28pq - 35q^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} &8p^2 - 10pq + 28pq - 35q^2 \\ &= 2p(4p - 5q) + 7q(4p - 5q) \quad \text{Remove the common binomial factor.} \\ &= (4p - 5q)(2p + 7q) \end{aligned}$$

So, $8p^2 + 18pq - 35q^2 = (4p - 5q)(2p + 7q)$

To verify, multiply:

$$\begin{aligned} (4p - 5q)(2p + 7q) &= 4p(2p + 7q) - 5q(2p + 7q) \\ &= 8p^2 + 28pq - 10pq - 35q^2 \\ &= 8p^2 + 18pq - 35q^2 \end{aligned}$$

Since this trinomial is the same as the original trinomial, the factors are correct.

12. a) $4x^2 + 28xy + 49y^2$

The first term is a perfect square since $4x^2 = (2x)(2x)$.

The third term is a perfect square since $49y^2 = (7y)(7y)$.

The second term is twice the product of $2x$ and $7y$:

$$28xy = 2(2x)(7y)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(2x + 7y)(2x + 7y), \text{ or } (2x + 7y)^2$$

b) $15m^2 + 7mn - 4n^2$

The first term is not a perfect square, so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?m + ?n)(?m - ?n)$

The product of the coefficients of m^2 and n^2 is: $(15)(-4) = -60$

Write $7mn$ as a sum of two terms whose coefficients have a product of -60 .

Factors of -60	Sum of Factors
4, -15	$4 - 15 = -11$
-4 , 15	$-4 + 15 = 11$
5, -12	$5 - 12 = -7$
-5 , 12	$-5 + 12 = 7$

The two coefficients are -5 and 12 , so write the trinomial $15m^2 + 7mn - 4n^2$ as $15m^2 - 5mn + 12mn - 4n^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$\begin{aligned} &15m^2 - 5mn + 12mn - 4n^2 \\ &= 5m(3m - n) + 4n(3m - n) \quad \text{Remove the common binomial factor.} \\ &= (3m - n)(5m + 4n) \end{aligned}$$

$$\text{So, } 15m^2 + 7mn - 4n^2 = (3m - n)(5m + 4n)$$

c) $16r^2 + 8rt + t^2$

The first term is a perfect square since $16r^2 = (4r)(4r)$.

The third term is a perfect square since $t^2 = (t)(t)$.

The second term is twice the product of $4r$ and t :

$$8rt = 2(4r)(t)$$

Since the second term is positive, the operations in the binomial factors must be addition. So, the trinomial is a perfect square trinomial and its factors are:

$$(4r + t)(4r + t), \text{ or } (4r + t)^2$$

d) $9a^2 - 42ab + 49b^2$

The first term is a perfect square since $9a^2 = (3a)(3a)$.

The third term is a perfect square since $49b^2 = (7b)(7b)$.

The second term is twice the product of $3a$ and $7b$:

$$42ab = 2(3a)(7b)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(3a - 7b)(3a - 7b), \text{ or } (3a - 7b)^2$$

e) $12h^2 + 25hk + 12k^2$

The first term is not a perfect square, so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is positive, these operations will be addition.

The two binomials will have the form: $(?h + ?k)(?h + ?k)$

The product of the coefficients of h^2 and k^2 is: $(12)(12) = 144$

Write $25hk$ as a sum of two terms whose coefficients have a product of 144.

Factors of 144	Sum of Factors
4, 36	$4 + 36 = 40$
6, 24	$6 + 24 = 30$
8, 18	$8 + 18 = 26$
9, 16	$9 + 16 = 25$

The two coefficients are 9 and 16, so write the trinomial $12h^2 + 25hk + 12k^2$ as $12h^2 + 9hk + 16hk + 12k^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$12h^2 + 9hk + 16hk + 12k^2$$

$$= 3h(4h + 3k) + 4k(4h + 3k) \quad \text{Remove the common binomial factor.}$$

$$= (4h + 3k)(3h + 4k)$$

$$\text{So, } 12h^2 + 25hk + 12k^2 = (4h + 3k)(3h + 4k)$$

f) $15f^2 - 31fg + 10g^2$

The first term is not a perfect square, so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?f - ?g)(?f - ?g)$

The product of the coefficients of f^2 and g^2 is: $(15)(10) = 150$

Write $-31fg$ as a sum of two terms whose coefficients have a product of 150.

Factors of 150	Sum of Factors
-3, -50	$-3 - 50 = -53$
-5, -30	$-5 - 30 = -35$
-6, -25	$-6 - 25 = -31$

The two coefficients are -6 and -25, so write the trinomial $15f^2 - 31fg + 10g^2$ as $15f^2 - 6fg - 25fg + 10g^2$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$15f^2 - 6fg - 25fg + 10g^2$$

$$= 3f(5f - 2g) - 5g(5f - 2g) \quad \text{Remove the common binomial factor.}$$

$$= (5f - 2g)(3f - 5g)$$

$$\text{So, } 15f^2 - 31fg + 10g^2 = (5f - 2g)(3f - 5g)$$

13. a) $8m^2 - 72n^2$

As written, each term of the binomial is not a perfect square. But the terms have a common factor 8. Remove this common factor.

$$8m^2 - 72n^2 = 8(m^2 - 9n^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= 8[(m)^2 - (3n)^2] \quad \text{Write these terms in binomial factors.}$$

$$= 8(m + 3n)(m - 3n)$$

b) $8z^2 + 8yz + 2y^2$

As written, each term of the trinomial has a common factor 2.
Remove this common factor.

$$8z^2 + 8yz + 2y^2 = 2(4z^2 + 4yz + y^2)$$

Look at the trinomial $4z^2 + 4yz + y^2$.

The first term is a perfect square since $4z^2 = (2z)(2z)$.

The third term is a perfect square since $y^2 = (y)(y)$.

The second term is twice the product of $2z$ and y :

$$4yz = 2(2z)(y)$$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$$(2z + y)(2z + y), \text{ or } (2z + y)^2$$

$$\text{So, } 8z^2 + 8yz + 2y^2 = 2(2z + y)^2$$

c) $12x^2 - 27y^2$

As written, each term of the binomial is not a perfect square. But the terms have a common factor 3. Remove this common factor.

$$12x^2 - 27y^2 = 3(4x^2 - 9y^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= 3[(2x)^2 - (3y)^2] \quad \text{Write these terms in binomial factors.}$$

$$= 3(2x + 3y)(2x - 3y)$$

d) $8p^2 + 40pq + 50q^2$

As written, each term of the trinomial has a common factor 2.
Remove this common factor.

$$8p^2 + 40pq + 50q^2 = 2(4p^2 + 20pq + 25q^2)$$

Look at the trinomial $4p^2 + 20pq + 25q^2$.

The first term is a perfect square since $4p^2 = (2p)(2p)$.

The third term is a perfect square since $25q^2 = (5q)(5q)$.

The second term is twice the product of $2p$ and $5q$:

$$20pq = 2(2p)(5q)$$

Since the second term is positive, the operations in the binomial factors must be addition.
So, the trinomial is a perfect square trinomial and its factors are:

$$(2p + 5q)(2p + 5q), \text{ or } (2p + 5q)^2$$

$$\text{So, } 8p^2 + 40pq + 50q^2 = 2(2p + 5q)^2$$

e) $-24u^2 - 6uv + 9v^2$

As written, each term of the trinomial has a common factor -3 .

Remove this common factor.

$$-24u^2 - 6uv + 9v^2 = -3(8u^2 + 2uv - 3v^2)$$

Look at the trinomial $8u^2 + 2uv - 3v^2$.

The first term is not a perfect square, so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is negative, the operations in the binomial factors will be addition and subtraction.

The two binomials will have the form: $(?u + ?v)(?u - ?v)$

The product of the coefficients of u^2 and v^2 is: $(8)(-3) = -24$

Write $2uv$ as a sum of two terms whose coefficients have a product of -24 .

Factors of -24	Sum of Factors
3, -8	$3 - 8 = -5$
-3 , 8	$-3 + 8 = 5$
4, -6	$4 - 6 = -2$
-4 , 6	$-4 + 6 = 2$

The two coefficients are -4 and 6 , so write the trinomial $8u^2 + 2uv - 3v^2$ as

$$8u^2 - 4uv + 6uv - 3v^2.$$

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$8u^2 - 4uv + 6uv - 3v^2$$

$$= 4u(2u - v) + 3v(2u - v) \quad \text{Remove the common binomial factor.}$$

$$= (2u - v)(4u + 3v)$$

$$\text{So, } 8u^2 + 2uv - 3v^2 = (2u - v)(4u + 3v)$$

$$\text{And, } -24u^2 - 6uv + 9v^2 = -3(2u - v)(4u + 3v)$$

f) $-18b^2 + 128c^2$

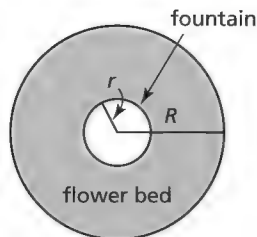
As written, each term of the binomial is not a perfect square. But the terms have a common factor -2 . Remove this common factor.

$$-18b^2 + 128c^2 = -2(9b^2 - 64c^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= -2[(3b)^2 - (8c)^2] \quad \text{Write these terms in binomial factors.}$$

$$= -2(3b + 8c)(3b - 8c)$$

14. a)



- b) The flower bed and the circular fountain form a large circle with radius R .
The area of the flower bed = the area of the large circle minus the area of the small circle.
The formula for the area of a circle, A , is $A = \pi r^2$, where r is the radius of the circle.

The area of the large circle is: $A = \pi R^2$

The area of the small circle is: $A = \pi r^2$

So, the area of the flower bed is: $\pi R^2 - \pi r^2$

As written, each term of the binomial is not a perfect square. But the terms have a common factor, π . Remove this common factor.

$$\pi R^2 - \pi r^2 = \pi(R^2 - r^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= \pi[(R)^2 - (r)^2] \quad \text{Write these terms in binomial factors.}$$

$$= \pi(R + r)(R - r)$$

An expression for the area of the flower bed is: $\pi(R + r)(R - r)$

- c) Use the expression: $\pi(R + r)(R - r)$

Substitute: $R = 350$ and $r = 150$

$$\pi(R + r)(R - r) = \pi(350 + 150)(350 - 150)$$

$$= \pi(500)(200)$$

$$= 100\,000\pi$$

$$= 314\,159.2654\dots$$

So, the area of the flower bed is approximately $314\,159 \text{ cm}^2$.

15. a) i) $x^2 + \square x + 49$

The first term is a perfect square since $x^2 = (x)(x)$.

The third term is a perfect square since $49 = (7)(7)$ or $49 = (-7)(-7)$.

For the trinomial to be a perfect square, the second term must be twice the product of x and 7 or twice the product of x and -7 :

$$\square x = 2(x)(7)$$

$$= 14x$$

$$\square x = 2(x)(-7)$$

$$= -14x$$

So, the integers that can replace $\square$ are 14 and -14 .

ii) $4a^2 + 20ab + \square b^2$

For the trinomial to be a perfect square, the coefficient of the second term, 20, must be twice the product of the square root of 4 and the square root of the number represented by the box.

I know the square root of 4 is 2.

Let the square root of the number represented by the box be x .

$$\text{So, } 20 = 2(2)(x)$$

$$20 = 4x$$

$$x = 5$$

$$\text{So, } \square = 5^2, \text{ or } 25$$

iii) $\square c^2 - 24cd + 16d^2$

For the trinomial to be a perfect square, the coefficient of the second term, -24 , must be negative two times the product of the square root of 16 and the square root of the number represented by the box.

I know the square root of 16 is 4.

Let the square root of the number represented by the box be x .

$$\text{So, } -24 = -2(4)(x)$$

$$-24 = -8x$$

$$x = 3$$

$$\text{So, } \square = 3^2, \text{ or } 9$$

- b) i) 2 integers; the middle term can be positive or negative. It can only have one value: twice the product of the square roots of its first and last terms.
- ii) 1 integer; the last term in a perfect square is positive. It can only have one value: the square of half the value of the middle term divided by the square root of the first term.
- iii) 1 integer; the first term in a perfect square is positive. It can only have one value: the square of half the value of the middle term divided by the square root of the last term.

16. Let's look at a few examples.

$$(x + 1)^2 = x^2 + 2x + 1$$

$$(2x + 3)^2 = 4x^2 + 12x + 9$$

When all the terms in the trinomial are positive, b is greater than c and b is greater than a .

This is not possible when a , b , and c are consecutive positive integers.

$$(x - 1)^2 = x^2 - 2x + 1$$

$$(2x - 3)^2 = 4x^2 - 12x + 9$$

When the middle terms in the trinomial is negative, b is less than c and b is less than a .

This is not possible when a , b , and c are consecutive negative integers.

So, there are three possibilities for a , b , and c when they are not all positive integers and not all negative integers:

$$a = 0, b = 1, c = 2$$

$$a = -1, b = 0, c = 1$$

$$a = -2, b = -1, c = 0$$

Check each possibility to determine whether the trinomial can be factored:

$$0x^2 + x + 2 = x + 2$$

This cannot be factored.

$$\begin{aligned} -x^2 + 0x + 1 &= -x^2 + 1 \\ &= -(x^2 - 1) \\ &= -(x - 1)(x + 1) \end{aligned}$$

This trinomial can be factored.

$$\begin{aligned} -2x^2 - x + 0 &= -2x^2 - x \\ &= -x(2x + 1) \end{aligned}$$

This trinomial can be factored.

So, there are 2 possibilities for consecutive integers a , b , and c :

$-1, 0, 1$ and $-2, -1, 0$

17. To determine $(199)(201)$, I can use the difference of squares.

I know $199 = 200 - 1$ and $201 = 200 + 1$

$$\begin{aligned} \text{So, } (199)(201) &= (200 - 1)(200 + 1) & \text{I know } (a - b)(a + b) &= a^2 - b^2 \\ &= 200^2 - 1^2 \\ &= 40\,000 - 1 \\ &= 39\,999 \end{aligned}$$

$$(199)(201) = 39\,999$$

18. The area of the shaded region is:

area of large square – area of small square

The formula for the area of a square is: $A = s^2$, where s is the side length of the square.

Area of large square:

Use the formula $A = s^2$. Substitute: $s = 3x + 5$

$$A_L = (3x + 5)^2$$

Area of small rectangle:

Use the formula $A = s^2$. Substitute: $s = 2x - 1$

$$A_s = (2x - 1)^2$$

So, area of the shaded region is:

$$\begin{aligned} A &= A_L - A_s \\ &= (3x + 5)^2 - (2x - 1)^2 \\ &= (3x + 5)(3x + 5) - (2x - 1)(2x - 1) \\ &= 3x(3x + 5) + 5(3x + 5) - [2x(2x - 1) - 1(2x - 1)] \\ &= 9x^2 + 15x + 15x + 25 - (4x^2 - 2x - 2x + 1) \\ &= 9x^2 + 30x + 25 - 4x^2 + 4x - 1 \\ &= 5x^2 + 34x + 24 \end{aligned}$$

A polynomial that represents the area of the shaded region is:

$$5x^2 + 34x + 24$$

C

19. a) i) $(x^2 + 5)^2$

This expression is not a difference of squares because it is not of the form $a^2 - b^2$.
It is not a perfect square trinomial because it does not have three terms.
So, this expression is neither a perfect square trinomial nor a difference of squares.
It is the square of a binomial, or a perfect square trinomial in factored form.

ii) $-100 + r^2$

This expression is a difference of squares because it can be written in the form $a^2 - b^2$: $r^2 - 100$, and $r^2 = (r)(r)$ and $100 = (10)(10)$

iii) $81a^2b^2 - 1$

This expression is a difference of squares because it is written in the form $a^2 - b^2$,
and $81a^2b^2 = (9ab)(9ab)$ and $1 = (1)(1)$.

iv) $16s^4 + 8s^2 + 1$

This expression is not a difference of squares because it is not of the form $a^2 - b^2$.

It has 3 terms, so it may be a perfect square trinomial.

Check: $16s^4 = (4s^2)(4s^2)$ and $1 = (1)(1)$

Since $2(4s^2)(1) = 8s^2$, the expression is a perfect square trinomial.

b) i) This expression represents a perfect square trinomial in factored form, and it cannot be factored further.

ii) $-100 + r^2$ can be written as: $r^2 - 100$

This is a difference of squares so it can be factored.

Write each term as a perfect square.

$$\begin{aligned} r^2 - 100 &= (r)^2 - (10)^2 && \text{Write these terms in binomial factors.} \\ &= (r + 10)(r - 10) \end{aligned}$$

iii) $81a^2b^2 - 1$ is a difference of squares so it can be factored.

Write each term as a perfect square.

$$\begin{aligned} 81a^2b^2 - 1 &= (9ab)^2 - (1)^2 && \text{Write these terms in binomial factors.} \\ &= (9ab + 1)(9ab - 1) \end{aligned}$$

iv) $16s^4 + 8s^2 + 1$ is a perfect square trinomial, so it can be factored.

The first term is a perfect square since $16s^4 = (4s^2)(4s^2)$.

The third term is a perfect square since $1 = (1)(1)$.

The second term is twice the product of $4s^2$ and 1: $8s^2 = 2(4s^2)(1)$

Since the second term is positive, the operations in the binomial factors must be addition.

So, its factors are: $(4s^2 + 1)(4s^2 + 1)$, or $(4s^2 + 1)^2$

20. a) $x^4 - 13x^2 + 36$

The first term is a perfect square since $x^4 = (x^2)(x^2)$.

The third term is a perfect square since $36 = (6)(6)$.

The second term is not twice the product of x^2 and 6 so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?x^2 - ?)(?x^2 - ?)$

The product of the coefficient of x^4 and the constant term is: $(1)(36) = 36$

Write $-13x^2$ as a sum of two terms whose coefficients have a product of 36.

Factors of 36	Sum of Factors
-2, -18	$-2 - 18 = -20$
-3, -12	$-3 - 12 = -15$
-4, -9	$-4 - 9 = -13$

The two coefficients are -4 and -9, so write the trinomial $x^4 - 13x^2 + 36$ as $x^4 - 4x^2 - 9x^2 + 36$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$x^4 - 4x^2 - 9x^2 + 36$$

$$= x^2(x^2 - 4) - 9(x^2 - 4) \quad \text{Remove the common binomial factor.}$$

$$= (x^2 - 4)(x^2 - 9)$$

$$\text{So, } x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$$

This can be factored further. Both $x^2 - 4$ and $x^2 - 9$ are difference of squares.

$$\begin{aligned} (x^2 - 4)(x^2 - 9) &= [(x)^2 - (2)^2][(x)^2 - (3)^2] \\ &= (x + 2)(x - 2)(x + 3)(x - 3) \end{aligned}$$

$$\text{So, } x^4 - 13x^2 + 36 = (x + 2)(x - 2)(x + 3)(x - 3)$$

b) $a^4 - 17a^2 + 16$

The first term is a perfect square since $a^4 = (a^2)(a^2)$.

The third term is a perfect square since $16 = (4)(4)$.

The second term is not twice the product of a^2 and 4 so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?a^2 - ?)(?a^2 - ?)$

The product of the coefficient of a^4 and the constant term is: $(1)(16) = 16$

Write $-17a^2$ as a sum of two terms whose coefficients have a product of 16.

Factors of 16	Sum of Factors
-1, -16	$-1 - 16 = -17$

The two coefficients are -1 and -16 , so write the trinomial $a^4 - 17a^2 + 16$ as

$$a^4 - a^2 - 16a^2 + 16$$

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$a^4 - a^2 - 16a^2 + 16$$

$$= a^2(a^2 - 1) - 16(a^2 - 1) \quad \text{Remove the common binomial factor.}$$

$$= (a^2 - 1)(a^2 - 16)$$

$$\text{So, } a^4 - 17a^2 + 16 = (a^2 - 1)(a^2 - 16)$$

This can be factored further. Both $a^2 - 1$ and $a^2 - 16$ are difference of squares.

$$(a^2 - 1)(a^2 - 16) = [(a)^2 - (1)^2][(a)^2 - (4)^2]$$

$$= (a + 1)(a - 1)(a + 4)(a - 4)$$

$$\text{So, } a^4 - 17a^2 + 16 = (a + 1)(a - 1)(a + 4)(a - 4)$$

c) $y^4 - 5y^2 + 4$

The first term is a perfect square since $y^4 = (y^2)(y^2)$.

The third term is a perfect square since $4 = (2)(2)$.

The second term is not twice the product of y^2 and 2 so the trinomial is not a perfect square.

So, I'll factor by decomposition.

Since the 3rd term in the trinomial is positive, the operations in the binomial factors will be the same. Since the second term is negative, these operations will be subtraction.

The two binomials will have the form: $(?y^2 - ?)(?y^2 - ?)$

The product of the coefficient of y^4 and the constant term is: $(1)(4) = 4$

Write $-5y^2$ as a sum of two terms whose coefficients have a product of 4.

Factors of 4	Sum of Factors
-1, -4	$-1 - 4 = -5$

The two coefficients are -1 and -4 , so write the trinomial $y^4 - 5y^2 + 4$ as

$$y^4 - y^2 - 4y^2 + 4$$

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$$y^4 - y^2 - 4y^2 + 4$$

$$= y^2(y^2 - 1) - 4(y^2 - 1) \quad \text{Remove the common binomial factor.}$$

$$= (y^2 - 1)(y^2 - 4)$$

$$\text{So, } y^4 - 5y^2 + 4 = (y^2 - 1)(y^2 - 4)$$

This can be factored further. Both $y^2 - 1$ and $y^2 - 4$ are difference of squares.

$$\begin{aligned}(y^2 - 1)(y^2 - 4) &= [(y)^2 - (1)^2][(y)^2 - (2)^2] \\ &= (y + 1)(y - 1)(y + 2)(y - 2)\end{aligned}$$

$$\text{So, } y^4 - 5y^2 + 4 = (y + 1)(y - 1)(y + 2)(y - 2)$$

21. a) $8d^2 - 32e^2$ Remove the common factor, 8.

$$8d^2 - 32e^2 = 8(d^2 - 4e^2)$$

$d^2 - 4e^2$ is a difference of squares.

$$\begin{aligned}8d^2 - 32e^2 &= 8((d)^2 - (2e)^2) \\ &= 8(d + 2e)(d - 2e)\end{aligned}$$

- b) $25m^2 - \frac{1}{4}n^2$ This is a difference of squares.

$$\begin{aligned}&= \left((5m)^2 - \left(\frac{1}{2}n \right)^2 \right) \\ &= \left(5m + \frac{1}{2}n \right) \left(5m - \frac{1}{2}n \right)\end{aligned}$$

- c) $18x^2y^2 - 50y^4$ Remove the common factor, $2y^2$.

$$18x^2y^2 - 50y^4 = 2y^2(9x^2 - 25y^2)$$

$9x^2 - 25y^2$ is a difference of squares.

$$\begin{aligned}18x^2y^2 - 50y^4 &= 2y^2((3x)^2 - (5y)^2) \\ &= 2y^2(3x + 5y)(3x - 5y)\end{aligned}$$

- d) $25s^2 + 49t^2$

This binomial cannot be factored because it is a sum of squares and not a difference of squares.

- e) $10a^2 - 7b^2$

This binomial is a difference of squares but it cannot be factored over whole numbers because the coefficients of a and b are not perfect squares.

f) $\frac{x^2}{16} - \frac{y^2}{49}$

This is a difference of squares.

$$= \left(\left(\frac{x}{4} \right)^2 - \left(\frac{y}{7} \right)^2 \right)$$

$$= \left(\frac{x}{4} + \frac{y}{7} \right) \left(\frac{x}{4} - \frac{y}{7} \right)$$

Review

(pages 198–200)

3.1

1. Use a calculator and repeated division by prime factors.

a) $594 \div 2 = 297$

$$297 \div 3 = 99$$

$$99 \div 3 = 33$$

$$33 \div 3 = 11$$

$$11 \div 11 = 1$$

$$\text{So, } 594 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 11$$

$$\text{Using powers: } 594 = 2 \cdot 3^3 \cdot 11$$

b) $2100 \div 2 = 1050$

$$1050 \div 2 = 525$$

$$525 \div 3 = 175$$

$$175 \div 5 = 35$$

$$35 \div 5 = 7$$

$$7 \div 7 = 1$$

$$\text{So, } 2100 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$$

$$\text{Using powers: } 2100 = 2^2 \cdot 3 \cdot 5^2 \cdot 7$$

c) $4875 \div 3 = 1625$

$$1625 \div 5 = 325$$

$$325 \div 5 = 65$$

$$65 \div 5 = 13$$

$$13 \div 13 = 1$$

$$\text{So, } 4875 = 3 \cdot 5 \cdot 5 \cdot 5 \cdot 13$$

$$\text{Using powers: } 4875 = 3 \cdot 5^3 \cdot 13$$

d) $9009 \div 3 = 3003$

$$3003 \div 3 = 1001$$

$$1001 \div 7 = 143$$

$$143 \div 11 = 13$$

$$13 \div 13 = 1$$

$$\text{So, } 9009 = 3 \cdot 3 \cdot 7 \cdot 11 \cdot 13$$

$$\text{Using powers: } 9009 = 3^2 \cdot 7 \cdot 11 \cdot 13$$

2. Write the prime factorization of each number.

Highlight the factors that appear in each prime factorization.

a) $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$$160 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$$

$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

$$\text{The greatest common factor is: } 2 \cdot 2 \cdot 5 = 20$$

b) $245 = 5 \cdot 7 \cdot 7$

$$280 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7$$

$$385 = 5 \cdot 7 \cdot 11$$

$$\text{The greatest common factor is: } 5 \cdot 7 = 35$$

c) $176 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11$

$320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

$368 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 23$

The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 = 16$

d) $484 = 2 \cdot 2 \cdot 11 \cdot 11$

$496 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 31$

$884 = 2 \cdot 2 \cdot 13 \cdot 17$

The greatest common factor is: $2 \cdot 2 = 4$

3. Write the prime factorization of each number. Highlight the greatest power of each prime factor in any list. The least common multiple is the product of the greatest power of each prime factor:

a) $70 = 2 \cdot 5 \cdot 7$

$90 = 2 \cdot 3 \cdot 3 \cdot 5$

$140 = 2 \cdot 2 \cdot 5 \cdot 7$

The greatest power of 2 in any list is: $2 \cdot 2 = 2^2$

The greatest power of 3 in any list is: $3 \cdot 3 = 3^2$

The greatest power of 5 in any list is 5.

The greatest power of 7 in any list is 7.

The least common multiple is: $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$

So, the least common multiple of 70, 90, and 140 is 1260.

b) $120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$130 = 2 \cdot 5 \cdot 13$

$309 = 3 \cdot 103$

The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 = 2^3$

The greatest power of 3 in any list is 3

The greatest power of 5 in any list is 5.

The greatest power of 13 in any list is 13.

The greatest power of 103 in any list is 103.

The least common multiple is: $2^3 \cdot 3 \cdot 5 \cdot 13 \cdot 103 = 160\,680$

So, the least common multiple of 120, 130, and 309 is 160 680.

c) $200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$250 = 2 \cdot 5 \cdot 5 \cdot 5$

$500 = 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 = 2^3$

The greatest power of 5 in any list is: $5 \cdot 5 \cdot 5 = 5^3$

The least common multiple is: $2^3 \cdot 5^3 = 1000$

So, the least common multiple of 200, 250, and 500 is 1000.

d) $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$$240 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

$$340 = 2 \cdot 2 \cdot 5 \cdot 17$$

The greatest power of 2 in any list is: $2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

The greatest power of 3 in any list is: $3 \cdot 3 = 3^2$

The greatest power of 5 in any list is: 5

The greatest power of 17 in any list is: 17

The least common multiple is: $2^4 \cdot 3^2 \cdot 5 \cdot 17 = 12\,240$

So, the least common multiple of 180, 240, and 340 is 12 240.

4. The least number of beads on any strand is 1 more than the least common multiple of 6, 4, and 10.

Factor each number.

$$6 = 2 \cdot 3$$

$$4 = 2 \cdot 2$$

$$10 = 2 \cdot 5$$

The least common multiple of 6, 4, and 10 is: $2 \cdot 2 \cdot 3 \cdot 5 = 60$

So, the least number of beads on each strand is 61.

5. For parts a, b, and c, a fraction is simplified when its numerator and denominator have no common factors. So, divide the numerator and denominator of each fraction by their greatest common factor.

a) $\frac{1015}{1305}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$1015 = 5 \cdot 7 \cdot 29$$

$$1305 = 3 \cdot 3 \cdot 5 \cdot 29$$

The greatest common factor is: $5 \cdot 29 = 145$

$$\frac{1015}{1305} = \frac{1015 \div 145}{1305 \div 145}$$

$$= \frac{7}{9}$$

b) $\frac{2475}{3825}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$2475 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 11$$

$$3825 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 17$$

The greatest common factor is: $3 \cdot 3 \cdot 5 \cdot 5 = 225$

$$\frac{2475}{3825} = \frac{2475 \div 225}{3825 \div 225}$$

$$= \frac{11}{17}$$

c) $\frac{6656}{7680}$

Write the prime factorization of the numerator and of the denominator.

Highlight the prime factors that appear in both lists.

$$6656 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 13$$

$$7680 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

The greatest common factor is: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 512$

$$\begin{aligned}\frac{6656}{7680} &= \frac{6656 \div 512}{7680 \div 512} \\ &= \frac{13}{15}\end{aligned}$$

For parts d, e, and f, add, divide, or subtract fractions with a common denominator by adding, dividing, or subtracting their numerators. So, write the fractions in each part with denominator equal to the least common multiple of the original denominators.

d) $\frac{7}{36} + \frac{15}{64}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

$$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

The least common multiple is: $2^6 \cdot 3^2 = 576$

$$\begin{aligned}\frac{7}{36} + \frac{15}{64} &= \frac{112}{576} + \frac{135}{576} \\ &= \frac{112 + 135}{576} \\ &= \frac{247}{576}\end{aligned}$$

e) $\frac{5}{9} \div \frac{3}{4}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$9 = 3 \cdot 3$$

$$4 = 2 \cdot 2$$

The least common multiple is: $3^2 \cdot 2^2 = 36$

$$\begin{aligned}\frac{5}{9} \div \frac{3}{4} &= \frac{20}{36} \div \frac{27}{36} \\ &= \frac{20}{27}\end{aligned}$$

f) $\frac{28}{128} - \frac{12}{160}$

Both fractions can be simplified before they are subtracted.

For $\frac{28}{128}$, the numerator and denominator have a common factor of 4, so divide

numerator and denominator by 4 to get $\frac{7}{32}$.

For $\frac{12}{160}$, the numerator and denominator have a common factor of 4, so divide

numerator and denominator by 4 to get $\frac{3}{40}$.

The expression becomes: $\frac{7}{32} - \frac{3}{40}$

Write the prime factorization of the denominators.

Highlight the greatest power of each prime factor in either list.

$$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$40 = 2 \cdot 2 \cdot 2 \cdot 5$$

The least common multiple is: $2^5 \cdot 5 = 160$

$$\begin{aligned} \frac{7}{32} - \frac{3}{40} &= \frac{35}{160} - \frac{12}{160} \\ &= \frac{23}{160} \end{aligned}$$

3.2

6. Determine the prime factors of each area. If the factors can be arranged in 2 equal groups, then the area is a perfect square. The side length of the square is the square root of the area.

a) $784 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
 $= (2 \cdot 2 \cdot 7)(2 \cdot 2 \cdot 7)$
 $= 28 \cdot 28$

This is a perfect square.

So, $\sqrt{784} = 28$

The side length of the square is 28 in.

b) $1024 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
 $= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
 $= 32 \cdot 32$

This is a perfect square.

So, $\sqrt{1024} = 32$

The side length of the square is 32 cm.

7. Determine the prime factors of each volume. If the factors can be arranged in 3 equal groups, then the volume is a perfect cube. The edge length of the cube is the cube root of the volume.

a) $1728 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$
 $= (2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)(2 \cdot 2 \cdot 3)$
 $= 12 \cdot 12 \cdot 12$

$$\text{So, } \sqrt[3]{1728} = 12$$

The edge length of the cube is 12 cm.

$$\begin{aligned}\text{b) } 2744 &= 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7 \\ &= (2 \cdot 7)(2 \cdot 7)(2 \cdot 7) \\ &= 14 \cdot 14 \cdot 14\end{aligned}$$

$$\text{So, } \sqrt[3]{2744} = 14$$

The edge length of the cube is 14 ft.

8. Write each number as a product of its prime factors. If the factors can be arranged in 2 equal groups, then the number is a perfect square. If the factors can be arranged in 3 equal groups, then the number is a perfect cube.

$$\begin{aligned}\text{a) } 256 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2) \\ &= 16 \cdot 16\end{aligned}$$

The factors can be arranged in 2 equal groups, so 256 is a perfect square.

$$\sqrt{256} = 16$$

$$\begin{aligned}\text{b) } 324 &= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (2 \cdot 3 \cdot 3)(2 \cdot 3 \cdot 3) \\ &= 18 \cdot 18\end{aligned}$$

The factors can be arranged in 2 equal groups, so 324 is a perfect square.

$$\sqrt{324} = 18$$

$$\begin{aligned}\text{c) } 729 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= (3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3), \text{ or } 27 \cdot 27 \\ &= (3 \cdot 3)(3 \cdot 3)(3 \cdot 3), \text{ or } 9 \cdot 9 \cdot 9\end{aligned}$$

The factors can be arranged in 2 equal groups and 3 equal groups, so 729 is both a perfect square and a perfect cube.

$$\sqrt{729} = 27$$

$$\sqrt[3]{729} = 9$$

$$\text{d) } 1298 = 2 \cdot 11 \cdot 59$$

The factors cannot be arranged in 2 equal groups or 3 equal groups, so 1298 is neither a perfect square nor a perfect cube.

$$\begin{aligned}\text{e) } 1936 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \\ &= (2 \cdot 2 \cdot 11)(2 \cdot 2 \cdot 11) \\ &= 44 \cdot 44\end{aligned}$$

The factors can be arranged in 2 equal groups, so 1936 is a perfect square.

$$\sqrt{1936} = 44$$

$$\begin{aligned}\text{f) } 9261 &= 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7 \\ &= (3 \cdot 7)(3 \cdot 7)(3 \cdot 7) \\ &= 21 \cdot 21 \cdot 21\end{aligned}$$

The factors can be arranged 3 equal groups, so 9261 is a perfect cube.

$$\sqrt[3]{9261} = 21$$

9. The area of a square is 18 225 square feet.

The side length of the square is the square root of the area.

$$\begin{aligned} 18\,225 &= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \\ &= (3 \cdot 3 \cdot 3 \cdot 5)(3 \cdot 3 \cdot 3 \cdot 5) \\ &= 135 \cdot 135 \end{aligned}$$

$$\sqrt{18\,225} = 135$$

The side length of the square is 135 ft.

The perimeter of the square is: $4(135 \text{ ft.}) = 540 \text{ ft.}$

10. The surface area of a cube is $11\,616 \text{ cm}^2$.

So, the area of one face of the cube is: $\frac{11\,616}{6} \text{ cm} = 1936$

The edge length of the cube is the square root of the area of one face.

$$\begin{aligned} 1936 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \\ &= (2 \cdot 2 \cdot 11)(2 \cdot 2 \cdot 11) \\ &= 44 \cdot 44 \end{aligned}$$

$$\text{So, } \sqrt{1936} = 44$$

The edge length of the cube is 44 cm.

3.3

11. For each binomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product.

a) $8m = 2 \cdot 2 \cdot 2 \cdot m$

$$4m^2 = 2 \cdot 2 \cdot m \cdot m$$

The greatest common factor is: $2 \cdot 2 \cdot m = 4m$

$$\begin{aligned} 8m - 4m^2 &= 4m(2) - 4m(m) \\ &= 4m(2 - m) \end{aligned}$$

I could arrange algebra tiles as a rectangle; the rectangle would have length $2 - m$ and width $4m$. I would have to be careful with the signs of the factors because the rectangle would show a width of $-4m$.

b) $3 = 3$

$$9g^2 = 3 \cdot 3 \cdot g \cdot g$$

The greatest common factor is: 3

$$\begin{aligned} -3 + 9g^2 &= 3(-1) + 3(3g^2) \\ &= 3(-1 + 3g^2) && \text{Remove } -1 \text{ as a common factor.} \\ &= -3(1 - 3g^2) \end{aligned}$$

I could arrange algebra tiles into equal groups. There would be 3 equal groups with 1 negative 1-tile and 3 g^2 -tiles in each group.

c) $28a^2 = 2 \cdot 2 \cdot 7 \cdot a \cdot a$

$$7a^3 = 7 \cdot a \cdot a \cdot a$$

The greatest common factor is: $7 \cdot a \cdot a = 7a^2$

$$\begin{aligned} 28a^2 - 7a^3 &= 7a^2(4) - 7a^2(a) \\ &= 7a^2(4 - a) \end{aligned}$$

I could not use algebra tiles because I do not have an a^3 -tile.

$$\begin{aligned} \text{d) } 6a^2b^3c &= 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \\ 15a^2b^2c^2 &= 3 \cdot 5 \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \end{aligned}$$

The greatest common factor is: $3 \cdot a \cdot a \cdot b \cdot b \cdot c = 3a^2b^2c$

$$\begin{aligned} 6a^2b^3c - 15a^2b^2c^2 &= 3a^2b^2c(2b) - 3a^2b^2c(5c) \\ &= 3a^2b^2c(2b - 5c) \end{aligned}$$

I could not use algebra tiles because I do not have tiles for terms with more than one variable.

$$\begin{aligned} \text{e) } 24m^2n &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot m \cdot m \cdot n \\ 6mn^2 &= 2 \cdot 3 \cdot m \cdot n \cdot n \end{aligned}$$

The greatest common factor is: $2 \cdot 3 \cdot m \cdot n = 6mn$

$$\begin{aligned} -24m^2n - 6mn^2 &= 6mn(-4m) + 6mn(-n) \\ &= 6mn(-4m - n) && \text{Remove } -1 \text{ as a common factor.} \\ &= -6mn(4m + n) \end{aligned}$$

I could not use algebra tiles because I do not have tiles for terms with more than one variable.

$$\begin{aligned} \text{f) } 14b^3c^2 &= 2 \cdot 7 \cdot b \cdot b \cdot b \cdot c \cdot c \\ 21a^3b^2 &= 3 \cdot 7 \cdot a \cdot a \cdot a \cdot b \cdot b \end{aligned}$$

The greatest common factor is: $7 \cdot b \cdot b = 7b^2$

$$\begin{aligned} 14b^3c^2 - 21a^3b^2 &= 7b^2(2bc^2) - 7b^2(3a^3) \\ &= 7b^2(2bc^2 - 3a^3) \end{aligned}$$

I could not use algebra tiles because I do not have tiles for terms with more than one variable.

12. For each trinomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product. Expand to verify.

$$\begin{aligned} \text{a) } 12 &= 2 \cdot 2 \cdot 3 \\ 6g &= 2 \cdot 3 \cdot g \\ 3g^2 &= 3 \cdot g \cdot g \end{aligned}$$

The greatest common factor is: 3

$$\begin{aligned} 12 + 6g - 3g^2 &= 3(4) + 3(2g) - 3(g^2) \\ &= 3(4 + 2g - g^2) \end{aligned}$$

$$\begin{aligned} \text{Check: } 3(4 + 2g - g^2) &= 3(4) + 3(2g) - 3(g^2) \\ &= 12 + 6g - 3g^2 \end{aligned}$$

$$\begin{aligned} \text{b) } 3c^2d &= 3 \cdot c \cdot c \cdot d \\ 10cd &= 2 \cdot 5 \cdot c \cdot d \\ 2d &= 2 \cdot d \end{aligned}$$

The greatest common factor is: d

$$\begin{aligned} 3c^2d - 10cd - 2d &= d(3c^2) - d(10c) - d(2) \\ &= d(3c^2 - 10c - 2) \end{aligned}$$

$$\begin{aligned} \text{Check: } d(3c^2 - 10c - 2) &= d(3c^2) - d(10c) - d(2) \\ &= 3c^2d - 10cd - 2d \end{aligned}$$

$$\begin{aligned} \text{c) } 8mn^2 &= 2 \cdot 2 \cdot 2 \cdot m \cdot n \cdot n \\ 12mn &= 2 \cdot 2 \cdot 3 \cdot m \cdot n \\ 16m^2n &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot m \cdot m \cdot n \end{aligned}$$

The greatest common factor is: $2 \cdot 2 \cdot m \cdot n = 4mn$

$$8mn^2 - 12mn - 16m^2n = 4mn(2n) - 4mn(3) - 4mn(4m)$$

$$= 4mn(2n - 3 - 4m)$$

$$\begin{aligned}\text{Check: } 4mn(2n - 3 - 4m) &= 4mn(2n) - 4mn(3) - 4mn(4m) \\ &= 8mn^2 - 12mn - 16m^2n\end{aligned}$$

$$\begin{aligned}\text{d) } y^4 &= y \cdot y \cdot y \cdot y \\ 12y^2 &= 2 \cdot 2 \cdot 3 \cdot y \cdot y \\ 24y &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot y\end{aligned}$$

The greatest common factor is: y

$$\begin{aligned}y^4 - 12y^2 + 24y &= y(y^3) - y(12y) + y(24) \\ &= y(y^3 - 12y + 24)\end{aligned}$$

$$\begin{aligned}\text{Check: } y(y^3 - 12y + 24) &= y(y^3) - y(12y) + y(24) \\ &= y^4 - 12y^2 + 24y\end{aligned}$$

$$\begin{aligned}\text{e) } 30x^2y &= 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \\ 20x^2y^2 &= 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot y \cdot y \\ 10x^3y^2 &= 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y\end{aligned}$$

The greatest common factor is: $2 \cdot 5 \cdot x \cdot x \cdot y = 10x^2y$

$$\begin{aligned}30x^2y - 20x^2y^2 + 10x^3y^2 &= 10x^2y(3) - 10x^2y(2y) + 10x^2y(xy) \\ &= 10x^2y(3 - 2y + xy)\end{aligned}$$

$$\begin{aligned}\text{Check: } 10x^2y(3 - 2y + xy) &= 10x^2y(3) - 10x^2y(2y) + 10x^2y(xy) \\ &= 30x^2y - 20x^2y^2 + 10x^3y^2\end{aligned}$$

$$\begin{aligned}\text{f) } 8b^3 &= 2 \cdot 2 \cdot 2 \cdot b \cdot b \cdot b \\ 20b^2 &= 2 \cdot 2 \cdot 5 \cdot b \cdot b \\ 4b &= 2 \cdot 2 \cdot b\end{aligned}$$

The greatest common factor is: $2 \cdot 2 \cdot b = 4b$

$$\begin{aligned}-8b^3 + 20b^2 - 4b &= 4b(-2b^2) + 4b(5b) + 4b(-1) \\ &= 4b(-2b^2 + 5b - 1) && \text{Remove } -1 \text{ as a common factor.} \\ &= -4b(2b^2 - 5b + 1)\end{aligned}$$

$$\begin{aligned}\text{Check: } -4b(2b^2 - 5b + 1) &= -4b(2b^2) - 4b(-5b) - 4b(1) \\ &= -8b^3 + 20b^2 - 4b\end{aligned}$$

13. For each polynomial, factor each term then identify the greatest common factor. Write each term as the product of the greatest common factor and another monomial. Then use the distributive property to write the expression as a product. Expand to verify.

$$\begin{aligned}\text{a) } 8x^2 &= 2 \cdot 2 \cdot 2 \cdot x \cdot x \\ 12x &= 2 \cdot 2 \cdot 3 \cdot x\end{aligned}$$

The greatest common factor is: $2 \cdot 2 \cdot x = 4x$

$$\begin{aligned}8x^2 - 12x &= 4x(2x) - 4x(3) \\ &= 4x(2x - 3)\end{aligned}$$

$$\begin{aligned}\text{Check: } 4x(2x - 3) &= 4x(2x) - 4x(3) \\ &= 8x^2 - 12x\end{aligned}$$

$$\begin{aligned}\text{b) } 3y^3 &= 3 \cdot y \cdot y \cdot y \\ 12y^2 &= 2 \cdot 2 \cdot 3 \cdot y \cdot y \\ 15y &= 3 \cdot 5 \cdot y\end{aligned}$$

The greatest common factor is: $3 \cdot y = 3y$

$$\begin{aligned}3y^3 - 12y^2 + 15y &= 3y(y^2) - 3y(4y) + 3y(5) \\ &= 3y(y^2 - 4y + 5)\end{aligned}$$

$$\begin{aligned}\text{Check: } 3y(y^2 - 4y + 5) &= 3y(y^2) - 3y(4y) + 3y(5) \\ &= 3y^3 - 12y^2 + 15y\end{aligned}$$

c) $4b^3 = 2 \cdot 2 \cdot b \cdot b \cdot b$

$$2b = 2 \cdot b$$

$$6b^2 = 2 \cdot 3 \cdot b \cdot b$$

The greatest common factor is: $2 \cdot b = 2b$

$$4b^3 - 2b - 6b^2 = 2b(2b^2) - 2b(1) - 2b(3b)$$

$$= 2b(2b^2 - 1 - 3b)$$

$$\begin{aligned} \text{Check: } 2b(2b^2 - 1 - 3b) &= 2b(2b^2) - 2b(1) - 2b(3b) \\ &= 4b^3 - 2b - 6b^2 \end{aligned}$$

d) $6m^3 = 2 \cdot 3 \cdot m \cdot m \cdot m$

$$12m = 2 \cdot 2 \cdot 3 \cdot m$$

$$24m^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot m \cdot m$$

The greatest common factor is: $2 \cdot 3 \cdot m = 6m$

$$6m^3 - 12m - 24m^2 = 6m(m^2) - 6m(2) - 6m(4m)$$

$$= 6m(m^2 - 2 - 4m)$$

$$\begin{aligned} \text{Check: } 6m(m^2 - 2 - 4m) &= 6m(m^2) - 6m(2) - 6m(4m) \\ &= 6m^3 - 12m - 24m^2 \end{aligned}$$

14. a) Look at the trinomial factor. Each term contains q , so the common factor of the given trinomial is $5q$. Remove q from each term in the trinomial factor. The correct solution is:
 $15p^2q + 25pq^2 - 35q^3 = 5q(3p^2 + 5pq - 7q^2)$

- b) The common factor, -3 , is correct, but it was not removed from the 2nd and 3rd terms of the given trinomial. Remove -3 from these terms. Also, when -3 was removed from the first term of the given trinomial, the sign should have been changed. The correct solution is:

$$-12mn + 15m^2 + 18n^2 = -3(4mn - 5m^2 - 6n^2)$$

3.4

15. a) Try to arrange 1 x^2 -tile, 8 x -tiles, and twelve 1-tiles in a rectangle. A rectangle is possible.



- b) Try to arrange 1 x^2 -tile, 7 x -tiles, and ten 1-tiles in a rectangle. A rectangle is possible.



- c) Try to arrange 1 x^2 -tile, 4 x -tiles, and one 1-tile in a rectangle. A rectangle is not possible.

- d) Try to arrange 1 x^2 -tile, 8 x -tiles, and fifteen 1-tiles in a rectangle. A rectangle is possible.



16. a) Try to arrange 2 k^2 -tiles, 3 k -tiles, and two 1-tiles in a rectangle. A rectangle is not possible.
- b) Try to arrange 3 g^2 -tiles, 4 g -tiles, and one 1-tile in a rectangle. A rectangle is possible.
- 
- c) Try to arrange 2 t^2 -tiles, 7 t -tiles, and six 1-tiles in a rectangle. A rectangle is possible.
- 
- d) Try to arrange 7 h^2 -tiles, 5 h -tiles, and one 1-tile in a rectangle. A rectangle is not possible.
17. Since the number of 1-tiles is a prime number, these tiles can only be arranged in a row of 5. Arrange the tiles this way, at the bottom right corner of the x^2 -tile. Complete a rectangle by adding x -tiles. Six x -tiles are needed.

3.5

18. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

$$\begin{aligned} \text{a) } (g + 5)(g - 4) &= g(g - 4) + 5(g - 4) \\ &= g^2 - 4g + 5g - 20 \\ &= g^2 + g - 20 \end{aligned}$$

g	$(g)(g) = g^2$	$(g)(-4) = -4g$
5	$(5)(g) = 5g$	$(5)(-4) = -20$

$$\begin{aligned} \text{b) } (h + 7)(h + 7) &= h(h + 7) + 7(h + 7) \\ &= h^2 + 7h + 7h + 49 \\ &= h^2 + 14h + 49 \end{aligned}$$

h	$(h)(h) = h^2$	$(h)(7) = 7h$
7	$(7)(h) = 7h$	$(7)(7) = 49$

$$\begin{aligned} \text{c) } (k - 4)(k + 11) &= k(k + 11) - 4(k + 11) \\ &= k^2 + 11k - 4k - 44 \end{aligned}$$

$$= k^2 + 7k - 44$$

	k	11
k	$(k)(k) = k^2$	$(k)(11) = 11k$
-4	$(-4)(k) = -4k$	$(-4)(11) = -44$

d) $(9 + s)(9 - s) = 9(9 - s) + s(9 - s)$
 $= 81 - 9s + 9s - s^2$
 $= 81 - s^2$

	9	$-s$
9	$(9)(9) = 81$	$(9)(-s) = -9s$
s	$(s)(9) = 9s$	$(s)(-s) = -s^2$

e) $(12 - t)(12 - t) = 12(12 - t) - t(12 - t)$
 $= 144 - 12t - 12t + t^2$
 $= 144 - 24t + t^2$

	12	$-t$
12	$(12)(12) = 144$	$(12)(-t) = -12t$
$-t$	$(-t)(12) = -12t$	$(-t)(-t) = t^2$

f) $(7 + r)(6 - r) = 7(6 - r) + r(6 - r)$
 $= 42 - 7r + 6r - r^2$
 $= 42 - r - r^2$

	6	$-r$
7	$(7)(6) = 42$	$(7)(-r) = -7r$
r	$(r)(6) = 6r$	$(r)(-r) = -r^2$

g) $(y - 3)(y - 11) = y(y - 11) - 3(y - 11)$
 $= y^2 - 11y - 3y + 33$
 $= y^2 - 14y + 33$

	y	-11
y	$(y)(y) = y^2$	$(y)(-11) = -11y$
-3	$(-3)(y) = -3y$	$(-3)(-11) = 33$

h) $(x - 5)(x + 5) = x(x + 5) - 5(x + 5)$
 $= x^2 + 5x - 5x - 25$

$$= x^2 - 25$$

	x	5
x	$(x)(x) = x^2$	$(x)(5) = 5x$
-5	$(-5)(x) = -5x$	$(-5)(5) = -25$

19. For each trinomial, use mental math to find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) $q^2 + 6q + 8$

Since all the terms are positive, consider only positive factors of 8.

The factors of 8 are: 1 and 8; 2 and 4

The two factors with a sum of 6 are 2 and 4.

So, $q^2 + 6q + 8 = (q + 2)(q + 4)$

Check by expanding.

$$\begin{aligned}(q + 2)(q + 4) &= q(q + 4) + 2(q + 4) \\ &= q^2 + 4q + 2q + 8 \\ &= q^2 + 6q + 8\end{aligned}$$

b) $n^2 - 4n - 45$

The factors of -45 are: 1 and -45 ; -1 and 45; 3 and -15 ; -3 and 15; 5 and -9 ; -5 and 9

The two factors with a sum of -4 are 5 and -9 .

So, $n^2 - 4n - 45 = (n + 5)(n - 9)$

Check by expanding.

$$\begin{aligned}(n + 5)(n - 9) &= n(n - 9) + 5(n - 9) \\ &= n^2 - 9n + 5n - 45 \\ &= n^2 - 4n - 45\end{aligned}$$

c) $54 - 15s + s^2$

Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of s is negative, both numbers are negative. So, list only the negative factors of 54.

The negative factors of 54 are: -1 and -54 ; -2 and -27 ; -3 and -18 ; -6 and -9

The two factors with a sum of -15 are -6 and -9 .

So, $54 - 15s + s^2 = (-6 + s)(-9 + s)$

Multiply each binomial by -1 ; which is the same as multiplying their product by 1, so its value is not changed.

So, the factors can be written as: $(6 - s)(9 - s)$

Check by expanding.

$$\begin{aligned}(6 - s)(9 - s) &= 6(9 - s) - s(9 - s) \\ &= 54 - 6s - 9s + s^2 \\ &= 54 - 15s + s^2\end{aligned}$$

d) $k^2 - 9k - 90$

The factors of -90 are: 1 and -90 ; -1 and 90; 2 and -45 ; -2 and 45; 3 and -30 ; -3 and 30; 5 and -18 ; -5 and 18; 6 and -15 ; -6 and 15; 9 and -10 ; -9 and 10

The two factors with a sum of -9 are 6 and -15 .

So, $k^2 - 9k - 90 = (k + 6)(k - 15)$

Check by expanding.

$$\begin{aligned}(k+6)(k-15) &= k(k-15) + 6(k-15) \\ &= k^2 - 15k + 6k - 90 \\ &= k^2 - 9k - 90\end{aligned}$$

e) $x^2 - x - 20$

The factors of -20 are: 1 and -20 ; -1 and 20 ; 2 and -10 ; -2 and 10 ; 4 and -5 ; -4 and 5

The two factors with a sum of -1 are 4 and -5 .

So, $x^2 - x - 20 = (x+4)(x-5)$

Check by expanding.

$$\begin{aligned}(x+4)(x-5) &= x(x-5) + 4(x-5) \\ &= x^2 - 5x + 4x - 20 \\ &= x^2 - x - 20\end{aligned}$$

f) $12 - 7y + y^2$

Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of y is negative, both numbers are negative. So, list only the negative factors of 12.

The negative factors of 12 are: -1 and -12 ; -2 and -6 ; -3 and -4

The two factors with a sum of -7 are -3 and -4 .

So, $12 - 7y + y^2 = (-3+y)(-4+y)$

Multiply each binomial by -1 ; which is the same as multiplying their product by 1, so its value is not changed.

So, the factors can be written as: $(3-y)(4-y)$

Check by expanding.

$$\begin{aligned}(3-y)(4-y) &= 3(4-y) - y(4-y) \\ &= 12 - 3y - 4y + y^2 \\ &= 12 - 7y + y^2\end{aligned}$$

20. For each trinomial, use mental math to find two numbers whose sum is equal to the coefficient of the middle term and whose product is equal to the constant term. These numbers are the constants in the binomial factors.

a) i) $m^2 + 7m + 12$

Since all the terms are positive, consider only positive factors of 12.

The factors of 12 are: 1 and 12; 2 and 6; 3 and 4

The two factors with a sum of 7 are 3 and 4.

So, $m^2 + 7m + 12 = (m+3)(m+4)$

a) ii) $m^2 + 8m + 12$

Use the factors of 12 from part i.

The two factors with a sum of 8 are 2 and 6.

So, $m^2 + 8m + 12 = (m+2)(m+6)$

a) iii) $m^2 + 13m + 12$

Use the factors of 12 from part i.

The two factors with a sum of 13 are 1 and 12.

So, $m^2 + 13m + 12 = (m+1)(m+12)$

a) iv) $m^2 - 7m + 12$

Since the constant term is positive, the two numbers in the binomials have the same sign. Since the coefficient of m is negative, both numbers are negative. So, list only the negative factors of 12.

The negative factors of 12 are: -1 and -12 ; -2 and -6 ; -3 and -4

The two factors with a sum of -7 are -3 and -4 .

$$\text{So, } m^2 - 7m + 12 = (m - 3)(m - 4)$$

a) v) $m^2 - 8m + 12$

Use the factors of 12 from part iv.

The two factors with a sum of -8 are -2 and -6 .

$$\text{So, } m^2 - 8m + 12 = (m - 2)(m - 6)$$

a) vi) $m^2 - 13m + 12$

Use the factors of 12 from part iv.

The two factors with a sum of -13 are -1 and -12 .

$$\text{So, } m^2 - 13m + 12 = (m - 1)(m - 12)$$

- b) There are no more trinomials that begin with m^2 , end in $+12$, and can be factored. In part a, there are only 6 pairs of factors of 12. All these pairs have already been used to factor the 6 trinomials in part a.

21. a) The constant terms in the binomials have a sum of 12 and a product of 27. Their sum should be -12 . So, these constant terms should be -3 and -9 . The correct solution is:
 $u^2 - 12u + 27 = (u - 3)(u - 9)$

- b) The constant terms in the binomials have a sum of 1 and a product of -20 . Their sum should be -1 . So, these constant terms should be 4 and -5 . The correct solution is:
 $v^2 - v - 20 = (v + 4)(v - 5)$

- c) The constant terms in the binomials have a sum of 10 and a product of 24. Their product should be -24 . So, the constant terms have opposite signs; they are factors of -24 with a sum of 10. The factors are -2 and 12. The correct solution is:
 $w^2 + 10w - 24 = (w - 2)(w + 12)$

3.6

22. a) $(h + 4)(2h + 2)$

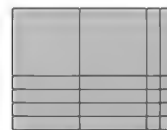
Use algebra tiles.

Make a rectangle with dimensions $h + 4$ and $2h + 2$.

Place tiles to represent each dimension, then fill the rectangle with tiles.

The tiles that form the product represent $2h^2 + 10h + 8$.

$$\text{So, } (h + 4)(2h + 2) = 2h^2 + 10h + 8$$



b) $(j + 5)(3j + 1)$

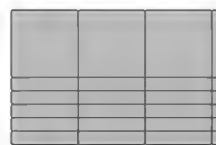
Use algebra tiles.

Make a rectangle with dimensions $j + 5$ and $3j + 1$.

Place tiles to represent each dimension, then fill the rectangle with tiles.

The tiles that form the product represent $3j^2 + 16j + 5$.

$$\text{So, } (j + 5)(3j + 1) = 3j^2 + 16j + 5$$



c) $(3k + 2)(2k + 1)$

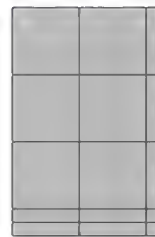
Use algebra tiles.

Make a rectangle with dimensions $3k + 2$ and $2k + 1$.

Place tiles to represent each dimension,
then fill the rectangle with tiles.

The tiles that form the product represent $6k^2 + 7k + 2$.

So, $(3k + 2)(2k + 1) = 6k^2 + 7k + 2$



d) $(4m + 1)(2m + 3)$

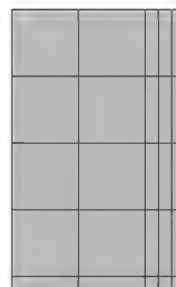
Use algebra tiles.

Make a rectangle with dimensions $4m + 1$ and $2m + 3$.

Place tiles to represent each dimension,
then fill the rectangle with tiles.

The tiles that form the product represent $8m^2 + 14m + 3$.

So, $(4m + 1)(2m + 3) = 8m^2 + 14m + 3$



23. a) i) There are $2x^2$ -tiles, $5x$ -tiles, and 3 unit tiles.

So, the algebra tiles represent the trinomial: $2x^2 + 5x + 3$

- ii) I arranged the tiles to form a rectangle:



- iii) I can use the rectangle to factor the trinomial.

The rectangle has side lengths $2x + 3$ and $x + 1$.

So, $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

- b) i) There are $3x^2$ -tiles, $10x$ -tiles, and 8 unit tiles.

So, the algebra tiles represent the trinomial: $3x^2 + 10x + 8$

- ii) I arranged the tiles to form a rectangle:



- iii) I can use the rectangle to factor the trinomial.

The rectangle has side lengths $3x + 4$ and $x + 2$.

So, $3x^2 + 10x + 8 = (3x + 4)(x + 2)$

24. Use the distributive property to expand. Draw a rectangle and label its sides with the binomial factors. Divide the rectangle into 4 smaller rectangles, and label each with a term in the expansion.

a) $(2r + 7)(3r + 5) = 2r(3r + 5) + 7(3r + 5)$

$$= 6r^2 + 10r + 21r + 35$$

$$= 6r^2 + 31r + 35$$

	$3r$	5
$2r$	$(2r)(3r) = 6r^2$	$(2r)(5) = 10r$
7	$(7)(3r) = 21r$	$(7)(5) = 35$

b) $(9y + 1)(y - 9) = 9y(y - 9) + 1(y - 9)$
 $= 9y^2 - 81y + y - 9$
 $= 9y^2 - 80y - 9$

	y	-9
$9y$	$(9y)(y) = 9y^2$	$(9y)(-9) = -81y$
1	$(1)(y) = y$	$(1)(-9) = -9$

c) $(2a - 7)(2a - 6) = 2a(2a - 6) - 7(2a - 6)$
 $= 4a^2 - 12a - 14a + 42$
 $= 4a^2 - 26a + 42$

	$2a$	-6
$2a$	$(2a)(2a) = 4a^2$	$(2a)(-6) = -12a$
-7	$(-7)(2a) = -14a$	$(-7)(-6) = 42$

d) $(3w - 2)(3w - 1) = 3w(3w - 1) - 2(3w - 1)$
 $= 9w^2 - 3w - 6w + 2$
 $= 9w^2 - 9w + 2$

	$3w$	-1
$3w$	$(3w)(3w) = 9w^2$	$(3w)(-1) = -3w$
-2	$(-2)(3w) = -6w$	$(-2)(-1) = 2$

e) $(4p + 5)(4p + 5) = 4p(4p + 5) + 5(4p + 5)$
 $= 16p^2 + 20p + 20p + 25$
 $= 16p^2 + 40p + 25$

	$4p$	5
$4p$	$(4p)(4p) = 16p^2$	$(4p)(5) = 20p$
5	$(5)(4p) = 20p$	$(5)(5) = 25$

$$\begin{aligned} \text{f) } (-y + 1)(-3y - 1) &= -y(-3y - 1) + 1(-3y - 1) \\ &= 3y^2 + y - 3y - 1 \\ &= 3y^2 - 2y - 1 \end{aligned}$$

	-3y	-1
-y	$(-y)(-3y) = 3y^2$	$(-y)(-1) = y$
1	$(1)(-3y) = -3y$	$(1)(-1) = -1$

25. Factor by decomposition. For each trinomial, write the middle term as the sum of two terms whose coefficients have a product equal to the product of the coefficient of the 1st term and the constant term. Remove common factors from the 1st two terms and from the last two terms, then remove a binomial as a common factor.

a) $4k^2 - 7k + 3$

The product is: $3(4) = 12$

Consider only the negative factors of 12 because the k -term is negative and the constant term is positive.

The negative factors of 12 are: -1 and -12 ; -2 and -6 ; -3 and -4

The factors with a sum of -7 are -3 and -4 .

$$\begin{aligned} \text{So, } 4k^2 - 7k + 3 &= 4k^2 - 3k - 4k + 3 \\ &= k(4k - 3) - 1(4k - 3) \\ &= (4k - 3)(k - 1) \end{aligned}$$

$$\begin{aligned} \text{Check: } (4k - 3)(k - 1) &= 4k(k - 1) - 3(k - 1) \\ &= 4k^2 - 4k - 3k + 3 \\ &= 4k^2 - 7k + 3 \end{aligned}$$

b) $6c^2 - 13c - 5$

The product is: $6(-5) = -30$

The factors of -30 are: 1 and -30 ; -1 and 30 ; 2 and -15 ; -2 and 15 ; 3 and -10 ; -3 and 10 ; 5 and -6 ; -5 and 6

The two factors with a sum of -13 are: 2 and -15

$$\begin{aligned} \text{So, } 6c^2 - 13c - 5 &= 6c^2 + 2c - 15c - 5 \\ &= 2c(3c + 1) - 5(3c + 1) \\ &= (3c + 1)(2c - 5) \end{aligned}$$

$$\begin{aligned} \text{Check: } (3c + 1)(2c - 5) &= 3c(2c - 5) + 1(2c - 5) \\ &= 6c^2 - 15c + 2c - 5 \\ &= 6c^2 - 13c - 5 \end{aligned}$$

c) $4b^2 - 5b - 6$

The product is: $4(-6) = -24$

The factors of -24 are: 1 and -24 ; -1 and 24 ; 2 and -12 ; -2 and 12 ; 3 and -8 ; -3 and 8 ; 4 and -6 ; -4 and 6

The two factors with a sum of -5 are: 3 and -8

$$\begin{aligned} \text{So, } 4b^2 - 5b - 6 &= 4b^2 + 3b - 8b - 6 \\ &= b(4b + 3) - 2(4b + 3) \\ &= (4b + 3)(b - 2) \end{aligned}$$

$$\begin{aligned} \text{Check: } (4b + 3)(b - 2) &= 4b(b - 2) + 3(b - 2) \\ &= 4b^2 - 8b + 3b - 6 \\ &= 4b^2 - 5b - 6 \end{aligned}$$

d) $6a^2 - 31a + 5$

The product is: $6(5) = 30$

Consider only the negative factors of 30 because the a -term is negative and the constant term is positive.

The negative factors of 30 are: -1 and -30 ; -2 and -15 ; -3 and -10 ; -5 and -6

The factors with a sum of -31 are -1 and -30 .

So, $6a^2 - 31a + 5 = 6a^2 - 1a - 30a + 5$

$$= a(6a - 1) - 5(6a - 1)$$

$$= (6a - 1)(a - 5)$$

Check: $(6a - 1)(a - 5) = 6a(a - 5) - 1(a - 5)$

$$= 6a^2 - 30a - 1a + 5$$

$$= 6a^2 - 31a + 5$$

e) $28x^2 + 9x - 4$

The product is: $28(-4) = -112$

The factors of -112 are: 1 and -112 ; -1 and 112 ; 2 and -56 ; 4 and -28 ; -4 and 28 ;

7 and -16 ; -7 and 16 ; 8 and -14 ; -8 and 14

The two factors with a sum of 9 are: -7 and 16

So, $28x^2 + 9x - 4 = 28x^2 - 7x + 16x - 4$

$$= 7x(4x - 1) + 4(4x - 1)$$

$$= (4x - 1)(7x + 4)$$

Check: $(4x - 1)(7x + 4) = 4x(7x + 4) - 1(7x + 4)$

$$= 28x^2 + 16x - 7x - 4$$

$$= 28x^2 + 9x - 4$$

f) $21x^2 + 8x - 4$

The product is: $21(-4) = -84$

The factors of -84 are: 1 and -84 ; -1 and 84 ; 2 and -42 ; -2 and 42 ; 3 and -28 ; -3 and 28 ;

4 and -21 ; -4 and 21 ; 6 and -14 ; -6 and 14 ; 7 and -12 ; -7 and 12

The two factors with a sum of 8 are: -6 and 14

So, $21x^2 + 8x - 4 = 21x^2 - 6x + 14x - 4$

$$= 3x(7x - 2) + 2(7x - 2)$$

$$= (7x - 2)(3x + 2)$$

Check: $(7x - 2)(3x + 2) = 7x(3x + 2) - 2(3x + 2)$

$$= 21x^2 + 14x - 6x - 4$$

$$= 21x^2 + 8x - 4$$

26. a) $6m^2 + 5m - 21 = (6m - 20)(m + 1)$

Consider how the mistake might have been made.

The product of the constant terms in the binomial factors is not correct.

These terms must have a product of -21 , so try -21 and 1 .

$$(6m - 21)(m + 1) = 6m(m + 1) - 21(m + 1)$$

$$= 6m^2 + 6m - 21m - 21$$

$$= 6m^2 - 15m - 21$$

This does not match the given trinomial.

Try transposing the constant terms in the binomial factors.

$$(6m - 1)(m + 21) = 6m(m + 21) - 1(m + 21)$$

$$= 6m^2 + 126m - 1m - 21$$

$$= 6m^2 + 125m - 21$$

This does not match the given trinomial.

Try transposing the signs in the binomial factors: $(6m + 1)(m - 21)$

The trinomial will be $6m^2 - 125m - 21$. This does not match the given trinomial.

So, factor the trinomial by decomposition.

The product is: $6(-21) = -126$

The factors of -126 are: 1 and -126 ; -1 and 126 ; 2 and -63 ; -2 and 63 ; 3 and -42 ; -3 and 42 ; 6 and -21 ; -6 and 21 ; 7 and -18 ; -7 and 18 ; 9 and -14 ; -9 and 14

The two factors with a sum of 5 are: -9 and 14

So, $6m^2 + 5m - 21 = 6m^2 - 9m + 14m - 21$

$$= 3m(2m - 3) + 7(2m - 3)$$

$$= (2m - 3)(3m + 7) \quad \leftarrow \text{These factors are correct.}$$

b) $12n^2 - 17n - 5 = (4n - 1)(3n + 5)$

The product of the constant terms is correct.

Try transposing the constant terms in the binomial factors.

$$(4n - 5)(3n + 1) = 4n(3n + 1) - 5(3n + 1)$$

$$= 12n^2 + 4n - 15n - 5$$

$$= 12n^2 - 11n - 5$$

This does not match the given trinomial.

Try transposing the signs in the binomial factors.

$$(4n + 1)(3n - 5) = 4n(3n - 5) + 1(3n - 5)$$

$$= 12n^2 - 20n + 3n - 5$$

$$= 12n^2 - 17n - 5$$

This matches the given trinomial, so the correct factors are: $(4n + 1)(3n - 5)$

Alternative solution:

Expand the given binomial factors.

$$(4n - 1)(3n + 5) = 4n(3n + 5) - 1(3n + 5)$$

$$= 12n^2 + 20n - 3n - 5$$

$$= 12n^2 + 17n - 5$$

Compare this trinomial with the given trinomial. The only difference is the sign of the middle term. So, transpose the signs in the binomial factors.

$$(4n + 1)(3n - 5) = 4n(3n - 5) + 1(3n - 5)$$

$$= 12n^2 - 20n + 3n - 5$$

$$= 12n^2 - 17n - 5$$

This matches the given trinomial, so the correct factors are: $(4n + 1)(3n - 5)$

c) $20p^2 - 9p - 20 = (4p + 4)(5p - 5)$

The product of the constant terms is correct.

Try transposing the constant terms in the binomial factors.

$$(4p + 5)(5p - 4) = 4p(5p - 4) + 5(5p - 4)$$

$$= 20p^2 - 16p + 25p - 20$$

$$= 20p^2 + 9p - 20$$

Compare this trinomial with the given trinomial. The only difference is the sign of the middle term. So, transpose the signs in the binomial factors.

$$(4p - 5)(5p + 4) = 4p(5p + 4) - 5(5p + 4)$$

$$= 20p^2 + 16p - 25p - 20$$

$$= 20p^2 - 9p - 20$$

This matches the given trinomial, so the correct factors are: $(4p - 5)(5p + 4)$

3.7

27. a) $(c + 1)(c^2 + 3c + 2)$

$$= c(c^2 + 3c + 2) + 1(c^2 + 3c + 2)$$

$$= c(c^2) + c(3c) + c(2) + c^2 + 3c + 2$$

$$= c^3 + 3c^2 + 2c + c^2 + 3c + 2 \quad \text{Collect like terms.}$$

$$= c^3 + 3c^2 + c^2 + 2c + 3c + 2 \quad \text{Combine like terms.}$$

$$= c^3 + 4c^2 + 5c + 2$$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$$(c + 1)(c^2 + 3c + 2) = c^3 + 4c^2 + 5c + 2$$

$$\text{Left side: } (c + 1)(c^2 + 3c + 2) = (2 + 1)(2^2 + 3(2) + 2)$$

$$= 3(4 + 6 + 2)$$

$$= 3(12)$$

$$= 36$$

$$\text{Right side: } c^3 + 4c^2 + 5c + 2 = 2^3 + 4(2)^2 + 5(2) + 2$$

$$= 8 + 16 + 10 + 2$$

$$= 36$$

Since the left side equals the right side, the product is likely correct.

b) $(5 - 4r)(6 + 3r - 2r^2)$

$$= 5(6 + 3r - 2r^2) - 4r(6 + 3r - 2r^2)$$

$$= 5(6) + 5(3r) + 5(-2r^2) - 4r(6) - 4r(3r) - 4r(-2r^2)$$

$$= 30 + 15r - 10r^2 - 24r - 12r^2 + 8r^3 \quad \text{Collect like terms.}$$

$$= 8r^3 - 10r^2 - 12r^2 + 15r - 24r + 30 \quad \text{Combine like terms.}$$

$$= 8r^3 - 22r^2 - 9r + 30$$

To check, substitute $r = 2$ into the trinomial product and its simplification.

$$(5 - 4r)(6 + 3r - 2r^2) = 8r^3 - 22r^2 - 9r + 30$$

$$\text{Left side: } (5 - 4r)(6 + 3r - 2r^2) = (5 - 4(2))(6 + 3(2) - 2(2)^2)$$

$$= -3(6 + 6 - 8)$$

$$= -3(4)$$

$$= -12$$

$$\text{Right side: } 8r^3 - 22r^2 - 9r + 30 = 8(2)^3 - 22(2)^2 - 9(2) + 30$$

$$= 64 - 88 - 18 + 30$$

$$= -12$$

Since the left side equals the right side, the product is likely correct.

c) $(-j^2 + 3j + 1)(2j + 11)$

$$= -j^2(2j + 11) + 3j(2j + 11) + 1(2j + 11)$$

$$= -j^2(2j) - j^2(11) + 3j(2j) + 3j(11) + 2j + 11$$

$$= -2j^3 - 11j^2 + 6j^2 + 33j + 2j + 11 \quad \text{Combine like terms.}$$

$$= -2j^3 - 5j^2 + 35j + 11$$

To check, substitute $j = 2$ into the trinomial product and its simplification.

$$(-j^2 + 3j + 1)(2j + 11) = -2j^3 - 5j^2 + 35j + 11$$

Left side: $(-j^2 + 3j + 1)(2j + 11) = (-(2)^2 + 3(2) + 1)(2(2) + 11)$

$$= (-4 + 6 + 1)(4 + 11)$$

$$= (3)(15)$$

$$= 45$$

Right side: $-2j^3 - 5j^2 + 35j + 11 = -2(2)^3 - 5(2)^2 + 35(2) + 11$

$$= -16 - 20 + 70 + 11$$

$$= 45$$

Since the left side equals the right side, the product is likely correct.

d) $(3x^2 + 7x + 2)(2x - 3)$

$$= 3x^2(2x - 3) + 7x(2x - 3) + 2(2x - 3)$$

$$= 3x^2(2x) + 3x^2(-3) + 7x(2x) + 7x(-3) + 2(2x) + 2(-3)$$

$$= 6x^3 - 9x^2 + 14x^2 - 21x + 4x - 6 \quad \text{Combine like terms.}$$

$$= 6x^3 + 5x^2 - 17x - 6$$

To check, substitute $x = 2$ into the trinomial product and its simplification.

$$(3x^2 + 7x + 2)(2x - 3) = 6x^3 + 5x^2 - 17x - 6$$

Left side: $(3x^2 + 7x + 2)(2x - 3) = (3(2)^2 + 7(2) + 2)(2(2) - 3)$

$$= (12 + 14 + 2)(1)$$

$$= 28$$

Right side: $6x^3 + 5x^2 - 17x - 6 = 6(2)^3 + 5(2)^2 - 17(2) - 6$

$$= 48 + 20 - 34 - 6$$

$$= 28$$

Since the left side equals the right side, the product is likely correct.

$$\begin{aligned}
 28. \text{ a) } & (4m - p)^2 \\
 & (4m - p)(4m - p) \\
 & = 4m(4m - p) - p(4m - p) \\
 & = 4m(4m) + 4m(-p) - p(4m) - p(-p) \\
 & = 16m^2 - 4mp - 4mp + p^2 && \text{Combine like terms.} \\
 & = 16m^2 - 8mp + p^2
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (3g - 4h)^2 \\
 & (3g - 4h)(3g - 4h) \\
 & = 3g(3g - 4h) - 4h(3g - 4h) \\
 & = 3g(3g) + 3g(-4h) - 4h(3g) - 4h(-4h) \\
 & = 9g^2 - 12gh - 12gh + 16h^2 && \text{Combine like terms.} \\
 & = 9g^2 - 24gh + 16h^2
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & (y - 2z)(y + z - 2) \\
 & = y(y + z - 2) - 2z(y + z - 2) \\
 & = y(y) + y(z) + y(-2) - 2z(y) - 2z(z) - 2z(-2) \\
 & = y^2 + yz - 2y - 2yz - 2z^2 + 4z && \text{Collect like terms.} \\
 & = y^2 + yz - 2yz - 2z^2 - 2y + 4z && \text{Combine like terms.} \\
 & = y^2 - yz - 2z^2 - 2y + 4z
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & (3c - 4d)(7 - 6c + 5d) \\
 & = 3c(7 - 6c + 5d) - 4d(7 - 6c + 5d) \\
 & = 3c(7) + 3c(-6c) + 3c(5d) - 4d(7) - 4d(-6c) - 4d(5d) \\
 & = 21c - 18c^2 + 15cd - 28d + 24cd - 20d^2 && \text{Collect like terms.} \\
 & = -18c^2 + 15cd + 24cd - 20d^2 + 21c - 28d && \text{Combine like terms.} \\
 & = -18c^2 + 39cd - 20d^2 + 21c - 28d
 \end{aligned}$$

$$\begin{aligned}
 29. \text{ a) } & (m^2 + 3m + 2)(2m^2 + m + 5) \\
 & = m^2(2m^2 + m + 5) + 3m(2m^2 + m + 5) + 2(2m^2 + m + 5) \\
 & = m^2(2m^2) + m^2(m) + m^2(5) + 3m(2m^2) + 3m(m) + 3m(5) + 2(2m^2) + 2(m) + 2(5) \\
 & = 2m^4 + m^3 + 5m^2 + 6m^3 + 3m^2 + 15m + 4m^2 + 2m + 10 && \text{Collect like terms.} \\
 & = 2m^4 + m^3 + 6m^3 + 5m^2 + 3m^2 + 4m^2 + 15m + 2m + 10 && \text{Combine like terms.} \\
 & = 2m^4 + 7m^3 + 12m^2 + 17m + 10
 \end{aligned}$$

To check, substitute $m = 2$ into the trinomial product and its simplification.

$$(m^2 + 3m + 2)(2m^2 + m + 5) = 2m^4 + 7m^3 + 12m^2 + 17m + 10$$

$$\begin{aligned}\text{Left side: } (m^2 + 3m + 2)(2m^2 + m + 5) &= [(2)^2 + 3(2) + 2][2(2)^2 + 2 + 5] \\ &= (4 + 6 + 2)(8 + 7) \\ &= (12)(15) \\ &= 180\end{aligned}$$

$$\begin{aligned}\text{Right side: } 2m^4 + 7m^3 + 12m^2 + 17m + 10 &= 2(2)^4 + 7(2)^3 + 12(2)^2 + 17(2) + 10 \\ &= 32 + 56 + 48 + 34 + 10 \\ &= 180\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

$$\begin{aligned}\text{b) } (1 - 3x + 2x^2)(5 + 4x - x^2) &= 1(5 + 4x - x^2) - 3x(5 + 4x - x^2) + 2x^2(5 + 4x - x^2) \\ &= 5 + 4x - x^2 - 3x(5) - 3x(4x) - 3x(-x^2) + 2x^2(5) + 2x^2(4x) + 2x^2(-x^2) \\ &= 5 + 4x - x^2 - 15x - 12x^2 + 3x^3 + 10x^2 + 8x^3 - 2x^4 \quad \text{Collect like terms.} \\ &= 5 + 4x - 15x - x^2 - 12x^2 + 10x^2 + 3x^3 + 8x^3 - 2x^4 \quad \text{Combine like terms.} \\ &= 5 - 11x - 3x^2 + 11x^3 - 2x^4\end{aligned}$$

To check, substitute $x = 2$ into the trinomial product and its simplification.

$$(1 - 3x + 2x^2)(5 + 4x - x^2) = 5 - 11x - 3x^2 + 11x^3 - 2x^4$$

$$\begin{aligned}\text{Left side: } (1 - 3x + 2x^2)(5 + 4x - x^2) &= [1 - 3(2) + 2(2)^2][5 + 4(2) - (2)^2] \\ &= (1 - 6 + 8)(5 + 8 - 4) \\ &= (3)(9) \\ &= 27\end{aligned}$$

Right side:

$$\begin{aligned}5 - 11x - 3x^2 + 11x^3 - 2x^4 &= 5 - 11(2) - 3(2)^2 + 11(2)^3 - 2(2)^4 \\ &= 5 - 22 - 12 + 88 - 32 \\ &= 27\end{aligned}$$

Since the left side equals the right side, the product is likely correct.

c)

$$\begin{aligned}
 & (-2k^2 + 7k + 6)(3k^2 - 2k - 3) \\
 &= -2k^2(3k^2 - 2k - 3) + 7k(3k^2 - 2k - 3) + 6(3k^2 - 2k - 3) \\
 &= -2k^2(3k^2) - 2k^2(-2k) - 2k^2(-3) + 7k(3k^2) + 7k(-2k) + 7k(-3) + 6(3k^2) + 6(-2k) + 6(-3) \\
 &= -6k^4 + 4k^3 + 6k^2 + 21k^3 - 14k^2 - 21k + 18k^2 - 12k - 18 && \text{Collect like terms.} \\
 &= -6k^4 + 4k^3 + 21k^3 + 6k^2 - 14k^2 + 18k^2 - 21k - 12k - 18 && \text{Combine like terms.} \\
 &= -6k^4 + 25k^3 + 10k^2 - 33k - 18
 \end{aligned}$$

To check, substitute $c = 2$ into the trinomial product and its simplification.

$$(-2k^2 + 7k + 6)(3k^2 - 2k - 3) = -6k^4 + 25k^3 + 10k^2 - 33k - 18$$

$$\begin{aligned}
 \text{Left side: } (-2k^2 + 7k + 6)(3k^2 - 2k - 3) &= [-2(2)^2 + 7(2) + 6][3(2)^2 - 2(2) - 3] \\
 &= (-8 + 14 + 6)(12 - 4 - 3) \\
 &= (12)(5) \\
 &= 60
 \end{aligned}$$

$$\begin{aligned}
 \text{Right side: } -6k^4 + 25k^3 + 10k^2 - 33k - 18 &= -6(2)^4 + 25(2)^3 + 10(2)^2 - 33(2) - 18 \\
 &= -96 + 200 + 40 - 66 - 18 \\
 &= 60
 \end{aligned}$$

Since the left side equals the right side, the product is likely correct.

d) $(-3 - 5h + 2h^2)(-1 + h + h^2)$

$$\begin{aligned}
 &= -3(-1 + h + h^2) - 5h(-1 + h + h^2) + 2h^2(-1 + h + h^2) \\
 &= 3 - 3h - 3h^2 + 5h - 5h^2 - 5h^3 - 2h^2 + 2h^3 + 2h^4 && \text{Collect like terms.} \\
 &= 3 - 3h + 5h - 3h^2 - 5h^2 - 2h^2 - 5h^3 + 2h^3 + 2h^4 && \text{Combine like terms.} \\
 &= 3 + 2h - 10h^2 - 3h^3 + 2h^4
 \end{aligned}$$

To check, substitute $h = 2$ into the trinomial product and its simplification.

$$(-3 - 5h + 2h^2)(-1 + h + h^2) = 3 + 2h - 10h^2 - 3h^3 + 2h^4$$

$$\begin{aligned}
 \text{Left side: } (-3 - 5h + 2h^2)(-1 + h + h^2) &= [-3 - 5(2) + 2(2)^2][-1 + 2 + (2)^2] \\
 &= (-3 - 10 + 8)(1 + 4) \\
 &= (-5)(5) \\
 &= -25
 \end{aligned}$$

Right side:

$$\begin{aligned}
 3 + 2h - 10h^2 - 3h^3 + 2h^4 &= 3 + 2(2) - 10(2)^2 - 3(2)^3 + 2(2)^4 \\
 &= 3 + 4 - 40 - 24 + 32 \\
 &= -25
 \end{aligned}$$

Since the left side equals the right side, the product is likely correct.

$$\begin{aligned}
 30. \text{ a) } & (5a + 1)(4a + 2) + (a - 5)(2a - 1) \\
 &= 5a(4a + 2) + 1(4a + 2) + a(2a - 1) - 5(2a - 1) \\
 &= 5a(4a) + 5a(2) + 4a + 2 + a(2a) + a(-1) - 5(2a) - 5(-1) \\
 &= 20a^2 + 10a + 4a + 2 + 2a^2 - a - 10a + 5 && \text{Collect like terms.} \\
 &= 20a^2 + 2a^2 + 10a + 4a - a - 10a + 2 + 5 && \text{Combine like terms.} \\
 &= 22a^2 + 3a + 7
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (6c - 2)(4c + 2) - (c + 7)^2 \\
 &= (6c - 2)(4c + 2) - (c + 7)(c + 7) \\
 &= 6c(4c + 2) - 2(4c + 2) - c(c + 7) - 7(c + 7) \\
 &= 6c(4c) + 6c(2) - 2(4c) - 2(2) - c(c) - c(7) - 7(c) - 7(7) \\
 &= 24c^2 + 12c - 8c - 4 - c^2 - 7c - 7c - 49 && \text{Collect like terms.} \\
 &= 24c^2 - c^2 + 12c - 8c - 7c - 7c - 4 - 49 && \text{Combine like terms.} \\
 &= 23c^2 - 10c - 53
 \end{aligned}$$

31. a) Suppose n represents an even integer.
Then, the next two consecutive even integers are: $n + 2$, $n + 4$

b) An expression for the product of the 3 integers is:

$$n(n + 2)(n + 4)$$

Simplify.

$$\begin{aligned}
 n(n + 2)(n + 4) &= n[n(n + 4) + 2(n + 4)] \\
 &= n[n(n) + n(4) + 2(n) + 2(4)] \\
 &= n(n^2 + 4n + 2n + 8) \\
 &= n(n^2 + 6n + 8) \\
 &= n^3 + 6n^2 + 8n
 \end{aligned}$$

3.8

$$32. \text{ a) } 81 - 4b^2$$

Write each term as a perfect square.

$$\begin{aligned}
 81 - 4b^2 &= (9)^2 - (2b)^2 && \text{Write these terms in binomial factors.} \\
 &= (9 + 2b)(9 - 2b)
 \end{aligned}$$

$$\text{b) } 16v^2 - 49$$

Write each term as a perfect square.

$$\begin{aligned}
 16v^2 - 49 &= (4v)^2 - (7)^2 && \text{Write these terms in binomial factors.} \\
 &= (4v + 7)(4v - 7)
 \end{aligned}$$

c) $64g^2 - 16h^2$

As written, each term of the binomial has a common factor 16. Remove this common factor.

$$64g^2 - 16h^2 = 16(4g^2 - h^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= 16[(2g)^2 - (h)^2] \quad \text{Write these terms in binomial factors.}$$

$$= 16(2g + h)(2g - h)$$

d) $18m^2 - 2n^2$

As written, each term of the binomial has a common factor 2. Remove this common factor.

$$18m^2 - 2n^2 = 2(9m^2 - n^2) \quad \text{Write each term in the binomial as a perfect square.}$$

$$= 2[(3m)^2 - (n)^2] \quad \text{Write these terms in binomial factors.}$$

$$= 2(3m + n)(3m - n)$$

33. a) $m^2 - 14m + 49$

The first term is a perfect square since $m^2 = (m)(m)$.

The third term is a perfect square since $49 = (7)(7)$.

The second term is twice the product of m and 7:

$$14m = 2(m)(7)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(m - 7)(m - 7), \text{ or } (m - 7)^2$$

To verify, multiply:

$$(m - 7)(m - 7) = m(m - 7) - 7(m - 7)$$

$$= m^2 - 7m - 7m + 49$$

$$= m^2 - 14m + 49$$

Since the trinomial is the same as the original trinomial, the factors are correct.

b) $n^2 + 10n + 25$

The first term is a perfect square since $n^2 = (n)(n)$.

The third term is a perfect square since $25 = (5)(5)$.

The second term is twice the product of n and 5:

$$10n = 2(n)(5)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(n + 5)(n + 5), \text{ or } (n + 5)^2$$

To verify, multiply:

$$\begin{aligned}(n+5)(n+5) &= n(n+5) + 5(n+5) \\ &= n^2 + 5n + 5n + 25 \\ &= n^2 + 10n + 25\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

c) $4p^2 + 12p + 9$

The first term is a perfect square since $4p^2 = (2p)(2p)$.

The third term is a perfect square since $9 = (3)(3)$.

The second term is twice the product of $2p$ and 3 :

$$12p = 2(2p)(3)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(2p+3)(2p+3), \text{ or } (2p+3)^2$$

To verify, multiply:

$$\begin{aligned}(2p+3)(2p+3) &= 2p(2p+3) + 3(2p+3) \\ &= 4p^2 + 6p + 6p + 9 \\ &= 4p^2 + 12p + 9\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

d) $16 - 40q + 25q^2$

The first term is a perfect square since $16 = (4)(4)$.

The third term is a perfect square since $25q^2 = (5q)(5q)$.

The second term is twice the product of 4 and $5q$:

$$40q = 2(4)(5q)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(4-5q)(4-5q), \text{ or } (4-5q)^2$$

To verify, multiply:

$$\begin{aligned}(4-5q)(4-5q) &= 4(4-5q) - 5q(4-5q) \\ &= 16 - 20q - 20q + 25q^2 \\ &= 16 - 40q + 25q^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

e) $4r^2 + 28r + 49$

The first term is a perfect square since $4r^2 = (2r)(2r)$.

The third term is a perfect square since $49 = (7)(7)$.

The second term is twice the product of $2r$ and 7 :

$$28r = 2(2r)(7)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(2r + 7)(2r + 7), \text{ or } (2r + 7)^2$$

To verify, multiply:

$$\begin{aligned}(2r + 7)(2r + 7) &= 2r(2r + 7) + 7(2r + 7) \\ &= 4r^2 + 14r + 14r + 49 \\ &= 4r^2 + 28r + 49\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

f) $36 - 132s + 121s^2$

The first term is a perfect square since $36 = (6)(6)$.

The third term is a perfect square since $121s^2 = (11s)(11s)$.

The second term is twice the product of 6 and $11s$:

$$132s = 2(6)(11s)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(6 - 11s)(6 - 11s), \text{ or } (6 - 11s)^2$$

To verify, multiply:

$$\begin{aligned}(6 - 11s)(6 - 11s) &= 6(6 - 11s) - 11s(6 - 11s) \\ &= 36 - 66s - 66s + 121s^2 \\ &= 36 - 132s + 121s^2\end{aligned}$$

Since the trinomial is the same as the original trinomial, the factors are correct.

34. a) $g^2 + 6gh + 9h^2$

The first term is a perfect square since $g^2 = (g)(g)$.

The third term is a perfect square since $9h^2 = (3h)(3h)$.

The second term is twice the product of g and $3h$:

$$6gh = 2(g)(3h)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(g + 3h)(g + 3h), \text{ or } (g + 3h)^2$$

b) $16j^2 - 24jk + 9k^2$

The first term is a perfect square since $16j^2 = (4j)(4j)$.

The third term is a perfect square since $9k^2 = (3k)(3k)$.

The second term is twice the product of $4j$ and $3k$:

$$24jk = 2(4j)(3k)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(4j - 3k)(4j - 3k), \text{ or } (4j - 3k)^2$$

c) $25t^2 + 20tu + 4u^2$

The first term is a perfect square since $25t^2 = (5t)(5t)$.

The third term is a perfect square since $4u^2 = (2u)(2u)$.

The second term is twice the product of $5t$ and $2u$:

$$20tu = 2(5t)(2u)$$

Since the second term is positive, the operations in the binomial factors must be addition.

So, the trinomial is a perfect square trinomial and its factors are:

$$(5t + 2u)(5t + 2u), \text{ or } (5t + 2u)^2$$

d) $9v^2 - 48vw + 64w^2$

The first term is a perfect square since $9v^2 = (3v)(3v)$.

The third term is a perfect square since $64w^2 = (8w)(8w)$.

The second term is twice the product of $3v$ and $8w$:

$$48vw = 2(3v)(8w)$$

Since the second term is negative, the operations in the binomial factors must be subtraction.

So, the trinomial is a perfect square trinomial and its factors are:

$$(3v - 8w)(3v - 8w), \text{ or } (3v - 8w)^2$$

35. The area of the shaded region is:

area of large square – area of small square

The formula for the area of a square is: $A = s^2$, where s is the side length of the square.

Area of large square:

Use the formula $A = s^2$. Substitute: $s = 2x + 5$

$$A_L = (2x + 5)^2$$

Area of small square:

Use the formula $A = s^2$. Substitute: $s = x + 3$

$$A_s = (x + 3)^2$$

So, area of the shaded region is:

$$A = A_L - A_s$$

$$= (2x + 5)^2 - (x + 3)^2$$

$$= (2x + 5)(2x + 5) - (x + 3)(x + 3)$$

$$= 2x(2x + 5) + 5(2x + 5) - [x(x + 3) + 3(x + 3)]$$

$$= 4x^2 + 10x + 10x + 25 - (x^2 + 3x + 3x + 9)$$

$$= 4x^2 + 20x + 25 - x^2 - 6x - 9$$

$$= 3x^2 + 14x + 16$$

A polynomial that represents the area of the shaded region is:

$$3x^2 + 14x + 16$$

Practice Test

(page 201)

1. A. This is not true because 64 has these factors: 1, 2, 4, 8, 16, 32, and 64
So, A is the correct answer.

2. Expand each product until the trinomial matches the given trinomial.

A. $(2x + 1)(x + 6) = 2x^2 + 12x + 1x + 6$
 $= 2x^2 + 13x + 6$

B. $(2x + 2)(x + 3) = 2x^2 + 6x + 2x + 6$
 $= 2x^2 + 8x + 6$

C. $(2x + 3)(x + 2) = 2x^2 + 4x + 3x + 6$
 $= 2x^2 + 7x + 6$

This trinomial matches the given trinomial, so C is the correct answer.

3. $20 = 2 \cdot 2 \cdot 5$
 $45 = 3 \cdot 3 \cdot 5$
 $50 = 2 \cdot 5 \cdot 5$

The least common multiple is the product of the greatest prime factor in each set of factors:

$$2^2 \cdot 3^2 \cdot 5^2 = 900$$

The least common multiple is 900.

The greatest common factor is the factor that occurs in every set of factors: 5

The greatest common factor is 5.

4. a) i) For a perfect square; its prime factors can be arranged into 2 equal groups.
Use the factors from question 3.

$$20 = 2 \cdot 2 \cdot 5$$

Multiply 20 by 5 to get 2 equal groups of factors: $2 \cdot 2 \cdot 5 \cdot 5$

$$20 \cdot 5 = 100$$

100 is a perfect square.

To get more perfect squares, multiply 20 by 5 times any perfect square, such as $20(5)(4)$ to get 400; and $20(5)(9)$ to get 900.

$$45 = 3 \cdot 3 \cdot 5$$

Multiply 45 by 5 to get 2 equal groups of factors: $3 \cdot 3 \cdot 5 \cdot 5$

$$45 \cdot 5 = 225$$

225 is a perfect square.

To get more perfect squares, multiply 45 by 5 times any perfect square, such as $45(5)(4)$ to get 900; and $45(5)(9)$ to get 2025.

$$50 = 2 \cdot 5 \cdot 5$$

Multiply 50 by 2 to get 2 equal groups of factors: $2 \cdot 2 \cdot 5 \cdot 5$

$$50 \cdot 2 = 100$$

100 is a perfect square.

To get more perfect squares, multiply 50 by 2 times any perfect square, such as $50(2)(4)$ to get 400; and $50(2)(9)$ to get 900.

- ii) For a perfect cube, its prime factors can be arranged in 3 equal groups.
Use the factors from question 3.

$$20 = 2 \cdot 2 \cdot 5$$

Multiply 20 by $2 \cdot 5 \cdot 5 = 50$ to get 3 equal groups of factors: $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

$$20 \cdot 50 = 1000$$

1000 is a perfect cube.

To get more perfect cubes, multiply 20 by 50 times any perfect cube, such as $20(50)(8)$ to get 8000; and $20(50)(27)$ to get 27 000.

$$45 = 3 \cdot 3 \cdot 5$$

Multiply 45 by $3 \cdot 5 \cdot 5 = 75$ to get 3 equal groups of factors: $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$

$$45 \cdot 75 = 3375$$

3375 is a perfect cube.

To get more perfect cubes, multiply 45 by 75 times any perfect cube, such as $45(75)(8)$ to get 27 000; and $45(75)(27)$ to get 91 125.

$$50 = 2 \cdot 5 \cdot 5$$

Multiply 50 by $2 \cdot 2 \cdot 5 = 20$ to get 3 equal groups of factors: $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

$$50 \cdot 20 = 1000$$

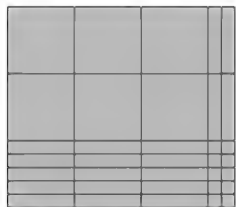
1000 is a perfect cube.

To get more perfect cubes, multiply 50 by 20 times any perfect cube, such as $50(20)(8)$ to get 8000; and $50(20)(27)$ to get 27 000.

- b) There is more than one answer in each of part a because a perfect square can be generated by multiplying any two perfect squares; and a perfect cube can be generated by multiplying any 2 perfect cubes.

5. a) $(2c + 5)(3c + 2)$

Use algebra tiles to make a rectangle with width $2c + 5$ and length $3c + 2$.

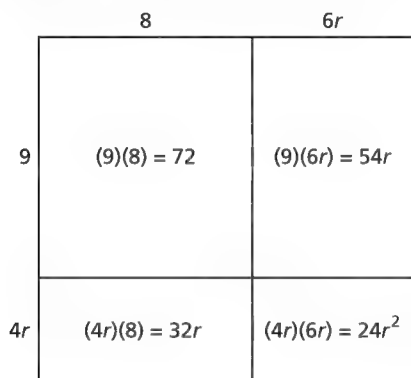


The tiles that form the product are: 6 c^2 -tiles, 19 c -tiles, and ten 1-tiles

$$\text{So, } (2c + 5)(3c + 2) = 6c^2 + 19c + 10$$

b) $(9 + 4r)(8 + 6r)$

Sketch a rectangle with dimensions $8 + 6r$ and $9 + 4r$. Divide it into 4 smaller rectangles and calculate the area of each.

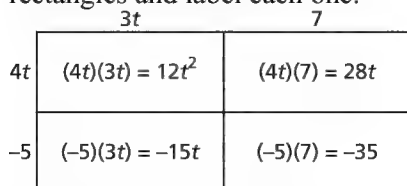


From the diagram:

$$\begin{aligned}(9 + 4r)(8 + 6r) &= 72 + 54r + 32r + 24r^2 \\ &= 72 + 86r + 24r^2\end{aligned}$$

c) $(4t - 5)(3t + 7)$

Sketch a rectangle. Label its dimensions $4t - 5$ and $3t + 7$. Divide it into 4 smaller rectangles and label each one.



From the diagram:

$$\begin{aligned}(4t - 5)(3t + 7) &= 12t^2 + 28t - 15t - 35 \\ &= 12t^2 + 13t - 35\end{aligned}$$

6. Use the distributive property.

a) $(2p - 1)(p^2 + 2p - 7) = 2p(p^2 + 2p - 7) - 1(p^2 + 2p - 7)$
 $= 2p^3 + 4p^2 - 14p - p^2 - 2p + 7$
 $= 2p^3 + 4p^2 - p^2 - 14p - 2p + 7$
 $= 2p^3 + 3p^2 - 16p + 7$

b) $(e + 2f)(2f^2 + 5f + 3e^2) = e(2f^2 + 5f + 3e^2) + 2f(2f^2 + 5f + 3e^2)$
 $= 2ef^2 + 5ef + 3e^3 + 4f^3 + 10f^2 + 6e^2f$

c) $(3y + 2z)(y + 4z) - (5y - 3z)(2y - 8z)$
 $= 3y(y + 4z) + 2z(y + 4z) - [5y(2y - 8z) - 3z(2y - 8z)]$
 $= 3y^2 + 12yz + 2yz + 8z^2 - [10y^2 - 40yz - 6yz + 24z^2]$
 $= 3y^2 + 12yz + 2yz + 8z^2 - 10y^2 + 40yz + 6yz - 24z^2$
 $= 3y^2 - 10y^2 + 12yz + 2yz + 40yz + 6yz - 24z^2 + 8z^2$
 $= -7y^2 + 60yz - 16z^2$

7. a) $f^2 + 17f + 16$

Find two numbers whose sum is 17 and whose product is 16.
The numbers are 1 and 16.

$$\text{So, } f^2 + 17f + 16 = (f + 1)(f + 16)$$

I could use these algebra tiles: 1 f^2 -tile, 17 f -tiles, and sixteen 1-tiles to make a rectangle with length $f + 16$ and width $f + 1$.

b) $c^2 - 13c + 22$

Find two numbers whose sum is -13 and whose product is 22 .

Since the constant term is positive and the c -term is negative, the numbers are negative.

The numbers are -2 and -11 .

$$\text{So, } c^2 - 13c + 22 = (c - 2)(c - 11)$$

I could use these algebra tiles: 1 c^2 -tile, 13 negative c -tiles, and twenty-two 1-tiles to make a rectangle with length $c - 11$ and width $c - 2$.

c) $4t^2 + 9t - 28$

Use decomposition.

$$\text{Multiply: } 4(-28) = -112$$

Find factors of -112 that have a sum of 9 .

Factors of -112 are: 1 and -112 ; -1 and 112 ; 2 and -56 ; -2 and 56 ; 4 and -28 ; -4 and 28 ; 7 and -16 ; -7 and 16

The factors of -112 that have a sum of 9 are -7 and 16 .

$$\begin{aligned}\text{So, } 4t^2 + 9t - 28 &= 4t^2 - 7t + 16t - 28 \\ &= t(4t - 7) + 4(4t - 7) \\ &= (4t - 7)(t + 4)\end{aligned}$$

I would not use algebra tiles to factor. I would need to use guess and check to find a combination of positive and negative t -tiles to form a rectangle with 4 t^2 -tiles, and 28 negative 1-tiles.

d) $4r^2 + 20rs + 25s^2$

This is a perfect square trinomial because:

the 1st term is a perfect square: $4r^2 = (2r)(2r)$

the 3rd term is a perfect square: $25s^2 = (5s)(5s)$

and the 2nd term is: $20rs = 2(2r)(5s)$

$$\text{So, } 4r^2 + 20rs + 25s^2 = (2r + 5s)(2r + 5s), \text{ or } (2r + 5s)^2$$

I could not use algebra tiles to factor the given trinomial because I do not have tiles for more than one variable. I could use tiles to factor $4r^2 + 20r + 25$, then include the variable s when I write the factors.

e) $6x^2 - 17xy + 5y^2$

Use decomposition.

$$\text{Multiply: } 6(5) = 30$$

Find factors of 30 that have a sum of -17 .

Since the coefficient of y^2 is positive and the xy -term is negative, the numbers are negative.

List negative factors of 30 : -1 and -30 ; -2 and -15

The factors of 30 that have a sum of -17 are -2 and -15 .

$$\begin{aligned}\text{So, } 6x^2 - 17xy + 5y^2 &= 6x^2 - 2xy - 15xy + 5y^2 \\ &= 2x(3x - y) - 5y(3x - y) \\ &= (3x - y)(2x - 5y)\end{aligned}$$

I could not use algebra tiles to factor the trinomial because I do not have tiles for more than one variable.

f) $h^2 - 25j^2$

This is a difference of squares.

$$h^2 = (h)(h)$$

$$25j^2 = (5j)(5j)$$

$$\text{So, } h^2 - 25j^2 = (h + 5j)(h - 5j)$$

I could not use algebra tiles to factor the binomial because I do not have tiles for more than one variable.

8. The remaining volume is the difference between the volume of the cube and the volume of the prism.

The volume of the cube is: $(2r + 1)^3$

The volume of the prism is: $r(r)(2r + 1)$

The remaining volume is:

$$\begin{aligned} (2r + 1)^3 - r(r)(2r + 1) &= (2r + 1)(2r + 1)^2 - [r^2(2r + 1)] \\ &= (2r + 1)(4r^2 + 4r + 1) - [2r^3 + r^2] \\ &= 2r(4r^2 + 4r + 1) + 1(4r^2 + 4r + 1) - 2r^3 - r^2 \\ &= 8r^3 + 8r^2 + 2r + 4r^2 + 4r + 1 - 2r^3 - r^2 \\ &= 8r^3 - 2r^3 + 8r^2 + 4r^2 - r^2 + 2r + 4r + 1 \\ &= 6r^3 + 11r^2 + 6r + 1 \end{aligned}$$

The volume that remains is: $6r^3 + 11r^2 + 6r + 1$

9. All the trinomials that begin with $8t^2$ and end with $+3$ have t -terms with coefficients that are the sum of the factors of $8(3) = 24$.

The factors of 24 are: 1 and 24; -1 and -24; 2 and 12; -2 and -12; 3 and 8; -3 and -8; 4 and 6; -4 and -6

The sums of the factors are: 25, -25; 14, -14; 11; -11; 10; -10

So, the possible trinomials are:

$$8t^2 + 25t + 3; 8t^2 - 25t + 3; 8t^2 + 14t + 3; 8t^2 - 14t + 3; 8t^2 + 11t + 3; 8t^2 - 11t + 3; 8t^2 + 10t + 3; 8t^2 - 10t + 3$$

I have found all the trinomials because there are no other pairs of factors of 24.

Lesson 4.1

Math Lab:
Estimating Roots

Assess Your Understanding (page 206)

1. a) Answers may vary; for example, $\sqrt{34}$, $\sqrt[3]{21}$, $\sqrt[4]{45}$, $\sqrt[5]{12}$
 - b) For $\sqrt{34}$, the radicand is 34 and the index is 2.
For $\sqrt[3]{21}$, the radicand is 21 and the index is 3.
For $\sqrt[4]{45}$, the radicand is 45 and the index is 4.
For $\sqrt[5]{12}$, the radicand is 12 and the index is 5.
 - c) When the index is 2, I take the square root of the number.
When the index is 3, I take the cube root of the number.
When the index is 4, I take the fourth root of the number.
When the index is 5, I take the fifth root of the number.
2. a) $\sqrt{36} = \sqrt{6 \cdot 6}$
 $= 6$
 - b) $\sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$
 $= 2$
 - c) $\sqrt[4]{10\,000} = \sqrt[4]{10 \cdot 10 \cdot 10 \cdot 10}$
 $= 10$
 - d) $\sqrt[5]{-32} = \sqrt[5]{(-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)}$
 $= -2$
 - e) $\sqrt[3]{\frac{27}{125}} = \sqrt[3]{\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}}$
 $= \frac{3}{5}$
 - f) $\sqrt{2.25} = \sqrt{1.5 \cdot 1.5}$
 $= 1.5$
 - g) $\sqrt[3]{0.125} = \sqrt[3]{0.5 \cdot 0.5 \cdot 0.5}$
 $= 0.5$
 - h) $\sqrt[4]{625} = \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5}$
 $= 5$

3. Use benchmarks with guess and check.
- a) 8 is between the perfect squares 4 and 9, but closer to 9.
 So, $\sqrt{8}$ is between 2 and 3, but closer to 3.
 Estimate to 1 decimal place: $\sqrt{8} \doteq 2.9$
 Square the estimate: $2.9^2 = 8.41$ (too large, but close)
 Revise the estimate: $\sqrt{8} \doteq 2.8$
 Square the estimate: $2.8^2 = 7.84$ (very close)
 7.84 is closer to 8, so $\sqrt{8}$ is approximately 2.8.
- b) 9 is between the perfect cubes 8 and 27, but closer to 8.
 So, $\sqrt[3]{9}$ is between 2 and 3, but closer to 2.
 Estimate to 1 decimal place: $\sqrt[3]{9} \doteq 2.2$
 Cube the estimate: $2.2^3 = 10.648$ (too large)
 Revise the estimate: $\sqrt[3]{9} \doteq 2.1$
 Cube the estimate: $2.1^3 = 9.261$ (very close)
 9.261 is closer to 9, so $\sqrt[3]{9}$ is approximately 2.1.
- c) 10 is between the perfect fourth powers 1 and 16, but closer to 16.
 So, $\sqrt[4]{10}$ is between 1 and 2, but closer to 2.
 Estimate to 1 decimal place: $\sqrt[4]{10} \doteq 1.7$
 Raise the estimate to the fourth power: $1.7^4 = 8.3521$ (too small)
 Revise the estimate: $\sqrt[4]{10} \doteq 1.8$
 Raise the estimate to the fourth power: $1.8^4 = 10.4976$ (very close)
 10.4976 is closer to 10, so $\sqrt[4]{10}$ is approximately 1.8.
- d) 13 is between the perfect squares 9 and 16, but closer to 16.
 So, $\sqrt{13}$ is between 3 and 4, but closer to 4.
 Estimate to 1 decimal place: $\sqrt{13} \doteq 3.6$
 Square the estimate: $3.6^2 = 12.96$ (very close)
 So, $\sqrt{13}$ is approximately 3.6.
- e) 15 is between the perfect cubes 8 and 27, but closer to 8.
 So, $\sqrt[3]{15}$ is between 2 and 3, but closer to 2.
 Estimate to 1 decimal place: $\sqrt[3]{15} \doteq 2.4$
 Cube the estimate: $2.4^3 = 13.824$ (too small)
 Revise the estimate: $\sqrt[3]{15} \doteq 2.5$
 Cube the estimate: $2.5^3 = 15.625$ (close)
 15.625 is closer to 15, so $\sqrt[3]{15}$ is approximately 2.5.

- f)** 17 is between the perfect fourth powers 16 and 81, but closer to 16.
 So, $\sqrt[4]{17}$ is between 2 and 3, but closer to 2.
 Estimate to 1 decimal place: $\sqrt[4]{17} \doteq 2.1$
 Raise the estimate to the fourth power: $2.1^4 = 19.4481$ (too large)
 Revise the estimate: $\sqrt[4]{17} \doteq 2.0$
 Raise the estimate to the fourth power: $2.0^4 = 16.0$ (close)
 16 is closer to 17, so $\sqrt[4]{17}$ is approximately 2.0.
- g)** 19 is between the perfect squares 16 and 25, but closer to 16.
 So, $\sqrt{19}$ is between 4 and 5, but closer to 4.
 Estimate to 1 decimal place: $\sqrt{19} \doteq 4.3$
 Square the estimate: $4.3^2 = 18.49$ (too small, but close)
 Revise the estimate: $\sqrt{19} \doteq 4.4$
 Square the estimate: $4.4^2 = 19.36$ (very close)
 19.36 is closer to 19, so $\sqrt{19}$ is approximately 4.4.
- h)** 20 is between the perfect cubes 8 and 27, but closer to 27.
 So, $\sqrt[3]{20}$ is between 2 and 3, but closer to 3.
 Estimate to 1 decimal place: $\sqrt[3]{20} \doteq 2.8$
 Cube the estimate: $2.8^3 = 21.952$ (too large)
 Revise the estimate: $\sqrt[3]{20} \doteq 2.7$
 Cube the estimate: $2.7^3 = 19.683$ (close)
 19.683 is closer to 20, so $\sqrt[3]{20}$ is approximately 2.7.
- 4. a)** When I try to determine the square root of -4 using a calculator, I get an error message. This makes sense because I cannot write a negative number as the product of two equal factors.
- b)** I get the same result with a negative radicand when the index is an even number.
- c) i)** When a radicand is negative, I can evaluate or estimate the value of the radical when the index is an odd number. This is because the product of an odd number of negative factors is negative.
- ii)** When a radicand is negative, I cannot evaluate or estimate the value of the radical when the index is an even number. This is because the product of an even number of negative factors is positive.
- 5. a) i)** $2^2 = 4$, so $2 = \sqrt{4}$
- ii)** $2^3 = 8$, so $2 = \sqrt[3]{8}$
- iii)** $2^4 = 16$, so $2 = \sqrt[4]{16}$

b) i) $3^2 = 9$, so $3 = \sqrt{9}$

ii) $3^3 = 27$, so $3 = \sqrt[3]{27}$

iii) $3^4 = 81$, so $3 = \sqrt[4]{81}$

c) i) $4^2 = 16$, so $4 = \sqrt{16}$

ii) $4^3 = 64$, so $4 = \sqrt[3]{64}$

iii) $4^4 = 256$, so $4 = \sqrt[4]{256}$

d) i) $10^2 = 100$, so $10 = \sqrt{100}$

ii) $10^3 = 1000$, so $10 = \sqrt[3]{1000}$

iii) $10^4 = 10\,000$, so $10 = \sqrt[4]{10\,000}$

e) i) $0.9^2 = 0.81$, so $0.9 = \sqrt{0.81}$

ii) $0.9^3 = 0.729$, so $0.9 = \sqrt[3]{0.729}$

iii) $0.9^4 = 0.6561$, so $0.9 = \sqrt[4]{0.6561}$

f) i) $0.2^2 = 0.04$, so $0.2 = \sqrt{0.04}$

ii) $0.2^3 = 0.008$, so $0.2 = \sqrt[3]{0.008}$

iii) $0.2^4 = 0.0016$, so $0.2 = \sqrt[4]{0.0016}$

6. Answers may vary.

- a) For $\sqrt[n]{x}$ to be a whole number, the radical must be a square root of a whole number that is a perfect square, the cube root of a whole number that is a perfect cube, the fourth root of a whole number that is a perfect fourth power, and so on.

For example, $\sqrt[4]{1296}$

$$\begin{aligned}\sqrt[4]{1296} &= \sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} \\ &= 6\end{aligned}$$

- b) For $\sqrt[n]{x}$ to be a negative integer, the radical must be a cube root of a negative integer that is a perfect cube, the fifth root of a negative integer that is a perfect fifth power, and so on.

For example, $\sqrt[5]{-243}$

$$\begin{aligned}\sqrt[5]{-243} &= \sqrt[5]{(-3)(-3)(-3)(-3)(-3)} \\ &= -3\end{aligned}$$

- c) For $\sqrt[n]{x}$ to be a rational number, the radical must be a square root of a rational number that is a perfect square, the cube root of a rational number that is a perfect cube, the fourth root of a rational number that is a perfect fourth power, and so on.

For example, $\sqrt[3]{0.64}$

$$\begin{aligned}\sqrt[3]{0.064} &= \sqrt[3]{(0.4)(0.4)(0.4)} \\ &= 0.4\end{aligned}$$

- d) For $\sqrt[n]{x}$ to be an approximate decimal, the radical must be a square root of a number that is not a perfect square, the cube root of a number that is not a perfect cube, the fourth root of a number that is not a perfect fourth power, and so on.

For example, $\sqrt[3]{13}$

$$\sqrt[3]{13} \doteq 2.4$$

Lesson 4.2 Irrational Numbers

Exercises (pages 211–212)

A

3. a) $\sqrt{12}$ is irrational because 12 is not a perfect square.
The decimal form of $\sqrt{12}$ neither terminates nor repeats.
- b) $\sqrt[4]{16}$ is rational because 16 is a perfect fourth power.
Its decimal form is 2.0, which terminates.
- c) $\sqrt[3]{-100}$ is irrational because -100 is not a perfect cube.
The decimal form of $\sqrt[3]{-100}$ neither terminates nor repeats.
- d) $\sqrt{\frac{4}{9}}$ is rational because $\frac{4}{9}$ is a perfect square.
 $\sqrt{\frac{4}{9}} = \frac{2}{3}$ or $0.\overline{6}$, which is a repeating decimal
- e) $\sqrt{1.25}$ is irrational because 1.25 is not a perfect square.
The decimal form of $\sqrt{1.25}$ neither terminates nor repeats.
- f) 1.25 is rational because it is a terminating decimal.
4. Natural numbers are the set of numbers 1, 2, 3, 4, 5, ...
Integers are the set of numbers ...-3, -2, -1, 0, 1, 2, 3, ...
A rational number is a number that can be written as the quotient of two integers; its decimal form either terminates or repeats.
An irrational number is a number that cannot be written as the quotient of two integers; its decimal form neither terminates nor repeats.
- $\frac{4}{3}$ is a quotient of integers, so $\frac{4}{3}$ is a rational number.
- $0.3\overline{4}$ is a repeating decimal, so $0.3\overline{4}$ is a rational number.
- 5 is both an integer and a rational number.
- $\sqrt[4]{9}$ is an irrational number because 9 is not a perfect fourth power. The decimal form of $\sqrt[4]{9}$ neither terminates nor repeats.
- 2.1538 is a terminating decimal, so -2.1538 is a rational number.
- $\sqrt[3]{27}$ is a rational number because 27 is a perfect cube. $\sqrt[3]{27} = 3$, which is a natural number and an integer
- 7 is an integer, a natural number, and a rational number.
- a) 7, $\sqrt[3]{27}$
- b) -5, $\sqrt[3]{27}$, 7

c) $\frac{4}{3}$, $0.3\overline{4}$, -5 , -2.1538 , $\sqrt[3]{27}$, 7

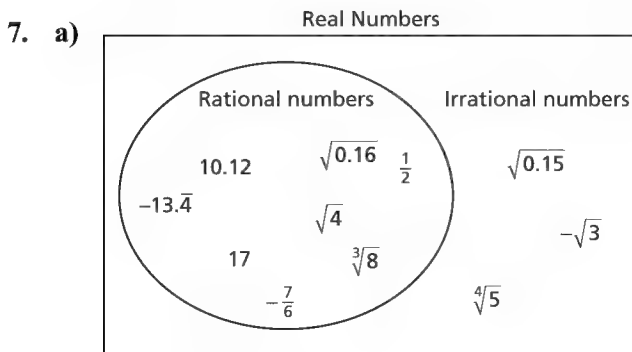
d) $\sqrt[4]{9}$

B

5. a) $\sqrt{49}$ is a rational number because 49 is a perfect square. $\sqrt{49} = 7$, which is a terminating decimal
 $\sqrt[4]{16}$ is a rational number because 16 is a perfect fourth power. $\sqrt[4]{16} = 2$, which is a terminating decimal

- b) $\sqrt{21}$ is an irrational number because 21 is not a perfect square.
 The decimal form of $\sqrt{21}$ neither terminates nor repeats.
 $\sqrt[3]{36}$ is an irrational number because 36 is not a perfect cube.
 The decimal form of $\sqrt[3]{36}$ neither terminates nor repeats.

6. a) 12.247 448 71 is a terminating decimal, so 12.247 448 71 is a rational number.
- b) Since 150 is not a perfect square, I know that $\sqrt{150}$ is an irrational number and that its decimal form neither terminates nor repeats. The calculator screen indicates that the decimal value of $\sqrt{150} = 12.247\ 448\ 71$. But, when I enter 12.247 448 71 in my calculator and square it, I get 149.9 instead of exactly 150. So, the value shown on the screen is a close approximation of $\sqrt{150}$, but not an exact value.



- b) $\frac{1}{2}$ and $-\frac{7}{6}$ are rational because each is a quotient of integers.
 $-\sqrt{3}$, $\sqrt{0.15}$, and $\sqrt[4]{5}$ are irrational because 3 and 0.15 are not perfect squares and 5 is not a perfect fourth power.
 $\sqrt{4}$, $\sqrt{0.16}$, and $\sqrt[3]{8}$ are rational because 4 and 0.16 are perfect squares and 8 is a perfect cube.
 10.12 is rational because it is a terminating decimal.
 $-13.\overline{4}$ is rational because it is a repeating decimal.
 17 is rational because it can be written as a quotient of integers, $\frac{17}{1}$.

8. a) $\sqrt[3]{8}$ is rational because 8 is a perfect cube.

I used a calculator: $\sqrt[3]{8} = 2$ and 2 can be written as a quotient of integers; $\frac{2}{1}$.

- b) $\sqrt[3]{64}$ is rational because 64 is a perfect cube.

I used a calculator: $\sqrt[3]{64} = 4$ and 4 can be written as a quotient of integers; $\frac{4}{1}$.

- c) $\sqrt[3]{30}$ is irrational because 30 is not a perfect cube.

I used a calculator: the decimal form of $\sqrt[3]{30}$ appears to neither terminate nor repeat.

- d) $\sqrt[3]{300}$ is irrational because 300 is not a perfect cube.

I used a calculator: the decimal form of $\sqrt[3]{300}$ appears to neither terminate nor repeat.

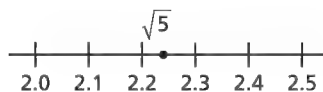
9. a) 5 is between the perfect squares 4 and 9, and is closer to 4.

$$\sqrt{4} \quad \sqrt{5} \quad \sqrt{9}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$$\sqrt{5} = 2.2360\dots$$



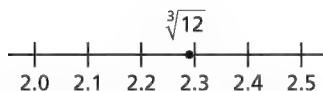
- b) 12 is between the perfect cubes 8 and 27, and is closer to 8.

$$\sqrt[3]{8} \quad \sqrt[3]{12} \quad \sqrt[3]{27}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$$\sqrt[3]{12} = 2.2894\dots$$



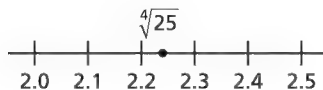
- c) 25 is between the perfect fourth powers 16 and 81, and is closer to 16.

$$\sqrt[4]{16} \quad \sqrt[4]{25} \quad \sqrt[4]{81}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$$\sqrt[4]{25} = 2.2360\dots$$



- d) -12 is between the perfect cubes -8 and -27 , and is closer to -8 .

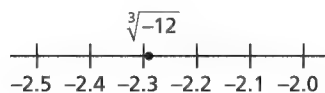
$$\sqrt[3]{-8} \quad \sqrt[3]{-12} \quad \sqrt[3]{-27}$$



-2

?

-3



Use a calculator.

$$\sqrt[3]{-12} = -2.2894\dots$$

10. a) 70 is between the perfect cubes 64 and 125 , and is closer to 64 .

$$\sqrt[3]{64} \quad \sqrt[3]{70} \quad \sqrt[3]{125}$$



4

?

5

Use a calculator.

$$\sqrt[3]{70} = 4.1212\dots$$

50 is between the perfect squares 49 and 64 , and is closer to 49 .

$$\sqrt{49} \quad \sqrt{50} \quad \sqrt{64}$$



7

?

8

Use a calculator.

$$\sqrt{50} = 7.0710\dots$$

100 is between the perfect fourth powers 81 and 256 , and is closer to 81 .

$$\sqrt[4]{81} \quad \sqrt[4]{100} \quad \sqrt[4]{256}$$



3

?

4

Use a calculator.

$$\sqrt[4]{100} = 3.1622\dots$$

400 is between the perfect cubes 343 and 512 , and is closer to 343 .

$$\sqrt[3]{343} \quad \sqrt[3]{400} \quad \sqrt[3]{512}$$



7

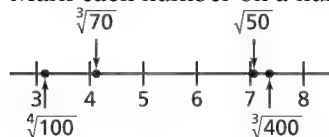
?

8

Use a calculator.

$$\sqrt[3]{400} = 7.3680\dots$$

Mark each number on a number line.



From greatest to least: $\sqrt[3]{400}$, $\sqrt{50}$, $\sqrt[3]{70}$, $\sqrt[4]{100}$

- b) 89 is between the perfect squares 81 and 100, and is closer to 81.

$$\sqrt{81} \quad \sqrt{89} \quad \sqrt{100}$$

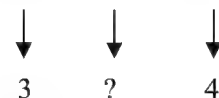


Use a calculator.

$$\sqrt{89} = 9.4339\dots$$

250 is between the perfect fourth powers 81 and 256, and is closer to 256.

$$\sqrt[4]{81} \quad \sqrt[4]{250} \quad \sqrt[4]{256}$$

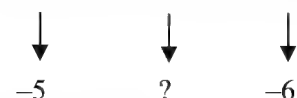


Use a calculator.

$$\sqrt[4]{250} = 3.9763\dots$$

-150 is between the perfect cubes -125 and -216, and is closer to -125.

$$\sqrt[3]{-125} \quad \sqrt[3]{-150} \quad \sqrt[3]{-216}$$

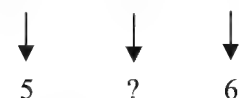


Use a calculator.

$$\sqrt[3]{-150} = -5.3132\dots$$

150 is between the perfect cubes 125 and 216, and is closer to 125.

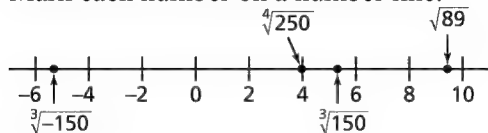
$$\sqrt[3]{125} \quad \sqrt[3]{150} \quad \sqrt[3]{216}$$



Use a calculator.

$$\sqrt[3]{150} = 5.3132\dots$$

Mark each number on a number line.



From greatest to least: $\sqrt{89}$, $\sqrt[3]{150}$, $\sqrt[4]{250}$, $\sqrt[3]{-150}$

11. 40 is between the perfect squares 36 and 49, and is closer to 36.

$$\begin{array}{ccc} \sqrt{36} & \sqrt{40} & \sqrt{49} \\ \downarrow & \downarrow & \downarrow \\ 6 & ? & 7 \end{array}$$

Use a calculator.

$$\sqrt{40} = 6.3245\dots$$

- 500 is between the perfect cubes 343 and 512, and is closer to 512.

$$\begin{array}{ccc} \sqrt[3]{343} & \sqrt[3]{500} & \sqrt[3]{512} \\ \downarrow & \downarrow & \downarrow \\ 7 & ? & 8 \end{array}$$

Use a calculator.

$$\sqrt[3]{500} = 7.9370\dots$$

- 98 is between the perfect squares 81 and 100, and is closer to 100.

$$\begin{array}{ccc} \sqrt{81} & \sqrt{98} & \sqrt{100} \\ \downarrow & \downarrow & \downarrow \\ 9 & ? & 10 \end{array}$$

Use a calculator.

$$\sqrt{98} = 9.8994\dots$$

- 98 is between the perfect cubes 64 and 125, and is closer to 125.

$$\begin{array}{ccc} \sqrt[3]{64} & \sqrt[3]{98} & \sqrt[3]{125} \\ \downarrow & \downarrow & \downarrow \\ 4 & ? & 5 \end{array}$$

Use a calculator.

$$\sqrt[3]{98} = 4.6104\dots$$

- 75 is between the perfect squares 64 and 81, and is closer to 81.

$$\begin{array}{ccc} \sqrt{64} & \sqrt{75} & \sqrt{81} \\ \downarrow & \downarrow & \downarrow \\ 8 & ? & 9 \end{array}$$

Use a calculator.

$$\sqrt{75} = 8.6602\dots$$

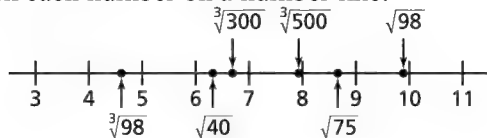
- 300 is between the perfect cubes 216 and 343, and is closer to 343.

$$\begin{array}{ccc} \sqrt[3]{216} & \sqrt[3]{300} & \sqrt[3]{343} \\ \downarrow & \downarrow & \downarrow \\ 6 & ? & 7 \end{array}$$

Use a calculator.

$$\sqrt[3]{300} = 6.6943\dots$$

Mark each number on a number line.



From least to greatest: $\sqrt[3]{98}$, $\sqrt{40}$, $\sqrt[3]{300}$, $\sqrt[3]{500}$, $\sqrt{75}$, $\sqrt{98}$

To check, I can subtract the number on the left from the number to its right each time and I should always get a positive difference.

12. Use a calculator.

$$\frac{-14}{5} = -2.8$$

Use a calculator.

$$\frac{123}{99} = 1.24$$

-10 is between the perfect cubes -8 and -27 , and is closer to -8 .

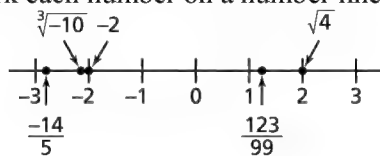
$$\begin{array}{ccc} \sqrt[3]{-8} & \sqrt[3]{-10} & \sqrt[3]{-27} \\ \downarrow & \downarrow & \downarrow \\ -2 & ? & -3 \end{array}$$

Use a calculator.

$$\sqrt[3]{-10} = -2.1544\dots$$

$$\sqrt{4} = 2$$

Mark each number on a number line.



From least to greatest: $\frac{-14}{5}$, $\sqrt[3]{-10}$, -2 , $\frac{123}{99}$, $\sqrt{4}$

$\frac{-14}{5}$ and $\frac{123}{99}$ are rational because each is a quotient of integers.

-2 is rational because it can be written as a quotient of integers, $\frac{-2}{1}$.

$\sqrt[3]{-10}$ is irrational because -10 is not a perfect cube.

$\sqrt{4}$ is rational because 4 is a perfect square.

13. Use the Pythagorean Theorem to determine the length of the hypotenuse.

In a right triangle with hypotenuse length h and legs of lengths a and b ,

$$h^2 = a^2 + b^2 \quad \text{Substitute: } a = 5 \text{ and } b = 3$$

$$h^2 = 5^2 + 3^2$$

$$h^2 = 25 + 9$$

$$h^2 = 34$$

$$h = \sqrt{34}$$

Because 34 is not a perfect square, $\sqrt{34}$ is an irrational number.

The length of the hypotenuse is $\sqrt{34}$ cm.

14. a) i) Natural numbers are the counting numbers: 1, 2, 3, 4, ...

Integers are the set of numbers ... -3, -2, -1, 0, 1, 2, 3, ...

So, all natural numbers are integers. The statement is true.

- ii) All integers are rational numbers because any integer n can be written as $\frac{n}{1}$.

The statement is true.

- iii) The set of whole numbers is the set of natural numbers with the number 0 included.

So, all whole numbers are not natural numbers. The statement is false.

- iv) Assume that we consider the root of a rational number.

π is irrational, and π is not a root of a rational number.

So, not all irrational numbers are roots.

The statement is false.

Assume that we consider the root of an irrational number.

π is the n th root of π^n . So, all irrational numbers are roots.

The statement is true.

- v) All rational numbers can be written as the quotient of two integers. Since all natural numbers can be written as a quotient with denominator 1, some rational numbers are natural numbers. The statement is true.

- b) iii) The number 0 is a whole number, but it is not a natural number.

- iv) π is irrational, and π is not a root of a rational number.

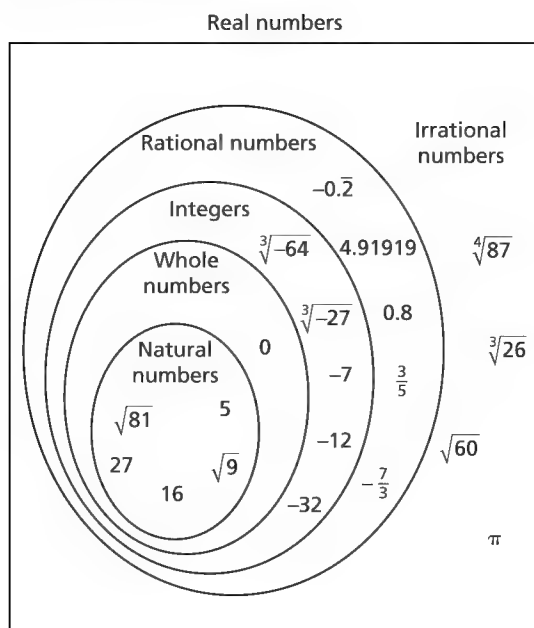
15. Answers may vary.

- a) The decimal form of a rational number either terminates or repeats. So, write a terminating decimal that is not a natural number; for example, 1.5.

- b) The set of whole numbers is the set of natural numbers with the number 0 included. So, the only whole number that is not a natural number is 0.

- c) $\sqrt{21}$ is an irrational number because 21 is not a perfect square.

16. a) Some numbers belong to more than one set.
Place these numbers in the smallest set to which it belongs.
- $\frac{3}{5}$ is a rational number because it is the quotient of two integers.
- 4.91919 is a rational number because it is a terminating decimal.
- 16 is a natural number.
- $\sqrt[3]{-64} = -4$, which is an integer
- $\sqrt{60}$ is an irrational number because 60 is not a perfect square.
- $\sqrt{9} = 3$, which is a natural number
- -7 is an integer.
- 0 is a whole number.



- b) Answers may vary.
- Natural numbers: I chose 2 counting numbers, 5 and 27, and the square root of a natural number that is a perfect square, $\sqrt{81}$.
- Whole numbers: I cannot choose any more whole numbers that are not natural numbers.
- Integers: I chose -12 and -32 , and the cube root of a negative integer that is a perfect cube, $\sqrt[3]{-27}$.
- Rational numbers: I chose a repeating decimal, $-0.\overline{2}$, a terminating decimal, 0.8 , and a fraction, $-\frac{7}{3}$.
- Irrational numbers: I chose the cube root of a number that is not a perfect cube, $\sqrt[3]{26}$, the fourth root of a number that is not a perfect fourth power, $\sqrt[4]{87}$, and π because its decimal form neither terminates nor repeats.

17. The formula for the volume, V , of a cube with edge length s units is:

$$V = s^3$$

To determine the value of s , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{s^3}$$

$$\sqrt[3]{V} = s$$

Answers may vary.

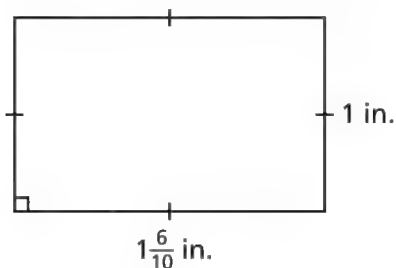
- a) For s to be irrational, V must be a positive number that is not a perfect cube.
For example, $V = 15$
- b) For s to be rational, V must be a positive number that is a perfect cube.
For example, $V = 64$

18. a) $\frac{1 + \sqrt{5}}{2} = 1.6180\dots$

This value to the nearest tenth is 1.6.

- b) Since the units are inches, write 1.6 as a fraction: $1\frac{6}{10}$

Draw a rectangle with length $1\frac{6}{10}$ in. and width 1 in.



- c) Answers may vary.
I measured a book with length 21 cm and width 13 cm.

The ratio of length to width is: $\frac{21}{13} = 1.6153\dots$

Since the ratio of length to width is approximately 1.6 to 1, the book approximates a golden rectangle.

19. The pyramid has base side length 755 ft. and height 481 ft.

The ratio of base side length to its height is: $\frac{755}{481} = 1.5696\dots$

Since I can write 1.5696... as $\frac{1.5696\dots}{1}$, the ratio is approximately 1.6:1.

From question 18, I know the golden ratio $\frac{1+\sqrt{5}}{2}:1$ is approximately 1.6:1.

So, the ratio of the base side length of the pyramid to its height approximates the golden ratio.

20. The formula for the area, A , of a square with side length s units is:

$$A = s^2$$

To determine the value of s , take the square root of each side.

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s$$

- a) Since 40 is not a perfect square, $\sqrt{40}$ is irrational.

So, the side length of the square is irrational.

The perimeter of a square is: $P = 4s$

Since the product of a rational number and an irrational number is irrational, the perimeter of the square is irrational.

- b) Since 81 is a perfect square, $\sqrt{81}$ is rational.

So, the side length of the square is rational.

The perimeter of a square is: $P = 4s$

Since the product of two rational numbers is rational, the perimeter of the square is rational.

C

21. Since $\sqrt[n]{\frac{a}{b}}$ is rational, when $n = 2$, $\frac{a}{b}$ is a perfect square; when $n = 3$, $\frac{a}{b}$ is a perfect cube;

when $n = 4$, $\frac{a}{b}$ is a perfect fourth power; and so on.

So, each prime factor of a and b must occur a multiple of n times.

For example, when $n = 2$:

$$\sqrt{\frac{16}{9}} = \sqrt{\frac{(2)(2)(2)(2)}{(3)(3)}}$$

The prime factors of 16 occur 4 times; and 4 is a multiple of 2.

The prime factors of 9 occur 2 times; and 2 is a multiple of 2.

When $n = 3$:

$$\sqrt[3]{\frac{8}{125}} = \sqrt[3]{\frac{(2)(2)(2)}{(5)(5)(5)}}$$

Each prime factor of 8 and 125 occurs 3 times; and 3 is a multiple of 3.

When $n = 4$:

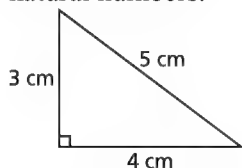
$$\sqrt[4]{\frac{256}{81}} = \sqrt[4]{\frac{(2)(2)(2)(2)(2)(2)(2)(2)}{(3)(3)(3)(3)}}$$

The prime factors of 256 occur 8 times; and 8 is a multiple of 4.

The prime factors of 81 occur 4 times; and 4 is a multiple of 4.

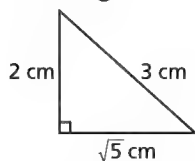
22. In a right triangle with hypotenuse length h and legs of lengths a and b , $h^2 = a^2 + b^2$.

- a) Since all natural numbers are rational numbers, I drew a triangle with side lengths that are natural numbers.



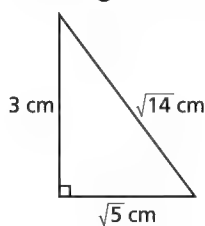
- b) I started with the irrational number $\sqrt{5}$ as the length of one leg in centimetres. I then determined a value for b so that $(\sqrt{5})^2 + b^2$ is a perfect square. Since $5 + 4 = 9$ and $2^2 = 4$, I know $b = 2$.

The length of the hypotenuse, in centimetres, is $\sqrt{9} = 3$.

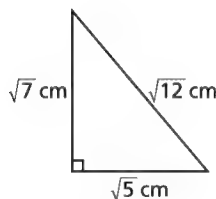


- c) I started with the irrational number $\sqrt{5}$ as the length of one leg, in centimetres. I then determined a value for b so that $(\sqrt{5})^2 + b^2$ is not a perfect square, with b a natural number. I chose $b = 3$.

The length of the hypotenuse, in centimetres, is $\sqrt{5 + 3^2} = \sqrt{14}$.



- d) I started with the irrational number $\sqrt{5}$ as the length of one leg, in centimetres. I then determined a value for b so that $(\sqrt{5})^2 + b^2$ is not a perfect square, with b an irrational number. I chose $b = \sqrt{7}$. The length of the hypotenuse, in centimetres, is $\sqrt{5 + 7} = \sqrt{12}$.



23. a) Yes, when a rational number is not a perfect square, its square root is irrational. For example, 31 is a rational number because it can be written as the quotient of two integers, $\frac{31}{1}$. But, $\sqrt{31}$ is an irrational number because 31 is not a perfect square.

- b) A rational number can be written in the form $\frac{a}{b}$, where a and b are integers. When a rational number is squared, the result is $\frac{a^2}{b^2}$. Since the square of an integer is another integer, it is impossible for the square of a rational number to be irrational. So, the square root of an irrational number cannot be rational.

24. A number is a perfect square when the exponent of each factor in its prime factorization is a multiple of 2; for example, $9 = 3^2$.

A number is a perfect cube when the exponent of each factor in its prime factorization is a multiple of 3; for example, $64 = 2^6$, or $2^{2(3)}$.

A number is a perfect fourth power when the exponent of each factor in its prime factorization is a multiple of 4; for example, $81 = 3^4$.

For a number to be a perfect square, a perfect cube, and a perfect fourth power, the exponent of each factor in its prime factorization must be a multiple of 2, 3, and 4.

List the multiples of 2, 3, and 4:

2: 2, 4, 6, 8, 10, 12, ...

3: 3, 6, 9, 12, ...

4: 4, 8, 12, ...

The least common multiple of 2, 3, and 4 is 12.

So, to generate numbers with the property that their square roots, cube roots, and fourth roots are all rational numbers, raise the number to the 12th power; for example, $2^{12} = 4096$.

For example

$$\sqrt{4096} = \sqrt{2^{12}} = \sqrt{2^{2(6)}} = 2^6 = 64$$

$$\sqrt[3]{4096} = \sqrt[3]{2^{12}} = \sqrt[3]{2^{3(4)}} = 2^4 = 16$$

$$\sqrt[4]{4096} = \sqrt[4]{2^{12}} = \sqrt[4]{2^{4(3)}} = 2^3 = 8$$

Lesson 4.3 Mixed and Entire Radicals

Exercises (pages 218–219)

A
3.

Square of Number	Perfect Square	Square Root
$1^2 = 1 \cdot 1$	1	1
$2^2 = 2 \cdot 2$	4	2
$3^2 = 3 \cdot 3$	9	3
$4^2 = 4 \cdot 4$	16	4
$5^2 = 5 \cdot 5$	25	5
$6^2 = 6 \cdot 6$	36	6
$7^2 = 7 \cdot 7$	49	7
$8^2 = 8 \cdot 8$	64	8
$9^2 = 9 \cdot 9$	81	9
$10^2 = 10 \cdot 10$	100	10
$11^2 = 11 \cdot 11$	121	11
$12^2 = 12 \cdot 12$	144	12
$13^2 = 13 \cdot 13$	169	13
$14^2 = 14 \cdot 14$	196	14
$15^2 = 15 \cdot 15$	225	15
$16^2 = 16 \cdot 16$	256	16
$17^2 = 17 \cdot 17$	289	17
$18^2 = 18 \cdot 18$	324	18
$19^2 = 19 \cdot 19$	361	19
$20^2 = 20 \cdot 20$	400	20

4. a) $\sqrt{8} = \sqrt{2 \cdot 4}$
 $= \sqrt{2 \cdot 2 \cdot 2}$
 $= \sqrt{(2 \cdot 2) \cdot 2}$
 $= \sqrt{2 \cdot 2} \cdot \sqrt{2}$
 $= 2 \cdot \sqrt{2}$
 $= 2\sqrt{2}$

b) $\sqrt{12} = \sqrt{4 \cdot 3}$
 $= \sqrt{2 \cdot 2 \cdot 3}$
 $= \sqrt{(2 \cdot 2) \cdot 3}$
 $= \sqrt{2 \cdot 2} \cdot \sqrt{3}$
 $= 2 \cdot \sqrt{3}$
 $= 2\sqrt{3}$

$$\begin{aligned}\text{c) } \sqrt{32} &= \sqrt{4 \cdot 8} \\ &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\ &= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2) \cdot 2} \\ &= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{2} \\ &= 2 \cdot 2 \cdot \sqrt{2} \\ &= 4\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{d) } \sqrt{50} &= \sqrt{25 \cdot 2} \\ &= \sqrt{5 \cdot 5 \cdot 2} \\ &= \sqrt{(5 \cdot 5) \cdot 2} \\ &= \sqrt{5 \cdot 5} \cdot \sqrt{2} \\ &= 5 \cdot \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{e) } \sqrt{18} &= \sqrt{9 \cdot 2} \\ &= \sqrt{3 \cdot 3 \cdot 2} \\ &= \sqrt{(3 \cdot 3) \cdot 2} \\ &= \sqrt{3 \cdot 3} \cdot \sqrt{2} \\ &= 3 \cdot \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{f) } \sqrt{27} &= \sqrt{9 \cdot 3} \\ &= \sqrt{3 \cdot 3 \cdot 3} \\ &= \sqrt{(3 \cdot 3) \cdot 3} \\ &= \sqrt{3 \cdot 3} \cdot \sqrt{3} \\ &= 3 \cdot \sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$$

$$\begin{aligned}
 \text{g) } \sqrt{48} &= \sqrt{4 \cdot 12} \\
 &= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} \\
 &= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2) \cdot 3} \\
 &= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{3} \\
 &= 2 \cdot 2 \cdot \sqrt{3} \\
 &= 4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } \sqrt{75} &= \sqrt{25 \cdot 3} \\
 &= \sqrt{5 \cdot 5 \cdot 3} \\
 &= \sqrt{(5 \cdot 5) \cdot 3} \\
 &= \sqrt{5 \cdot 5} \cdot \sqrt{3} \\
 &= 5 \cdot \sqrt{3} \\
 &= 5\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{5. a) Write 5 as: } \sqrt{5 \cdot 5} &= \sqrt{25} \\
 5\sqrt{2} &= \sqrt{25} \cdot \sqrt{2} \\
 &= \sqrt{25 \cdot 2} \\
 &= \sqrt{50}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) Write 6 as: } \sqrt{6 \cdot 6} &= \sqrt{36} \\
 6\sqrt{2} &= \sqrt{36} \cdot \sqrt{2} \\
 &= \sqrt{36 \cdot 2} \\
 &= \sqrt{72}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) Write 7 as: } \sqrt{7 \cdot 7} &= \sqrt{49} \\
 7\sqrt{2} &= \sqrt{49} \cdot \sqrt{2} \\
 &= \sqrt{49 \cdot 2} \\
 &= \sqrt{98}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) Write 8 as: } \sqrt{8 \cdot 8} &= \sqrt{64} \\
 8\sqrt{2} &= \sqrt{64} \cdot \sqrt{2} \\
 &= \sqrt{64 \cdot 2} \\
 &= \sqrt{128}
 \end{aligned}$$

e) Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$
 $5\sqrt{3} = \sqrt{25} \cdot \sqrt{3}$
 $= \sqrt{25 \cdot 3}$
 $= \sqrt{75}$

f) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$
 $6\sqrt{3} = \sqrt{36} \cdot \sqrt{3}$
 $= \sqrt{36 \cdot 3}$
 $= \sqrt{108}$

g) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$
 $7\sqrt{3} = \sqrt{49} \cdot \sqrt{3}$
 $= \sqrt{49 \cdot 3}$
 $= \sqrt{147}$

h) Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$
 $8\sqrt{3} = \sqrt{64} \cdot \sqrt{3}$
 $= \sqrt{64 \cdot 3}$
 $= \sqrt{192}$

6. a)

Cube of Number	Perfect Cube	Cube Root
$1^3 = 1 \cdot 1 \cdot 1$	1	1
$2^3 = 2 \cdot 2 \cdot 2$	8	2
$3^3 = 3 \cdot 3 \cdot 3$	27	3
$4^3 = 4 \cdot 4 \cdot 4$	64	4
$5^3 = 5 \cdot 5 \cdot 5$	125	5
$6^3 = 6 \cdot 6 \cdot 6$	216	6
$7^3 = 7 \cdot 7 \cdot 7$	343	7
$8^3 = 8 \cdot 8 \cdot 8$	512	8
$9^3 = 9 \cdot 9 \cdot 9$	729	9
$10^3 = 10 \cdot 10 \cdot 10$	1000	10

b)

Fourth Power of Number	Perfect Fourth Power	Fourth Root
$1^4 = 1 \cdot 1 \cdot 1 \cdot 1$	1	1
$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$	16	2
$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$	81	3
$4^4 = 4 \cdot 4 \cdot 4 \cdot 4$	256	4
$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$	625	5

B

7. a) Use the Pythagorean Theorem in right $\triangle CBA$.

$$AC^2 + AB^2 = BC^2 \quad \text{Substitute: } AC = 1, AB = 2$$

$$1^2 + 2^2 = BC^2$$

$$1 + 4 = BC^2$$

$$5 = BC^2$$

$$BC = \sqrt{5}$$

Use the Pythagorean Theorem in right $\triangle DEA$.

$$AD^2 + AE^2 = DE^2 \quad \text{Substitute: } AD = 3, AE = 6$$

$$3^2 + 6^2 = DE^2$$

$$9 + 36 = DE^2$$

$$45 = DE^2$$

$$DE = \sqrt{45}$$

Each side of $\triangle DEA$ is 3 times the length of the corresponding side in $\triangle CBA$.

So, $DE = 3(CB)$

$$\sqrt{45} = 3\sqrt{5}$$

b) $\sqrt{45} = \sqrt{9 \cdot 5}$

$$= \sqrt{3 \cdot 3 \cdot 5}$$

$$= \sqrt{(3 \cdot 3) \cdot 5}$$

$$= \sqrt{3 \cdot 3} \cdot \sqrt{5}$$

$$= 3 \cdot \sqrt{5}$$

$$= 3\sqrt{5}$$

8. a) Use the Pythagorean Theorem in right $\triangle QRS$.

$$QR^2 + RS^2 = QS^2 \quad \text{Substitute: } QR = 3, RS = 1$$

$$3^2 + 1^2 = QS^2$$

$$9 + 1 = QS^2$$

$$10 = QS^2$$

$$QS = \sqrt{10}$$

Use the Pythagorean Theorem in right $\triangle PRT$.

$$PR^2 + RT^2 = PT^2 \quad \text{Substitute: } PR = 6, RT = 2$$

$$6^2 + 2^2 = PT^2$$

$$36 + 4 = PT^2$$

$$40 = PT^2$$

$$PT = \sqrt{40}$$

Each side of $\triangle PRT$ is 2 times the length of the corresponding side in $\triangle QRS$.

So, $PT = 2(QS)$

$$\sqrt{40} = 2\sqrt{10}$$

$$\begin{aligned}
 \text{b) } \sqrt{40} &= \sqrt{4 \cdot 10} \\
 &= \sqrt{2 \cdot 2 \cdot 10} \\
 &= \sqrt{(2 \cdot 2) \cdot 10} \\
 &= \sqrt{2 \cdot 2} \cdot \sqrt{10} \\
 &= 2 \cdot \sqrt{10} \\
 &= 2\sqrt{10}
 \end{aligned}$$

9. When I simplify $\sqrt{50}$, it helps to rewrite $\sqrt{50}$ as $\sqrt{25} \cdot \sqrt{2}$ because 25 is a perfect square.

I can write $\sqrt{25}$ as 5. Then $\sqrt{50} = 5\sqrt{2}$.

When I simplify $\sqrt{50}$, it does not help to rewrite $\sqrt{50}$ as $\sqrt{10} \cdot \sqrt{5}$ because neither 10 nor 5 is a perfect square.

10. a) The factors of 90 are: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
The greatest perfect square is $9 = 3 \cdot 3$, so write 90 as $9 \cdot 10$.

$$\begin{aligned}
 \sqrt{90} &= \sqrt{9 \cdot 10} \\
 &= \sqrt{9} \cdot \sqrt{10} \\
 &= 3 \cdot \sqrt{10} \\
 &= 3\sqrt{10}
 \end{aligned}$$

- b) The factors of 73 are 1 and 73.
There are no perfect square factors other than 1.
So, $\sqrt{73}$ cannot be simplified.

- c) The factors of 108 are: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
The greatest perfect square is $36 = 6 \cdot 6$, so write 108 as $36 \cdot 3$.

$$\begin{aligned}
 \sqrt{108} &= \sqrt{36 \cdot 3} \\
 &= \sqrt{36} \cdot \sqrt{3} \\
 &= 6 \cdot \sqrt{3} \\
 &= 6\sqrt{3}
 \end{aligned}$$

- d) The factors of 600 are: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100, 120, 150, 200, 300, 600
The greatest perfect square is $100 = 10 \cdot 10$, so write 600 as $100 \cdot 6$.

$$\begin{aligned}
 \sqrt{600} &= \sqrt{100 \cdot 6} \\
 &= \sqrt{100} \cdot \sqrt{6} \\
 &= 10 \cdot \sqrt{6} \\
 &= 10\sqrt{6}
 \end{aligned}$$

- e) The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54
The greatest perfect square is $9 = 3 \cdot 3$, so write 54 as $9 \cdot 6$.

$$\begin{aligned}\sqrt{54} &= \sqrt{9 \cdot 6} \\ &= \sqrt{9} \cdot \sqrt{6} \\ &= 3 \cdot \sqrt{6} \\ &= 3\sqrt{6}\end{aligned}$$

- f) The factors of 91 are 1, 7, 13, and 91.
There are no perfect square factors other than 1.
So, $\sqrt{91}$ cannot be simplified.

- g) The factors of 28 are: 1, 2, 4, 7, 14, 28
The greatest perfect square is $4 = 2 \cdot 2$, so write 28 as $4 \cdot 7$.

$$\begin{aligned}\sqrt{28} &= \sqrt{4 \cdot 7} \\ &= \sqrt{4} \cdot \sqrt{7} \\ &= 2 \cdot \sqrt{7} \\ &= 2\sqrt{7}\end{aligned}$$

- h) The factors of 33 are 1, 3, 11, and 33.
There are no perfect square factors other than 1.
So, $\sqrt{33}$ cannot be simplified.

- i) The factors of 112 are: 1, 2, 4, 7, 8, 14, 16, 28, 56, 112
The greatest perfect square is $16 = 4 \cdot 4$, so write 112 as $16 \cdot 7$.

$$\begin{aligned}\sqrt{112} &= \sqrt{16 \cdot 7} \\ &= \sqrt{16} \cdot \sqrt{7} \\ &= 4 \cdot \sqrt{7} \\ &= 4\sqrt{7}\end{aligned}$$

11. a) The factors of 16 are: 1, 2, 4, 8, 16
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 16 as $8 \cdot 2$.

$$\begin{aligned}\sqrt[3]{16} &= \sqrt[3]{8 \cdot 2} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{2} \\ &= 2 \cdot \sqrt[3]{2} \\ &= 2\sqrt[3]{2}\end{aligned}$$

- b) The factors of 81 are: 1, 3, 9, 27, 81

The greatest perfect cube is $27 = 3 \cdot 3 \cdot 3$, so write 81 as $27 \cdot 3$.

$$\begin{aligned}\sqrt[3]{81} &= \sqrt[3]{27 \cdot 3} \\ &= \sqrt[3]{27} \cdot \sqrt[3]{3} \\ &= 3 \cdot \sqrt[3]{3} \\ &= 3\sqrt[3]{3}\end{aligned}$$

- c) The factors of 256 are: 1, 2, 4, 8, 16, 32, 64, 128, 256

The greatest perfect cube is $64 = 4 \cdot 4 \cdot 4$, so write 256 as $64 \cdot 4$.

$$\begin{aligned}\sqrt[3]{256} &= \sqrt[3]{64 \cdot 4} \\ &= \sqrt[3]{64} \cdot \sqrt[3]{4} \\ &= 4 \cdot \sqrt[3]{4} \\ &= 4\sqrt[3]{4}\end{aligned}$$

- d) The factors of 128 are: 1, 2, 4, 8, 16, 32, 64, 128

The greatest perfect cube is $64 = 4 \cdot 4 \cdot 4$, so write 128 as $64 \cdot 2$.

$$\begin{aligned}\sqrt[3]{128} &= \sqrt[3]{64 \cdot 2} \\ &= \sqrt[3]{64} \cdot \sqrt[3]{2} \\ &= 4 \cdot \sqrt[3]{2} \\ &= 4\sqrt[3]{2}\end{aligned}$$

- e) The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

There are no perfect cube factors other than 1.

So, $\sqrt[3]{60}$ cannot be simplified.

- f) The factors of 192 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, 192

The greatest perfect cube is $64 = 4 \cdot 4 \cdot 4$, so write 192 as $64 \cdot 3$.

$$\begin{aligned}\sqrt[3]{192} &= \sqrt[3]{64 \cdot 3} \\ &= \sqrt[3]{64} \cdot \sqrt[3]{3} \\ &= 4 \cdot \sqrt[3]{3} \\ &= 4\sqrt[3]{3}\end{aligned}$$

- g) The factors of 135 are: 1, 3, 5, 9, 15, 27, 45, 135

The greatest perfect cube is $27 = 3 \cdot 3 \cdot 3$, so write 135 as $27 \cdot 5$.

$$\begin{aligned}\sqrt[3]{135} &= \sqrt[3]{27 \cdot 5} \\ &= \sqrt[3]{27} \cdot \sqrt[3]{5} \\ &= 3 \cdot \sqrt[3]{5} \\ &= 3\sqrt[3]{5}\end{aligned}$$

- h) The factors of 100 are: 1, 2, 4, 5, 10, 20, 25, 50, 100

There are no perfect cube factors other than 1.

So, $\sqrt[3]{100}$ cannot be simplified.

- i) The factors of 500 are: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500

The greatest perfect cube is $125 = 5 \cdot 5 \cdot 5$, so write 500 as $125 \cdot 4$.

$$\begin{aligned}\sqrt[3]{500} &= \sqrt[3]{125 \cdot 4} \\ &= \sqrt[3]{125} \cdot \sqrt[3]{4} \\ &= 5 \cdot \sqrt[3]{4} \\ &= 5\sqrt[3]{4}\end{aligned}$$

- j) The factors of 375 are: 1, 3, 5, 15, 25, 75, 125, 375

The greatest perfect cube is $125 = 5 \cdot 5 \cdot 5$, so write 375 as $125 \cdot 3$.

$$\begin{aligned}\sqrt[3]{375} &= \sqrt[3]{125 \cdot 3} \\ &= \sqrt[3]{125} \cdot \sqrt[3]{3} \\ &= 5 \cdot \sqrt[3]{3} \\ &= 5\sqrt[3]{3}\end{aligned}$$

12. a) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{2} &= \sqrt{9} \cdot \sqrt{2} \\ &= \sqrt{9 \cdot 2} \\ &= \sqrt{18}\end{aligned}$$

- b) Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned}4\sqrt{2} &= \sqrt{16} \cdot \sqrt{2} \\ &= \sqrt{16 \cdot 2} \\ &= \sqrt{32}\end{aligned}$$

- c) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned}6\sqrt{5} &= \sqrt{36} \cdot \sqrt{5} \\ &= \sqrt{36 \cdot 5} \\ &= \sqrt{180}\end{aligned}$$

- d) Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$\begin{aligned}5\sqrt{6} &= \sqrt{25} \cdot \sqrt{6} \\ &= \sqrt{25 \cdot 6} \\ &= \sqrt{150}\end{aligned}$$

e) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$\begin{aligned} 7\sqrt{7} &= \sqrt{49} \cdot \sqrt{7} \\ &= \sqrt{49 \cdot 7} \\ &= \sqrt{343} \end{aligned}$$

f) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$$\begin{aligned} 2\sqrt[3]{2} &= \sqrt[3]{8} \cdot \sqrt[3]{2} \\ &= \sqrt[3]{8 \cdot 2} \\ &= \sqrt[3]{16} \end{aligned}$$

g) Write 3 as: $\sqrt[3]{3 \cdot 3 \cdot 3} = \sqrt[3]{27}$

$$\begin{aligned} 3\sqrt[3]{3} &= \sqrt[3]{27} \cdot \sqrt[3]{3} \\ &= \sqrt[3]{27 \cdot 3} \\ &= \sqrt[3]{81} \end{aligned}$$

h) Write 4 as: $\sqrt[3]{4 \cdot 4 \cdot 4} = \sqrt[3]{64}$

$$\begin{aligned} 4\sqrt[3]{3} &= \sqrt[3]{64} \cdot \sqrt[3]{3} \\ &= \sqrt[3]{64 \cdot 3} \\ &= \sqrt[3]{192} \end{aligned}$$

i) Write 5 as: $\sqrt[3]{5 \cdot 5 \cdot 5} = \sqrt[3]{125}$

$$\begin{aligned} 5\sqrt[3]{2} &= \sqrt[3]{125} \cdot \sqrt[3]{2} \\ &= \sqrt[3]{125 \cdot 2} \\ &= \sqrt[3]{250} \end{aligned}$$

j) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$$\begin{aligned} 2\sqrt[3]{9} &= \sqrt[3]{8} \cdot \sqrt[3]{9} \\ &= \sqrt[3]{8 \cdot 9} \\ &= \sqrt[3]{72} \end{aligned}$$

13. a) Yes, every mixed radical can be expressed as an entire radical. To express a mixed radical as an entire radical, I write the number in front of the radical as the square root of its square, or the cube root of its cube, or the fourth root of its perfect fourth power, and so on, depending on the index of the radical.

For example, to write $3\sqrt{3}$ as an entire radical, I write 3 as $\sqrt{3 \cdot 3} = \sqrt{9}$, then multiply:

$$\sqrt{9} \cdot \sqrt{3} = \sqrt{27}$$

To write $2\sqrt[4]{3}$ as an entire radical, I write 2 as $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$, then multiply:

$$\sqrt[4]{16} \cdot \sqrt[4]{3} = \sqrt[4]{48}$$

Every mixed radical can be expressed as an entire radical.

- b) No, every entire radical cannot be expressed as a mixed radical. An entire radical can be written as a mixed radical when one of the factors of the radicand is a perfect square, or a perfect cube, or a perfect fourth power, and so on, depending on the index of the radical. For example, $\sqrt[3]{16}$ can be written as a mixed radical because $16 = 8 \cdot 2$, and 8 is a perfect cube; so $\sqrt[3]{16} = 2\sqrt[3]{2}$.

$\sqrt[3]{21}$ cannot be written as a mixed radical because the factors of 21 are 1, 3, 7, 21, and there are no cube factors other than 1.

14. The formula for the area, A , of a square with side length s units is:

$$A = s^2$$

To determine the value of s , take the square root of each side.

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s \quad \text{Substitute: } A = 252$$

$$s = \sqrt{252}$$

The factors of 252 are: 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252

The greatest perfect square is $36 = 6 \cdot 6$, so write 252 as $36 \cdot 7$.

$$\begin{aligned} s &= \sqrt{36 \cdot 7} \\ &= \sqrt{36} \cdot \sqrt{7} \\ &= 6 \cdot \sqrt{7} \\ &= 6\sqrt{7} \end{aligned}$$

The square has side length $6\sqrt{7}$ ft.

15. The formula for the volume, V , of a cube with edge length e units is:

$$V = e^3$$

To determine the value of e , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{e^3}$$

$$\sqrt[3]{V} = e \quad \text{Substitute: } V = 200$$

$$e = \sqrt[3]{200}$$

The factors of 200 are: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 200 as $8 \cdot 25$.

$$\begin{aligned} e &= \sqrt[3]{8 \cdot 25} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{25} \\ &= 2 \cdot \sqrt[3]{25} \\ &= 2\sqrt[3]{25} \end{aligned}$$

The cube has edge length $2\sqrt[3]{25}$ cm.

16. The formula for the area, A , of a square with side length s units is:

$$A = s^2$$

To determine the value of s , take the square root of each side.

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s \quad \text{Substitute: } A = 54$$

$$s = \sqrt{54}$$

The factors of 54 are: 1, 2, 3, 6, 9, 18, 27, 54

The greatest perfect square is $9 = 3 \cdot 3$, so write 54 as $9 \cdot 6$.

$$s = \sqrt{9 \cdot 6}$$

$$= \sqrt{9} \cdot \sqrt{6}$$

$$= 3 \cdot \sqrt{6}$$

$$= 3\sqrt{6}$$

The formula for the perimeter, P , of a square with side length s units is:

$$P = 4s \quad \text{Substitute: } s = 3\sqrt{6}$$

$$= 4(3\sqrt{6})$$

$$= 12\sqrt{6}$$

The perimeter of the square is $12\sqrt{6}$ in.

17. a) The factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$, so write 48 as $16 \cdot 3$.

$$\sqrt[4]{48} = \sqrt[4]{16 \cdot 3}$$

$$= \sqrt[4]{16} \cdot \sqrt[4]{3}$$

$$= 2 \cdot \sqrt[4]{3}$$

$$= 2\sqrt[4]{3}$$

- b) The factors of 405 are: 1, 3, 5, 9, 15, 27, 45, 81, 135, 405

The greatest perfect fourth power is $81 = 3 \cdot 3 \cdot 3 \cdot 3$, so write 405 as $81 \cdot 5$.

$$\sqrt[4]{405} = \sqrt[4]{81 \cdot 5}$$

$$= \sqrt[4]{81} \cdot \sqrt[4]{5}$$

$$= 3 \cdot \sqrt[4]{5}$$

$$= 3\sqrt[4]{5}$$

- c) The factors of 1250 are: 1, 2, 5, 10, 25, 50, 125, 250, 625, 1250

The greatest perfect fourth power is $625 = 5 \cdot 5 \cdot 5 \cdot 5$, so write 1250 as $625 \cdot 2$.

$$\sqrt[4]{1250} = \sqrt[4]{625 \cdot 2}$$

$$= \sqrt[4]{625} \cdot \sqrt[4]{2}$$

$$= 5 \cdot \sqrt[4]{2}$$

$$= 5\sqrt[4]{2}$$

- d) The factors of 176 are: 1, 2, 4, 8, 11, 16, 22, 44, 88, 176
The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$, so write 176 as $16 \cdot 11$.

$$\begin{aligned}\sqrt[4]{176} &= \sqrt[4]{16 \cdot 11} \\ &= \sqrt[4]{16} \cdot \sqrt[4]{11} \\ &= 2 \cdot \sqrt[4]{11} \\ &= 2\sqrt[4]{11}\end{aligned}$$

18. a) Write 6 as: $\sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6} = \sqrt[4]{1296}$

$$\begin{aligned}6\sqrt[4]{3} &= \sqrt[4]{1296} \cdot \sqrt[4]{3} \\ &= \sqrt[4]{1296 \cdot 3} \\ &= \sqrt[4]{3888}\end{aligned}$$

- b) Write 7 as: $\sqrt[4]{7 \cdot 7 \cdot 7 \cdot 7} = \sqrt[4]{2401}$

$$\begin{aligned}7\sqrt[4]{2} &= \sqrt[4]{2401} \cdot \sqrt[4]{2} \\ &= \sqrt[4]{2401 \cdot 2} \\ &= \sqrt[4]{4802}\end{aligned}$$

- c) Write 3 as: $\sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[5]{243}$

$$\begin{aligned}3\sqrt[5]{4} &= \sqrt[5]{243} \cdot \sqrt[5]{4} \\ &= \sqrt[5]{243 \cdot 4} \\ &= \sqrt[5]{972}\end{aligned}$$

- d) Write 4 as: $\sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \sqrt[5]{1024}$

$$\begin{aligned}4\sqrt[5]{3} &= \sqrt[5]{1024} \cdot \sqrt[5]{3} \\ &= \sqrt[5]{1024 \cdot 3} \\ &= \sqrt[5]{3072}\end{aligned}$$

19. a) Use the Pythagorean Theorem in each right triangle, moving counterclockwise.
Each hypotenuse becomes a leg of the next triangle.

$$(h_1)^2 = 1^2 + 1^2$$

$$(h_1)^2 = 1 + 1$$

$$(h_1)^2 = 2$$

$$h_1 = \sqrt{2}$$

$$(h_2)^2 = (h_1)^2 + 1^2$$

$$(h_2)^2 = (\sqrt{2})^2 + 1$$

$$(h_2)^2 = 2 + 1$$

$$(h_2)^2 = 3$$

$$h_2 = \sqrt{3}$$

$$(h_3)^2 = (h_2)^2 + 1^2$$

$$(h_3)^2 = (\sqrt{3})^2 + 1$$

$$(h_3)^2 = 3 + 1$$

$$(h_3)^2 = 4$$

$$h_3 = \sqrt{4}$$

The pattern continues. The lengths of the hypotenuses, in units, are:

$$\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}, \sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}, \sqrt{14}$$

b) i) The radicand starts at 2 and increases by 1 each time.

ii) The 1st triangle has an hypotenuse of length $\sqrt{2}$ units.

The 2nd triangle has an hypotenuse of length $\sqrt{3}$ units.

The 3rd triangle has an hypotenuse of length $\sqrt{4}$ units.

The radicand is always 1 greater than the number of the triangle.

So, the 50th triangle has an hypotenuse of length $\sqrt{50+1}$, or $\sqrt{51}$ units.

iii) The radicands of the hypotenuse lengths of the first 100 triangles start at 2 and end at 101.

To be able to write an entire radical as a mixed radical, the radicand must be divisible by a perfect square, and not be a perfect square.

- The radicand could be divisible by 4, and not be a perfect square. So, list the multiples of 4 and cross out the perfect squares.

4, 8, 12, ~~16~~, 20, 24, 28, 32, ~~36~~, 40, 44, 48, 52, 56, 60, ~~64~~, 68, 72, 76, 80, 84, 88, 92, 96, ~~100~~

OR

- The radicand could be divisible by 9, and not be a perfect square. So, list the multiples of 9 and cross out the perfect squares.

9, 18, 27, ~~36~~, 45, 54, 63, 72, ~~81~~, 90, 99 (72 is in both lists, so it only counts once)

OR

- The radicand could be divisible by 25, and not be a perfect square. So, list the multiples of 25 and cross out the perfect squares.

~~25~~, 50, 75, ~~100~~

OR

- The radicand could be divisible by 49, and not be a perfect square. So, list the multiples of 49 and cross out the perfect squares.

~~49~~, 98

We do not have to find radicands that are divisible by 16, 36, and 64 because they are divisible by 4.

Count how many numbers have not been crossed out.

So, 30 of the first 100 triangles have hypotenuse lengths that can be written as mixed radicals.

20. In the second line, to write 8 as the cube root of a perfect cube, the student took the cube root of 8 instead of cubing 8.

A correct solution is:

$$\begin{aligned}
 8\sqrt[3]{2} &= 8 \cdot \sqrt[3]{2} \\
 &= \sqrt[3]{512} \cdot \sqrt[3]{2} \\
 &= \sqrt[3]{512 \cdot 2} \\
 &= \sqrt[3]{1024}
 \end{aligned}$$

21. In the first line, the student wrote $\sqrt{96}$ as $\sqrt{4} \cdot \sqrt{48}$ instead of $\sqrt{4} \cdot \sqrt{24}$. In the third line, the student wrote $\sqrt{48}$ as $\sqrt{8} \cdot \sqrt{6}$, which does not help in the simplification because neither 8 nor 6 is a perfect square. In the fourth line, the student wrote $\sqrt{8}$ as 4, which is not correct; 8 is not a perfect square.

A correct solution is:

The factors of 96 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

The greatest perfect square is $16 = 4 \cdot 4$, so write 96 as $16 \cdot 6$.

$$\begin{aligned}
 \sqrt{96} &= \sqrt{16 \cdot 6} \\
 &= \sqrt{16} \cdot \sqrt{6} \\
 &= 4 \cdot \sqrt{6} \\
 &= 4\sqrt{6}
 \end{aligned}$$

22. Since all the radicals are square roots, I will rewrite each mixed radical as an entire radical, then order the entire radicals from the greatest radicand to the least radicand.

a) Write 9 as: $\sqrt{9 \cdot 9} = \sqrt{81}$

$$\begin{aligned}
 9\sqrt{2} &= \sqrt{81} \cdot \sqrt{2} \\
 &= \sqrt{81 \cdot 2} \\
 &= \sqrt{162}
 \end{aligned}$$

Write 2 as: $\sqrt{2 \cdot 2} = \sqrt{4}$

$$\begin{aligned}
 2\sqrt{6} &= \sqrt{4} \cdot \sqrt{6} \\
 &= \sqrt{4 \cdot 6} \\
 &= \sqrt{24}
 \end{aligned}$$

Write 8 as: $\sqrt{8 \cdot 8} = \sqrt{64}$

$$\begin{aligned}
 8\sqrt{3} &= \sqrt{64} \cdot \sqrt{3} \\
 &= \sqrt{64 \cdot 3} \\
 &= \sqrt{192}
 \end{aligned}$$

Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned} 4\sqrt{5} &= \sqrt{16} \cdot \sqrt{5} \\ &= \sqrt{16 \cdot 5} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} \text{Write 6 as: } \sqrt{6 \cdot 6} &= \sqrt{36} \\ 6\sqrt{2} &= \sqrt{36} \cdot \sqrt{2} \\ &= \sqrt{36 \cdot 2} \\ &= \sqrt{72} \end{aligned}$$

From greatest to least, the entire radicals are: $\sqrt{192}$, $\sqrt{162}$, $\sqrt{80}$, $\sqrt{72}$, $\sqrt{24}$
So, from greatest to least, the mixed radicals are: $8\sqrt{3}$, $9\sqrt{2}$, $4\sqrt{5}$, $6\sqrt{2}$, $2\sqrt{6}$

b) Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned} 4\sqrt{7} &= \sqrt{16} \cdot \sqrt{7} \\ &= \sqrt{16 \cdot 7} \\ &= \sqrt{112} \end{aligned}$$

$$\begin{aligned} \text{Write 8 as: } \sqrt{8 \cdot 8} &= \sqrt{64} \\ 8\sqrt{3} &= \sqrt{64} \cdot \sqrt{3} \\ &= \sqrt{64 \cdot 3} \\ &= \sqrt{192} \end{aligned}$$

$$\begin{aligned} \text{Write 2 as: } \sqrt{2 \cdot 2} &= \sqrt{4} \\ 2\sqrt{13} &= \sqrt{4} \cdot \sqrt{13} \\ &= \sqrt{4 \cdot 13} \\ &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} \text{Write 6 as: } \sqrt{6 \cdot 6} &= \sqrt{36} \\ 6\sqrt{5} &= \sqrt{36} \cdot \sqrt{5} \\ &= \sqrt{36 \cdot 5} \\ &= \sqrt{180} \end{aligned}$$

From greatest to least, the entire radicals are: $\sqrt{192}$, $\sqrt{180}$, $\sqrt{112}$, $\sqrt{52}$
So, from greatest to least, the mixed radicals are: $8\sqrt{3}$, $6\sqrt{5}$, $4\sqrt{7}$, $2\sqrt{13}$

c) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$\begin{aligned} 7\sqrt{3} &= \sqrt{49} \cdot \sqrt{3} \\ &= \sqrt{49 \cdot 3} \\ &= \sqrt{147} \end{aligned}$$

$$\begin{aligned} \text{Write 9 as: } \sqrt{9 \cdot 9} &= \sqrt{81} \\ 9\sqrt{2} &= \sqrt{81} \cdot \sqrt{2} \\ &= \sqrt{81 \cdot 2} \\ &= \sqrt{162} \end{aligned}$$

$$\begin{aligned} \text{Write 5 as: } \sqrt{5 \cdot 5} &= \sqrt{25} \\ 5\sqrt{6} &= \sqrt{25} \cdot \sqrt{6} \\ &= \sqrt{25 \cdot 6} \\ &= \sqrt{150} \end{aligned}$$

$\sqrt{103}$ is an entire radical.

$$\begin{aligned} \text{Write 3 as: } \sqrt{3 \cdot 3} &= \sqrt{9} \\ 3\sqrt{17} &= \sqrt{9} \cdot \sqrt{17} \\ &= \sqrt{9 \cdot 17} \\ &= \sqrt{153} \end{aligned}$$

From greatest to least, the entire radicals are: $\sqrt{162}$, $\sqrt{153}$, $\sqrt{150}$, $\sqrt{147}$, $\sqrt{103}$

So, from greatest to least, the radicals are: $9\sqrt{2}$, $3\sqrt{17}$, $5\sqrt{6}$, $7\sqrt{3}$, $\sqrt{103}$

23. a) I know $\sqrt{4} = 2$.

$$\begin{aligned} \sqrt{400} &= \sqrt{4 \cdot 100} \\ &= \sqrt{4} \cdot \sqrt{100} \\ &= 2 \cdot 10 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \sqrt{40\,000} &= \sqrt{4 \cdot 10\,000} \\ &= \sqrt{4} \cdot \sqrt{10\,000} \\ &= 2 \cdot 100 \\ &= 200 \end{aligned}$$

In the entire radicals, the radicand starts at 4 and is multiplied by 100 each time.

The square root starts at 2, and is multiplied by 10 each time, which is the square root of 100.

The next two radicals are:

$$\begin{aligned}\sqrt{40\,000 \cdot 100} &= \sqrt{4\,000\,000} \\ \sqrt{4\,000\,000 \cdot 100} &= \sqrt{400\,000\,000}\end{aligned}$$

b) I know $\sqrt[3]{27} = 3$.

$$\begin{aligned}\sqrt[3]{27\,000} &= \sqrt[3]{27 \cdot 1000} \\ &= \sqrt[3]{27} \cdot \sqrt[3]{1000} \\ &= 3 \cdot 10 \\ &= 30 \\ \sqrt[3]{27\,000\,000} &= \sqrt[3]{27 \cdot 1\,000\,000} \\ &= \sqrt[3]{27} \cdot \sqrt[3]{1\,000\,000} \\ &= 3 \cdot 100 \\ &= 300\end{aligned}$$

In the entire radicals, the radicand starts at 27 and is multiplied by 1000 each time. The cube root starts at 3, and is multiplied by 10 each time, which is the cube root of 1000.

The next two radicals are:

$$\begin{aligned}\sqrt[3]{27\,000\,000 \cdot 1000} &= \sqrt[3]{27\,000\,000\,000} \\ \sqrt[3]{27\,000\,000\,000 \cdot 1000} &= \sqrt[3]{27\,000\,000\,000\,000}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{8} &= \sqrt{4 \cdot 2} \\ &= \sqrt{4} \cdot \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{800} &= \sqrt{400 \cdot 2} \\ &= \sqrt{400} \cdot \sqrt{2} \\ &= 20\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{80\,000} &= \sqrt{40\,000 \cdot 2} \\ &= \sqrt{40\,000} \cdot \sqrt{2} \\ &= 200\sqrt{2}\end{aligned}$$

In the entire radicals, the radicand starts at 8 and is multiplied by 100 each time. In the mixed radicals, the number in front of the radical starts at 2, and is multiplied by 10 each time, which is the square root of 100. The radicand is always 2.

The next two radicals are:

$$\begin{aligned}\sqrt{80\,000 \cdot 100} &= \sqrt{8\,000\,000} \\ \sqrt{8\,000\,000 \cdot 100} &= \sqrt{800\,000\,000}\end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt[3]{24} &= \sqrt[3]{8 \cdot 3} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{3} \\ &= 2\sqrt[3]{3} \end{aligned}$$

$$\begin{aligned} \sqrt[3]{24\,000} &= \sqrt[3]{8000 \cdot 3} \\ &= \sqrt[3]{8000} \cdot \sqrt[3]{3} \\ &= 20\sqrt[3]{3} \end{aligned}$$

$$\begin{aligned} \sqrt[3]{24\,000\,000} &= \sqrt[3]{8\,000\,000 \cdot 3} \\ &= \sqrt[3]{8\,000\,000} \cdot \sqrt[3]{3} \\ &= 200\sqrt[3]{3} \end{aligned}$$

In the entire radicals, the radicand starts at 24 and is multiplied by 1000 each time.

In the mixed radicals, the number in front of the radical starts at 2, and is multiplied by 10 each time, which is the cube root of 1000.

The next two radicals are:

$$\begin{aligned} \sqrt[3]{24\,000\,000 \cdot 1000} &= \sqrt[3]{24\,000\,000\,000} \\ \sqrt[3]{24\,000\,000\,000 \cdot 1000} &= \sqrt[3]{24\,000\,000\,000\,000} \end{aligned}$$

C

24. Label the vertices of the squares.

In $\triangle DEH$, $DE = DH$

$$\begin{aligned} DE &= \frac{1}{2}(DA) \\ &= \frac{1}{2}(8) \\ &= 4 \end{aligned}$$

Use the Pythagorean Theorem in right $\triangle DEH$ to determine EH , the side length of square $EFGH$.

$$EH^2 = DE^2 + DH^2$$

$$EH^2 = 4^2 + 4^2$$

$$EH^2 = 16 + 16$$

$$EH^2 = 32$$

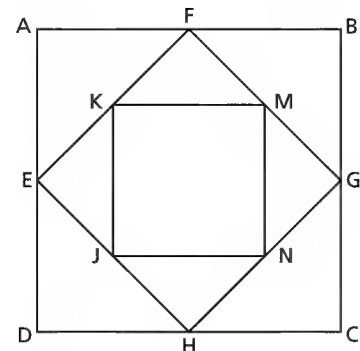
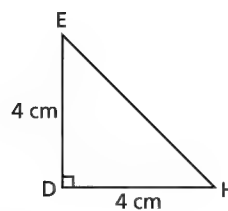
$$EH = \sqrt{32}$$

$$= \sqrt{16 \cdot 2}$$

$$= \sqrt{16} \cdot \sqrt{2}$$

$$= 4\sqrt{2}$$

Square $EFGH$ has side length $4\sqrt{2}$ cm.



Use the formula for the area, A , of a square with side length s :

$$A = s^2$$

$$= (4\sqrt{2})^2$$

$$= 4\sqrt{2} \cdot 4\sqrt{2}$$

$$= 16 \cdot 2$$

$$= 32$$

Square EFGH has area 32 cm^2 .

In $\triangle JEK$, $JE = EK$

$$JE = \frac{1}{2}(EH)$$

$$= \frac{1}{2}(4\sqrt{2})$$

$$= 2\sqrt{2}$$

Use the Pythagorean Theorem in right $\triangle JEK$ to determine KJ , the side length of square JKMN.

$$KJ^2 = JE^2 + EK^2$$

$$KJ^2 = (2\sqrt{2})^2 + (2\sqrt{2})^2$$

$$KJ^2 = (4 \cdot 2) + (4 \cdot 2)$$

$$KJ^2 = 8 + 8$$

$$KJ^2 = 16$$

$$KJ = \sqrt{16}$$

$$KJ = 4$$

Square JKMN has side length 4 cm.

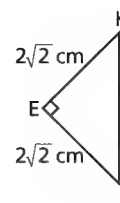
Use the formula for the area, A , of a square with side length s :

$$A = s^2$$

$$= 4^2$$

$$= 16$$

Square JKMN has area 16 cm^2 .



25. a) i)

$$\sqrt{200} = \sqrt{100 \cdot 2}$$

$$= \sqrt{100} \cdot \sqrt{2}$$

$$= 10\sqrt{2}$$

$$\text{Since } \sqrt{2} \doteq 1.4142, 10\sqrt{2} \doteq 10(1.4142)$$

$$= 14.142$$

$$\text{So, } \sqrt{200} \doteq 14.142$$

$$\begin{aligned}\text{ii) } \sqrt{20\,000} &= \sqrt{10\,000 \cdot 2} \\ &= \sqrt{10\,000} \cdot \sqrt{2} \\ &= 100\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Since } \sqrt{2} \doteq 1.4142, \quad 100\sqrt{2} &\doteq 100(1.4142) \\ &= 141.42\end{aligned}$$

$$\text{So, } \sqrt{20\,000} \doteq 141.42$$

$$\begin{aligned}\text{b) i) } \sqrt{8} &= \sqrt{4 \cdot 2} \\ &= \sqrt{4} \cdot \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Since } \sqrt{2} \doteq 1.4142, \quad 2\sqrt{2} &\doteq 2(1.4142) \\ &= 2.8284\end{aligned}$$

$$\text{So, } \sqrt{8} \doteq 2.8284$$

$$\begin{aligned}\text{ii) } \sqrt{18} &= \sqrt{9 \cdot 2} \\ &= \sqrt{9} \cdot \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Since } \sqrt{2} \doteq 1.4142, \quad 3\sqrt{2} &\doteq 3(1.4142) \\ &= 4.2426\end{aligned}$$

$$\text{So, } \sqrt{18} \doteq 4.2426$$

$$\begin{aligned}\text{iii) } \sqrt{32} &= \sqrt{16 \cdot 2} \\ &= \sqrt{16} \cdot \sqrt{2} \\ &= 4\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Since } \sqrt{2} \doteq 1.4142, \quad 4\sqrt{2} &\doteq 4(1.4142) \\ &= 5.6568\end{aligned}$$

$$\text{So, } \sqrt{32} \doteq 5.6568$$

$$\begin{aligned}\text{iv) } \sqrt{50} &= \sqrt{25 \cdot 2} \\ &= \sqrt{25} \cdot \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Since } \sqrt{2} \doteq 1.4142, \quad 5\sqrt{2} &\doteq 5(1.4142) \\ &= 7.071\end{aligned}$$

$$\text{So, } \sqrt{50} \doteq 7.071$$

Checkpoint 1

Assess Your Understanding (page 221)

4.1

$$1. \quad a) \quad \sqrt{81} = \sqrt{9 \cdot 9} \\ = 9$$

$$b) \quad \sqrt[3]{-125} = \sqrt[3]{(-5)(-5)(-5)} \\ = -5$$

$$c) \quad \sqrt[4]{256} = \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4} \\ = 4$$

$$d) \quad \sqrt[5]{243} = \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} \\ = 3$$

I wrote the radicand as the product of the same number of equal factors as the index of the radical. For example, when the index was 3, I wrote the radicand as the product of 3 equal factors.

2. Instead of using the root keys on a calculator, I can use benchmarks with guess and check.

- a) 10 is between the perfect squares 9 and 16, but closer to 9.

So, $\sqrt{10}$ is between 3 and 4, but closer to 3.

Estimate to 1 decimal place: $\sqrt{10} \doteq 3.1$

Square the estimate: $3.1^2 = 9.61$ (too small, but close)

Revise the estimate: $\sqrt{10} \doteq 3.2$

Square the estimate: $3.2^2 = 10.24$ (too large, but close)

$\sqrt{10}$ is between 3.1 and 3.2, but closer to 3.2.

Now, estimate to 2 decimal places.

Revise the estimate: $\sqrt{10} \doteq 3.16$

Square the estimate: $3.16^2 = 9.9856$ (too small, very close)

Revise the estimate: $\sqrt{10} \doteq 3.17$

Square the estimate: $3.17^2 = 10.0489$ (too large, very close)

9.9856 is closer to 10, so $\sqrt{10}$ is approximately 3.16.

- b) 15 is between the perfect cubes 8 and 27, but closer to 8.

So, $\sqrt[3]{15}$ is between 2 and 3, but closer to 2.

Estimate to 1 decimal place: $\sqrt[3]{15} \doteq 2.4$

Cube the estimate: $2.4^3 = 13.824$ (too small)

Revise the estimate: $\sqrt[3]{15} \doteq 2.5$

Cube the estimate: $2.5^3 = 15.625$ (too large, but close)

$\sqrt[3]{15}$ is between 2.4 and 2.5, but closer to 2.5.

Now, estimate to 2 decimal places.

Revise the estimate: $\sqrt[3]{15} \doteq 2.46$

Cube the estimate: $2.46^3 = 14.886\,936$ (too small, very close)

Revise the estimate: $\sqrt[3]{15} \doteq 2.47$

Cube the estimate: $2.47^3 = 15.069\,223$ (too large, very close)

15.069 223 is closer to 15, so $\sqrt[3]{15}$ is approximately 2.47.

- c) 9 is between the perfect fourth powers 1 and 16, but closer to 16.

So, $\sqrt[4]{9}$ is between 1 and 2, but closer to 2.

Estimate to 1 decimal place: $\sqrt[4]{9} \doteq 1.7$

Raise the estimate to the fourth power: $1.7^4 = 8.3521$ (too small)

Revise the estimate: $\sqrt[4]{9} \doteq 1.8$

Raise the estimate to the fourth power: $1.8^4 = 10.4976$ (too large)

$\sqrt[4]{9}$ is between 1.7 and 1.8, but closer to 1.7.

Now, estimate to 2 decimal places.

Revise the estimate: $\sqrt[4]{9} \doteq 1.73$

Raise the estimate to the fourth power: $1.73^4 = 8.957\,450\,41\dots$ (too small, very close)

Revise the estimate: $\sqrt[4]{9} \doteq 1.74$

Raise the estimate to the fourth power: $1.74^4 = 9.166\,361\,76\dots$ (too large, close)

8.957 450 41... is closer to 9, so $\sqrt[4]{9}$ is approximately 1.73.

- d) 23 is between the perfect fifth powers 1 and 32, but closer to 32.

So, $\sqrt[5]{23}$ is between 1 and 2, but closer to 2.

Estimate to 1 decimal place: $\sqrt[5]{23} \doteq 1.8$

Raise the estimate to the fifth power: $1.8^5 = 18.895\,68$ (too small)

Revise the estimate: $\sqrt[5]{23} \doteq 1.9$

Raise the estimate to the fifth power: $1.9^5 = 24.760\,99$ (too large, but close)

$\sqrt[5]{23}$ is between 1.8 and 1.9, but closer to 1.9.

Now, estimate to 2 decimal places.

Revise the estimate: $\sqrt[5]{23} \doteq 1.88$

Raise the estimate to the fifth power: $1.88^5 = 23.484\,928\,72\dots$ (too large, close)

Revise the estimate: $\sqrt[5]{23} \doteq 1.87$

Raise the estimate to the fifth power: $1.87^5 = 22.866\,938\,97\dots$ (too small, very close)

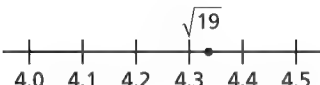
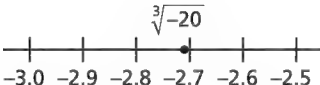
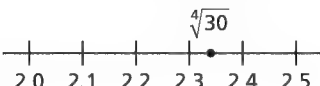
22.866 938 97... is closer to 23, so $\sqrt[5]{23}$ is approximately 1.87.

3. I used my calculator. $\sqrt[4]{60} \doteq 2.783\,157\,684$

The decimal representation fills the calculator screen.

The decimal does not appear to repeat and it does not appear to terminate. The decimal representation appears to go on forever.

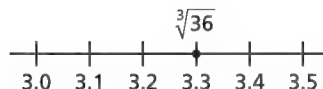
4.2

4. a) $\sqrt{11}$ is irrational because 11 is not a perfect square.
The decimal form of $\sqrt{11}$ neither terminates nor repeats.
- b) $\sqrt[3]{16}$ is irrational because 16 is not a perfect cube.
The decimal form of $\sqrt[3]{16}$ neither terminates nor repeats.
- c) $\sqrt[3]{-16}$ is irrational because -16 is not a perfect cube.
The decimal form of $\sqrt[3]{-16}$ neither terminates nor repeats.
- d) $\sqrt{121}$ is rational because 121 is a perfect square.
Its decimal form is 11.0, which terminates.
- e) $\sqrt{\frac{121}{16}}$ is rational because $\frac{121}{16}$ is a perfect square.
 $\sqrt{\frac{121}{16}} = \frac{11}{4}$ or 2.75, which is a terminating decimal
- f) $\sqrt{12.1}$ is irrational because 12.1 is not a perfect square.
The decimal form of $\sqrt{12.1}$ neither terminates nor repeats.
5. I will use benchmarks to estimate the position of each irrational number, then use a calculator to refine the estimate.
- a) 19 is between the perfect squares 16 and 25, and is closer to 16.
- | | | |
|-------------|-------------|-------------|
| $\sqrt{16}$ | $\sqrt{19}$ | $\sqrt{25}$ |
| ↓ | ↓ | ↓ |
| 4 | ? | 5 |
- Use a calculator.
 $\sqrt{19} = 4.3588\dots$
- 
- b) -20 is between the perfect cubes -8 and -27 , and is closer to -27 .
- | | | |
|----------------|-----------------|-----------------|
| $\sqrt[3]{-8}$ | $\sqrt[3]{-20}$ | $\sqrt[3]{-27}$ |
| ↓ | ↓ | ↓ |
| -2 | ? | -3 |
- Use a calculator.
 $\sqrt[3]{-20} = -2.7144\dots$
- 
- c) 30 is between the perfect fourth powers 16 and 81, and is closer to 16.
- | | | |
|----------------|----------------|----------------|
| $\sqrt[4]{16}$ | $\sqrt[4]{30}$ | $\sqrt[4]{81}$ |
| ↓ | ↓ | ↓ |
| 2 | ? | 3 |
- Use a calculator.
- 

$$\sqrt[4]{30} = 2.3403\dots$$

- d) 36 is between the perfect cubes 27 and 64, and is closer to 27.

$$\begin{array}{ccc} \sqrt[3]{27} & \sqrt[3]{36} & \sqrt[3]{64} \\ \downarrow & \downarrow & \downarrow \\ 3 & ? & 4 \end{array}$$



Use a calculator.

$$\sqrt[3]{36} = 3.3019\dots$$

6. a) Some numbers belong to more than one set.

Place each of these numbers in the smallest set to which it belongs.

- i) $3\frac{1}{3}$ is a rational number because it can be written as the quotient of two integers: $\frac{10}{3}$

- ii) -42 is an integer.

- iii) 4.5 is a rational number because it is a terminating decimal.

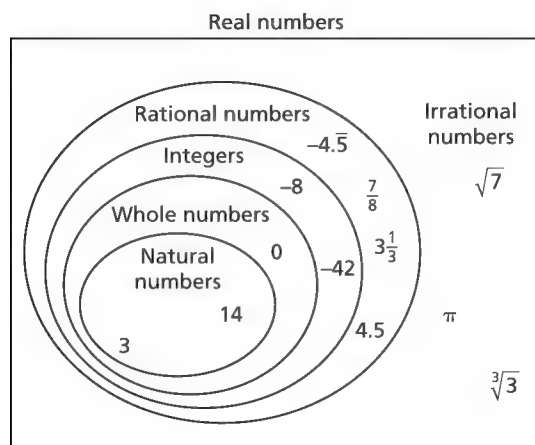
- iv) $-4.\bar{5}$ is a rational number because it is a repeating decimal.

- v) 0 is a whole number.

- vi) 14 is a natural number.

- vii) $\sqrt{7}$ is an irrational number because 7 is not a perfect square.

- viii) π is an irrational number because its decimal representation neither terminates nor repeats.



- b) Answers may vary.

Natural number: I chose a counting number, 3.

Whole number: I cannot choose another whole number that is not a natural number.

Integer: I chose -8 .

Rational number: I chose the fraction $\frac{7}{8}$ because it is the quotient of two integers.

Irrational number: I chose the cube root of a number that is not a perfect cube, $\sqrt[3]{3}$.

7. a) i) 32 is between the perfect squares 25 and 36, and is closer to 36.

$$\begin{array}{ccc} \sqrt{25} & \sqrt{32} & \sqrt{36} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$5 \quad ? \quad 6$$

Use a calculator.

$$\sqrt{32} = 5.6568\dots$$

- ii) 72 is between the perfect cubes 64 and 125, and is closer to 64.

$$\begin{array}{ccc} \sqrt[3]{64} & \sqrt[3]{72} & \sqrt[3]{125} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$4 \quad ? \quad 5$$

Use a calculator.

$$\sqrt[3]{72} = 4.1601\dots$$

- iii) 100 is between the perfect fourth powers 81 and 256, and is closer to 81.

$$\begin{array}{ccc} \sqrt[4]{81} & \sqrt[4]{100} & \sqrt[4]{256} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$3 \quad ? \quad 4$$

Use a calculator.

$$\sqrt[4]{100} = 3.1622\dots$$

- iv) 50 is between the perfect cubes 27 and 64, and is closer to 64.

$$\begin{array}{ccc} \sqrt[3]{27} & \sqrt[3]{50} & \sqrt[3]{64} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$3 \quad ? \quad 4$$

Use a calculator.

$$\sqrt[3]{50} = 3.6840\dots$$

- v) 65 is between the perfect squares 64 and 81, and is closer to 64.

$$\begin{array}{ccc} \sqrt{64} & \sqrt{65} & \sqrt{81} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$8 \quad ? \quad 9$$

Use a calculator.

$$\sqrt{65} = 8.0622\dots$$

- vi) 60 is between the perfect fourth powers 16 and 81, and is closer to 81.

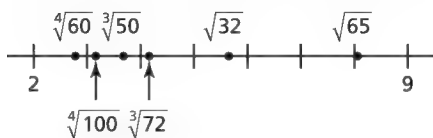
$$\begin{array}{ccc} \sqrt[4]{16} & \sqrt[4]{60} & \sqrt[4]{81} \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$2 \quad ? \quad 3$$

Use a calculator.

$$\sqrt[4]{60} = 2.7831\dots$$

Mark each number on a number line.



b) From greatest to least, the numbers are: $\sqrt{65}$, $\sqrt{32}$, $\sqrt[3]{72}$, $\sqrt[3]{50}$, $\sqrt[4]{100}$, $\sqrt[4]{60}$

8. Answers may vary.

- a) For the perimeter of the square to be rational, the side length of the square must be rational. Choose a rational number for the side length, 1.2 m.

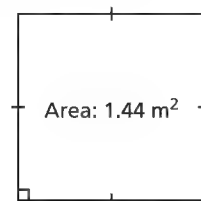
Use the formula for the area, A , of a square with side length s :

$$A = s^2 \quad \text{Substitute: } s = 1.2$$

$$A = 1.2^2$$

$$A = 1.44$$

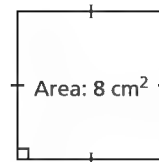
So, the square could have area 1.44 m^2 .



- b) For the perimeter of the square to be irrational, the side length of the square must be irrational. Since the side length, s , of a square with area A is:

$$s = \sqrt{A}, A \text{ cannot be a perfect square.}$$

Since 8 is not a perfect square, the square could have area 8 cm^2 .



4.3

9. a) The factors of 45 are: 1, 3, 5, 9, 15, 45
The greatest perfect square is $9 = 3 \cdot 3$, so write 45 as $9 \cdot 5$.

$$\begin{aligned} \sqrt{45} &= \sqrt{9 \cdot 5} \\ &= \sqrt{9} \cdot \sqrt{5} \\ &= 3 \cdot \sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

- b) The factors of 96 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 96 as $8 \cdot 12$.

$$\begin{aligned} \sqrt[3]{96} &= \sqrt[3]{8 \cdot 12} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{12} \\ &= 2 \cdot \sqrt[3]{12} \\ &= 2\sqrt[3]{12} \end{aligned}$$

- c) The factors of 17 are: 1, 17
There are no perfect square factors other than 1.
So, $\sqrt{17}$ cannot be simplified.
- d) The factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$, so write 48 as $16 \cdot 3$.

$$\begin{aligned}\sqrt[4]{48} &= \sqrt[4]{16 \cdot 3} \\ &= \sqrt[4]{16} \cdot \sqrt[4]{3} \\ &= 2 \cdot \sqrt[4]{3} \\ &= 2\sqrt[4]{3}\end{aligned}$$
- e) The factors of 80 are: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$, so write 80 as $8 \cdot 10$.

$$\begin{aligned}\sqrt[3]{80} &= \sqrt[3]{8 \cdot 10} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{10} \\ &= 2 \cdot \sqrt[3]{10} \\ &= 2\sqrt[3]{10}\end{aligned}$$
- f) The factors of 50 are: 1, 2, 5, 10, 25, 50
There are no perfect fourth power factors other than 1.
So, $\sqrt[4]{50}$ cannot be simplified.

10. Answers may vary.

I chose $\sqrt{45}$.

List all the factors of 45.

Look for the greatest factor that is a perfect square.

Write the radicand as the product of two factors, one of which is the greatest perfect square factor.

Use the multiplication property of radicals to write the radical as the product of two radicals.

Evaluate the radical whose radicand is a perfect square.

The radical is now in simplest form.

11. a) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{7} &= \sqrt{9} \cdot \sqrt{7} \\ &= \sqrt{9 \cdot 7} \\ &= \sqrt{63}\end{aligned}$$

b) Write 2 as: $\sqrt[3]{2 \cdot 2 \cdot 2} = \sqrt[3]{8}$

$$\begin{aligned}2\sqrt[3]{4} &= \sqrt[3]{8} \cdot \sqrt[3]{4} \\ &= \sqrt[3]{8 \cdot 4} \\ &= \sqrt[3]{32}\end{aligned}$$

c) Write 7 as: $\sqrt{7 \cdot 7} = \sqrt{49}$

$$\begin{aligned} 7\sqrt{3} &= \sqrt{49} \cdot \sqrt{3} \\ &= \sqrt{49 \cdot 3} \\ &= \sqrt{147} \end{aligned}$$

d) Write 2 as: $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$

$$\begin{aligned} 2\sqrt[4]{12} &= \sqrt[4]{16} \cdot \sqrt[4]{12} \\ &= \sqrt[4]{16 \cdot 12} \\ &= \sqrt[4]{192} \end{aligned}$$

e) Write 3 as: $\sqrt[3]{3 \cdot 3 \cdot 3} = \sqrt[3]{27}$

$$\begin{aligned} 3\sqrt[3]{10} &= \sqrt[3]{27} \cdot \sqrt[3]{10} \\ &= \sqrt[3]{27 \cdot 10} \\ &= \sqrt[3]{270} \end{aligned}$$

f) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned} 6\sqrt{11} &= \sqrt{36} \cdot \sqrt{11} \\ &= \sqrt{36 \cdot 11} \\ &= \sqrt{396} \end{aligned}$$

Lesson 4.4 Fractional Exponents and Radicals

Exercises (pages 227–228)

A

3. The denominator of the exponent is the index of the radical.

$$\begin{aligned}\text{a) } 16^{\frac{1}{2}} &= \sqrt{16} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{b) } 36^{\frac{1}{2}} &= \sqrt{36} \\ &= 6\end{aligned}$$

$$\begin{aligned}\text{c) } 64^{\frac{1}{3}} &= \sqrt[3]{64} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{d) } 32^{\frac{1}{5}} &= \sqrt[5]{32} \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{e) } (-27)^{\frac{1}{3}} &= \sqrt[3]{-27} \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{f) } (-1000)^{\frac{1}{3}} &= \sqrt[3]{-1000} \\ &= -10\end{aligned}$$

4. Write each decimal exponent as a fraction.

$$\text{a) } 0.5 = \frac{1}{2}$$

$$\text{b) } 0.25 = \frac{1}{4}$$

$$\begin{aligned}\text{So, } 100^{0.5} &= 100^{\frac{1}{2}} \\ &= \sqrt{100} \\ &= 10\end{aligned}$$

$$\begin{aligned}\text{So, } 81^{0.25} &= 81^{\frac{1}{4}} \\ &= \sqrt[4]{81} \\ &= 3\end{aligned}$$

$$\text{c) } 0.2 = \frac{1}{5}$$

$$\text{d) } 0.2 = \frac{1}{5}$$

$$\begin{aligned}\text{So, } 1024^{0.2} &= 1024^{\frac{1}{5}} \\ &= \sqrt[5]{1024} \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{So, } (-32)^{0.2} &= (-32)^{\frac{1}{5}} \\ &= \sqrt[5]{-32} \\ &= -2\end{aligned}$$

5. Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$\text{a) } 36^{\frac{1}{3}} = \sqrt[3]{36}$$

$$\text{b) } 48^{\frac{1}{2}} = \sqrt{48}$$

$$\text{c) } (-30)^{\frac{1}{5}} = \sqrt[5]{-30}$$

6. Use the rule: $\sqrt[n]{x} = x^{\frac{1}{n}}$

$$\text{a) } \sqrt{39} = 39^{\frac{1}{2}}$$

$$\text{b) } \sqrt[4]{90} = 90^{\frac{1}{4}}$$

$$\text{c) } \sqrt[3]{29} = 29^{\frac{1}{3}}$$

$$\text{d) } \sqrt[5]{100} = 100^{\frac{1}{5}}$$

7. a) Any number raised to the exponent 0 is 1.

$$8^0 = 1$$

b) $8^{\frac{1}{3}} = \sqrt[3]{8}$
 $= 2$

c) $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$
 $= \left(\sqrt[3]{8}\right)^2$
 $= 2^2$
 $= 4$

d) $8^{\frac{3}{3}} = 8^1$
 $= 8$

e) $8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4$
 $= \left(\sqrt[3]{8}\right)^4$
 $= 2^4$
 $= 16$

f) $8^{\frac{5}{3}} = \left(8^{\frac{1}{3}}\right)^5$
 $= \left(\sqrt[3]{8}\right)^5$
 $= 2^5$
 $= 32$

B

8. Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

a) $4^{\frac{2}{3}} = \left(\sqrt[3]{4}\right)^2$ or $\sqrt[3]{4^2}$

b) $(-10)^{\frac{3}{5}} = \left(\sqrt[5]{-10}\right)^3$ or $\sqrt[5]{(-10)^3}$

c) $2.3^{\frac{3}{2}} = \left(\sqrt{2.3}\right)^3$ or $\sqrt{2.3^3}$

9. The formula for the volume, V , of a cube with edge length e units is:
 $V = e^3$

To determine the value of e , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{e^3}$$

$$\sqrt[3]{V} = e \quad \text{Substitute: } V = 350$$

$$e = \sqrt[3]{350} \text{ or } e = 350^{\frac{1}{3}}$$

The cube has edge length $\sqrt[3]{350}$ cm or $350^{\frac{1}{3}}$ cm.

10. Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

a) $48^{\frac{2}{3}} = \left(\sqrt[3]{48}\right)^2$ or $\sqrt[3]{48^2}$

b) $(-1.8)^{\frac{5}{3}} = \left(\sqrt[3]{-1.8}\right)^5$ or $\sqrt[3]{(-1.8)^5}$

c) $2.5 = \frac{5}{2}$

$$\begin{aligned} \text{So, } \left(\frac{3}{8}\right)^{2.5} &= \left(\frac{3}{8}\right)^{\frac{5}{2}} \\ &= \left(\sqrt{\frac{3}{8}}\right)^5 \text{ or } \sqrt{\left(\frac{3}{8}\right)^5} \end{aligned}$$

d) $0.75 = \frac{3}{4}$

$$\begin{aligned} \text{So, } 0.75^{0.75} &= (0.75)^{\frac{3}{4}} \\ &= \left(\sqrt[4]{0.75}\right)^3 \text{ or } \sqrt[4]{0.75^3} \end{aligned}$$

$$\text{e) } \left(-\frac{5}{9}\right)^{\frac{2}{5}} = \left(\sqrt[5]{-\frac{5}{9}}\right)^2 \text{ or } \sqrt[5]{\left(-\frac{5}{9}\right)^2}$$

$$\text{f) } 1.5 = \frac{3}{2}$$

$$\begin{aligned} \text{So, } 1.25^{1.5} &= 1.25^{\frac{3}{2}} \\ &= \left(\sqrt{1.25}\right)^3 \text{ or } \sqrt{1.25^3} \end{aligned}$$

11. Use: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$$\text{a) } \sqrt{3.8^3} = 3.8^{\frac{3}{2}}$$

$$\text{Or, since } \frac{3}{2} = 1.5, \sqrt{3.8^3} = 3.8^{1.5}$$

$$\text{b) } \left(\sqrt[3]{-1.5}\right)^2 = (-1.5)^{\frac{2}{3}}$$

$$\text{c) } \sqrt[4]{\left(\frac{9}{5}\right)^5} = \left(\frac{9}{5}\right)^{\frac{5}{4}}$$

$$\text{Or, since } \frac{5}{4} = 1.25, \sqrt[4]{\left(\frac{9}{5}\right)^5} = \left(\frac{9}{5}\right)^{1.25}$$

$$\text{d) } \sqrt[3]{\left(\frac{3}{8}\right)^4} = \left(\frac{3}{8}\right)^{\frac{4}{3}}$$

$$\text{e) } \left(\sqrt{\frac{5}{4}}\right)^3 = \left(\frac{5}{4}\right)^{\frac{3}{2}}$$

$$\text{Or, since } \frac{3}{2} = 1.5, \left(\sqrt{\frac{5}{4}}\right)^3 = \left(\frac{5}{4}\right)^{1.5}$$

$$\text{f) } \sqrt[5]{(-2.5)^3} = (-2.5)^{\frac{3}{5}}$$

$$\text{Or, since } \frac{3}{5} = 0.6, \sqrt[5]{(-2.5)^3} = (-2.5)^{0.6}$$

$$\begin{aligned} 12. \text{ a) } 9^{\frac{3}{2}} &= \left(9^{\frac{1}{2}}\right)^3 \\ &= (\sqrt{9})^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{27}{8}\right)^{\frac{2}{3}} &= \left[\left(\frac{27}{8}\right)^{\frac{1}{3}}\right]^2 \\ &= \left(\sqrt[3]{\frac{27}{8}}\right)^2 \\ &= \left(\frac{3}{2}\right)^2 \\ &= \frac{9}{4} \end{aligned}$$

$$\begin{aligned} \text{c) } (-27)^{\frac{2}{3}} &= \left[(-27)^{\frac{1}{3}}\right]^2 \\ &= \left(\sqrt[3]{-27}\right)^2 \\ &= (-3)^2 \\ &= 9 \end{aligned}$$

$$\text{d) The exponent } 1.5 = \frac{3}{2}$$

$$\begin{aligned} \text{So, } 0.36^{1.5} &= 0.36^{\frac{3}{2}} \\ &= \left(0.36^{\frac{1}{2}}\right)^3 \\ &= (\sqrt{0.36})^3 \\ &= 0.6^3 \\ &= 0.216 \end{aligned}$$

$$\begin{aligned} \text{e) } (-64)^{\frac{2}{3}} &= \left[(-64)^{\frac{1}{3}}\right]^2 \\ &= \left(\sqrt[3]{-64}\right)^2 \\ &= (-4)^2 \\ &= 16 \end{aligned}$$

$$\begin{aligned}
 \text{f) } \left(\frac{4}{25}\right)^3 &= \left[\left(\frac{4}{25}\right)^{\frac{1}{2}}\right]^3 \\
 &= \left(\sqrt{\frac{4}{25}}\right)^3 \\
 &= \left(\frac{2}{5}\right)^3 \\
 &= \frac{8}{125}
 \end{aligned}$$

13. Raising a number to the exponent $\frac{1}{2}$ is equivalent to taking the square root of the number.

So, to write an equivalent form of each number using a power with exponent $\frac{1}{2}$, square the number, then write it as a square root.

a) $2^2 = 4$

So, $2 = 4^{\frac{1}{2}}$, or $\sqrt{4}$

b) $4^2 = 16$

So, $4 = 16^{\frac{1}{2}}$, or $\sqrt{16}$

c) $10^2 = 100$

So, $10 = 100^{\frac{1}{2}}$, or $\sqrt{100}$

d) $3^2 = 9$

So, $3 = 9^{\frac{1}{2}}$, or $\sqrt{9}$

e) $5^2 = 25$

So, $5 = 25^{\frac{1}{2}}$, or $\sqrt{25}$

14. Raising a number to the exponent $\frac{1}{3}$ is equivalent to taking the cube root of the number.

So, to write an equivalent form of each number using a power with exponent $\frac{1}{3}$, cube the number, then write it as a cube root.

a) $(-1)^3 = -1$

So, $-1 = (-1)^{\frac{1}{3}}$, or $\sqrt[3]{-1}$

b) $2^3 = 8$

So, $2 = 8^{\frac{1}{3}}$, or $\sqrt[3]{8}$

c) $3^3 = 27$

So, $3 = 27^{\frac{1}{3}}$, or $\sqrt[3]{27}$

d) $(-4)^3 = -64$

So, $-4 = (-64)^{\frac{1}{3}}$, or $\sqrt[3]{-64}$

e) $4^3 = 64$

So, $4 = 64^{\frac{1}{3}}$, or $\sqrt[3]{64}$

15. Since 4 is a perfect square, and $4^{\frac{3}{2}}$ and $\left(\frac{1}{4}\right)^2$ involve the square root of 4, I will evaluate

these numbers without a calculator. I can evaluate 4^2 using mental math. Because 4 is not a perfect cube, I will use a calculator to evaluate $\sqrt[3]{4}$.

Use a calculator: $\sqrt[3]{4} = 1.5874\dots$

$$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3$$

$$= (\sqrt{4})^3$$

$$= 2^3$$

$$= 8$$

$$4^2 = 16$$

$$\left(\frac{1}{4}\right)^2 = \left[\left(\frac{1}{4}\right)^{\frac{1}{2}}\right]^3$$

$$= \left(\sqrt{\frac{1}{4}}\right)^3$$

$$= \left(\frac{1}{2}\right)^3$$

$$= \frac{1}{8}$$

So, from least to greatest, the numbers are: $\left(\frac{1}{4}\right)^2$, $\sqrt[3]{4}$, $4^{\frac{3}{2}}$, 4^2

16. a) i) The exponent $1.5 = \frac{3}{2}$

$$\text{So, } 16^{1.5} = 16^{\frac{3}{2}}$$

$$= \left(16^{\frac{1}{2}}\right)^3$$

$$= (\sqrt{16})^3$$

$$= 4^3$$

$$= 64$$

ii) The exponent $0.75 = \frac{3}{4}$

$$\begin{aligned}\text{So, } 81^{0.75} &= 81^{\frac{3}{4}} \\ &= \left(81^{\frac{1}{4}}\right)^3 \\ &= \left(\sqrt[4]{81}\right)^3 \\ &= 3^3 \\ &= 27\end{aligned}$$

iii) The exponent $0.8 = \frac{4}{5}$

$$\begin{aligned}\text{So, } (-32)^{0.8} &= (-32)^{\frac{4}{5}} \\ &= \left[(-32)^{\frac{1}{5}}\right]^4 \\ &= \left(\sqrt[5]{-32}\right)^4 \\ &= (-2)^4 \\ &= 16\end{aligned}$$

iv) The exponent $0.5 = \frac{1}{2}$

$$\begin{aligned}\text{So, } 35^{0.5} &= 35^{\frac{1}{2}} \\ &= \sqrt{35} \quad \text{Since 35 is not a perfect square, use a calculator.} \\ &= 5.9160...\end{aligned}$$

v) The exponent $1.5 = \frac{3}{2}$

$$\begin{aligned}\text{So, } 1.21^{1.5} &= 1.21^{\frac{3}{2}} \\ &= \left(1.21^{\frac{1}{2}}\right)^3 \\ &= \left(\sqrt{1.21}\right)^3 \\ &= 1.1^3 \\ &= 1.331\end{aligned}$$

vi) The exponent $0.6 = \frac{3}{5}$

So,

$$\left(\frac{3}{4}\right)^{0.6} = \left(\frac{3}{4}\right)^{\frac{3}{5}}$$

$$= \left(\sqrt[5]{\frac{3}{4}}\right)^3$$

Since $\frac{3}{4}$ is not a perfect fifth power, use a calculator.
= 0.8414...

b) I was able to evaluate the powers in parts i, ii, iii, and v without a calculator. I can tell before I evaluate because: in part i, 16 is a perfect square; in part ii, 81 is a perfect fourth power; in part iii, -32 is a perfect fifth power; and in part v, 1.21 is a perfect square.

17. Use the formula: $h = 35d^{\frac{2}{3}}$ Substitute: $d = 3.2$

$$h = 35(3.2)^{\frac{2}{3}}$$

$$= 35\left(\sqrt[3]{3.2}\right)^2$$

Since 3.2 is not a perfect cube, use a calculator.

$$= 76.0036...$$

A fir tree with base diameter 3.2 m is approximately 76 m tall.

18. The rule for a power with a rational exponent is:

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

In the first line, the student should have written $1.96^{\frac{3}{2}}$ as $\left(\sqrt{1.96}\right)^3$

The correct solution is:

$$1.96^{\frac{3}{2}} = \left(\sqrt{1.96}\right)^3$$

$$= (1.4)^3$$

$$= 2.744$$

19. Use the formula: $SA = 0.096m^{0.7}$ Substitute: $m = 40$

$$SA = 0.096(40)^{0.7}$$

Use a calculator.

$$= 1.2697...$$

The surface area of a child with mass 40 kg is approximately 1.3 m².

20. a) Use the expression: $100(0.5)^{\frac{n}{5}}$

Substitute: $n = \frac{1}{2}$

$$\begin{aligned} 100(0.5)^{\frac{n}{5}} &= 100(0.5)^{\frac{\frac{1}{2}}{5}} \\ &= 100(0.5)^{\frac{1}{2} \cdot \frac{1}{5}} \\ &= 100(0.5)^{\frac{1}{10}} \quad \text{Use a calculator.} \\ &= 93.3032... \end{aligned}$$

After $\frac{1}{2}$ h, approximately 93% of caffeine remains in your body.

Use a calculator to evaluate the expression: $100(0.87)^{\frac{1}{2}} = 93.2737...$

After $\frac{1}{2}$ h, approximately 93% of caffeine remains in your body.

Both expressions give the same result, to the nearest whole number.

- b) Use the expression: $100(0.5)^{\frac{n}{5}}$

Substitute: $n = 1.5$

$$\begin{aligned} 100(0.5)^{\frac{n}{5}} &= 100(0.5)^{\frac{1.5}{5}} \\ &= 100(0.5)^{0.3} \quad \text{Use a calculator.} \\ &= 81.2252... \end{aligned}$$

After 1.5 h, approximately 81% of caffeine remains in your body.

- c) Use the expression: $100(0.5)^{\frac{n}{5}}$

Equate the expression to 50, the percent of caffeine that remains.

$$100(0.5)^{\frac{n}{5}} = 50 \quad \text{Solve for } n. \text{ Divide both sides by 100.}$$

$$0.5^{\frac{n}{5}} = \frac{50}{100}$$

$$0.5^{\frac{n}{5}} = 0.5$$

$$0.5^{\frac{n}{5}} = 0.5^1 \quad \text{Equate the exponents.}$$

$$\frac{n}{5} = 1$$

$$n = 5$$

So, after 5 h, 50% of caffeine remains in your body.

21. Use the formula: $T \doteq 0.2R^{\frac{3}{2}}$

To calculate the period for Earth, substitute: $R = 149$

$$T \doteq 0.2(149)^{\frac{3}{2}} \quad \text{Use a calculator.}$$

$$\doteq 363.7553\dots$$

It takes approximately 363.8 Earth days for Earth to orbit the sun.

To calculate the period for Mars, substitute: $R = 228$

$$T \doteq 0.2(228)^{\frac{3}{2}} \quad \text{Use a calculator.}$$

$$\doteq 688.5449\dots$$

It takes approximately 688.5 Earth days for Mars to orbit the sun.

So, Mars has the longer period.

C

22. Karen is correct. You can only multiply a number by itself a whole number of times.

Lesson 4.5 Negative Exponents and Reciprocals

Exercises (pages 233–234)

A

3. a) Use the rule: $\frac{1}{x^n} = x^{-n}$

$$\frac{1}{5^4} = 5^{-4}$$

- b) To write with a positive exponent, write the reciprocal of the fraction.

$$\left(-\frac{1}{2}\right)^{-3} = (-2)^3$$

- c) Use the rule: $\frac{1}{x^{-n}} = x^n$

$$\frac{1}{3^{-2}} = 3^2$$

- d) Use the rule: $\frac{1}{x^{-n}} = x^n$

$$\frac{1}{4^{-2}} = 4^2$$

4. Use: $x^{-n} = \frac{1}{x^n}$

- a) $4^2 = 16$

$$\begin{aligned}\text{So, } 4^{-2} &= \frac{1}{4^2} \\ &= \frac{1}{16}\end{aligned}$$

- b) $2^4 = 16$

$$\begin{aligned}\text{So, } 2^{-4} &= \frac{1}{2^4} \\ &= \frac{1}{16}\end{aligned}$$

- c) $6^1 = 6$

$$\begin{aligned}\text{So, } 6^{-1} &= \frac{1}{6^1} \\ &= \frac{1}{6}\end{aligned}$$

d) $4^3 = 64$

$$\begin{aligned}\text{So, } 4^{-3} &= \frac{1}{4^3} \\ &= \frac{1}{64}\end{aligned}$$

Each pair of answers involves the same numbers. The number in the numerator of one answer is the number in the denominator of the other answer, and the number in the denominator of one answer is the number in the numerator of the other answer. The answers in each pair are different in that they are reciprocals.

5. Use: $x^{-n} = \frac{1}{x^n}$

Since $2^{10} = 1024$,

$$\begin{aligned}\text{then } 2^{-10} &= \frac{1}{2^{10}} \\ &= \frac{1}{1024}\end{aligned}$$

6. Use: $x^{-n} = \frac{1}{x^n}$

a) $2^{-3} = \frac{1}{2^3}$

b) $3^{-5} = \frac{1}{3^5}$

c) $(-7)^{-2} = \frac{1}{(-7)^2}$

Since the square of a negative number is positive, this can be written as:

$$(-7)^{-2} = \frac{1}{7^2}$$

7. To write each power with a positive exponent, write the reciprocal of the fraction.

a) $\left(\frac{1}{2}\right)^{-2} = 2^2$

b) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

c) $\left(-\frac{6}{5}\right)^{-4} = \left(-\frac{5}{6}\right)^4$

Since a negative number raised to the fourth power is positive, this can be written as:

$$\left(-\frac{6}{5}\right)^{-4} = \left(\frac{5}{6}\right)^4$$

$$\begin{aligned} 8. \quad \text{a)} \quad 3^{-2} &= \frac{1}{3^2} \\ &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad 2^{-4} &= \frac{1}{2^4} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad (-2)^{-5} &= \frac{1}{(-2)^5} \\ &= \frac{1}{-32} \\ &= -\frac{1}{32} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad \left(\frac{1}{3}\right)^{-3} &= \left(\frac{3}{1}\right)^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad \left(-\frac{2}{3}\right)^{-2} &= \left(-\frac{3}{2}\right)^2 \\ &= \frac{9}{4} \end{aligned}$$

$$\begin{aligned} \text{f)} \quad \frac{1}{5^{-3}} &= 5^3 \\ &= 125 \end{aligned}$$

B

$$\begin{aligned} 9. \quad \text{a)} \quad 4^{-\frac{1}{2}} &= \frac{1}{4^{\frac{1}{2}}} && \text{Write with a positive exponent.} \\ &= \frac{1}{\sqrt{4}} && \text{Take the square root.} \\ &= \frac{1}{2} \end{aligned}$$

- b) Write 0.09 as a fraction: $\frac{9}{100}$

Then,

$$0.09^{-\frac{1}{2}} = \left(\frac{9}{100}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{100}{9}\right)^{\frac{1}{2}}$$

Write with a positive exponent.

$$= \sqrt{\frac{100}{9}}$$

Take the square root.

$$= \frac{10}{3}$$

- c) $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$ Write with a positive exponent.

$$= \frac{1}{\sqrt[3]{27}}$$

Take the cube root.

$$= \frac{1}{3}$$

- d) $(-64)^{-\frac{1}{3}} = \frac{1}{(-64)^{\frac{1}{3}}}$ Write with a positive exponent.

$$= \frac{1}{\sqrt[3]{-64}}$$

Take the cube root.

$$= \frac{1}{-4}$$

$$= -\frac{1}{4}$$

- e) Write -0.027 as a fraction: $-\frac{27}{1000}$

Then,

$$\begin{aligned} (-0.027)^{-\frac{2}{3}} &= \left(-\frac{27}{1000}\right)^{-\frac{2}{3}} \\ &= \left(-\frac{1000}{27}\right)^{\frac{2}{3}} && \text{Write with a positive exponent.} \\ &= \left(\sqrt[3]{-\frac{1000}{27}}\right)^2 && \text{Take the cube root.} \\ &= \left(-\frac{10}{3}\right)^2 && \text{Square the result.} \\ &= \frac{100}{9} \end{aligned}$$

- f) $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}}$ Write with a positive exponent.

$$\begin{aligned} &= \frac{1}{\left(\sqrt[5]{32}\right)^2} && \text{Take the fifth root.} \\ &= \frac{1}{2^2} && \text{Square the result.} \\ &= \frac{1}{4} \end{aligned}$$

- g) $9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}}$ Write with a positive exponent.

$$\begin{aligned} &= \frac{1}{\left(\sqrt{9}\right)^3} && \text{Take the square root.} \\ &= \frac{1}{3^3} && \text{Cube the result.} \\ &= \frac{1}{27} \end{aligned}$$

- h) Write 0.04 as a fraction: $\frac{4}{100}$

Then,

$$0.04^{-\frac{3}{2}} = \left(\frac{4}{100}\right)^{-\frac{3}{2}}$$

$$= \left(\frac{100}{4}\right)^{\frac{3}{2}}$$

Write with a positive exponent.

$$= \left(\sqrt{\frac{100}{4}}\right)^3$$

Take the square root.

$$= \left(\frac{10}{2}\right)^3$$

Cube the result.

$$= \frac{1000}{8}$$

$$= 125$$

10. Answers may vary.

a) $\frac{1}{9} = \frac{1}{3^2}$ Use: $\frac{1}{x^n} = x^{-n}$
 $= 3^{-2}$

b) $\frac{1}{5} = \frac{1}{\sqrt{25}}$
 $= \frac{1}{25^{\frac{1}{2}}}$ Use: $\frac{1}{x^n} = x^{-n}$
 $= 25^{-\frac{1}{2}}$

c) $4 = 2^2$
 $= \left(\frac{2}{1}\right)^2$ To write with a negative exponent, write the reciprocal of the fraction.
 $= \left(\frac{1}{2}\right)^{-2}$

$$\begin{aligned}
 \text{d) } -3 &= \sqrt[3]{-27} \\
 &= (-27)^{\frac{1}{3}} \\
 &= \left(\frac{-27}{1}\right)^{\frac{1}{3}} && \text{To write with a negative exponent, write the reciprocal of the fraction.} \\
 &= \left(\frac{1}{-27}\right)^{-\frac{1}{3}}
 \end{aligned}$$

11. Use a calculator to evaluate:

$$\begin{aligned}
 P &= 3000(1.025)^{-5} \\
 &= 2651.5628\dots
 \end{aligned}$$

To have \$3000 in 5 years, \$2651.56 must be invested now.

12. In the first line of the solution, to write the power with a positive exponent, the student wrote the fraction inside the brackets as a positive fraction instead of writing the reciprocal of the fraction.

A correct solution is:

$$\begin{aligned}
 \left(-\frac{64}{125}\right)^{-\frac{5}{3}} &= \left(-\frac{125}{64}\right)^{\frac{5}{3}} \\
 &= \left(\sqrt[3]{-\frac{125}{64}}\right)^5 \\
 &= \left(-\frac{5}{4}\right)^5 \\
 &= -\frac{3125}{1024}
 \end{aligned}$$

$$\text{13. a) } 27^{-\frac{4}{3}} = \frac{1}{27^{\frac{4}{3}}} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{\left(\sqrt[3]{27}\right)^4} \quad \text{Take the cube root.}$$

$$= \frac{1}{3^4} \quad \text{Raise the result to the fourth power.}$$

$$= \frac{1}{81}$$

b) The exponent $-1.5 = -\frac{3}{2}$

$$16^{-1.5} = 16^{-\frac{3}{2}}$$

$$= \frac{1}{16^{\frac{3}{2}}}$$

Write with a positive exponent.

$$= \frac{1}{(\sqrt{16})^3}$$

Take the square root.

$$= \frac{1}{4^3}$$

Cube the result.

$$= \frac{1}{64}$$

c) The exponent $-0.4 = -\frac{4}{10}$, or $-\frac{2}{5}$

$$32^{-0.4} = 32^{-\frac{2}{5}}$$

$$= \frac{1}{32^{\frac{2}{5}}}$$

Write with a positive exponent.

$$= \frac{1}{(\sqrt[5]{32})^2}$$

Take the fifth root.

$$= \frac{1}{2^2}$$

Square the result.

$$= \frac{1}{4}$$

d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}} = \left(-\frac{27}{8}\right)^{\frac{2}{3}}$ Write with a positive exponent.

$$= \left(\sqrt[3]{-\frac{27}{8}}\right)^2$$

Take the cube root.

$$= \left(-\frac{3}{2}\right)^2$$

Square the result.

$$= \frac{9}{4}$$

e) $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$ Write with a positive exponent.

$= \left(\sqrt[4]{\frac{16}{81}}\right)^3$ Take the fourth root.

$= \left(\frac{2}{3}\right)^3$ Cube the result.
 $= \frac{8}{27}$

f) $\left(\frac{9}{4}\right)^{-\frac{5}{2}} = \left(\frac{4}{9}\right)^{\frac{5}{2}}$ Write with a positive exponent.

$= \left(\sqrt{\frac{4}{9}}\right)^5$ Take the square root.

$= \left(\frac{2}{3}\right)^5$ Raise the result to the fifth power.
 $= \frac{32}{243}$

14. Use a calculator to evaluate:

$$P = \frac{150[1 - 1.032^{-10}]}{0.032}$$

$$= 1266.5690\dots$$

Michelle must invest \$1266.57 on January 1st.

15. Use the formula: $I = 100d^{-2}$

Substitute: $d = 23$

$I = 100(23)^{-2}$ Use a calculator.
 $= 0.1890\dots$

The intensity of the light 23 cm from the source is approximately 0.19%.

16. Write each power with a positive exponent.

$$2^{-5} = \frac{1}{2^5} \qquad 5^{-2} = \frac{1}{5^2}$$

$$= \frac{1}{32} \qquad = \frac{1}{25}$$

Since $\frac{1}{25} > \frac{1}{32}$, 5^{-2} is greater.

17. a) The numbers at the left are divided by 2 each time.
The exponents in the powers at the right decrease by 1 each time.

b)

$$16 = 2^4$$

$$8 = 2^3$$

$$4 = 2^2$$

$$2 = 2^1$$

$$1 = 2^0$$

$$\frac{1}{2} = 2^{-1}$$

$$\frac{1}{4} = 2^{-2}$$

$$\frac{1}{8} = 2^{-3}$$

c) Look at the last 3 rows.

$$2^{-1} = \frac{1}{2}$$

$$= \frac{1}{2^1}$$

$$2^{-2} = \frac{1}{4}$$

$$= \frac{1}{2^2}$$

$$2^{-3} = \frac{1}{8}$$

$$= \frac{1}{2^3}$$

So, this pattern shows that $a^{-n} = \frac{1}{a^n}$.

$$18. \quad 3^3 = 27 \qquad 3^{-5} = \frac{1}{3^5}$$

$$= \frac{1}{243}$$

Divide the greater number by the smaller number:

$$\frac{3^3}{3^{-5}} = \frac{27}{\frac{1}{243}}$$

$$= 27 \left(\frac{243}{1} \right)$$

$$= 6561$$

So, 3^3 is 6561 times as great as 3^{-5} .

Or, divide the powers:

$$\frac{3^3}{3^{-5}} = 3^3 \cdot \frac{1}{3^{-5}}$$

$$= 3^3 \cdot 3^5$$

$$= 3^8$$

So, 3^3 is 3^8 times as great as 3^{-5} .

19. Make an organized list to test powers of 3.

Power of 3	Value
3^3	27
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{3^2}$, or $\frac{1}{9}$
3^{-3}	$\frac{1}{3^3}$, or $\frac{1}{27}$

- a) From the table, when the exponent is greater than 0, the value is greater than 1. So, when the sign of the exponent is positive, $3^x > 1$.
- b) From the table, when the exponent is less than 0, the value is less than 1. So, when the sign of the exponent is negative, $3^x < 1$.
- c) From the table, when the exponent is 0, the value is 1. So, when the exponent is 0, $3^x = 1$.

C

20. Try different bases and different negative exponents.

$$8^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} \quad (-4)^{-2} = \frac{1}{(-4)^2} \quad \left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}} \quad \left(-\frac{1}{2}\right)^{-2} = (-2)^2$$

$$= \frac{1}{2} \quad = \frac{1}{16} \quad = 2 \quad = 4$$

The values of $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ and $\left(-\frac{1}{2}\right)^{-2}$ are greater than 1.

So, when a number is raised to a negative exponent, the value of the power is not always less than 1.

21. a) Use the formula: $F = (6.67 \times 10^{-11}) Mmr^{-2}$

Write the distance between Earth and the moon in metres:

$$382\,260\text{ km} = 382\,260\,000\text{ m}$$

Substitute: $M = 5.9736 \times 10^{24}$, $m = 7.349 \times 10^{22}$, and $r = 382\,260\,000$

$$F = (6.67 \times 10^{-11})(5.9736 \times 10^{24})(7.349 \times 10^{22})(382\,260\,000)^{-2}$$

Use a calculator.

$$F = 2.0038 \dots \times 10^{20}$$

The gravitational force between Earth and the moon is about 2.0×10^{20} N.

- b) Answers may vary. They depend on the diameters of Earth and the moon.

The diameter of Earth is about 12 756 km.

The diameter of the moon is about 3475 km.

So, the distance between the centres of Earth and the moon is:

$$382\,260 \text{ km} + \frac{1}{2} \text{diameter of Earth} + \frac{1}{2} \text{diameter of the moon}$$

$$= 382\,260 \text{ km} + \frac{1}{2}(12\,756 \text{ km}) + \frac{1}{2}(3475 \text{ km})$$

$$= 382\,260 \text{ km} + 6378 \text{ km} + 1737.5 \text{ km}$$

$$= 390\,375.5 \text{ km}$$

Write this distance in metres: $390\,375.5 \text{ km} = 390\,375\,500 \text{ m}$

Use the formula: $F = (6.67 \times 10^{-11})Mmr^{-2}$

Substitute: $M = 5.9736 \times 10^{24}$, $m = 7.349 \times 10^{22}$, and $r = 390\,375\,500$

$$F = (6.67 \times 10^{-11})(5.9736 \times 10^{24})(7.349 \times 10^{22})(390\,375\,500)^{-2}$$

Use a calculator.

$$F = 1.9214... \times 10^{20}$$

The gravitational force between Earth and the moon is about $1.9 \times 10^{20} \text{ N}$.

Checkpoint 2

Assess Your Understanding (page 236)

4.4

$$1. \quad \text{a) } 16^{\frac{1}{4}} = \sqrt[4]{16} \\ = 2$$

$$\text{b) } 0.5 = \frac{1}{2}$$

$$\text{So, } 49^{0.5} = 49^{\frac{1}{2}} \\ = \sqrt{49} \\ = 7$$

$$\text{c) } (-64)^{\frac{2}{3}} = \left(\sqrt[3]{-64}\right)^2 \\ = (-4)^2 \\ = 16$$

$$\text{d) } 1.5 = \frac{3}{2}$$

$$\text{So, } \left(\frac{49}{9}\right)^{1.5} = \left(\frac{49}{9}\right)^{\frac{3}{2}} \\ = \left(\sqrt{\frac{49}{9}}\right)^3 \\ = \left(\frac{7}{3}\right)^3 \\ = \frac{343}{27}$$

$$\text{e) } (-8)^{\frac{5}{3}} = \left(\sqrt[3]{-8}\right)^5 \\ = (-2)^5 \\ = -32$$

2. a) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

i) $35^{\frac{2}{3}} = \left(\sqrt[3]{35}\right)^2$ or $\sqrt[3]{35^2}$

ii) $32^{\frac{3}{2}} = \left(\sqrt{32}\right)^3$ or $\sqrt{32^3}$

iii) $(-32)^{\frac{2}{5}} = \left(\sqrt[5]{-32}\right)^2$ or $\sqrt[5]{(-32)^2}$

iv) $1.5 = \frac{3}{2}$

$$\begin{aligned}\text{So, } 400^{1.5} &= 400^{\frac{3}{2}} \\ &= \left(\sqrt{400}\right)^3 \text{ or } \sqrt{400^3}\end{aligned}$$

v) $(-125)^{\frac{1}{3}} = \sqrt[3]{-125}$

vi) $\left(\frac{8}{125}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{125}}\right)^2$ or $\sqrt[3]{\left(\frac{8}{125}\right)^2}$

b) i) $35^{\frac{2}{3}} = \left(\sqrt[3]{35}\right)^2$

I cannot evaluate this radical without using a calculator because 35 is not a perfect cube.

ii) $32^{\frac{3}{2}} = \left(\sqrt{32}\right)^3$

I cannot evaluate this radical without using a calculator because 32 is not a perfect square.

$$\begin{aligned}\text{iii) } (-32)^{\frac{2}{5}} &= \left(\sqrt[5]{-32}\right)^2 \\ &= (-2)^2 \\ &= 4\end{aligned}$$

$$\begin{aligned}\text{iv) } 400^{1.5} &= (\sqrt{400})^3 \\ &= (20)^3 \\ &= 8000\end{aligned}$$

$$\begin{aligned}\text{v) } (-125)^{\frac{1}{3}} &= \sqrt[3]{-125} \\ &= -5\end{aligned}$$

$$\begin{aligned}\text{vi) } \left(\frac{8}{125}\right)^{\frac{2}{3}} &= \left(\sqrt[3]{\frac{8}{125}}\right)^2 \\ &= \left(\frac{2}{5}\right)^2 \\ &= \frac{4}{25}\end{aligned}$$

3. Use $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ or $(\sqrt[n]{a})^m = a^{\frac{m}{n}}$.

$$\text{a) } \sqrt[3]{4} = 4^{\frac{1}{3}}$$

$$\text{b) } \sqrt{9} = 9^{\frac{1}{2}}$$

$$\text{Or, since } \frac{1}{2} = 0.5, \sqrt{9} = 9^{0.5}$$

$$\text{c) } \sqrt[4]{18} = 18^{\frac{1}{4}}$$

$$\text{Or, since } \frac{1}{4} = 0.25, \sqrt[4]{18} = 18^{0.25}$$

$$\text{d) } (\sqrt{10})^3 = 10^{\frac{3}{2}}$$

$$\text{Or, since } \frac{3}{2} = 1.5, (\sqrt{10})^3 = 10^{1.5}$$

$$\text{e) } (\sqrt[3]{-10})^2 = (-10)^{\frac{2}{3}}$$

4. Use the formula: $T \doteq 17.4m^{\frac{1}{4}}$ Substitute: $m = 85$

$$\begin{aligned}T &\doteq 17.4(85)^{\frac{1}{4}} \quad \text{Since 85 is not a perfect fourth power, use a calculator.} \\ &= 52.8328...\end{aligned}$$

The circulation time for a mammal with mass 85 kg is about 53 s.

5. Since every base is 3, write each number with a fractional exponent.

$$3^2$$

$$\sqrt[3]{3} = 3^{\frac{1}{3}}$$

$$(\sqrt{3})^5 = 3^{\frac{5}{2}}$$

$$3^{\frac{2}{3}}$$

$$(\sqrt[3]{3})^4 = 3^{\frac{4}{3}}$$

Order the exponents from least to greatest: $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{3}{2}, \frac{5}{2}$

So, from least to greatest, the numbers are: $\sqrt[3]{3}, 3^{\frac{2}{3}}, (\sqrt[3]{3})^4, 3^{\frac{3}{2}}, (\sqrt{3})^5$

6. The formula for the volume, V , of a cube with edge length e units is:

$$V = e^3$$

To determine the value of e , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{e^3}$$

$$\sqrt[3]{V} = e$$

Substitute: $V = 421\,875$

$$e = \sqrt[3]{421\,875} \text{ or } e = 421\,875^{\frac{1}{3}}$$

The cube has edge length $\sqrt[3]{421\,875}$ mm or $421\,875^{\frac{1}{3}}$ mm.

Use a calculator to determine the edge length: $\sqrt[3]{421\,875} = 75$

The cube has edge length 75 mm.

4.5

7. a) $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$ Write with a positive exponent.

$$= \frac{81}{16}$$

b) Write 0.5 as a fraction: $\frac{1}{2}$

Then,

$$\begin{aligned} 0.5^{-2} &= \left(\frac{1}{2}\right)^{-2} \\ &= \left(\frac{2}{1}\right)^2 \\ &= 4 \end{aligned}$$

c) $(-1000)^{-\frac{2}{3}} = \left(\frac{1}{-1000}\right)^{\frac{2}{3}}$ Write with a positive exponent.

$$= \left(\sqrt[3]{\frac{1}{-1000}}\right)^2$$
 Take the cube root.

$$= \left(-\frac{1}{10}\right)^2$$
 Square the result.

$$= \frac{1}{100}$$

d) $\left(\frac{1}{4}\right)^{-\frac{1}{2}} = 4^{\frac{1}{2}}$ Write with a positive exponent.

$$\begin{aligned} &= \sqrt{4} \quad \text{Take the square root.} \\ &= 2 \end{aligned}$$

e) $\left(\frac{1}{10}\right)^{-2} = 10^2$ Write with a positive exponent.

$$= 100$$

- f) Write -0.008 as a fraction: $-\frac{8}{1000}$

Then,

$$(-0.008)^{-\frac{4}{3}} = \left(-\frac{8}{1000}\right)^{-\frac{4}{3}}$$

$$= \left(-\frac{1000}{8}\right)^{\frac{4}{3}}$$

Write with a positive exponent.

$$= \left(\sqrt[3]{-\frac{1000}{8}}\right)^4$$

Take the cube root.

$$= \left(-\frac{10}{2}\right)^4$$

$$= (-5)^4$$

Raise the result to the fourth power.

$$= 625$$

8. Use a calculator to evaluate:

$$P = 5000(1.029)^{-3}$$

$$= 4589.0615\dots$$

To have \$5000 in 3 years, \$4589.06 must be invested now.

Lesson 4.6 Applying the Exponent Laws

Exercises (pages 241–243)

A

3. Use the product of powers law:
When the bases are the same, add the exponents.

$$\begin{aligned}\text{a) } x^3 \cdot x^4 &= x^{3+4} \\ &= x^7\end{aligned}$$

$$\begin{aligned}\text{b) } a^2 \cdot a^{-5} &= a^{2+(-5)} \\ &= a^{-3} \quad \text{Write with a positive exponent.} \\ &= \frac{1}{a^3}\end{aligned}$$

$$\begin{aligned}\text{c) } b^{-3} \cdot b^5 &= b^{-3+5} \\ &= b^2\end{aligned}$$

$$\begin{aligned}\text{d) } m^2 \cdot m^{-3} &= m^{2+(-3)} \\ &= m^{-1} \quad \text{Write with a positive exponent.} \\ &= \frac{1}{m}\end{aligned}$$

$$\begin{aligned}4. \text{ a) } 0.5^2 \cdot 0.5^3 \\ \text{Use the product of powers law:} \\ \text{When the bases are the same, add the exponents.} \\ 0.5^2 \cdot 0.5^3 &= 0.5^{2+3} \\ &= 0.5^5\end{aligned}$$

$$\begin{aligned}\text{b) } 0.5^2 \cdot 0.5^{-3} \\ \text{Use the product of powers law:} \\ \text{When the bases are the same, add the exponents.} \\ 0.5^2 \cdot 0.5^{-3} &= 0.5^{2+(-3)} \\ &= 0.5^{-1} \quad \text{Write with a positive exponent.} \\ &= \frac{1}{0.5}\end{aligned}$$

c) $\frac{0.5^2}{0.5^3}$

Use the quotient of powers law:

When the bases are the same, subtract the exponents.

$$\frac{0.5^2}{0.5^3} = 0.5^{2-3}$$

$$= 0.5^{-1} \text{ Write with a positive exponent.}$$

$$= \frac{1}{0.5}$$

d) $\frac{0.5^2}{0.5^{-3}}$

Use the quotient of powers law:

When the bases are the same, subtract the exponents.

$$\frac{0.5^2}{0.5^{-3}} = 0.5^{2-(-3)}$$

$$= 0.5^5$$

5. Use the quotient of powers law:

When the bases are the same, subtract the exponents.

a) $\frac{x^4}{x^2} = x^{4-2}$
 $= x^2$

b) $\frac{x^2}{x^5} = x^{2-5}$
 $= x^{-3} \text{ Write with a positive exponent.}$
 $= \frac{1}{x^3}$

c) $n^6 \div n^5 = n^{6-5}$
 $= n^1$
 $= n$

d) $\frac{a^2}{a^6} = a^{2-6}$
 $= a^{-4} \text{ Write with a positive exponent.}$
 $= \frac{1}{a^4}$

6. Use the power of a power law: multiply the exponents

$$\begin{aligned}\text{a) } (n^2)^3 &= n^{(2)(3)} \\ &= n^6\end{aligned}$$

$$\begin{aligned}\text{b) } (z^2)^{-3} &= z^{(2)(-3)} \\ &= z^{-6} \quad \text{Write with a positive exponent.} \\ &= \frac{1}{z^6}\end{aligned}$$

$$\begin{aligned}\text{c) } (n^{-4})^{-3} &= n^{(-4)(-3)} \\ &= n^{12}\end{aligned}$$

$$\begin{aligned}\text{d) } (c^{-2})^2 &= c^{(-2)(2)} \\ &= c^{-4} \quad \text{Write with a positive exponent.} \\ &= \frac{1}{c^4}\end{aligned}$$

7. Use the power of a power law: multiply the exponents

$$\begin{aligned}\text{a) } \left[\left(\frac{3}{5}\right)^3\right]^4 &= \left(\frac{3}{5}\right)^{(3)(4)} \\ &= \left(\frac{3}{5}\right)^{12}\end{aligned}$$

$$\begin{aligned}\text{b) } \left[\left(\frac{3}{5}\right)^3\right]^{-4} &= \left(\frac{3}{5}\right)^{(3)(-4)} \\ &= \left(\frac{3}{5}\right)^{-12} \quad \text{Write with a positive exponent.} \\ &= \left(\frac{5}{3}\right)^{12}\end{aligned}$$

$$\begin{aligned}\text{c) } \left[\left(\frac{3}{5}\right)^{-3}\right]^{-4} &= \left(\frac{3}{5}\right)^{(-3)(-4)} \\ &= \left(\frac{3}{5}\right)^{12}\end{aligned}$$

$$\begin{aligned} \text{d) } \left[\left(-\frac{3}{5} \right)^{-3} \right]^{-4} &= \left(-\frac{3}{5} \right)^{(-3)(-4)} \\ &= \left(-\frac{3}{5} \right)^{12} \end{aligned}$$

8. a) Use the power of a quotient law.

$$\left(\frac{a}{b} \right)^2 = \frac{a^2}{b^2}$$

- b) Use the power of a quotient law.

$$\begin{aligned} \left(\frac{n^2}{m} \right)^3 &= \frac{(n^2)^3}{m^3} && \text{Use the power of a power law.} \\ &= \frac{n^{(2)(3)}}{m^3} \\ &= \frac{n^6}{m^3} \end{aligned}$$

$$\text{c) } \left(\frac{c^2}{d^2} \right)^{-4} = \left(\frac{d^2}{c^2} \right)^4 \quad \text{Writing with a positive exponent}$$

$$= \frac{(d^2)^4}{(c^2)^4} \quad \text{Using the power of a quotient law}$$

$$= \frac{d^{(2)(4)}}{c^{(2)(4)}} \quad \text{Using the power of a power law}$$

$$= \frac{d^8}{c^8}$$

- d) Use the power of a quotient law.

$$\left(\frac{2b}{5c} \right)^2 = \frac{(2b)^2}{(5c)^2} \quad \text{Use the power of a product law.}$$

$$= \frac{2^2 b^2}{5^2 c^2}$$

$$= \frac{4b^2}{25c^2}$$

e) $(ab)^2 = a^2b^2$ Using the power of a product law

f) $(n^2m)^3 = (n^2)^3 \cdot m^3$ Using the power of a product law
 $= n^{(2)(3)}m^3$ Using the power of a power law
 $= n^6m^3$

g) $(c^3d^2)^{-4} = (c^3)^{-4} \cdot (d^2)^{-4}$ Using the power of a product law
 $= c^{(3)(-4)}d^{(2)(-4)}$ Using the power of a power law
 $= c^{-12}d^{-8}$ Write with positive exponents.
 $= \frac{1}{c^{12}d^8}$

h) $(xy^{-1})^3 = (x)^3 \cdot (y^{-1})^3$ Using the power of a product law
 $= x^3y^{(-1)(3)}$ Using the power of a power law
 $= x^3y^{-3}$ Write with a positive exponent.
 $= \frac{x^3}{y^3}$

B

9. a) $x^{-3} \cdot x^4 = x^{-3+4}$ Using the product of powers law
 $= x^1$
 $= x$

b) $a^{-4} \cdot a^{-1} = a^{-4+(-1)}$ Using the product of powers law
 $= a^{-5}$ Write with a positive exponent.
 $= \frac{1}{a^5}$

c) $b^4 \cdot b^{-3} \cdot b^2 = b^{4+(-3)+2}$ Using the product of powers law
 $= b^3$

d) $m^8 \cdot m^{-2} \cdot m^{-6} = m^{8+(-2)+(-6)}$ Using the product of powers law
 $= m^0$
 $= 1$

e) $\frac{x^{-5}}{x^2} = x^{-5-2}$ Using the quotient of powers law
 $= x^{-7}$ Write with a positive exponent.
 $= \frac{1}{x^7}$

f) $\frac{s^5}{s^{-5}} = s^{5-(-5)}$ Using the quotient of powers law
 $= s^{10}$

g) $\frac{b^{-8}}{b^{-3}} = b^{-8-(-3)}$ Using the quotient of powers law
 $= b^{-5}$ Write with a positive exponent.
 $= \frac{1}{b^5}$

h) $\frac{t^{-4}}{t^{-4}} = t^{-4-(-4)}$ Using the quotient of powers law
 $= t^0$
 $= 1$

10. a) $1.5^3 \cdot 1.5^1 = 1.5^{3+1}$ Using the product of powers law
 $= 1.5^4$
 $= 1.5^2$
 $= 2.25$

b) $\left(\frac{3}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^{3+5}$ Using the product of powers law
 $= \left(\frac{3}{4}\right)^8$
 $= \left(\frac{3}{4}\right)^2$
 $= \frac{9}{16}$

$$\begin{aligned} \text{c) } (-0.6)^{\frac{1}{3}} \cdot (-0.6)^{\frac{5}{3}} &= (-0.6)^{\frac{1}{3} + \frac{5}{3}} && \text{Using the product of powers law} \\ &= (-0.6)^{\frac{6}{3}} \\ &= (-0.6)^2 \\ &= 0.36 \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\frac{4}{5}\right)^{-\frac{4}{3}} &= \left(\frac{4}{5}\right)^{\frac{4}{3} + \left(-\frac{4}{3}\right)} && \text{Using the product of powers law} \\ &= \left(\frac{4}{5}\right)^0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}} &= 0.6^{\frac{1}{2} - \frac{3}{2}} && \text{Using the quotient of powers law} \\ &= 0.6^{-1} && \text{Write with a positive exponent.} \\ &= \frac{1}{0.6} \\ &= \frac{1}{\frac{3}{5}} \\ &= \frac{10}{3} \\ &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{\left(-\frac{3}{8}\right)^{\frac{2}{3}}}{\left(-\frac{3}{8}\right)^{-\frac{1}{3}}} &= \left(-\frac{3}{8}\right)^{\frac{2}{3} - \left(-\frac{1}{3}\right)} && \text{Using the quotient of powers law.} \\ &= \left(-\frac{3}{8}\right)^{\frac{3}{3}} \\ &= \left(-\frac{3}{8}\right)^1 \\ &= -\frac{3}{8} \end{aligned}$$

g)

$$\frac{0.49^{\frac{5}{2}}}{0.49^4} = 0.49^{\frac{5}{2} - 4} \quad \text{Using the quotient of powers law}$$

$$= 0.49^{\frac{5}{2} - \frac{8}{2}}$$

$$= 0.49^{-\frac{3}{2}} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{0.49^{\frac{3}{2}}}$$

$$= \frac{1}{(\sqrt{0.49})^3}$$

$$= \frac{1}{0.7^3}$$

$$= \frac{1}{0.343}$$

$$= \frac{1}{343}$$

$$\frac{1000}{1000}$$

$$= \frac{1000}{343}$$

$$\text{h) } \frac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}} = 0.027^{\frac{5}{3} - \frac{4}{3}} \quad \text{Using the quotient of powers law}$$

$$= 0.027^{\frac{1}{3}}$$

$$= \sqrt[3]{0.027}$$

$$= 0.3, \text{ or } \frac{3}{10}$$

$$11. \text{ a) } (x^{-1}y^{-2})^{-3} = (x^{-1})^{-3} \cdot (y^{-2})^{-3} \quad \text{Using the power of a product law}$$

$$= x^{(-1)(-3)} \cdot y^{(-2)(-3)} \quad \text{Using the power of a power law}$$

$$= x^3 \cdot y^6$$

$$= x^3 y^6$$

$$\begin{aligned}
 \text{b) } (2a^{-2}b^2)^{-2} &= (2^1)^{-2} \cdot (a^{-2})^{-2} \cdot (b^2)^{-2} \\
 &= 2^{(1)(-2)} \cdot a^{(-2)(-2)} \cdot b^{(2)(-2)} \\
 &= 2^{-2} \cdot a^4 \cdot b^{-4} \\
 &= \frac{a^4}{2^2 \cdot b^4} \\
 &= \frac{a^4}{4b^4}
 \end{aligned}$$

Using the power of a product law

Using the power of a power law

Write with positive exponents.

$$\begin{aligned}
 \text{c) } (4m^2n^3)^{-3} &= (4^1)^{-3} \cdot (m^2)^{-3} \cdot (n^3)^{-3} \\
 &= 4^{(1)(-3)} \cdot m^{(2)(-3)} \cdot n^{(3)(-3)} \\
 &= 4^{-3} \cdot m^{-6} \cdot n^{-9} \\
 &= \frac{1}{4^3 \cdot m^6 \cdot n^9} \\
 &= \frac{1}{64m^6n^9}
 \end{aligned}$$

Using the power of a product law

Using the power of a power law

Write with positive exponents.

$$\begin{aligned}
 \text{d) } \left(\frac{3}{2}m^{-2}n^{-3}\right)^{-4} &= \left(\frac{3^1}{2}\right)^{-4} \cdot (m^{-2})^{-4} \cdot (n^{-3})^{-4} \\
 &= \left(\frac{3}{2}\right)^{(1)(-4)} \cdot m^{(-2)(-4)} \cdot n^{(-3)(-4)} \\
 &= \left(\frac{3}{2}\right)^{-4} \cdot m^8 \cdot n^{12} \\
 &= \left(\frac{2}{3}\right)^4 \cdot m^8 \cdot n^{12} \\
 &= \frac{16}{81} \cdot m^8 \cdot n^{12} \\
 &= \frac{16m^8n^{12}}{81}
 \end{aligned}$$

Using the power of a product law

Using the power of a power law

Write with a positive exponent.

12. The volume of a cone with base radius r and height h is given by the formula:

$$V = \frac{1}{3}\pi r^2 h$$

The cone has equal height and radius.

So, substitute: $r = h$

$$V = \frac{1}{3}\pi h^2 h$$

$$= \frac{1}{3}\pi h^2 h^1 \quad \text{Use the product of powers law.}$$

$$= \frac{1}{3}\pi h^{2+1}$$

$$= \frac{1}{3}\pi h^3$$

Substitute $V = 1234$, then solve for h .

$$1234 = \frac{1}{3}\pi h^3 \quad \text{Multiply each side by 3.}$$

$$3(1234) = 3\left(\frac{1}{3}\pi h^3\right)$$

$$3702 = \pi h^3 \quad \text{Divide each side by } \pi.$$

$$\frac{3702}{\pi} = \frac{\pi h^3}{\pi}$$

$$\frac{3702}{\pi} = h^3$$

To solve for h , take the cube root of each side
by raising each side to the one-third power.

$$\left(\frac{3702}{\pi}\right)^{\frac{1}{3}} = (h^3)^{\frac{1}{3}} \quad \text{Use the power of a power law.}$$

$$\left(\frac{3702}{\pi}\right)^{\frac{1}{3}} = h \quad \text{Use a calculator.}$$

$$h = 10.5623...$$

The height of the cone is approximately 10.6 cm.

13. The volume, V , of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

Substitute $V = 375$, then solve for r .

$$375 = \frac{4}{3}\pi r^3 \quad \text{Multiply each side by 3.}$$

$$3(375) = 3\left(\frac{4}{3}\pi r^3\right)$$

$$1125 = 4\pi r^3 \quad \text{Divide each side by } 4\pi.$$

$$\frac{1125}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\frac{1125}{4\pi} = r^3 \quad \text{To solve for } r, \text{ take the cube root of each side by raising each side to the one-third power.}$$

$$\left(\frac{1125}{4\pi}\right)^{\frac{1}{3}} = (r^3)^{\frac{1}{3}} \quad \text{Use the power of a power law.}$$

$$\left(\frac{1125}{4\pi}\right)^{\frac{1}{3}} = r \quad \text{Use a calculator.}$$

$$r = 4.4735\dots$$

The surface area, SA , of a sphere with radius r is given by the formula:

$$SA = 4\pi r^2 \quad \text{Substitute: } r = 4.4735\dots$$

$$= 4\pi(4.4735\dots)^2$$

$$= 251.4808\dots$$

The surface area of the sphere is approximately 251 square feet.

14. a) $\frac{(a^2b^{-1})^{-2}}{(a^{-3}b)^3} = \frac{a^{2(-2)} \cdot b^{(-1)(-2)}}{a^{(-3)(3)} \cdot b^{(1)(3)}} \quad \text{Using the power of a power law}$

$$= \frac{a^{-4} \cdot b^2}{a^{-9} \cdot b^3} \quad \text{Use the quotient of powers law.}$$

$$= a^{-4 - (-9)} \cdot b^{2-3}$$

$$= a^5 \cdot b^{-1} \quad \text{Write with a positive exponent.}$$

$$= \frac{a^5}{b}$$

b)

$$\left(\frac{(c^{-3}d)^{-1}}{c^2d} \right)^{-2} = \left(\frac{c^{(-3)(-1)} \cdot d^{(1)(-1)}}{c^2d} \right)^{-2}$$

Using the power of a power law inside the large brackets

$$= \left(\frac{c^3 \cdot d^{-1}}{c^2d} \right)^{-2}$$

Use the quotient of powers law.

$$= (c^{3-2} \cdot d^{-1-1})^{-2}$$

$$= (c^1 \cdot d^{-2})^{-2}$$

Use the power of a power law.

$$= c^{1(-2)} \cdot d^{(-2)(-2)}$$

$$= c^{-2}d^4$$

Write with a positive exponent.

$$= \frac{d^4}{c^2}$$

15. a) Simplify first.

$$(a^3b^2)(a^2b^3) = a^3 \cdot b^2 \cdot a^2 \cdot b^3$$

$$= a^3 \cdot a^2 \cdot b^2 \cdot b^3$$

Use the product of powers law.

$$= a^{3+2} \cdot b^{2+3}$$

$$= a^5b^5$$

Substitute: $a = -2$, $b = 1$

$$a^5b^5 = (-2)^5(1)^5$$

$$= (-32)(1)$$

$$= -32$$

b) Simplify first.

$$(a^{-1}b^{-2})(a^{-2}b^{-3}) = a^{-1} \cdot b^{-2} \cdot a^{-2} \cdot b^{-3}$$

$$= a^{-1} \cdot a^{-2} \cdot b^{-2} \cdot b^{-3}$$

Use the product of powers law.

$$= a^{-1+(-2)} \cdot b^{-2+(-3)}$$

$$= a^{-3}b^{-5}$$

Write with positive exponents.

$$= \frac{1}{a^3b^5}$$

Substitute: $a = -2$, $b = 1$

$$\frac{1}{a^3b^5} = \frac{1}{(-2)^3(1)^5}$$

$$= \frac{1}{(-8)(1)}$$

$$= -\frac{1}{8}$$

c) Simplify first.

$$\frac{a^{-4}b^5}{ab^3} = a^{-4-1} \cdot b^{5-3}$$

Using the quotient of powers law

$$= a^{-5} \cdot b^2$$

Write with a positive exponent.

$$= \frac{b^2}{a^5}$$

Substitute: $a = -2$, $b = 1$

$$\frac{b^2}{a^5} = \frac{1^2}{(-2)^5}$$

$$= -\frac{1}{32}$$

d) Simplify first.

$$\left(\frac{a^{-7}b^7}{a^{-9}b^{10}} \right)^{-5} = \left(a^{-7-(-9)} \cdot b^{7-10} \right)^{-5}$$

Using the quotient of powers law inside the brackets

$$= \left(a^2 \cdot b^{-3} \right)^{-5}$$

Write the expression inside the brackets with a positive exponent.

$$= \left(\frac{a^2}{b^3} \right)^{-5}$$

Write with a positive exponent.

$$= \left(\frac{b^3}{a^2} \right)^5$$

Use the power of a power law.

$$= \frac{b^{3(5)}}{a^{2(5)}}$$

$$= \frac{b^{15}}{a^{10}}$$

Substitute: $a = -2$, $b = 1$

$$\frac{b^{15}}{a^{10}} = \frac{1^{15}}{(-2)^{10}}$$

$$= \frac{1}{1024}$$

16. a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}} = m^{\frac{2}{3} + \frac{4}{3}}$

Using the product of powers law

$$= m^{\frac{6}{3}}$$

$$= m^2$$

b) $x^{-\frac{3}{2}} \div x^{-\frac{1}{4}} = x^{-\frac{3}{2} - (-\frac{1}{4})}$ Using the quotient of powers law

$$= x^{-\frac{6}{4} - (-\frac{1}{4})}$$

$$= x^{-\frac{5}{4}}$$

Write with a positive exponent.

$$= \frac{1}{x^{\frac{5}{4}}}$$

c)

$$\frac{-9a^{-4}b^{\frac{3}{4}}}{3a^{\frac{1}{2}}b^{\frac{1}{4}}} = \frac{-9}{3} \cdot \frac{a^{-4}}{a^{\frac{1}{2}}} \cdot \frac{b^{\frac{3}{4}}}{b^{\frac{1}{4}}}$$

Use the quotient of powers law.

$$= -3 \cdot a^{-4 - \frac{1}{2}} \cdot b^{\frac{3}{4} - \frac{1}{4}}$$

$$= -3 \cdot a^{-\frac{8}{2} - \frac{1}{2}} \cdot b^{\frac{2}{4}}$$

$$= -3 \cdot a^{-\frac{9}{2}} \cdot b^{\frac{1}{2}}$$

Write with a positive exponent.

$$= -\frac{3b^{\frac{1}{2}}}{a^{\frac{9}{2}}}$$

d)

$$\left(\frac{-64c^6}{a^9b^{-\frac{1}{2}}} \right)^{\frac{1}{3}} = \left(-64 \cdot \frac{1}{a^9} \cdot \frac{1}{b^{-\frac{1}{2}}} \cdot c^6 \right)^{\frac{1}{3}}$$

Simplify inside the brackets first.

$$= \left(-64 \cdot a^{-9} \cdot b^{\frac{1}{2}} \cdot c^6 \right)^{\frac{1}{3}}$$

Use the power of a power law.

$$= (-64)^{\frac{1}{3}} \cdot a^{-9 \cdot \frac{1}{3}} \cdot b^{\frac{1}{2} \cdot \frac{1}{3}} \cdot c^{6 \cdot \frac{1}{3}}$$

$$= (-64)^{\frac{1}{3}} \cdot a^{-3} \cdot b^{\frac{1}{6}} \cdot c^2$$

Write with a positive exponent.

$$= -4 \cdot a^{-3} \cdot b^{\frac{1}{6}} \cdot c^2$$

$$= \frac{-4b^{\frac{1}{6}}c^2}{a^3}$$

17. a) In the second line, the exponents were multiplied instead of added.

A correct solution is:

$$(x^2 y^{-3}) \left(x^{\frac{1}{2}} y^{-1} \right) = x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1} \quad \text{Use the product of powers law.}$$

$$= x^{2 + \frac{1}{2}} \cdot y^{-3 + (-1)}$$

$$= x^{2\frac{4}{2} + \frac{1}{2}} \cdot y^{-3 + (-1)}$$

$$= x^2 y^{-4}$$

Write with a positive exponent.

$$= \frac{x^2}{y^4}$$

- b) In the first line, -5 was multiplied by -2 instead of being raised to the power -2 .

A correct solution is:

$$\left(\frac{-5a^2}{b^{\frac{1}{2}}} \right)^{-2} = \frac{(-5)^{-2} \cdot a^{2(-2)}}{b^{\left(\frac{1}{2}\right)(-2)}} \quad \text{Using the power of a power law}$$

$$= \frac{(-5)^{-2} \cdot a^{-4}}{b^{-1}} \quad \text{Write with positive exponents.}$$

$$= \frac{b^1}{(-5)^2 \cdot a^4}$$

$$= \frac{b}{25a^4}$$

18. I record the volume of water in the measuring cylinder, in millilitres. Then I carefully place the marble in the cylinder. The water level rises; this is the total volume of the water and the marble. I record this volume. I subtract the volume of the water alone from the total volume of the water and the marble to determine the volume of the marble in millilitres. Since $1 \text{ mL} = 1 \text{ cm}^3$, I can write the volume of the marble in cubic centimetres.

The volume, V , of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

I substitute the volume of the marble in cubic centimetres for V .

I then multiply both sides of the equation by 3, then divide both sides by 4π . To solve for r ,

I raise each side to the one-third power. I then use the power of a power law to write $(r^3)^{\frac{1}{3}}$ as r . Once I have determined r , I multiply the radius by 2 to get the diameter of the marble.

19. a) There are two errors in the first line: the quotient of powers law was used before the power of a power law, and when using the quotient of powers law, the powers outside the brackets were subtracted. There is an error in the second line: when using the power of a power law, the product of 5 and -6 should have been -30 .

A correct solution is:

$$\begin{aligned} \frac{(m^{-3} \cdot n^2)^{-4}}{(m^2 \cdot n^{-3})^2} &= \frac{m^{(-3)(-4)} \cdot n^{2(-4)}}{m^{2(2)} \cdot n^{-3(2)}} && \text{Using the power of a power law} \\ &= \frac{m^{12} \cdot n^{-8}}{m^4 \cdot n^{-6}} && \text{Use the quotient of powers law.} \\ &= m^{12-4} \cdot n^{-8-(-6)} \\ &= m^8 n^{-2} && \text{Write with a positive exponent.} \\ &= \frac{m^8}{n^2} \end{aligned}$$

- b) There is an error in the first line: when using the power of a power law, the exponents were added when they should have been multiplied.

A correct solution is:

$$\begin{aligned} \left(r^{\frac{1}{2}} \cdot s^{-\frac{3}{2}}\right)^2 \cdot \left(r^{-\frac{1}{4}} \cdot s^{\frac{1}{2}}\right)^{-1} &= r^{\left(\frac{1}{2}\right)\left(2\right)} \cdot s^{\left(-\frac{3}{2}\right)\left(2\right)} \cdot r^{\left(-\frac{1}{4}\right)(-1)} \cdot s^{\left(\frac{1}{2}\right)(-1)} && \text{Using the power of a power law} \\ &= r^{\frac{1}{2}} \cdot s^{-\frac{3}{2}} \cdot r^{\frac{1}{4}} \cdot s^{-\frac{1}{2}} && \text{Use the product of powers law.} \\ &= r^{\frac{1}{2} + \frac{1}{4}} \cdot s^{-\frac{3}{2} - \frac{1}{2}} \\ &= r^{\frac{3}{4}} \cdot s^{-\frac{4}{2}} \\ &= r^{\frac{3}{4}} \cdot s^{-\frac{5}{4}} && \text{Write with a positive exponent.} \\ &= \frac{r^{\frac{3}{4}}}{s^{\frac{5}{4}}} \end{aligned}$$

20. Width, in metres, of a piece of A_n paper: $2^{-\frac{2n+1}{4}}$

Length, in metres, of a piece of A_n paper: $2^{-\frac{2n-1}{4}}$

a) i) To determine the dimensions of a piece of A3 paper, substitute: $n = 3$

$$\begin{aligned}\text{Width: } 2^{-\frac{2n+1}{4}} &= 2^{-\frac{2(3)+1}{4}} \\ &= 2^{-\frac{7}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{7}{4}}}\end{aligned}$$

This is the width in metres. Since 1 m = 1000 mm, to write the width in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{7}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{7}{4}}}$$

So, the width of the paper is $\frac{1000}{2^{\frac{7}{4}}}$ mm.

Use a calculator to evaluate: $\frac{1000}{2^{\frac{7}{4}}} = 297.3017\dots$

The width of a piece of A3 paper is about 297 mm.

$$\begin{aligned}\text{Length: } 2^{-\frac{2n-1}{4}} &= 2^{-\frac{2(3)-1}{4}} \\ &= 2^{-\frac{5}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{5}{4}}}\end{aligned}$$

This is the length in metres. Since 1 m = 1000 mm, to write the length in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{5}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{5}{4}}}$$

So, the length of the paper is $\frac{1000}{2^{\frac{5}{4}}}$ mm.

Use a calculator to evaluate: $\frac{1000}{2^{\frac{5}{4}}} = 420.4482\dots$

The length of a piece of A3 paper is about 420 mm.

A piece of A3 paper has dimensions $\frac{1000}{2^{\frac{5}{4}}}$ mm by $\frac{1000}{2^{\frac{7}{4}}}$ mm, or 297 mm by 420 mm.

ii) To determine the dimensions of a piece of A4 paper, substitute: $n = 4$

$$\begin{aligned}\text{Width: } 2^{-\frac{2n+1}{4}} &= 2^{-\frac{2(4)+1}{4}} \\ &= 2^{-\frac{9}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{9}{4}}}\end{aligned}$$

This is the width in metres. To write the width in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{9}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{9}{4}}}$$

So, the width of the paper is $\frac{1000}{2^{\frac{9}{4}}}$ mm.

Use a calculator to evaluate: $\frac{1000}{2^{\frac{9}{4}}} = 210.2241\dots$

The width of a piece of A4 paper is about 210 mm.

$$\begin{aligned}\text{Length: } 2^{-\frac{2n-1}{4}} &= 2^{-\frac{2(4)-1}{4}} \\ &= 2^{-\frac{7}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{7}{4}}}\end{aligned}$$

From part i, $\frac{1000}{2^{\frac{7}{4}}} = 297.3017\dots$

The length of a piece of A4 paper is about 297 mm.

A piece of A4 paper has dimensions $\frac{1000}{2^{\frac{7}{4}}}$ mm by $\frac{1000}{2^{\frac{9}{4}}}$ mm, or 210 mm by 297 mm.

iii) To determine the dimensions of a piece of A5 paper, substitute: $n = 5$

$$\begin{aligned}\text{Width: } 2^{-\frac{2n+1}{4}} &= 2^{-\frac{2(5)+1}{4}} \\ &= 2^{-\frac{11}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{11}{4}}}\end{aligned}$$

This is the width in metres. To write the width in millimetres, multiply by 1000.

$$\frac{1}{2^{\frac{11}{4}}} \cdot 1000 = \frac{1000}{2^{\frac{11}{4}}}$$

So, the width of the paper is $\frac{1000}{2^{\frac{11}{4}}}$ mm.

Use a calculator to evaluate: $\frac{1000}{2^{\frac{11}{4}}} = 148.6508\dots$

The width of a piece of A5 paper is about 149 mm.

$$\begin{aligned} \text{Length: } 2^{\frac{2n-1}{4}} &= 2^{\frac{2(5)-1}{4}} \\ &= 2^{\frac{9}{4}} && \text{Write with a positive exponent.} \\ &= \frac{1}{2^{\frac{4}{4}}} \end{aligned}$$

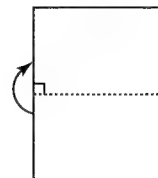
From part ii, $\frac{1000}{2^{\frac{9}{4}}} = 210.2241\dots$

The length of a piece of A5 paper is about 210 mm.

A piece of A5 paper has dimensions $\frac{1000}{2^{\frac{9}{4}}}$ mm by $\frac{1000}{2^{\frac{11}{4}}}$ mm, or 149 mm by 210 mm.

- b) i)** The paper is folded along a line perpendicular to its length.
So, one dimension of a folded piece of A3 paper, in millimetres,
is one-half the original length:

$$\begin{aligned} \frac{1000}{2^{\frac{5}{4}}} \cdot \frac{1}{2} &= \frac{1000}{2^{\frac{5}{4} + 1}} \\ &= \frac{1000}{2^{\frac{5}{4} + \frac{4}{4}}} \\ &= \frac{1000}{2^{\frac{9}{4}}} \end{aligned}$$



The other dimension is the original width: $\frac{1000}{2^4}$ mm

So, a folded piece of A3 paper has dimensions $\frac{1000}{2^4}$ mm by $\frac{1000}{2^4}$ mm.

ii) The paper is folded along a line perpendicular to its length.

So, one dimension of a folded piece of A4 paper, in millimetres, is one-half the original length:

$$\begin{aligned}\frac{1000}{2^4} \cdot \frac{1}{2} &= \frac{1000}{2^4 \cdot 2^1} \\ &= \frac{1000}{2^{4+1}} \\ &= \frac{1000}{2^{4+4}} \\ &= \frac{1000}{2^{11}}\end{aligned}$$

The other dimension is the original width: $\frac{1000}{2^4}$ mm

So, a folded piece of A4 paper has dimensions $\frac{1000}{2^4}$ mm by $\frac{1000}{2^{11}}$ mm.

iii) The paper is folded along a line perpendicular to its length.

So, one dimension of a folded piece of A5 paper, in millimetres, is one-half the original length:

$$\begin{aligned}\frac{1000}{2^4} \cdot \frac{1}{2} &= \frac{1000}{2^4 \cdot 2^1} \\ &= \frac{1000}{2^{4+1}} \\ &= \frac{1000}{2^{4+\frac{4}{2}}} \\ &= \frac{1000}{2^{13}}\end{aligned}$$

The other dimension is the original width: $\frac{1000}{2^{11}}$ mm

So, a folded piece of A5 paper has dimensions $\frac{1000}{2^{11}}$ mm by $\frac{1000}{2^{13}}$ mm.

- c) I noticed that a piece of A4 paper has the same dimensions as a folded piece of A3 paper, and a piece of A5 paper has the same dimensions as a folded piece of A4 paper.

C

21. a)

$$\begin{aligned} \left(\frac{a^{-3}b}{c^2}\right)^{-4} \cdot \left(\frac{c^5}{a^4b^{-3}}\right)^{-1} &= \left(\frac{a^{(-3)(-4)} \cdot b^{1(-4)}}{c^{2(-4)}}\right) \cdot \left(\frac{c^{5(-1)}}{a^{4(-1)} \cdot b^{(-3)(-1)}}\right) \\ &= \left(\frac{a^{12} \cdot b^{-4}}{c^{-8}}\right) \cdot \left(\frac{c^{-5}}{a^{-4} \cdot b^3}\right) \\ &= a^{12}b^{-4}c^8 \cdot a^4b^{-3}c^{-5} \\ &= a^{12+4}b^{-4-3}c^{8-5} \\ &= a^{16}b^{-7}c^3 \\ &= \frac{a^{16}c^3}{b^7} \end{aligned}$$

Using the power of a power law

Rewrite without using fractions.

Use the product of powers law.

Write with a positive exponent.

b)

$$\begin{aligned} \frac{(2a^{-1}b^4c^{-3})^{-2}}{(4a^2bc^{-4})^2} &= \frac{2^{-2} \cdot a^{(-1)(-2)} \cdot b^{4(-2)} \cdot c^{(-3)(-2)}}{4^2 \cdot a^{2(2)} \cdot b^2 \cdot c^{(-4)(2)}} \\ &= \frac{2^{-2} \cdot a^2 \cdot b^{-8} \cdot c^6}{16 \cdot a^4 \cdot b^2 \cdot c^{-8}} \\ &= \frac{2^{-2} \cdot a^2 \cdot b^{-8} \cdot c^6}{2^4 \cdot a^4 \cdot b^2 \cdot c^{-8}} \\ &= 2^{-2-4} \cdot a^{2-4} \cdot b^{-8-2} \cdot c^{6-(-8)} \\ &= 2^{-6} \cdot a^{-2} \cdot b^{-10} \cdot c^{14} \\ &= \frac{c^{14}}{2^6 a^2 b^{10}} \\ &= \frac{c^{14}}{64a^2b^{10}} \end{aligned}$$

Using the power of a power law

Rewrite 16 as a power of 2.

Use the quotient of powers law.

Write with positive exponents.

22. a)

$$\left(x^{\frac{1}{2}}y^{\frac{2}{3}}\right)^2 = x^{\frac{1}{2}(2)} \cdot y^{\frac{2}{3}(2)} \quad \text{Using the power of a power law}$$

$$= x \cdot y^{\frac{4}{3}} \quad \text{Substitute: } x = a^{-2}, y = a^{\frac{2}{3}}$$

$$= a^{-2} \cdot \left(a^{\frac{2}{3}}\right)^{\frac{4}{3}} \quad \text{Use the power of a power law.}$$

$$= a^{-2} \cdot a^{\frac{2}{3} \cdot \frac{4}{3}}$$

$$= a^{-2} \cdot a^{\frac{8}{9}} \quad \text{Use the product of powers law.}$$

$$= a^{-2 + \frac{8}{9}}$$

$$= a^{-\frac{18}{9} + \frac{8}{9}}$$

$$= a^{-\frac{10}{9}} \quad \text{Write with a positive exponent.}$$

$$= \frac{1}{a^{\frac{10}{9}}}$$

b)

$$\left(x^{\frac{3}{4}} \div y^{-\frac{1}{2}}\right)^3 = \left(\frac{x^{\frac{3}{4}}}{y^{-\frac{1}{2}}}\right)^3$$

Rewriting as a fraction

$$= \frac{x^{\frac{3}{4}(3)}}{y^{\left(-\frac{1}{2}\right)^3}}$$

Using the power of a power law

$$= \frac{x^{\frac{9}{4}}}{y^{-\frac{3}{2}}}$$

Substitute: $x = a^{-2}$, $y = a^{\frac{2}{3}}$

$$= \frac{(a^{-2})^{\frac{9}{4}}}{\left(a^{\frac{2}{3}}\right)^{-\frac{3}{2}}}$$

Use the power of a power law.

$$= \frac{a^{(-2)^{\frac{9}{4}}}}{a^{\frac{2}{3} \cdot \left(-\frac{3}{2}\right)}}$$

$$= \frac{a^{-\frac{9}{2}}}{a^{-1}}$$

Use the quotient of powers law.

$$= a^{-\frac{9}{2} - (-1)}$$

$$= a^{-\frac{9}{2} + \frac{2}{2}}$$

$$= a^{-\frac{7}{2}}$$

Write with a positive exponent.

$$= \frac{1}{a^{\frac{7}{2}}}$$

23. a) The product of powers law is: $x^m \cdot x^n = x^{m+n}$

So, for the product to be $x^{\frac{3}{2}}$, I need to find two exponents whose sum is $\frac{3}{2}$.

For example:

$$\begin{aligned} x^{\frac{1}{2}} \cdot x^1 &= x^{\frac{1}{2} + 1} \\ &= x^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} x^{\frac{1}{4}} \cdot x^{\frac{5}{4}} &= x^{\frac{1}{4} + \frac{5}{4}} \\ &= x^{\frac{6}{4}} \\ &= x^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} x^{\frac{11}{8}} \cdot x^{\frac{1}{8}} &= x^{\frac{11}{8} + \frac{1}{8}} \\ &= x^{\frac{12}{8}} \\ &= x^{\frac{3}{2}} \end{aligned}$$

- b) The quotient of powers law is: $x^m \div x^n = x^{m-n}$

So, for the quotient to be $x^{\frac{3}{2}}$, I need to find two exponents whose difference is $\frac{3}{2}$.

For example:

$$x^2 \div x^{\frac{1}{2}} = x^{2-\frac{1}{2}}$$

$$= x^{\frac{3}{2}}$$

$$x^5 \div x^{\frac{5}{2}} = x^{5-\frac{5}{2}}$$

$$= x^{\frac{5}{2}}$$

$$x^7 \div x^{\frac{7}{2}} = x^{7-\frac{7}{2}}$$

$$= x^{\frac{7}{2}}$$

- c) The power of a power law is: $(x^m)^n = x^{mn}$

So, for the result to be $x^{\frac{3}{2}}$, I need to find two exponents whose product is $\frac{3}{2}$.

For example:

$$(x^3)^{\frac{1}{2}} = x^{3(\frac{1}{2})}$$

$$= x^{\frac{3}{2}}$$

$$\left(x^{\frac{1}{3}}\right)^{\frac{9}{2}} = x^{\left(\frac{1}{3}\right)\left(\frac{9}{2}\right)}$$

$$= x^{\frac{9}{6}}$$

$$= x^{\frac{3}{2}}$$

$$(x^9)^{\frac{1}{6}} = x^{9\left(\frac{1}{6}\right)}$$

$$= x^{\frac{9}{6}}$$

$$= x^{\frac{3}{2}}$$

24. In $\frac{AC}{AB} = \frac{AD}{AE}$, substitute $AB = \left(\frac{2}{3}\right)^{\frac{1}{2}}$, $AC = \frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}}$, $AE = \left(\frac{1}{3}\right)^{\frac{1}{2}}$, then solve for AD.

$$\frac{\frac{3\left(\frac{1}{3}\right)^{\frac{1}{2}}}{2\left(\frac{2}{3}\right)^{\frac{1}{2}}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = \frac{AD}{\left(\frac{1}{3}\right)^{\frac{1}{2}}}$$

Multiply both sides by $\left(\frac{1}{3}\right)^{\frac{1}{2}}$.

$$\left(\frac{1}{3}\right)^{\frac{1}{2}} \left(\frac{\frac{3\left(\frac{1}{3}\right)^{\frac{1}{2}}}{2\left(\frac{2}{3}\right)^{\frac{1}{2}}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} \right) = \left(\frac{1}{3}\right)^{\frac{1}{2}} \left(\frac{AD}{\left(\frac{1}{3}\right)^{\frac{1}{2}}} \right)$$

$$\frac{\frac{3\left(\frac{1}{3}\right)^{\frac{1}{2}}\left(\frac{1}{3}\right)^{\frac{1}{2}}}{2\left(\frac{2}{3}\right)^{\frac{1}{2}}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = AD$$

Use the product of powers law.

$$\frac{\frac{3\left(\frac{1}{3}\right)^{\frac{1}{2}+\frac{1}{2}}}{2\left(\frac{2}{3}\right)^{\frac{1}{2}}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = AD$$

$$\frac{\frac{3\left(\frac{1}{3}\right)}{2\left(\frac{2}{3}\right)^{\frac{1}{2}}}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = AD$$

$$\frac{\frac{1}{2}}{\left(\frac{2}{3}\right)^{\frac{1}{2}}} = AD$$

Use a negative exponent to move the denominator to the numerator.

$$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{-\frac{1}{2}} = AD$$

Write with a positive exponent.

$$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}} = AD$$

Use a calculator to evaluate:

$$\begin{aligned} AD &= \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}} \\ &= 0.6123... \end{aligned}$$

The length of AD is $\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{\frac{1}{2}}$ cm, or approximately 0.6 cm.

Review

(pages 246–248)

4.1

$$1. \text{ a) } \sqrt[3]{1000} = \sqrt[3]{10 \cdot 10 \cdot 10} \\ = 10$$

$$\text{b) } \sqrt{0.81} = \sqrt{0.9 \cdot 0.9} \\ = 0.9$$

$$\text{c) } \sqrt[6]{64} = \sqrt[6]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\ = 2$$

$$\text{d) } \sqrt[4]{\frac{81}{625}} = \sqrt[4]{\frac{3 \cdot 3 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 5 \cdot 5}} \\ = \frac{3}{5}$$

I did not need a calculator because 1000 is a perfect cube, 0.81 is a perfect square, 64 is a perfect sixth power, and $\frac{81}{625}$ is a perfect fourth power.

2. The index of a radical tells me which root I have to take.

For example, the index of $\sqrt[3]{27}$ is 3, so I take the cube root.

The index of $\sqrt[4]{16}$ is 4, so I take the fourth root.

The index of $\sqrt[5]{243}$ is 5, so I take the fifth root.

When there is no index written, it is assumed to be 2, and I take the square root; for example, $\sqrt{25}$.

3. Use benchmarks with guess and check.

- a) 11 is between the perfect squares 9 and 16, but closer to 9.

So, $\sqrt{11}$ is between 3 and 4, but closer to 3.

Estimate to 1 decimal place: $\sqrt{11} \doteq 3.4$

Square the estimate: $3.4^2 = 11.56$ (too large, but close)

Revise the estimate: $\sqrt{11} \doteq 3.3$

Square the estimate: $3.3^2 = 10.89$ (very close)

10.89 is closer to 11, so $\sqrt{11}$ is approximately 3.3.

- b) -12 is between the perfect cubes -8 and -27, but closer to -8.

So, $\sqrt[3]{-12}$ is between -2 and -3, but closer to -2.

Estimate to 1 decimal place: $\sqrt[3]{-12} \doteq -2.2$

Cube the estimate: $(-2.2)^3 = -10.648$ (too large)

Revise the estimate: $\sqrt[3]{-12} \doteq -2.3$

Cube the estimate: $(-2.3)^3 = -12.167$ (very close)

-12.167 is closer to -12 , so $\sqrt[3]{-12}$ is approximately -2.3 .

- c) 15 is between the perfect fourth powers 1 and 16, but closer to 16.

So, $\sqrt[4]{15}$ is between 1 and 2, but closer to 2.

Estimate to 1 decimal place: $\sqrt[4]{15} \doteq 1.9$

Raise the estimate to the fourth power: $1.9^4 = 13.0321$ (too small)

Revise the estimate: $\sqrt[4]{15} \doteq 2.0$

Raise the estimate to the fourth power: $2.0^4 = 16.0$ (closer)

16 is closer to 15, so $\sqrt[4]{15}$ is approximately 2.0.

4. a) $5^2 = 25$, so $5 = \sqrt{25}$

The number is 25.

- b) $6^3 = 216$, so $6 = \sqrt[3]{216}$

The number is 216.

- c) $7^4 = 2401$, so $7 = \sqrt[4]{2401}$

The number is 2401.

5. $\sqrt[3]{35}$ is irrational because 35 is not a perfect cube.

The decimal form of $\sqrt[3]{35}$ neither terminates nor repeats.

4.2

6. a) -2 is rational because it can be written as a quotient of integers, $\frac{-2}{1}$.

- b) 17 is rational because it can be written as a quotient of integers, $\frac{17}{1}$.

- c) $\sqrt{16}$ is rational because 16 is a perfect square.

Its decimal form is 4.0, which terminates.

- d) $\sqrt{32}$ is irrational because 32 is not a perfect square.

The decimal form of $\sqrt{32}$ neither terminates nor repeats.

- e) 0.756 is rational because it is a terminating decimal.

- f) $12.\bar{3}$ is rational because it is a repeating decimal.

- g) 0 is rational because it can be written as a quotient of integers, $\frac{0}{1}$.

h) $\sqrt[3]{81}$ is irrational because 81 is not a perfect cube.

The decimal form of $\sqrt[3]{81}$ neither terminates nor repeats.

i) π is irrational because the decimal form of π neither terminates nor repeats.

7. The formula for the area, A , of a square with side length s units is:

$$A = s^2$$

To determine the value of s , take the square root of each side.

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s \quad \text{Substitute: } A = 23$$

$$s = \sqrt{23}$$

Use a calculator.

$$\sqrt{23} = 4.7958\dots$$

The side length of the square is approximately 4.8 cm.

I could check my answer by squaring the side length.

If the result is close to 23 cm^2 , my answer is correct.

8. a) 3.141 592 654 is a terminating decimal, so 3.141 592 654 is a rational number.

b) The number π is irrational because it cannot be written as a quotient of integers.
The calculator screen shows only the first 10 digits for the value of π .

9. 30 is between the perfect cubes 27 and 64, and is closer to 27.

$$\sqrt[3]{27} \quad \sqrt[3]{30} \quad \sqrt[3]{64}$$



$$3 \quad ? \quad 4$$

Use a calculator.

$$\sqrt[3]{30} = 3.1072\dots$$

20 is between the perfect squares 16 and 25, and is closer to 16.

$$\sqrt{16} \quad \sqrt{20} \quad \sqrt{25}$$



$$4 \quad ? \quad 5$$

Use a calculator.

$$\sqrt{20} = 4.4721\dots$$

18 is between the perfect fourth powers 16 and 81, and is closer to 16.

$$\begin{array}{ccc} \sqrt[4]{16} & \sqrt[4]{18} & \sqrt[4]{81} \\ \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$$\sqrt[4]{18} = 2.0597\dots$$

-30 is between the perfect cubes -27 and -64, and is closer to -27.

$$\begin{array}{ccc} \sqrt[3]{-27} & \sqrt[3]{-30} & \sqrt[3]{-64} \\ \downarrow & \downarrow & \downarrow \\ -3 & ? & -4 \end{array}$$

Use a calculator.

$$\sqrt[3]{-30} = -3.1072\dots$$

30 is between the perfect squares 25 and 36, and is closer to 25.

$$\begin{array}{ccc} \sqrt{25} & \sqrt{30} & \sqrt{36} \\ \downarrow & \downarrow & \downarrow \\ 5 & ? & 6 \end{array}$$

Use a calculator.

$$\sqrt{30} = 5.4772\dots$$

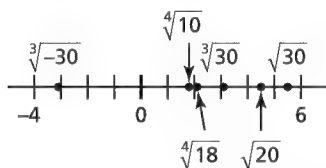
10 is between the perfect fourth powers 1 and 16, and is closer to 16.

$$\begin{array}{ccc} \sqrt[4]{1} & \sqrt[4]{10} & \sqrt[4]{16} \\ \downarrow & \downarrow & \downarrow \\ 1 & ? & 2 \end{array}$$

Use a calculator.

$$\sqrt[4]{10} = 1.7782\dots$$

Mark each number on a number line.



From least to greatest: $\sqrt[3]{-30}$, $\sqrt[4]{10}$, $\sqrt[4]{18}$, $\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt{30}$

10. Use the formula: $T = 2\pi\sqrt{\frac{L}{9.8}}$

Substitute: $L = 0.25$

$$\begin{aligned} T &= 2\pi\sqrt{\frac{0.25}{9.8}} && \text{Use a calculator.} \\ &= 1.0035... \end{aligned}$$

It takes the pendulum approximately 1 s to complete one swing.

4.3

11. a)
$$\begin{aligned} \sqrt{150} &= \sqrt{25 \cdot 6} \\ &= \sqrt{5 \cdot 5 \cdot 6} \\ &= \sqrt{(5 \cdot 5) \cdot 6} \\ &= \sqrt{5 \cdot 5} \cdot \sqrt{6} \\ &= 5 \cdot \sqrt{6} \\ &= 5\sqrt{6} \end{aligned}$$

b)
$$\begin{aligned} \sqrt[3]{135} &= \sqrt[3]{27 \cdot 5} \\ &= \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 5} \\ &= \sqrt[3]{(3 \cdot 3 \cdot 3) \cdot 5} \\ &= \sqrt[3]{(3 \cdot 3 \cdot 3)} \cdot \sqrt[3]{5} \\ &= 3 \cdot \sqrt[3]{5} \\ &= 3\sqrt[3]{5} \end{aligned}$$

c)
$$\begin{aligned} \sqrt{112} &= \sqrt{16 \cdot 7} \\ &= \sqrt{4 \cdot 4 \cdot 7} \\ &= \sqrt{(4 \cdot 4) \cdot 7} \\ &= \sqrt{4 \cdot 4} \cdot \sqrt{7} \\ &= 4 \cdot \sqrt{7} \\ &= 4\sqrt{7} \end{aligned}$$

d)
$$\begin{aligned} \sqrt[4]{162} &= \sqrt[4]{81 \cdot 2} \\ &= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2} \\ &= \sqrt[4]{(3 \cdot 3 \cdot 3 \cdot 3) \cdot 2} \\ &= \sqrt[4]{(3 \cdot 3 \cdot 3 \cdot 3)} \cdot \sqrt[4]{2} \\ &= 3 \cdot \sqrt[4]{2} \\ &= 3\sqrt[4]{2} \end{aligned}$$

12. a) Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned} 6\sqrt{5} &= \sqrt{36} \cdot \sqrt{5} \\ &= \sqrt{36 \cdot 5} \\ &= \sqrt{180} \end{aligned}$$

b) Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned} 3\sqrt{14} &= \sqrt{9} \cdot \sqrt{14} \\ &= \sqrt{9 \cdot 14} \\ &= \sqrt{126} \end{aligned}$$

c) Write 4 as: $\sqrt[3]{4 \cdot 4 \cdot 4} = \sqrt[3]{64}$

$$\begin{aligned} 4\sqrt[3]{3} &= \sqrt[3]{64} \cdot \sqrt[3]{3} \\ &= \sqrt[3]{64 \cdot 3} \\ &= \sqrt[3]{192} \end{aligned}$$

d) Write 2 as: $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16}$

$$\begin{aligned} 2\sqrt[4]{2} &= \sqrt[4]{16} \cdot \sqrt[4]{2} \\ &= \sqrt[4]{16 \cdot 2} \\ &= \sqrt[4]{32} \end{aligned}$$

13. The formula for the volume, V , of a cube with edge length e units is:

$$V = e^3$$

To determine the value of e , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{e^3}$$

$$\sqrt[3]{V} = e$$

To determine the edge length of a cube with volume 32 cm^3 , substitute: $V = 32$

$$e = \sqrt[3]{32}$$

Write 32 as $8 \cdot 4$.

$$\begin{aligned} e &= \sqrt[3]{8 \cdot 4} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{4} \\ &= 2 \cdot \sqrt[3]{4} \\ &= 2\sqrt[3]{4} \end{aligned}$$

The cube has edge length $2\sqrt[3]{4} \text{ cm}$.

Use a calculator to evaluate: $2\sqrt[3]{4} = 3.1748\dots$

The edge length of the cube is approximately 3.2 cm.

To determine the edge length of a cube with volume 11 cm^3 ,

in $e = \sqrt[3]{V}$ substitute: $V = 11$

$$e = \sqrt[3]{11}$$

$\sqrt[3]{11}$ cannot be simplified.

Use a calculator to evaluate: $\sqrt[3]{11} = 2.2239\dots$

The edge length of the cube is approximately 2.2 cm.

The difference in the edge lengths is approximately $3.2 \text{ cm} - 2.2 \text{ cm} = 1.0 \text{ cm}$.

14. In the second line, the student wrote $\sqrt{100}$ as $\sqrt{50} \cdot \sqrt{50}$; the student should have written $\sqrt{100}$ as 10 because 100 is a perfect square. From the third line to the fourth line, the student wrote $\sqrt{3} \cdot \sqrt{2}$ as 3, which is not correct.

A correct solution is:

$$\begin{aligned}\sqrt{300} &= \sqrt{100 \cdot 3} \\ &= \sqrt{100} \cdot \sqrt{3} \\ &= 10 \cdot \sqrt{3} \\ &= 10\sqrt{3}\end{aligned}$$

15. Since all the radicals are square roots, I will rewrite each mixed radical as an entire radical then order the entire radicals from the greatest radicand to the least radicand.

Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$\begin{aligned}5\sqrt{2} &= \sqrt{25} \cdot \sqrt{2} \\ &= \sqrt{25 \cdot 2} \\ &= \sqrt{50}\end{aligned}$$

Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$\begin{aligned}4\sqrt{3} &= \sqrt{16} \cdot \sqrt{3} \\ &= \sqrt{16 \cdot 3} \\ &= \sqrt{48}\end{aligned}$$

Write 3 as: $\sqrt{3 \cdot 3} = \sqrt{9}$

$$\begin{aligned}3\sqrt{6} &= \sqrt{9} \cdot \sqrt{6} \\ &= \sqrt{9 \cdot 6} \\ &= \sqrt{54}\end{aligned}$$

Write 2 as: $\sqrt{2 \cdot 2} = \sqrt{4}$

$$\begin{aligned} 2\sqrt{7} &= \sqrt{4} \cdot \sqrt{7} \\ &= \sqrt{4 \cdot 7} \\ &= \sqrt{28} \end{aligned}$$

Write 6 as: $\sqrt{6 \cdot 6} = \sqrt{36}$

$$\begin{aligned} 6\sqrt{2} &= \sqrt{36} \cdot \sqrt{2} \\ &= \sqrt{36 \cdot 2} \\ &= \sqrt{72} \end{aligned}$$

From greatest to least, the entire radicals are: $\sqrt{72}$, $\sqrt{54}$, $\sqrt{50}$, $\sqrt{48}$, $\sqrt{28}$

So, from greatest to least, the mixed radicals are: $6\sqrt{2}$, $3\sqrt{6}$, $5\sqrt{2}$, $4\sqrt{3}$, $2\sqrt{7}$

4.4

16. The product of powers law is: $a^m \cdot a^n = a^{m+n}$

A multiplication property of radicals is: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

$$\begin{aligned} \text{So, } 5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} &= 5^{\frac{1}{2} + \frac{1}{2}} & \text{and} & & \sqrt{5} \cdot \sqrt{5} &= \sqrt{25} \\ &= 5^1 & & & &= 5 \\ &= 5 & & & & \end{aligned}$$

So, $5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

$$\begin{aligned} \text{Similarly, } 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} &= 5^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} & \text{and} & & \sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} &= \sqrt[3]{125} \\ &= 5^1 & & & &= 5 \\ &= 5 & & & & \end{aligned}$$

So, $5^{\frac{1}{3}}$ and $\sqrt[3]{5}$ are equivalent expressions; that is, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

So, when n is a natural number and a is a rational number, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

17. a) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$12^{\frac{1}{4}} = \sqrt[4]{12}$$

b) Use $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$.

$$(-50)^{\frac{5}{3}} = \left(\sqrt[3]{-50}\right)^5, \text{ or } \sqrt[3]{(-50)^5}$$

c) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$\begin{aligned} 1.2^{0.5} &= 1.2^{\frac{1}{2}} \\ &= \sqrt{1.2} \end{aligned}$$

d) Use the rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

$$\left(\frac{3}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{3}{8}}$$

18. a) Use: $\sqrt[n]{x} = x^{\frac{1}{n}}$

$$\sqrt{1.4} = 1.4^{\frac{1}{2}}$$

b) Use: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$$\sqrt[3]{13^2} = 13^{\frac{2}{3}}$$

c) Use $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$.

$$\left(\sqrt[5]{2.5}\right)^4 = 2.5^{\frac{4}{5}}$$

d) Use: $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$

$$\left(\sqrt[4]{\frac{2}{5}}\right)^3 = \left(\frac{2}{5}\right)^{\frac{3}{4}}$$

19. a) The exponent $0.25 = \frac{1}{4}$

$$\begin{aligned} \text{So, } 16^{0.25} &= 16^{\frac{1}{4}} \\ &= \sqrt[4]{16} \\ &= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} \\ &= 2 \end{aligned}$$

$$\begin{aligned}\text{b) } 1.44^{\frac{1}{2}} &= \sqrt{1.44} \\ &= \sqrt{1.2 \cdot 1.2} \\ &= 1.2\end{aligned}$$

$$\begin{aligned}\text{c) } (-8)^{\frac{5}{3}} &= \left[(-8)^{\frac{1}{3}} \right]^5 \\ &= \left(\sqrt[3]{-8} \right)^5 \\ &= (-2)^5 \\ &= -32\end{aligned}$$

$$\begin{aligned}\text{d) } \left(\frac{9}{16} \right)^{\frac{3}{2}} &= \left[\left(\frac{9}{16} \right)^{\frac{1}{2}} \right]^3 \\ &= \left(\sqrt{\frac{9}{16}} \right)^3 \\ &= \left(\frac{3}{4} \right)^3 \\ &= \frac{27}{64}\end{aligned}$$

20. In the formula: $P = 100(0.5)^{\frac{t}{20}}$, substitute: $t = 30$

$$\begin{aligned}P &= 100(0.5)^{\frac{30}{20}} \quad \text{Use a calculator.} \\ &= 35.3553...\end{aligned}$$

Approximately 35% of polonium remains after 30 weeks.

21. The radicand of each number is 5, so write each number with a fractional exponent.
To order the numbers from greatest to least, order the exponents from greatest to least.
The number with the greatest exponent is the greatest number.
The number with the least exponent is the least number.

$$\sqrt[4]{5} = 5^{\frac{1}{4}} \qquad 5^{\frac{2}{3}} \qquad \sqrt[3]{5} = 5^{\frac{1}{3}} \qquad 5^{\frac{3}{4}} \qquad (\sqrt{5})^3 = 5^{\frac{3}{2}}$$

From greatest to least, the exponents are: $\frac{3}{2}, \frac{3}{4}, \frac{2}{3}, \frac{1}{3}, \frac{1}{4}$

So, from greatest to least, the numbers are: $(\sqrt{5})^3, 5^{\frac{3}{4}}, 5^{\frac{2}{3}}, \sqrt[3]{5}, \sqrt[4]{5}$

22. a) In the formula $q = 70M^{\frac{3}{4}}$, substitute: $M = 475$

$$q = 70(475)^{\frac{3}{4}}$$

$$= 7122.2669...$$

Use a calculator.

The metabolic rate of a cow with mass 475 kg is approximately 7122 Calories per day.

- b) In the formula $q = 70M^{\frac{3}{4}}$, the mass must be in kilograms.

To write 25 g in kilograms, divide by 1000: $25 \text{ g} = 0.025 \text{ kg}$

Substitute: $M = 0.025$

$$q = 70(0.025)^{\frac{3}{4}}$$

$$= 4.4010...$$

Use a calculator.

The metabolic rate of a mouse with mass 25 g is approximately 4 Calories per day.

4.5

23. a) The numbers at the left are divided by 3 each time.
The exponents in the powers at the right decrease by 1 each time.

b)

$$81 = 3^4$$

$$27 = 3^3$$

$$9 = 3^2$$

$$3 = 3^1$$

$$1 = 3^0$$

$$\frac{1}{3} = 3^{-1}$$

$$\frac{1}{9} = 3^{-2}$$

$$\frac{1}{27} = 3^{-3}$$

- c) Look at the last 3 rows.

$$3^{-1} = \frac{1}{3}$$

$$= \frac{1}{3^1}$$

$$3^{-2} = \frac{1}{9}$$

$$= \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{27}$$

$$= \frac{1}{3^3}$$

So, this pattern shows that $a^{-n} = \frac{1}{a^n}$.

$$\begin{aligned} 24. \text{ a) } 2^{-2} &= \frac{1}{2^2} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{2}{3}\right)^{-3} &= \left(\frac{3}{2}\right)^3 && \text{Writing with a positive exponent} \\ &= \frac{3^3}{2^3} && \text{Cubing the result} \\ &= \frac{27}{8} \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{4}{25}\right)^{-\frac{3}{2}} &= \left(\frac{25}{4}\right)^{\frac{3}{2}} && \text{Writing with a positive exponent} \\ &= \left(\sqrt{\frac{25}{4}}\right)^3 && \text{Taking the square root} \\ &= \left(\frac{5}{2}\right)^3 && \text{Cubing the result} \\ &= \frac{125}{8} \end{aligned}$$

25. Use a calculator to evaluate:

$$\begin{aligned} P &= 1000(1.0325)^{-3} \\ &= 908.5102... \end{aligned}$$

To have \$1000 in 3 years, Kyle should invest \$908.51 today.

26. In the formula: $h = 2000(3)^{-\frac{1}{2}}s^{-2}$, substitute: $s = 8.0$

$$\begin{aligned} h &= 2000(3)^{-\frac{1}{2}}(8.0)^{-2} && \text{Use a calculator.} \\ &= 18.0421... \end{aligned}$$

The height of a container with base side length 8.0 cm is approximately 18.0 cm.

27. Use the formula: $F = 440\left(\sqrt[12]{2}\right)^n$

Middle C is 9 semitones below the concert pitch, so substitute $n = -9$.

$$\begin{aligned} F &= 440\left(\sqrt[12]{2}\right)^{-9} && \text{Use a calculator.} \\ &= 261.6255... \end{aligned}$$

The frequency of middle C is approximately 262 Hz.

4.6

$$\begin{aligned}
 28. \text{ a) } (3m^4n)^2 &= (3^1)^2 \cdot (m^4)^2 \cdot (n^1)^2 \\
 &= 3^{(1)(2)} \cdot m^{(4)(2)} \cdot n^{(1)(2)} \\
 &= 3^2 \cdot m^8 \cdot n^2 \\
 &= 9m^8n^2
 \end{aligned}$$

Using the power of a product law

$$\begin{aligned}
 \text{b) } \left(\frac{x^2y}{y^2}\right)^2 &= (x^2 \cdot y^{1-2})^2 \\
 &= (x^2 \cdot y^{-1})^2 \\
 &= x^{2(2)} \cdot y^{3(-2)} \\
 &= x^4 \cdot y^{-6} \\
 &= x^4y^{-6} \\
 &= \frac{1}{x^4y^6}
 \end{aligned}$$

Using the quotient of powers law inside the brackets

Use the power of a product law.

Write with positive exponents.

c)

$$\begin{aligned}
 (16a^2b^6)^{-\frac{1}{2}} &= \frac{1}{(16a^2b^6)^{\frac{1}{2}}} \\
 &= \frac{1}{16^{\frac{1}{2}} \cdot a^{2(\frac{1}{2})} \cdot b^{6(\frac{1}{2})}} \\
 &= \frac{1}{16^{\frac{1}{2}} \cdot a^1 \cdot b^3} \\
 &= \frac{1}{4ab^3}
 \end{aligned}$$

Writing with a positive exponent

Using the power of a product law

d)

$$\begin{aligned}
 \left(\frac{r^3s^{-1}}{s^{-2}r^{-2}}\right)^{-\frac{2}{3}} &= \left(\frac{r^3}{r^{-2}} \cdot \frac{s^{-1}}{s^{-2}}\right)^{-\frac{2}{3}} \\
 &= (r^{3-(-2)} \cdot s^{-1-(-2)})^{-\frac{2}{3}} \\
 &= (r^5 \cdot s^1)^{-\frac{2}{3}} \\
 &= r^{5(-\frac{2}{3})} \cdot s^{1(-\frac{2}{3})} \\
 &= r^{-\frac{10}{3}} \cdot s^{-\frac{2}{3}} \\
 &= \frac{1}{r^{\frac{10}{3}}s^{\frac{2}{3}}}
 \end{aligned}$$

Simplify inside the brackets.

Using the quotient of powers law

Use the power of a product law.

Write with positive exponents.

29. a) $(a^3b)(a^{-1}b^4) = a^3 \cdot a^{-1} \cdot b^1 \cdot b^4$ Use the product of powers law.

$$= a^{3-1} \cdot b^{1+4}$$

$$= a^2 \cdot b^5$$

$$= a^2b^5$$

b)

$$\left(x^{\frac{1}{2}}y\right)\left(x^{\frac{3}{2}}y^{-2}\right) = x^{\frac{1}{2}} \cdot x^{\frac{3}{2}} \cdot y^1 \cdot y^{-2}$$

Use the product of powers law.

$$= x^{\frac{1}{2} + \frac{3}{2}} \cdot y^{1+(-2)}$$

$$= x^2 \cdot y^{-1}$$

Write with a positive exponent.

$$= x^2y^{-1}$$

$$= \frac{x^2}{y}$$

c) $\frac{a^3}{a^5} \cdot a^{-3} = a^{3-5} \cdot a^{-3}$ Using the quotient of powers law

$$= a^{-2} \cdot a^{-3}$$

Use the product of powers law.

$$= a^{-2+(-3)}$$

Write with a positive exponent.

$$= a^{-5}$$

$$= \frac{1}{a^5}$$

d) $\frac{x^2y}{x^{\frac{1}{2}}y^{-2}} = \frac{x^2}{x^{\frac{1}{2}}} \cdot \frac{y}{y^{-2}}$ Use the quotient of powers law.

$$= x^{2-\frac{1}{2}} \cdot y^{1-(-2)}$$

$$= x^{\frac{3}{2}} \cdot y^3$$

$$= x^{\frac{3}{2}}y^3$$

30. a) $\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \left(\frac{3}{2}\right)^{\frac{1}{2}} = \left(\frac{3}{2}\right)^{\frac{3}{2} + \frac{1}{2}}$ Using the product of powers law

$$= \left(\frac{3}{2}\right)^2$$

$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}$$

b) $\frac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}} = (-5.5)^{\frac{2}{3} - (-\frac{4}{3})}$ Using the quotient of powers law

$$= (-5.5)^{\frac{6}{3}}$$

$$= (-5.5)^2$$

$$= 30.25$$

c) $\left[\left(-\frac{12}{5}\right)^{\frac{1}{3}}\right]^6 = \left(-\frac{12}{5}\right)^{\frac{1}{3}(6)}$ Using the power of a power law

$$= \left(-\frac{12}{5}\right)^2$$

$$= \frac{144}{25}$$

d) $\frac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}} = 0.16^{\frac{3}{4} - \frac{1}{4}}$ Using the quotient of powers law

$$= 0.16^{\frac{2}{4}}$$

$$= 0.16^{\frac{1}{2}}$$

$$= \sqrt{0.16}$$

$$= 0.4$$

31. The volume, V , of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$

Substitute $V = 1100$, then solve for r .

$$1100 = \frac{4}{3}\pi r^3 \quad \text{Multiply each side by 3.}$$

$$3(1100) = 3\left(\frac{4}{3}\pi r^3\right)$$

$$3300 = 4\pi r^3 \quad \text{Divide each side by } 4\pi.$$

$$\frac{3300}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\frac{3300}{4\pi} = r^3 \quad \text{To solve for } r, \text{ take the cube root of each side by raising each side to the one-third power.}$$

$$\left(\frac{3300}{4\pi}\right)^{\frac{1}{3}} = (r^3)^{\frac{1}{3}} \quad \text{Use the power of a power law.}$$

$$\left(\frac{3300}{4\pi}\right)^{\frac{1}{3}} = r \quad \text{Use a calculator.}$$

$$r = 6.4037...$$

The radius of the sphere is approximately 6.4 cm.

32. a) In the second line, the exponents were multiplied instead of added.

A correct solution is:

$$\left(s^{-1}t^{\frac{1}{3}}\right)\left(s^4t^3\right) = s^{-1} \cdot s^4 \cdot t^{\frac{1}{3}} \cdot t^3 \quad \text{Use the product of powers law.}$$

$$= s^{-1+4} \cdot t^{\frac{1}{3}+3}$$

$$= s^3 \cdot t^{\frac{10}{3}}$$

$$= s^3 t^{\frac{10}{3}}$$

- b) In the first line, 4 was multiplied by -3 , instead of being raised to the power of -3 ; and, to raise d^3 to the power -3 , the exponents were added instead of multiplied.

In the third line, -12 was incorrectly written as $\frac{1}{12}$.

A correct solution is:

$$\left(\frac{4c^{\frac{1}{3}}}{d^3} \right)^{-3} = \frac{4^{-3} \cdot c^{\left(\frac{1}{3}\right)(-3)}}{d^{3(-3)}}$$

Using the power of a power law

$$= \frac{4^{-3} \cdot c^{-1}}{d^{-9}}$$

Write with positive exponents.

$$= \frac{d^9}{4^3 \cdot c}$$

$$= \frac{d^9}{64c}$$

Practice Test

(page 249)

1. The formula for the volume, V , of a cube with edge length s units is:

$$V = s^3$$

To determine the value of s , take the cube root of each side.

$$\sqrt[3]{V} = \sqrt[3]{s^3}$$

$$\sqrt[3]{V} = s$$

For s to be irrational, V must be a positive number that is not a perfect cube.

Try to write each given volume as a perfect cube.

$$5^3 = 125, 4^3 = 64, 6^3 = 216$$

75 is not a perfect cube.

This is the volume of the cube in choice B.

So, choice B is correct.

2. $\sqrt{0.09}$ is rational because 0.09 is a perfect square; $0.3^2 = 0.09$

$\sqrt{50}$ is irrational because 50 is not a perfect square.

$\sqrt[3]{-\frac{64}{121}}$ is irrational because 121 is not a perfect cube.

π is irrational because it cannot be written as a quotient of integers.

So, choice A is correct.

3. a) Since both radicals have an index of 2, the greater radical has the greater radicand.

Rewrite $5\sqrt{3}$ as an entire radical.

Write 5 as: $\sqrt{5 \cdot 5} = \sqrt{25}$

$$5\sqrt{3} = \sqrt{25} \cdot \sqrt{3}$$

$$= \sqrt{25 \cdot 3}$$

$$= \sqrt{75}$$

Compare $\sqrt{75}$ and $\sqrt{70}$.

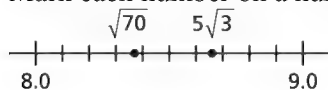
Since $75 > 70$, $\sqrt{75} > \sqrt{70}$

So, $\sqrt{75}$ or $5\sqrt{3}$ is the greater radical.

- b) Use a calculator.

$$5\sqrt{3} = 8.6602... \quad \sqrt{70} = 8.3666...$$

Mark each number on a number line.



4. a)
$$\sqrt[4]{\frac{256}{81}} = \sqrt[4]{\frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3}}$$
$$= \frac{4}{3}$$

$$\begin{aligned}\text{b) } (-4)^{-2} &= \frac{1}{(-4)^2} \\ &= \frac{1}{16}\end{aligned}$$

$$\begin{aligned}\text{c) } 0.81^{\frac{3}{2}} &= (\sqrt{0.81})^3 \\ &= 0.9^3 \\ &= 0.729\end{aligned}$$

$$\begin{aligned}\text{d) } 16^{-\frac{1}{2}} &= \frac{1}{16^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{16}} \\ &= \frac{1}{4}\end{aligned}$$

$$5. \quad 44^{\frac{1}{2}} = \sqrt{44}$$

The factors of 44 are: 1, 2, 4, 11, 22, 44

The greatest perfect square is $4 = 2 \cdot 2$, so write 44 as $4 \cdot 11$.

$$\begin{aligned}\sqrt{44} &= \sqrt{4 \cdot 11} \\ &= \sqrt{4} \cdot \sqrt{11} \\ &= 2 \cdot \sqrt{11} \\ &= 2\sqrt{11}\end{aligned}$$

6. The student is incorrect. When using the quotient of powers law, the exponents should be subtracted, not added.

A correct solution is:

$$\frac{x^{-1}y^3}{xy^{-2}} = x^{-1-1} \cdot y^{3-(-2)}$$

Using the quotient of powers law

$$= x^{-2} \cdot y^5$$

$$= x^{-2}y^5$$

Write with a positive exponent.

$$= \frac{y^5}{x^2}$$

7. a) $(p^{-2}q^{-1})^2(pq^2)^2 = (p^{(-2)(2)}q^{-1(2)})(p^2q^{2(2)})$ Using the power of a product law

$$= (p^{-4}q^{-2})(p^2q)$$

$$= p^{-4} \cdot q^{-2} \cdot p^2 \cdot q$$
 Use the product of powers law.
$$= p^{-4+2} \cdot q^{-2+1}$$

$$= p^{-2}q^{-1}$$
 Write with positive exponents.
$$= \frac{1}{p^2q}$$

b)

$$\left(\frac{c^6d^5}{c^3d^4}\right)^{\frac{1}{3}} = \left(\frac{c^3d^4}{c^6d^5}\right)^{\frac{1}{3}}$$
 Writing with a positive exponent
$$= \frac{c^{3\left(\frac{1}{3}\right)} \cdot d^{4\left(\frac{1}{3}\right)}}{c^{6\left(\frac{1}{3}\right)} \cdot d^{5\left(\frac{1}{3}\right)}}$$
 Using the power of a product law
$$= \frac{c^1 \cdot d^{\frac{4}{3}}}{c^2 \cdot d^{\frac{5}{3}}}$$
 Use the quotient of powers law.
$$= c^{1-2} \cdot d^{\frac{4}{3}-\frac{5}{3}}$$

$$= c^{-1}d^{-\frac{1}{3}}$$
 Write with positive exponents.
$$= \frac{1}{cd^{\frac{1}{3}}}$$

8. In the formula: $d = 0.099m^{\frac{9}{10}}$, substitute: $m = 550$

$$d = 0.099(550)^{\frac{9}{10}}$$
 Use a calculator.
$$= 28.9708...$$

A 550-kg moose should drink approximately 29 L of water in one day.

Lesson 5.1 Representing Relations

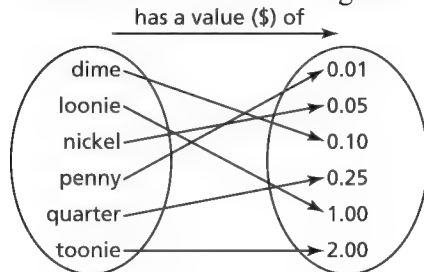
Exercises (pages 262–263)

A

3. a) i) The relation shows the association “has a value, in dollars, of” from a set of coins to a set of numbers. For example, a penny has a value of \$0.01.

- ii) The relation as a set of ordered pairs:
 $\{(\text{penny}, 0.01), (\text{nickel}, 0.05), (\text{dime}, 0.10), (\text{quarter}, 0.25), (\text{loonie}, 1.00), (\text{toonie}, 2.00)\}$

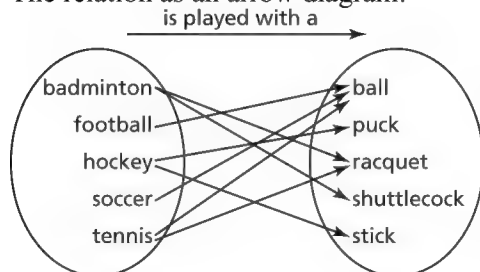
The relation as an arrow diagram:



- b) i) The relation shows the association “is played with a” from a set of sports to a set of equipment. For example, tennis is played with a racquet.

- ii) The relation as a set of ordered pairs:
 $\{(\text{badminton}, \text{shuttlecock}), (\text{badminton}, \text{racquet}), (\text{hockey}, \text{puck}), (\text{hockey}, \text{stick}), (\text{tennis}, \text{ball}), (\text{tennis}, \text{racquet}), (\text{soccer}, \text{ball})\}$

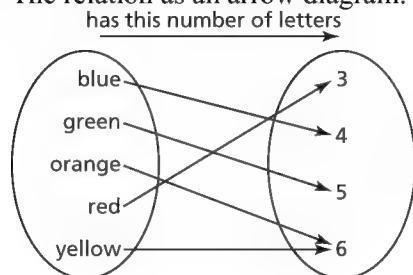
The relation as an arrow diagram:



4. a) The relation as a table:

Word	Number of letters
blue	4
green	5
orange	6
red	3
yellow	6

- b) The relation as an arrow diagram:

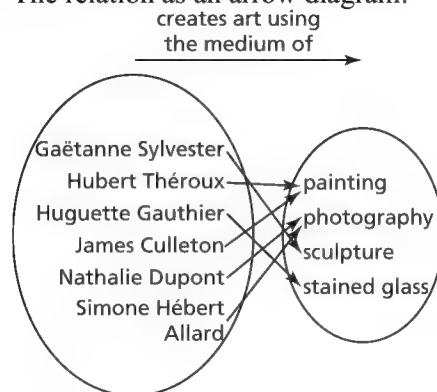


B

5. a) The relation shows the association “creates art using the medium of” from a set of francophone artists from Manitoba to a set of artistic media. For example, Nathalie Dupont creates art using the medium of photography.

- b) i) The relation as a set of ordered pairs:
 $\{(Ga\u00e9tanne\ Sylvester, sculpture), (Hubert\ Th\u00e9roux, painting), (Huguette\ Gauthier, stained\ glass), (James\ Culleton, painting), (Nathalie\ Dupont, photography), (Simone\ H\u00e9bert\ Allard, photography)\}$

- ii) The relation as an arrow diagram:



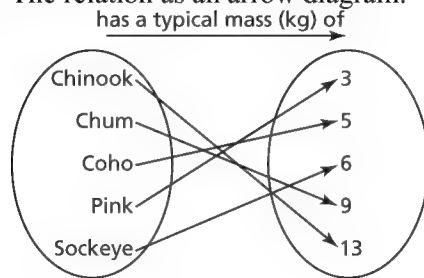
6. a) The relation shows the association “has a typical mass, in kilograms, of” from a set of salmon species to a set of masses. For example, a coho salmon has a typical mass of 5 kg.

- b) In the ordered pairs, the first item is the type of salmon. The second item is the typical mass of the salmon.

The relation as a set of ordered pairs:

$\{(\text{Chinook}, 13), (\text{Chum}, 9), (\text{Coho}, 5), (\text{Pink}, 3), (\text{Sockeye}, 6)\}$

- c) The relation as an arrow diagram:



7. a) The arrow diagram shows a relation with the association “is the number of letters in” from a set of numbers to a set of words beginning with the letter Z. For example, 6 is the number of letters in *zombie*.

- b) Sample response:

The relation as a set of ordered pairs:

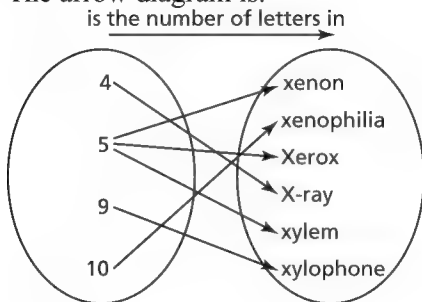
$\{(3, \text{Zen}), (4, \text{zany}), (4, \text{zero}), (5, \text{zebra}), (6, \text{zombie}), (7, \text{Zamboni}), (8, \text{zeppelin})\}$

The relation as a table:

Number of letters	Word beginning with Z
3	Zen
4	zany
4	zero
5	zebra
6	zombie
7	Zamboni
8	zeppelin

- c) Sample response:

The arrow diagram is:



The relation as a set of ordered pairs:

$\{(4, \text{X-ray}), (5, \text{xenon}), (5, \text{Xerox}), (5, \text{xylem}), (9, \text{xylophone}), (10, \text{xenophilia})\}$

The relation as a table:

Number of letters	Word beginning with X
4	X-ray
5	xenon
5	Xerox
5	xylem
9	xylophone
10	xenophilia

8. a) The arrow diagram shows a relation with the association “translates to” from a set of French words to a set of English words.
- b) Two ordered pairs that belong to the relation are: (oui, yes) and (et, and)

9. a) Sketch each digit:

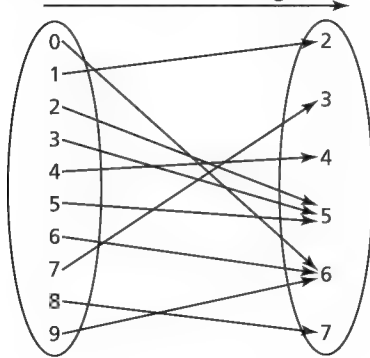


The ordered pairs are:

$\{(0, 6), (1, 2), (2, 5), (3, 5), (4, 4), (5, 5), (6, 6), (7, 3), (8, 7), (9, 6)\}$

- b) Sample response:

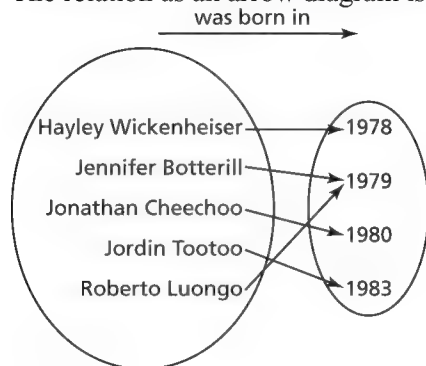
The relation as an arrow diagram:
has this number of lit segments



The relation as a table of values:

Digit	Number of lit segments
0	6
1	2
2	5
3	5
4	4
5	5
6	6
7	3
8	7
9	6

10. a) The relation as an arrow diagram is:



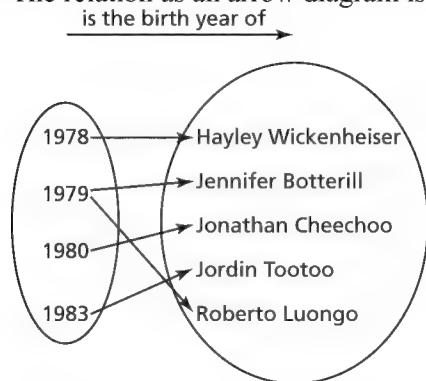
The relation as a set of ordered pairs:

$\{(Hayley\ Wickenheiser, 1978), (Jennifer\ Botterill, 1979), (Jonathan\ Cheechoo, 1980), (Jordin\ Tootoo, 1983), (Roberto\ Luongo, 1979)\}$

The relation as a table:

Hockey player	Birth year
Hayley Wickenheiser	1978
Jennifer Botterill	1979
Jonathan Cheechoo	1980
Jordin Tootoo	1983
Roberto Luongo	1979

b) The relation as an arrow diagram is:



The relation as a set of ordered pairs:

$\{(1978, Hayley\ Wickenheiser), (1979, Jennifer\ Botterill), (1979, Roberto\ Luongo), (1980, Jonathan\ Cheechoo), (1983, Jordin\ Tootoo)\}$

The relation as a table:

Birth year	Hockey player
1978	Hayley Wickenheiser
1979	Jennifer Botterill
1979	Roberto Luongo
1980	Jonathan Cheechoo
1983	Jordin Tootoo

11. Answers will vary.

a) Ordered pairs should be in the form: (older person, younger person)

b) Other associations include:

“is taller than”

“is involved in more school groups than”

“usually wakes up earlier than”

C

12. a) i) Create a table to organize the possible dice rolls.

Highlight the even sums:

Sums						
Roll	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The set of ordered pairs is:

$\{(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)\}$

ii) Use the table from part i. Subtract the lesser number from the greater number.

Highlight the differences that are prime numbers:

Differences						
Roll	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The set of ordered pairs is:

$\{(1, 3), (1, 4), (1, 6), (2, 4), (2, 5), (3, 1), (3, 5), (3, 6), (4, 1), (4, 2), (4, 6), (5, 2), (5, 3), (6, 1), (6, 3), (6, 4)\}$

b) In part a, the order doesn't matter because the sum of 2 numbers is the same regardless of the order of the numbers, and the difference doesn't matter for 2 dice because I subtract the lesser number from the greater number since there is no way of telling the dice apart.

- 13. a)** Each arrow goes from a parent to a child.
Arrows point to 6 different dots, so 6 children are shown.
- b)** Arrows come from 4 different dots, so 4 parents are shown.
- c)** Grandparents are parents of parents. There are 2 parents of parents, so 2 grandparents are shown.
- 14. a)** Each arrow goes from a sister to her sibling.
Arrows go from 2 different dots, so 2 females are shown.
- b)** The dots that are not female are male. There are 5 dots in total and 2 of them are female, so the remaining 3 dots are male.

Lesson 5.2 Properties of Functions

Exercises (pages 270–273)

A

4.
 - a) This relation is a function because each number in the first set associates with exactly 1 number in the second set.
 - b) This relation is not a function because the number 4 in the first set associates with 4 numbers in the second set.
 - c) This relation is a function because each number in the first set associates with exactly 1 number in the second set.
5.
 - a) Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is: $\{1, 2, 3, 4\}$
The range is: $\{3, 6, 9, 12\}$
 - b) The ordered pairs $(0, 1)$ and $(0, -1)$ have the same first element, 0. So, the relation is not a function.
The domain is: $\{1, 0, -1\}$
The range is: $\{0, 1, -1\}$
 - c) Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is: $\{2, 4, 6, 8\}$
The range is: $\{3, 5, 7, 9\}$
 - d) The ordered pairs $(0, 1)$, $(0, 2)$, and $(0, 3)$ have the same first element, 0; and the ordered pairs $(1, 2)$ and $(1, 3)$ have the same first element, 1. So, the relation is not a function.
The domain is: $\{0, 1, 2\}$
The range is: $\{1, 2, 3\}$
6.
 - a) $C(n) = 20n + 8$
 - b) $P(n) = n - 3$
 - c) $t(d) = 5d$
 - d) Use f to name the function when the variable is y :
 $f(x) = -x$
7.
 - a) $d = 3t - 5$
 - b) Use y for the variable when the function name is f and the other variable is x :
 $y = -6x + 4$
 - c) $C = 5n$
 - d) $P = 2n - 7$

B

8. a) Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the relation is a function.
The domain is the set of first elements:
 $\{1, 2, 3, 4\}$
The range is the set of second elements:
 $\{1, 8, 27, 64\}$
- b) The ordered pairs $(3, 4)$, $(3, 5)$, $(3, 6)$, and $(3, 7)$ have the same first element, 3.
So, the relation is not a function.
The domain is: $\{3\}$
The range is: $\{4, 5, 6, 7\}$
9. a) i) For each number in the first column, there is only one number in the second column.
So, the relation is a function.
- ii) From an understanding of the situation, the cost, C , depends on the number of cans of juice purchased, n . So, C is the dependent variable and n is the independent variable.
- iii) The domain is the set of numbers in the first column of the table:
 $\{1, 2, 3, 4, 5, 6, \dots\}$
The range is the set of numbers in the second column of the table:
 $\{2.39, 4.00, 6.39, 8.00, 10.39, 12.00, \dots\}$
- b) i) For each number in the first column, there is only one number in the second column.
So, the relation is a function.
- ii) From an understanding of the situation, the temperature, T , depends on the altitude, A . So, T is the dependent variable and A is the independent variable.
- iii) The domain is the set of numbers in the first column of the table:
 $\{610, 1220, 1830, 2440, 3050, 3660, \dots\}$
The range is the set of numbers in the second column of the table:
 $\{15.0, 11.1, 7.1, 3.1, -0.8, -4.8, \dots\}$

10. a) The ordered pairs (3, isosceles triangle), (3, equilateral triangle), (3, right triangle), and (3, scalene triangle) have the same first element, 3; and the ordered pairs (4, square), (4, rectangle), (4, rhombus), (4, trapezoid), and (4, parallelogram) have the same first element, 4. So, the relation is not a function.
- b) Write the new relation as a set of ordered pairs and list the first elements in alphabetical order: {(equilateral triangle, 3), (hexagon, 6), (isosceles triangle, 3), (parallelogram, 4), (pentagon, 5), (rectangle, 4), (rhombus, 4), (right triangle, 3), (scalene triangle, 3), (square, 4), (trapezoid, 4)}
Each ordered pair has a different first element, so for every first element there is exactly one second element. So, the new relation is a function.
- c) For the relation in part a:
The domain is the set of first elements: {3, 4, 5, 6}
The range is the set of second elements: {equilateral triangle, isosceles triangle, right triangle, scalene triangle, square, rectangle, rhombus, trapezoid, parallelogram, pentagon, hexagon}
For the function in part b:
The domain is the set of first elements:
{equilateral triangle, hexagon, isosceles triangle, parallelogram, pentagon, rectangle, rhombus, right triangle, scalene triangle, square, trapezoid}
The range is the set of second elements:
{3, 4, 5, 6}
11. a) For a function, each element in the first column is associated with exactly one element in the second column. So, each element in the first column only occurs once in that column. The only column with no repetition is: "Name"
So, any relation written as a table where the first column is "Name" will be a function. For example, these two relations are functions:

Name	From
Marie	Edmonton
Gabriel	Falher
Élise	Bonnyville
Christophe	Calgary
Jean	Edmonton
Mélanie	Edmonton
Nicole	Red Deer
Marc	Légal

Name	Age
Marie	13
Gabriel	16
Élise	14
Christophe	13
Jean	15
Mélanie	15
Nicole	17
Marc	13

- b) Any relation written as a table where the first column is not "Name" will be a relation that is not a function. For example, these two relations are not functions:

Age	Name
13	Marie
16	Gabriel
14	Élise
13	Christophe
15	Jean
15	Mélanie
17	Nicole
13	Marc

From	Age
Edmonton	13
Falher	16
Bonnyville	14
Calgary	13
Edmonton	15
Edmonton	15
Red Deer	17
Légal	13

12. The statement in part a is true. A function is a special type of relation, so it is still a relation. For example, $\{(1, 2), (2, 3), (3, 4)\}$ is a function because each first element in the ordered pairs occurs exactly once. It is also a relation because it is a set of ordered pairs. However, a relation such as $\{(0, 1), (0, 2), (0, 3)\}$ is not a function because the first element 0 occurs in each ordered pair.

13. a) Create one table for a relation with the association “is worth this number of points” from a set of letters to a set of numbers. Create a second table for a relation with the association “points are associated with the letter” from a set of numbers to a set of letters.

Letter	Number
A	1
D	2
F	4
G	2
M	3
Q	10
T	1
X	8
Z	10

Number	Letter
1	A
1	T
2	D
2	G
3	M
4	F
8	X
10	Q
10	Z

b) The first table represents a function; each letter in the first column is associated with exactly one number in the second column.
In the second table, the numbers 1, 2, and 10 in the first column are associated with more than 1 letter in the second column, so this does not represent a function.

14. a) To determine $f(1)$, use:

$$f(x) = -5x + 11 \quad \text{Substitute: } x = 1$$

$$f(1) = -5(1) + 11$$

$$f(1) = -5 + 11$$

$$f(1) = 6$$

- b) To determine $f(-3)$, use:

$$f(x) = -5x + 11 \quad \text{Substitute: } x = -3$$

$$f(-3) = -5(-3) + 11$$

$$f(-3) = 15 + 11$$

$$f(-3) = 26$$

- c) To determine $f(0)$, use:

$$f(x) = -5x + 11 \quad \text{Substitute: } x = 0$$

$$f(0) = -5(0) + 11$$

$$f(0) = 0 + 11$$

$$f(0) = 11$$

- d) To determine $f(1.2)$, use:

$$f(x) = -5x + 11 \quad \text{Substitute: } x = 1.2$$

$$f(1.2) = -5(1.2) + 11$$

$$f(1.2) = -6 + 11$$

$$f(1.2) = 5$$

15. a) i) To determine the value of n when $f(n) = 11$, use:

$$f(n) = 2n - 7 \quad \text{Substitute: } f(n) = 11$$

$$11 = 2n - 7 \quad \text{Solve for } n.$$

$$11 + 7 = 2n$$

$$18 = 2n$$

$$n = 9$$

- ii) To determine the value of n when $f(n) = -6$, use:

$$f(n) = 2n - 7 \quad \text{Substitute: } f(n) = -6$$

$$-6 = 2n - 7 \quad \text{Solve for } n.$$

$$-6 + 7 = 2n$$

$$1 = 2n$$

$$n = \frac{1}{2}, \text{ or } 0.5$$

- b) i)** To determine the value of x when $g(x) = 41$, use:

$$g(x) = -5x + 1 \quad \text{Substitute: } g(x) = 41$$

$$41 = -5x + 1 \quad \text{Solve for } x.$$

$$41 - 1 = -5x$$

$$40 = -5x$$

$$x = -8$$

- ii)** To determine the value of x when $g(x) = -16$, use:

$$g(x) = -5x + 1 \quad \text{Substitute: } g(x) = -16$$

$$-16 = -5x + 1 \quad \text{Solve for } x.$$

$$-16 - 1 = -5x$$

$$-17 = -5x$$

$$x = \frac{17}{5}, \text{ or } 3.4$$

- 16. a)** A measure in centimetres is a function of the measure in inches.

In two variables, $C = 2.54i$

- b)** To determine $C(12)$, use:

$$C(i) = 2.54i \quad \text{Substitute: } i = 12$$

$$C(12) = 2.54(12)$$

$$C(12) = 30.48$$

This means that a length of 12 in. is 30.48 cm.

- c)** To determine the value of i when $C(i) = 100$, use:

$$C(i) = 2.54i \quad \text{Substitute: } C(i) = 100$$

$$100 = 2.54i \quad \text{Solve for } i.$$

$$\frac{100}{2.54} = i$$

$$i = 39.3700\dots$$

This means that a length of 100 cm is approximately 39 in.

- 17. a)** The distance to Meadow Lake is a function of the travelling time.

In function notation, $D(t) = -80t + 300$

- b)** At the start of the journey, 0 h have passed, so $t = 0$.

Determine $D(0)$.

$$D(t) = -80t + 300 \quad \text{Substitute: } t = 0$$

$$D(0) = -80(0) + 300$$

$$D(0) = 300$$

The car was 300 km from Meadow Lake at the start its journey.

18. a) i) To determine $f(15)$, use:

$$f(l) = 2.754l + 71.475$$

Substitute: $l = 15$

$$f(15) = 2.754(15) + 71.475$$

$$f(15) = 112.785$$

This means that a female with humerus 15 cm long is approximately 113 cm tall.

- ii) To determine $m(20)$, use:

$$m(l) = 2.894l + 70.641$$

Substitute: $l = 20$

$$m(20) = 2.894(20) + 70.641$$

$$m(20) = 128.521$$

This means that a male with humerus 20 cm long is approximately 129 cm tall.

- b) i) To determine the value of l when $f(l) = 142$, use:

$$f(l) = 2.754l + 71.475$$

Substitute: $f(l) = 142$

$$142 = 2.754l + 71.475$$

$$142 - 71.475 = 2.754l$$

$$70.525 = 2.754l$$

$$\frac{70.525}{2.754} = l$$

$$l = 25.6082\dots$$

This means that a female 142 cm tall has a humerus approximately 26 cm long.

- ii) To determine the value of l when $m(l) = 194$, use:

$$m(l) = 2.894l + 70.641$$

Substitute: $m(l) = 194$

$$194 = 2.894l + 70.641$$

$$194 - 70.641 = 2.894l$$

$$123.359 = 2.894l$$

$$\frac{123.359}{2.894} = l$$

$$l = 42.6257\dots$$

This means that a male 194 cm tall has a humerus approximately 43 cm long.

- c) Sample response:

My humerus is approximately 33 cm long.

Use the formula for a female:

$$f(l) = 2.754l + 71.475$$

Substitute: $l = 33$

$$f(15) = 2.754(33) + 71.475$$

$$f(15) = 162.357$$

The formula estimates my height as approximately 162 cm.

I am approximately 164 cm tall, so the height given by the formula is close to my actual height.

19. a) i) To determine $C(50)$, use:

$$C(f) = \frac{5}{9}(f - 32) \quad \text{Substitute: } f = 50$$

$$C(50) = \frac{5}{9}(50 - 32)$$

$$C(50) = \frac{5}{9}(18)$$

$$C(50) = 10$$

ii) To determine $C(-13)$, use:

$$C(f) = \frac{5}{9}(f - 32) \quad \text{Substitute: } f = -13$$

$$C(-13) = \frac{5}{9}(-13 - 32)$$

$$C(-13) = \frac{5}{9}(-45)$$

$$C(-13) = -25$$

b) i) To determine the value of f when $C(f) = 20$, use:

$$C(f) = \frac{5}{9}(f - 32) \quad \text{Substitute: } C(f) = 20$$

$$20 = \frac{5}{9}(f - 32)$$

$$\frac{9}{5}(20) = f - 32$$

$$36 = f - 32$$

$$32 + 36 = f$$

$$f = 68$$

ii) To determine the value of f when $C(f) = -35$, use:

$$C(f) = \frac{5}{9}(f - 32) \quad \text{Substitute: } C(f) = -35$$

$$-35 = \frac{5}{9}(f - 32)$$

$$\frac{9}{5}(-35) = f - 32$$

$$-63 = f - 32$$

$$32 - 63 = f$$

$$f = -31$$

- c) i) $C(32) = 0$
 ii) $C(212) = 100$
 iii) $C(356) = 180$

C

20. The temperature in degrees Fahrenheit, F , is determined from the temperature in degrees Celsius, c . So, F is the dependent variable and c is the independent variable.

Multiply the Celsius temperature by $\frac{9}{5}$: $\frac{9}{5}c$

Add 32: $\frac{9}{5}c + 32$

So, the equation in function notation is:

$$F(c) = \frac{9}{5}c + 32$$

21. An equation for the area, A , of the rectangle is:

$$A = lw$$

$$\text{Substitute: } A = 9$$

$$9 = lw$$

Solve for w .

$$w = \frac{9}{l}$$

An equation for the perimeter, P , of the rectangle is:

$$P = 2(l + w)$$

$$\text{Substitute: } w = \frac{9}{l}$$

$$P = 2\left(l + \frac{9}{l}\right)$$

$$P = 2l + \frac{18}{l}$$

In function notation:

$$P(l) = 2l + \frac{18}{l}$$

22. An equation for the perimeter, P , of the rectangle is:

$$P = 2(l + w)$$

$$\text{Substitute: } P = 12$$

$$12 = 2(l + w)$$

Solve for l .

$$\frac{12}{2} = l + w$$

$$6 = l + w$$

$$l = 6 - w$$

In function notation:

$$l(w) = 6 - w$$

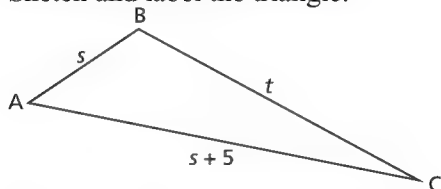
The domain is:

$$0 < w < 6$$

The range is:

$$0 < l < 6$$

23. Sketch and label the triangle:



An equation for the perimeter, P , of the triangle is:

$$P = s + (s + 5) + t \quad \text{Substitute: } P = 16$$

$$16 = 2s + 5 + t \quad \text{Solve for } t.$$

$$16 - 2s - 5 = t$$

$$t = 11 - 2s$$

In function notation:

$$t(s) = 11 - 2s$$

In a triangle, the length of a side is less than the sum of the lengths of the other two sides.

So, we get these inequalities:

$$AB < AC + BC$$

$$AC < AB + BC$$

$$BC < AB + AC$$

$$s < (s + 5) + t \quad \text{and} \quad s + 5 < s + t \quad \text{and} \quad t < s + (s + 5)$$

$$-5 < t$$

$$5 < t$$

$$t < 2s + 5$$

So:

$$t > 5 \quad \text{and} \quad t < 2s + 5$$

Use $t > 5$ to determine a corresponding inequality for s :

$$t > 5 \quad \text{Substitute: } t = 11 - 2s$$

$$11 - 2s > 5$$

$$2s < 6$$

$$s < 3$$

Use $t < 2s + 5$ to determine inequalities for s and t :

$$t < 2s + 5 \quad \text{Substitute: } t = 11 - 2s$$

$$11 - 2s < 2s + 5$$

$$11 - 5 < 2s + 2s$$

$$6 < 4s$$

$$s > \frac{6}{4}$$

$$s > 1.5$$

Write the equation $t = 11 - 2s$ in terms of s :

$$t = 11 - 2s$$

$$2s = 11 - t$$

$$s = \frac{1}{2}(11 - t)$$

So, when $s > 1.5$:

$$s > 1.5$$

$$\text{Substitute: } s = \frac{1}{2}(11 - t)$$

$$\frac{1}{2}(11 - t) > 1.5$$

$$11 - t > 2(1.5)$$

$$11 - t > 3$$

$$t < 11 - 3$$

$$t < 8$$

So, $t > 5$, $s < 3$, $s > 1.5$, and $t < 8$

Combine these inequalities to determine the domain and range of the function:

The domain is: $1.5 < s < 3$

The range is: $5 < t < 8$

Checkpoint 1

Assess Your Understanding (page 275)

5.1

1. Sample response:

	Description in words	Set of ordered pairs	Arrow diagram	Table or graph														
a)	This relation shows the association “is used to make” from a set of materials to a set of objects.	$\{(skin, drum), (skin, kayak), (bark, basket), (stone, inukshuk), (stone, carving)\}$	is used to make	<table><tr><th>Material</th><th>Object</th></tr><tr><td>bark</td><td>basket</td></tr><tr><td>skin</td><td>drum</td></tr><tr><td>skin</td><td>kayak</td></tr><tr><td>stone</td><td>carving</td></tr><tr><td>stone</td><td>inukshuk</td></tr></table>	Material	Object	bark	basket	skin	drum	skin	kayak	stone	carving	stone	inukshuk		
Material	Object																	
bark	basket																	
skin	drum																	
skin	kayak																	
stone	carving																	
stone	inukshuk																	
b)	This relation shows the association “has this many factors” from the natural numbers from 1 to 4 to a set of natural numbers.	$\{(1, 1), (2, 2), (3, 2), (4, 3)\}$	has this many factors	<table><tr><th>Number</th><th>Number of Factors</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>3</td></tr></table>	Number	Number of Factors	1	1	2	2	3	2	4	3				
Number	Number of Factors																	
1	1																	
2	2																	
3	2																	
4	3																	
c)	This relation shows the association “is usually coloured” from a set of objects to a set of colours.	$\{(grass, green), (sea, blue), (sky, blue), (snow, white)\}$	is usually coloured	<table><tr><th>Object</th><th>Colour</th></tr><tr><td>grass</td><td>green</td></tr><tr><td>sea</td><td>blue</td></tr><tr><td>sky</td><td>blue</td></tr><tr><td>snow</td><td>white</td></tr></table>	Object	Colour	grass	green	sea	blue	sky	blue	snow	white				
Object	Colour																	
grass	green																	
sea	blue																	
sky	blue																	
snow	white																	
d)	For the numbers 1 to 4, the first number in an ordered pair is greater than the second number.	$\{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$	is greater than	<table><tr><th>Number</th><th>Number</th></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>1</td></tr><tr><td>3</td><td>2</td></tr><tr><td>4</td><td>1</td></tr><tr><td>4</td><td>2</td></tr><tr><td>4</td><td>3</td></tr></table>	Number	Number	2	1	3	1	3	2	4	1	4	2	4	3
Number	Number																	
2	1																	
3	1																	
3	2																	
4	1																	
4	2																	
4	3																	

5.2

2. a) Justifications may vary. For example:

The relation in part a is not a function because two different ordered pairs have skin as a first element and two different ordered pairs have stone as a first element.

The relation in part b is a function because the ordered pairs have different first elements.

The relation in part c is a function because the ordered pairs have different first elements.

The relation in part d is not a function because two different ordered pairs have 3 as a first element and three different ordered pairs have 4 as a first element.

- b) In part b,
the domain is: $\{1, 2, 3, 4\}$
the range is: $\{1, 2, 3\}$

In part c,
the domain is: $\{\text{grass, sea, sky, snow}\}$
the range is: $\{\text{blue, green, white}\}$

3. Sample response:

- a) i) A relation that is not a function has at least two different ordered pairs with the same first element.

So, this relation is not a function:

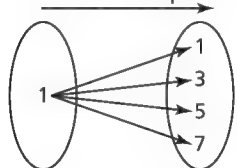
$\{(1, 1), (1, 3), (1, 5), (1, 7)\}$

- ii) For a function, each first element is associated with exactly one second element.

So, this is a function:

$\{(1, 1), (3, 3), (5, 5), (7, 7)\}$

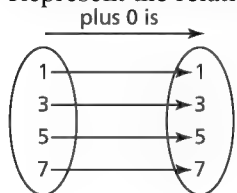
- b) i) Represent the relation as an arrow diagram:
is less than or equal to



Represent the relation as a table of values:

Number	Number
1	1
1	3
1	5
1	7

- ii) Represent the relation as an arrow diagram:



Represent the relation as a table of values:

Number	Number
1	1
3	3
5	5
7	7

4. a) From an understanding of the situation, the temperature, T , depends on the distance below Earth's surface, d . So, T is the dependent variable and d is the independent variable.

- b) In two variables, $T = 10d + 20$

- c) To determine $T(5)$, use:

$$T(d) = 10d + 20$$

Substitute: $d = 5$

$$T(5) = 10(5) + 20$$

$$T(5) = 70$$

This means that at a depth of 5 km below Earth's surface, the temperature is 70°C .

- d) To determine the value of d when $T(d) = 50$, use:

$$T(d) = 10d + 20$$

Substitute: $T(d) = 50$

$$50 = 10d + 20$$

$$50 - 20 = 10d$$

$$30 = 10d$$

$$\frac{30}{10} = d$$

$$d = 3$$

This means that, at a depth of 3 km below Earth's surface, the temperature is 50°C .

Lesson 5.3 Interpreting and Sketching Graphs

Exercises (pages 281–283)

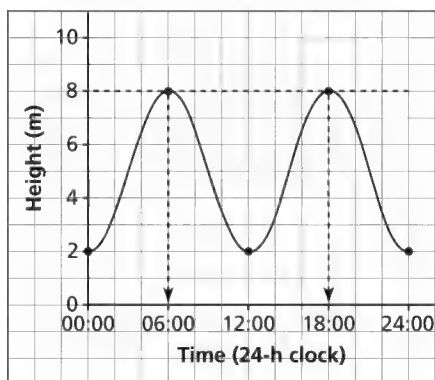
A

3. a) Bear F has the greatest mass because it is represented by the point on the graph farthest to the right and the horizontal axis represents mass. Its mass is approximately 650 kg.
- b) Bear A is the shortest because it is represented by the lowest point on the graph and the vertical axis represents height. Its height is approximately 0.7 m.
- c) Bears D and E have the same mass because the points that represent them lie on the same vertical line that passes through 400 on the *Mass* axis. The mass is 400 kg.
- d) Bears D and H have the same height because the points that represent them lie on the same horizontal line that passes through approximately 2.25 on the *Height* axis. The height is approximately 2.25 m.

B

4. a) Draw a horizontal line through the highest points on the graph. This line intersects the *Height* axis at 8 m. Draw vertical lines from the points where the horizontal line intersects the graph. These lines intersect the *Time* axis at 06:00 and 18:00. So, the greatest height is 8 m. It occurs at 06:00 and 18:00.

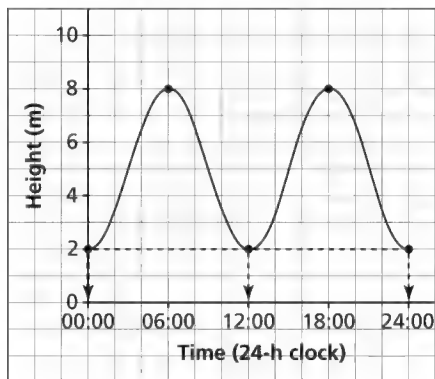
Height of the Tide in a Harbour



- b) Draw a horizontal line through the lowest points on the graph. This line intersects the *Height* axis at 2 m. Draw vertical lines from the points where the horizontal line intersects the graph. These lines intersect the *Time* axis at 00:00 (midnight), 12:00 (noon), and 24:00 (midnight).

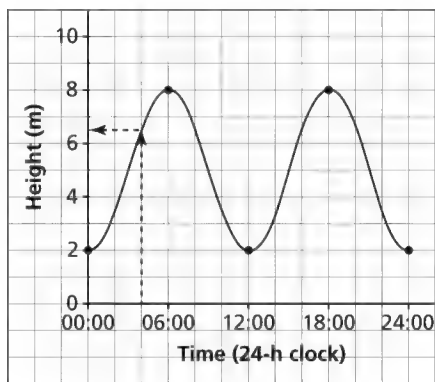
So, the least height is 2 m. It occurs at 00:00, 12:00, and 24:00.

Height of the Tide in a Harbour



- c) Draw a vertical line through 04:00 on the *Time* axis. From the point where this line intersects the graph, draw a horizontal line, which intersects the *Height* axis at approximately 6.5 m. So, at 04:00, the tide is approximately 6.5 m.

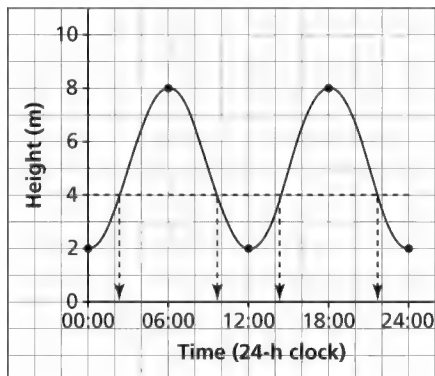
Height of the Tide in a Harbour



- d) Draw a horizontal line through 4 m on the *Height* axis. From the point where this line intersects the graph, draw vertical lines to intersect the *Time* axis.

The tide is 4 m high at approximately 02:20, 09:40, 14:20, and 21:40.

Height of the Tide in a Harbour



5. When Sepideh pulls the rope, the height of the flag increases. As she moves her hands up the rope, the height of the flag does not change. So, Graph B best represents the situation because it has segments that go up to the right and horizontal segments.

6.	Segment	Graph	Run
	OA	The graph goes up to the right, so as time increases, Gill's distance from home increases.	In the first 5 min, Gill leaves home and runs 1 km.
	AB	The graph is horizontal, so as time increases the distance stays the same.	Gill stops running for 5 min.
	BC	The graph goes up to the right, so as time increases, Gill's distance from home increases.	Gill runs 1 km farther from home during the next 10 min.
	CD	The graph goes down to the right, so as time increases, Gill's distance from home decreases.	Gill runs 2 km back home during the next 10 min.

7.	Segment	Graph	Dive
	OA	The graph goes up to the right, so as time increases, Katanya's depth increases.	In the first 4 min, Katanya dives down 15 m below the surface of the water.
	AB	The graph is horizontal, so as time increases Katanya's depth stays the same.	Katanya explores her environment at a depth of 15 m below the surface of the water for 6 min.
	BC	The graph goes up to the right, so as time increases, Katanya's depth increases.	Katanya descends another 10 m during the next 4 min.
	CD	The graph is horizontal, so as time increases Katanya's depth stays the same.	Katanya explores her environment at a depth of 25 m below the surface of the water for 4 min.
	DE	The graph goes down to the right, so as time increases, Katanya's depth decreases.	Katanya ascends 25 m in 10 min to reach the surface of the water.

8. a) Look at the graph of Cost against Age.
Helicopter B is older because it is represented by the point on the graph farther to the right and the horizontal axis represents age.
Helicopter B costs less to operate because it is represented by the lower point on the graph and the vertical axis represents cost to operate.
So, the statement “The older helicopter is cheaper to operate” is true.
- b) Look at the graph of Maximum Speed against Number of Seats.
Helicopter A has more seats because it is represented by the point on the graph farther to the right and the horizontal axis represents the number of seats.
Helicopter A is faster because it is represented by the higher point on the graph and the vertical axis represents maximum speed.
So, the statement “The helicopter with more seats has the lower maximum speed” is false.
- c) Look at the graph of Maximum Speed against Number of Seats.
Helicopter B is slower because it is represented by the lower point on the graph and the vertical axis represents maximum speed.
Look at the graph of Cost against Age.
Helicopter B is cheaper to operate because it is represented by the lower point on the graph and the vertical axis represents cost to operate.
So, the statement “The helicopter with the lower maximum speed is cheaper to operate” is true.
- d) Look at the graph of Maximum Speed against Number of Seats.
Helicopter A has the greater maximum speed because it is represented by the higher point on the graph and the vertical axis represents maximum speed.
Look at the graph of Cost against Age.
Helicopter A is newer because it is represented by the point on the graph farther to the left and the horizontal axis represents age.
So, the statement “The helicopter with the greater maximum speed is older” is false.
- e) Look at the graph of Maximum Speed against Number of Seats.
Helicopter B has fewer seats because it is represented by the point on the graph farther to the left and the horizontal axis represents the number of seats.
Look at the graph of Cost against Age.
Helicopter B is older because it is represented by the point on the graph farther to the right and the horizontal axis represents age.
So, the statement “The helicopter with fewer seats is newer” is false.

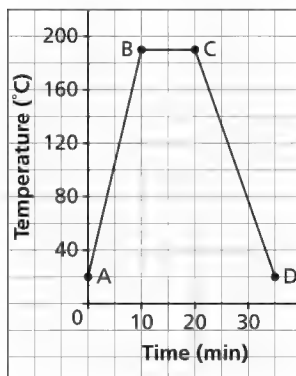
9. a)

Segment	Graph	Journey
AB	The graph goes down to the right, so as time increases, the volume of gas decreases.	At the start of the journey, there is 25 L of gas in the snowmobile. After 2 h of travel, there is 15 L of gas remaining in the tank.
BC	The graph goes up steeply to the right, so as time increases, the volume of gas increases.	The tank is filled until it contains 30 L of gas.
CD	The graph goes down to the right, so as time increases, the volume of gas decreases.	After nearly 2 h of travel, there is 20 L of gas remaining in the tank.
DE	The graph is horizontal, so as time increases the volume of gas in the tank stays the same.	The snowmobile stops for 2 h.
EF	The graph goes down to the right, so as time increases, the volume of gas decreases.	After 3 h of travel, there is 5 L of gas remaining in the tank.
FG	The graph goes up steeply to the right, so as time increases, the volume of gas increases.	The tank is filled until it contains 30 L of gas.

- b) At the start of the journey, there was 25 L of gas in the snowmobile because the journey starts at point A with coordinates (0, 25). At two points during the journey, the tank contained 30 L of gas, so the tank was not full at the start of the journey.

10. Draw and label axes on a grid. The horizontal axis represents time in minutes, and the vertical axis represents temperature in degrees Celsius.

Temperature of an Oven



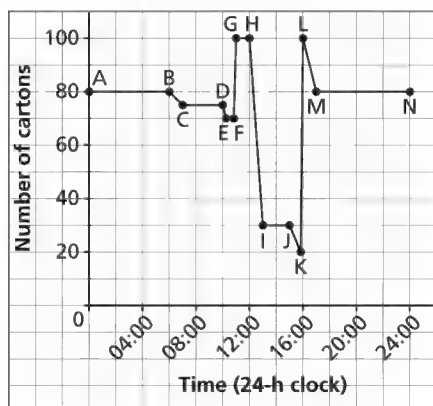
Segment	Oven
AB	The oven is turned on and increases in temperature from 20°C to 190°C in 10 min, so the segment goes up to the right.
BC	The cookies are placed in the oven and baked for 10 min. Since the temperature doesn't change, the segment is horizontal.
CD	The oven is turned off and the temperature decreases to 20°C in 15 min, so the segment goes down to the right.

11. Distance from home	Journey
0 km to 0.5 km	The car leaves home and accelerates to 50 km/h.
0.5 km to 1.5 km	The car travels at a constant speed of 50 km/h.
1.5 km to 2 km	The car decelerates to 40 km/h, then accelerates to 50 km/h.
2 km to 3 km	The car travels at a constant speed of 50 km/h.
3 km to 3.5 km	The car accelerates to 80 km/h.
3.5 km to 5.5 km	The car travels at a constant speed of 80 km/h.
5.5 km to 5.6 km	The car decelerates to a stop.
5.6 km to 6 km	The car accelerates to 60 km/h.
6 km to 7.4 km	The car travels at a constant speed of 60 km/h.
7.4 km to 7.5 km	The car decelerates to a stop at the person's work.

12. Graphs may vary. For example:

Draw and label axes on a grid. The horizontal axis represents time on a 24-h clock, and the vertical axis represents the number of cartons in the machine.

Number of Cartons in the School
Vending Machine



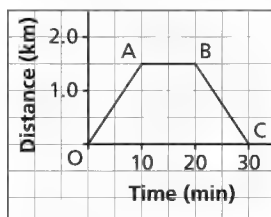
Segment	Vending machine
AB	The number of cartons remains constant until approximately 6 A.M., when students and teachers begin arriving at school.
BC	From 6 A.M. to 7 A.M., 5 cartons are purchased, so the graph goes down to the right.
CD	No cartons are purchased from 7 A.M. to 8 A.M. or while students are in class from 8 A.M. to 10 A.M., so the number of cartons in the machine remains constant. There are 75 cartons in the machine during this time period.
DE	From 10 A.M. to 10:15 A.M., 5 cartons are purchased, so the graph goes down to the right.
EF	From 10:15 A.M. to 10:55 A.M., students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
FG	From 10:55 A.M. to 11 A.M., the machine is filled to 100 cartons.
GH	From 11 A.M. to noon, students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
HI	From noon to 1 P.M., students purchase 70 cartons, so the graph goes down to the right.
IJ	From 1 P.M. to 3 P.M., students are in class, so no cartons are purchased and the number of cartons in the machine remains constant.
JK	From 3 P.M. to 3:55 P.M., 10 cartons are purchased, so the graph goes down to the right.
KL	From 3:55 P.M. to 4 P.M., the machine is filled to 100 cartons.
LM	From 4 P.M. to 5 P.M., students purchase 20 cartons, so the graph goes down to the right.
MN	No cartons are purchased after 5 P.M., so the number of cartons in the machine remains constant.

13. There are 2 errors in the graph.

When Jonah's mom asks him a question, he turns the volume down. So, the graph should lie below a volume level of 40 for the time period from 3 min to 4 min.

When Jonah presses the mute button, the volume level should drop to 0 immediately, so the graph should show a vertical line segment from a volume level of 80 to 0 at 9 min.

14. a) Copy and label the graph.
Distance from Home



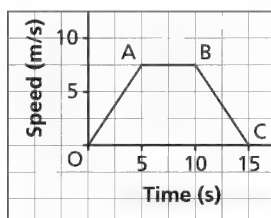
Situation: A person jogs from home to a park 1.5 km away in 10 min. He sits on a park bench and reads for 10 min. Then he jogs 1.5 km home in 10 min.

Justification: Line segment OA goes up to the right, so the person's distance from home is increasing with time.

Segment AB is horizontal, so the person's distance from home remains constant.

Segment BC goes down to the right, so the person's distance from home decreases with time.

- b) Copy and label the graph.
Speed while Sprinting



Situation: A person sprints down a track starting from a standstill. It takes the person 5 s to reach a speed of 7.5 m/s. After 5 s of running at 7.5 m/s, the person slows down and stops after another 5 s.

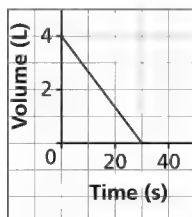
Justification: Line segment OA goes up to the right, so the person's speed is increasing with time.

Segment AB is horizontal, so the person's speed remains constant.

Segment BC goes down to the right, so the person's speed decreases with time.

15. a) Copy and label the graph. The dependent variable is the volume in litres.

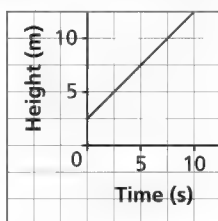
Emptying a
Watering Can



Situation: A watering can contains 4 L of water. It is poured at a steady rate so the watering can is empty after 30 s.

- b) Copy and label the graph. The dependent variable is height in metres.

Height of a Helium Balloon

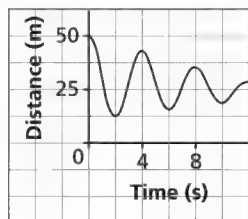


Situation: A person releases a helium balloon. It starts at a height of 2.5 m above the ground. After 10 s, it is at a height of 12.5 m above the ground.

C

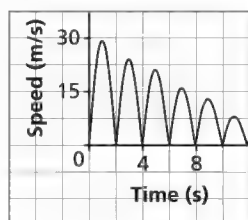
16. a) i) Sketch a graph. The distance decreases until the bungee chord is fully extended, then increases; this pattern continues until the person stops bouncing.

Distance above the Ground
while Bungee Jumping



- ii) Sketch a graph. The speed increases and decreases during the time a person descends to come to an instantaneous stop. This pattern continues for the ascent, and each repeated cycle of descent and ascent.

Speed while Bungee Jumping



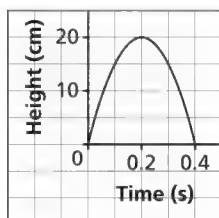
- b) **Similarities:** Both graphs are curves because on the first graph, speed is not constant, and on the second graph, acceleration is not constant.

Differences: The top graph is a continuous curve whose highest and lowest points get closer together. The bottom graph is a series of upside down U-shaped curves, whose highest points decrease.

The curves in the bottom graph always start and end at a speed of 0 m/s.

17. a) This graph could represent the height of a grasshopper during one hop. The independent variable is time, and the dependent variable is the height of the grasshopper.

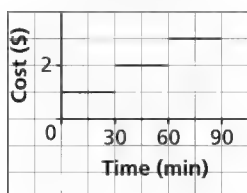
Height of a Jump



It takes 0.2 s for a grasshopper to jump 20 cm high, and another 0.2 s for it to return to the ground.

- b) The graph could represent the cost of parking in a parking garage. The independent variable is time, and the dependent variable is the cost.

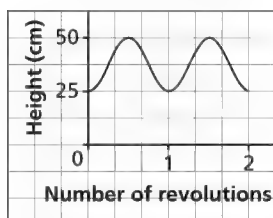
Cost of Parking in
a Parking Garage



It costs \$1 to park for up to 30 min, \$2 to park from 30 min to 60 min, and \$3 to park from 60 min to 90 min.

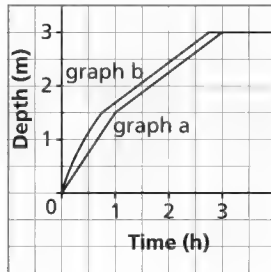
- c) This graph could represent the height of a point on the rim of a tire on a truck over time. The independent variable is time, and the dependent variable is the height of the point.

Height of a Point on
the Rim of a Tire



The point starts at the lowest point on the rim, 25 cm above the ground. As the wheel goes around, the point moves up to a maximum height of 50 cm, then down, then up again.

18. Sketch the graphs on the same grid.
Depth of Water in Two Pools

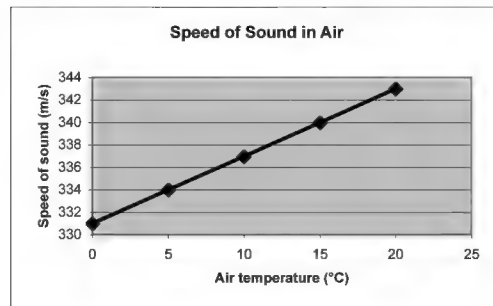


In part a, the water will fill the deeper depression at a constant rate, then the second depression at a slower rate since it is wider. When the pool is full, the tap will be shut off, and the volume of water in the pool will remain constant.

In part b, as the pool width increases, the water will fill the pool less and less rapidly, so the graph is a curve. When the width is constant, the water will fill the pool at a slower, but constant rate. When the pool is full, the tap will be shut off, and the volume of water in the pool will remain constant.

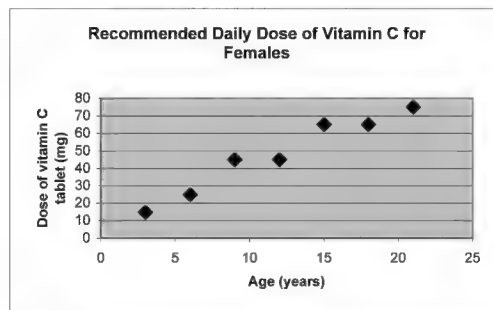
Lesson 5.4 Math Lab: Graphing Data Assess Your Understanding (page 286)

1. a) i) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are joined because the air temperature could be any number of degrees Celsius between 0°C and 20°C and the speed of sound could be any number of metres per second between 331 m/s and 343 m/s.

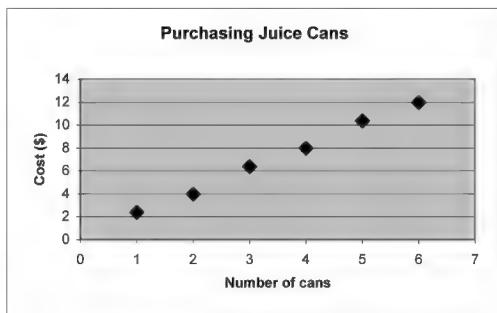
- ii) Every number in the first column of the table appears exactly once, so the relation is a function.
- b) i) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are not joined because the ages are whole numbers of years and it is not clear whether any number is permissible for the dose of vitamin C in a tablet.

- ii) Every number in the first column of the table appears exactly once, so the relation is a function.

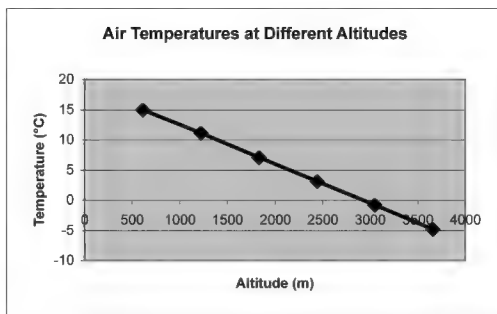
2. a) Graph the data using a computer spreadsheet program. Label the axes.



The points are not joined because only whole numbers are permissible for the number of juice cans purchased.

The relation is a function because a vertical line drawn on the graph passes through at most 1 point on the graph.

- b) Graph the data using a computer spreadsheet program. Label the axes.



The points on the graph are joined because the altitude could be any number of metres between 610 m and 3660 m and the temperature could be any number of degrees Celsius between -5°C and 15°C .

The relation is a function because a vertical line drawn on the graph passes through at most 1 point on the graph.

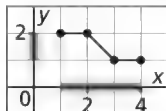
Lesson 5.5 Graphs of Relations and Functions

Exercises (pages 294–297)

A

4. a) The domain is the set of x -coordinates of the points on the graph:
 $\{-2, -1, 0, 1, 2\}$
The range is the set of y -coordinates of the points on the graph:
 $\{-4, -2, 0, 2, 4\}$
- b) The domain is the set of x -coordinates of the points on the graph:
 $\{-3, -1, 0, 2, 3\}$
The range is the set of y -coordinates of the points on the graph:
 $\{-2, 0, 1, 2, 3\}$
- c) The domain is the set of x -coordinates of the points on the graph:
 $\{-3, -2, -1, 0, 1, 2, 3\}$
The range is the set of y -coordinates of the points on the graph:
 $\{2\}$
5. A vertical line drawn on each graph intersects the graph at 0 points or 1 point.
6. a) The graph of $y = 1$ represents a function because each point on the line has a different x -coordinate.
- b) The graph of $x = 1$ does not represent a function because each point on the line has the same x -coordinate, 1.

7. a) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

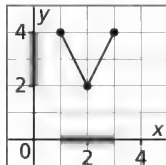
$$1 \leq x \leq 4$$

The range is the set of y -values of the graph:

$$1 \leq y \leq 2$$

So, match part a with part iv.

- b) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

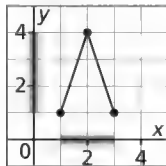
$$1 \leq x \leq 3$$

The range is the set of y -values of the graph:

$$2 \leq y \leq 4$$

So, match part b with part i.

- c) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

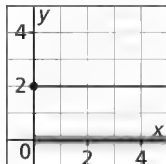
$$1 \leq x \leq 3$$

The range is the set of y -values of the graph:

$$1 \leq y \leq 4$$

So, match part c with part ii.

- d) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

$$x \geq 0$$

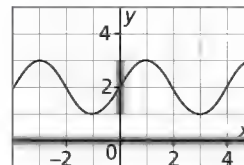
The range is the set of y -values of the graph:

$$y = 2$$

So, match part d with part iii.

B

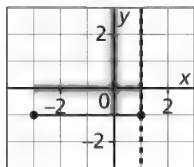
8. a) This graph is a function.
Any vertical line drawn on the graph passes through exactly 1 point.
Visualize the shadow of the graph on the x - and y -axes.



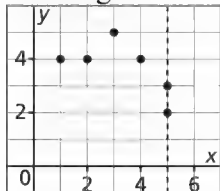
The domain is the set of x -values of the graph, which is the set of all real numbers.

The range is the set of y -values of the graph:
 $1 \leq y \leq 3$

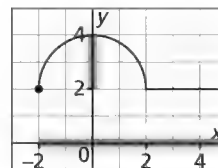
- b) This graph is not a function. The vertical line $x = 1$ passes through many points on the graph. Visualize the shadow of the graph on the x - and y -axes.
The domain is the set of x -values of the graph: $-3 \leq x \leq 1$
The range is the set of y -values of the graph: $y \geq -1$



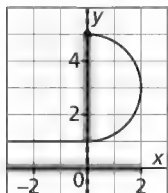
- c) This graph is not a function. The vertical line $x = 5$ passes through the points (5, 2) and (5, 3) on the graph.
The domain is the set of x -values of the graph: $\{1, 2, 3, 4, 5\}$
The range is the set of y -values of the graph: $\{2, 3, 4, 5\}$



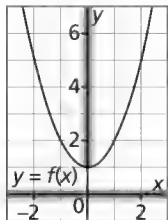
- d) This graph is a function. It is not possible to draw a vertical line on the graph that passes through 2 points on the graph.
Visualize the shadow of the graph on the x - and y -axes.
The domain is the set of x -values of the graph:
 $x \geq -2$
The range is the set of y -values of the graph:
 $2 \leq y \leq 4$



- e) This graph is not a function. The vertical line $x = 0$ passes through the points (0, 1) and (0, 5) on the graph. Visualize the shadow of the graph on the x - and y -axes.
The domain is the set of x -values of the graph: $x \leq 2$
The range is the set of y -values of the graph: $1 \leq y \leq 5$



9. a) Visualize the shadow of the graph on the x - and y -axes.

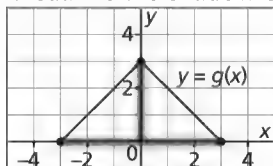


The domain is the set of x -values of the graph, which is the set of all real numbers.

The range is the set of y -values of the graph:

$$y \geq 1$$

- b) Visualize the shadow of the graph on the x - and y -axes.



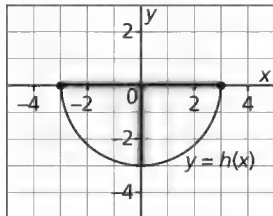
The domain is the set of x -values of the graph:

$$-3 \leq x \leq 3$$

The range is the set of y -values of the graph:

$$0 \leq y \leq 3$$

- c) Visualize the shadow of the graph on the x - and y -axes.



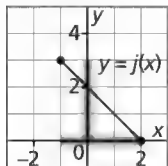
The domain is the set of x -values of the graph:

$$-3 \leq x \leq 3$$

The range is the set of y -values of the graph:

$$-3 \leq y \leq 0$$

- d) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

$$-1 \leq x \leq 2$$

The range is the set of y -values of the graph:

$$0 \leq y \leq 3$$

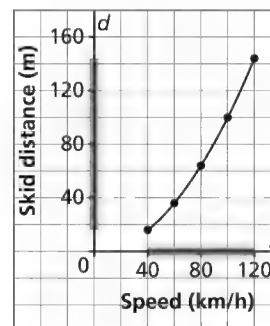
10. a) The points on the graph should not be connected because the price is calculated per letter, and only whole numbers of letters are possible.
- b) The points on the graph should be connected because the plane's altitude and travel time could be any positive number.
- c) The points on the graph should be connected because the baby's mass and age could be any positive number.
- d) The points on the graph should be connected because both a number and its cube root can be any real number.
11. a) i) The data in the graph represent the distance of a school bus from the school from 8:00 to 9:00.
- ii) The data in the graph represent the number of students on a school bus from 8:00 to 9:00.
- b) i) The independent variable is the time.
The dependent variable is the distance from the school.
- ii) The independent variable is the time.
The dependent variable is the number of students.
- c) The points on graph A are connected because all values of time and distance are permissible between the indicated plotted points.
The points on graph B are not connected because it is impossible to have only part of a student on a bus.
12. a) The points on the graph are connected because the car's speed in kilometres per hour and the skid length in metres can be any positive number between the plotted points.
- b) Visualize the shadow of the graph on the x - and y -axes.

The domain is the set of s -values of the graph:
 $40 \leq s \leq 120$

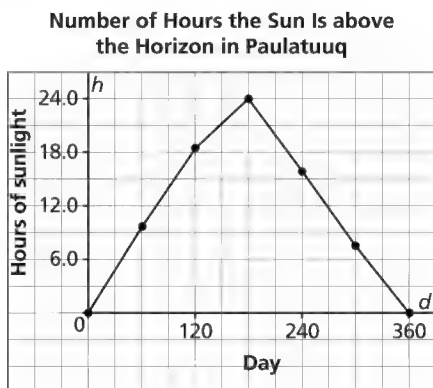
The range is the set of d -values of the graph:
approximately $16 \leq d \leq 144$

The domain and range cannot contain negative numbers because it is impossible to have a negative skid distance or a negative speed. The domain is also restricted because the relationship shown on the graph may not be true for speeds less than 40 km/h and greater than 120 km/h.

Skid Distance of a Car



13. a) The number of cars in the parking lot depends on the time of day. So, t is the independent variable and n is the dependent variable.
- b) The points are not connected because it is impossible to have part of a car in a parking lot.
- c) The domain is the set of x -coordinates of the points:
 $\{8:00, 10:00, 12:00, 14:00, 16:00\}$
 The range is the set of y -coordinates of the points, approximately:
 $\{4, 25, 31, 64, 65\}$
 The domain can be any time between 00:00 and 24:00, all the possible times in one day.
 The range can be any whole number up to the number of parking spaces in the lot.
14. a) The hours of sunlight depends on the time of year, or the day. So, the number of days after January 1 is the independent variable and the number of hours the sun is above the horizon, h , is the dependent variable.
- b) Graph the data.



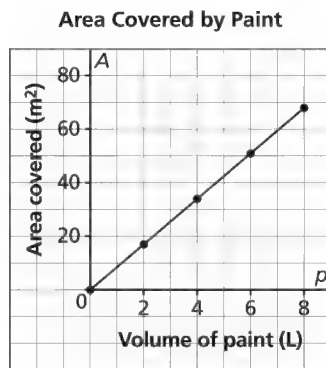
I connected the points because the relationship shown on the graph is true for days represented by points between the ones plotted.

- c) The relation is a function because each number in the first column of the table is associated with exactly one number in the second column in the table. In the graph, any vertical line passes through at most one point.

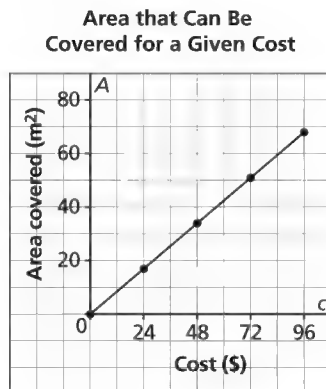
15. a) Copy and complete the table.

Volume of paint, p (L)	0	2	4	6	8
Cost, c (\$)	0	24	48	72	96
Area covered, A (m^2)	0	17	34	51	68

b) Graph the data.



c) Graph the data.



d) In part b:

The domain is the set of p -values of the graph:

$$0 \leq p \leq 8$$

The range is the set of A -values of the graph:

$$0 \leq A \leq 68$$

In part c:

The domain is the set of c -values of the graph:

$$0 \leq c \leq 96$$

The range is the set of A -values of the graph:

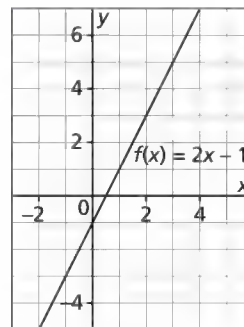
$$0 \leq A \leq 68$$

16. The domain value is a value of x . The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = 0$:

When $x = 0$, $f(x)$ is the y -intercept, which is -1 .

When the domain value is 0 , the range value is -1 .



b) To determine the value of x when $f(x) = 5$:

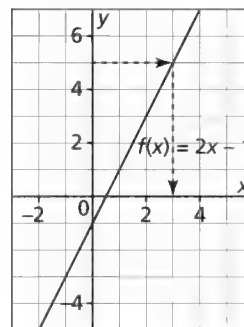
Begin at $f(x) = 5$ on the y -axis.

Draw a horizontal line to the graph,
then a vertical line to the x -axis.

The line intersects the x -axis at 3 .

So, when $f(x) = 5$, $x = 3$

When the range value is 5 , the domain value is 3 .



17. The domain value is a value of x . The range value is a value of $g(x)$.

a) To determine the value of $g(x)$ when $x = -2$:

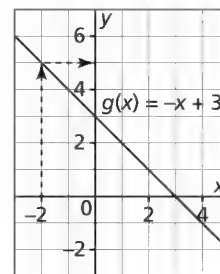
Begin at $x = -2$ on the x -axis.

Draw a vertical line to the graph,
then a horizontal line to the y -axis.

The line intersects the y -axis at 5 .

So, $g(-2) = 5$

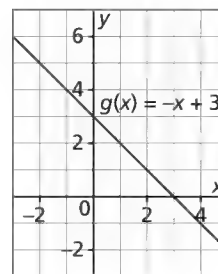
When the domain value is -2 , the range value is 5 .



b) To determine the value of x when $g(x) = 0$:

When $g(x) = 0$, x is the x -intercept, which is 3 .

When the range value is 0 , the domain value is 3 .



18. Sample response:

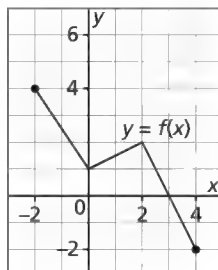
A possible function is:

The domain of this function is:

$$-2 \leq x \leq 4$$

The range of this function is:

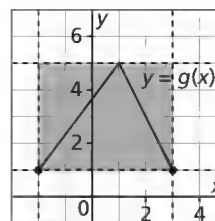
$$-2 \leq y \leq 4$$



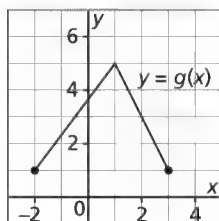
19. Answers will vary. For example:

- a) On a grid, draw faint vertical lines through $x = -2$ and $x = 3$; and faint horizontal lines through $y = 1$ and $y = 5$.

Draw line segments inside the region enclosed by the faint lines so that there is exactly one point on the graph for every x -value between -2 and 3 , and there is at least one point on the graph for every y -value between 1 and 5 .

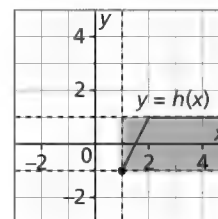


A function is:

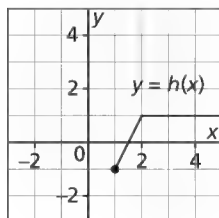


- b) On a grid, draw a faint vertical line through $x = 1$; and faint horizontal lines through $y = -1$ and $y = 1$.

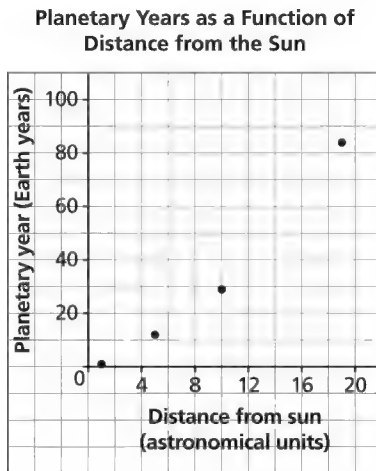
Draw line segments inside the region between the horizontal lines and to the right of the vertical line so that there is exactly one point on the graph for every x -value greater than or equal to 1 , and there is at least one point on the graph for every y -value between -1 and 1 .



A function is:



20. a) Sketch a graph.

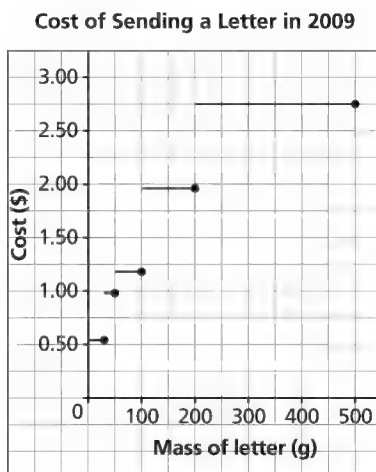


I didn't connect the points because there are only a few planets between the ones shown and I am not sure if the relationship shown in the graph is true for them.

- b) The domain of this function is the distances from the sun:
 $\{1, 5, 10, 19\}$
 The range of this function is the numbers of planetary years:
 $\{1, 12, 29, 84\}$

C

21. a) Sketch a graph.



I connected the points in each interval, but not the points at the endpoints of the intervals because otherwise my graph would have vertical line segments and would not have been a function.

- b) The domain of this function is the masses of letters:
 All real numbers between 0 and 500.
 The range of this function is the costs to send the letters:
 $\{0.54, 0.98, 1.18, 1.96, 2.75\}$

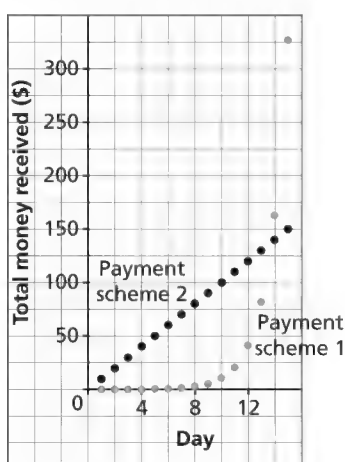
22. Yes, the points should have been connected because the patient has a temperature at all times, even if it is not measured, and the patient's temperature at a certain time was most likely between the temperatures measured immediately before and after that time.
23. A graph with time as its independent variable and a discrete measure as its dependent variable should not have its points connected. For example, a graph of the number of people on a bus over 4 h should not have its points connected because it's impossible to have only part of a person on the bus. So, the statement is false.

24. a) Tables of values:

Payment Scheme 1		
Day	Money each day (\$)	Total money received (\$)
1	0.01	0.01
2	0.02	$0.01 + 0.02 = 0.03$
3	0.04	$0.03 + 0.04 = 0.07$
4	0.08	$0.07 + 0.08 = 0.15$
5	0.16	$0.15 + 0.16 = 0.31$
6	0.32	$0.31 + 0.32 = 0.63$
7	0.64	$0.63 + 0.64 = 1.27$
8	1.28	$1.27 + 1.28 = 2.55$
9	2.56	$2.55 + 2.56 = 5.11$
10	5.12	$5.11 + 5.12 = 10.23$
11	10.24	$10.23 + 10.24 = 20.47$
12	20.48	$20.47 + 20.48 = 40.95$
13	40.96	$40.95 + 40.96 = 81.91$
14	81.92	$81.91 + 81.92 = 163.83$
15	163.84	$163.83 + 163.84 = 327.67$

Payment Scheme 2	
Day	Total money received (\$)
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100
11	110
12	120
13	130
14	140
15	150

- b) Graph the data.
Total Money Received Under
Two Payment Schemes



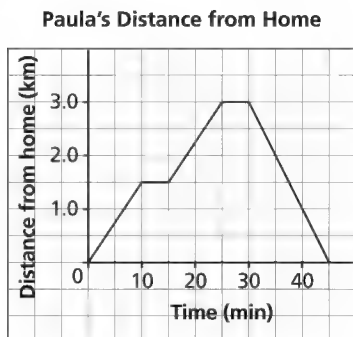
- c) I would choose Payment Scheme 1 because after 13 days, the money received is greater and increases at a faster rate.

Checkpoint 2

Assess Your Understanding (page 299)

5.3

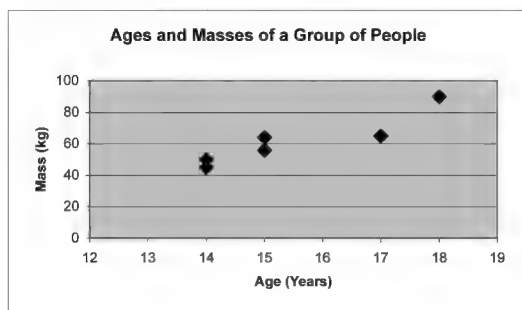
1. Copy and label the graph.



Situation: Paula jogs from her house to a store 1.5 km away in 10 min. She spends 5 min in the store, then jogs another 1.5 km away from her house to another store. It takes Paula 10 min to get to the second store. She spends 5 min in the second store, then returns home. It takes Paula 15 min to travel 3 km to her home.

5.4

2. a) Graph the data using a computer spreadsheet program. Label the axes.



- b) The points should not be joined because each person's age was measured in a whole number of years.
- c) The domain is the set of numbers in the first column of the table of values:
 $\{14, 15, 17, 18\}$
 The range is the set of numbers in the second column of the table of values:
 $\{45, 50, 56, 64, 65, 90\}$
- d) The domain is restricted to ages in whole numbers of years from 0 to 18 years. The range will then be the corresponding masses in kilograms.

5.5

3. a) The vertical line $x = 0$ passes through the points $(0, 1)$ and $(0, 5)$ on the graph.
So, the graph does not represent a function.
Its domain is:
 $0 \leq x \leq 2$
Its range is:
 $1 \leq y \leq 5$
- b) Any vertical line drawn on the graph passes through at most 1 point.
So, the graph is a function.
Its domain is:
 $x \geq -3$
Its range is:
 $y \geq 0$
- c) Any vertical line drawn on the graph passes through at most 1 point.
So, the graph is a function.
Its domain is:
 $-2 \leq x \leq 2$
Its range is:
 $-8 \leq y \leq 8$

Lesson 5.6 Properties of Linear Relations

Exercises (pages 308–310)

A

3. a) The terms in the first column are in numerical order.
So, calculate the change in each variable.

Time (min)	Change in Time (min)	Distance (m)	Change in Distance (m)
0		10	
2	$2 - 0 = 2$	50	$50 - 10 = 40$
4	$4 - 2 = 2$	90	$90 - 50 = 40$
6	$6 - 4 = 2$	130	$130 - 90 = 40$

Since the changes in both variables are constant, the table of values represents a linear relation.

- b) The terms in the first column are in numerical order.
So, calculate the change in each variable.

Time (s)	Change in Time (s)	Speed (m/s)	Change in Speed (m/s)
0		10	
1	$1 - 0 = 1$	20	$20 - 10 = 10$
2	$2 - 1 = 1$	40	$40 - 20 = 20$
3	$3 - 2 = 1$	80	$80 - 40 = 40$

The changes in time are constant, but the changes in speed are not constant.
So, the table of values does not represent a linear relation.

- c) Arrange the terms in the first column in numerical order.
Then calculate the change in each variable.

Speed (m/s)	Change in Speed (m/s)	Time (s)	Change in Time (s)
0		0	
5	$5 - 0 = 5$	2.5	$2.5 - 0 = 2.5$
10	$10 - 5 = 5$	5	$5 - 2.5 = 2.5$
15	$15 - 10 = 5$	7.5	$7.5 - 5 = 2.5$

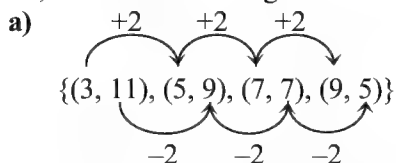
Since the changes in both variables are constant, the table of values represents a linear relation.

- d) Arrange the terms in the first column in numerical order.
Then calculate the change in each variable.

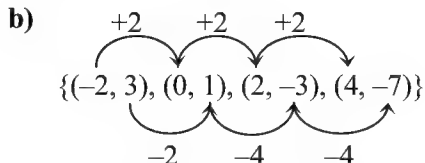
Distance (m)	Change in Distance (m)	Speed (m/s)	Change in Speed (m/s)
1		1	
4	$4 - 1 = 3$	2	$2 - 1 = 1$
9	$9 - 4 = 5$	3	$3 - 2 = 1$
16	$16 - 9 = 7$	4	$4 - 3 = 1$

The changes in speed are constant, but the changes in distance are not constant.
So, the table of values does not represent a linear relation.

4. The first elements are in numerical order.
So, calculate the change in each variable.



Since the changes in both elements are constant, the set of ordered pairs represents a linear relation.



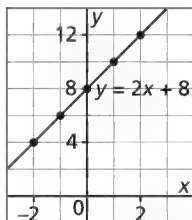
The changes in the first elements are constant, but the changes in the second elements are not constant. So, the set of ordered pairs does not represent a linear relation.

- c) This relation has two ordered pairs with first element 1 and two ordered pairs with first element 2.
So, the changes in the first elements are not constant, and the set of ordered pairs does not represent a linear relation.
5. a) The graph is a line, so it represents a linear relation.
- b) The graph is a line, so it represents a linear relation.
- c) All the points on the graph do not lie on the same line, so the graph does not represent a linear relation.
- d) The graph is a curve, so it does not represent a linear relation.

B

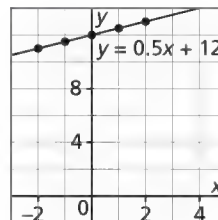
6. a) i) $y = 2x + 8$

x	y
-2	4
-1	6
0	8
1	10
2	12



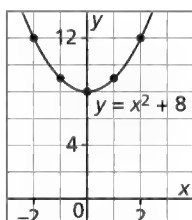
ii) $y = 0.5x + 12$

x	y
-2	11
-1	11.5
0	12
1	12.5
2	13



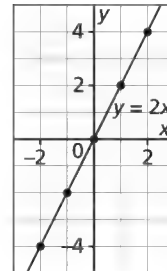
iii) $y = x^2 + 8$

x	y
-2	12
-1	9
0	8
1	9
2	12



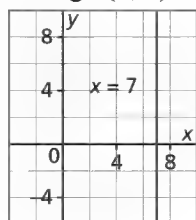
iv) $y = 2x$

x	y
-2	-4
-1	-2
0	0
1	2
2	4



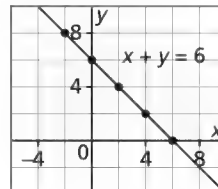
v) $x = 7$

This is a vertical line that passes through (7, 0).



vi) $x + y = 6$

x	y
-2	8
0	6
2	4
4	2
6	0



- b) The relations in part a, i, ii, iv, v, and vi have graphs that are straight lines, so the relations are linear.
The relation in part a, iii has a graph that is a curve, so the relation is not linear.

7. a) i) From an understanding of the situation, the braking distance, d , depends on the speed of the car, s . So, d is the dependent variable and s is the independent variable.

- ii) Look at the change in each variable.

	s	d	
	50	13	
+10	60	20	+7
+10	70	27	+7
+10	80	35	+8

The change in the first variable is constant, but the change in the second variable is not constant. So, the relation is not linear.

- b) i) From an understanding of the situation, the altitude of the plane, a , depends on the time, t , that has elapsed since it started its descent. So, a is the dependent variable and t is the independent variable.

- ii) Look at the change in each variable.

	t	a	
	0	12 000	
+2	2	11 600	-400
+2	4	11 200	-400
+2	6	10 800	-400
+2	8	10 400	-400

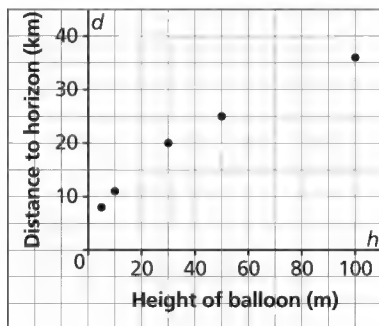
The changes in both variables are constant, so the relation is linear.

- iii) The rate of change is:

$$\frac{-400 \text{ m}}{2 \text{ min}} = -200 \text{ m/min}$$

8. a) Graph the data.

Distance to the Horizon for a Given
Height in a Hot-Air Balloon



- b) The relation is not linear because the points on the graph do not lie on a straight line.

9. I could examine the changes in the first and second coordinates. If both changes are constant, the relation is linear.

I could graph the ordered pairs. If the points lie on a straight line, the relation is linear.

10. Create a table of values, then check to see if the relation is linear.

	Number of people	Cost (\$)
+1	2	95
+1	3	105
+1	4	115
+1	5	125

There is a constant change of 1 in the first column, and a constant change of 10 in the second column. So, the relation is linear.

11. The first elements are in numerical order. So, calculate the change in each variable.

	+4	+4	+4
	↘	↘	↘
{(0, 3600), (4, 3526), (8, 3353.5), (12, 3147.5)}			
	↗	↗	↗
	-74	-172.5	-206

The change in the first elements is constant, but the change in the second elements is not constant. So, this set of ordered pairs does not represent a linear relation.

	+6	+6	+6
	↘	↘	↘
{(15, 2988.5), (21, 2670.5), (27, 2352.5), (33, 2034.5)}			
	↗	↗	↗
	-318	-318	-318

The changes in both elements are constant, so this set of ordered pairs represents a linear relation.

12. a) The equation represents a linear relation because it equates the dependent variable C to the sum of a constant and the product of the rate of change and the independent variable.
- b) The rate of change is 15. It represents the cost per guest, \$15, after the fixed cost of \$550 has been paid.
13. Sample response: I could create a table of values for the relation. Then I could either check the differences in the numbers in each column or I could plot the points in the table. If the differences are constant or the points lie on a line, the relation is linear. Otherwise, it is not linear.
14. a) From an understanding of the situation, the cost of the phone call, C , depends on the time, t , that has elapsed. So, C is the dependent variable and t is the independent variable.
- b) Sample response:
Choose two points on the line: (5, 0.40) and (15, 1.20)
Calculate the change in each variable from one point to the other.
The change in time is:
 $15 \text{ min} - 5 \text{ min} = 10 \text{ min}$
The change in cost is:
 $\$1.20 - \$0.40 = \$0.80$
The rate of change is:
 $\frac{\$0.80}{10 \text{ min}} = \$0.08/\text{min}$

The rate of change is positive, so the cost is increasing with time.
Every minute, the cost of the phone call increases by \$0.80.

15. Sample response:

Choose two points on the line: (1, 9.20) and (3, 7.60)

Calculate the change in each variable from one point to the other.

The change in the number of toll booths is:

3 booths – 1 booth = 2 booths

The change in the amount Kashala has in change is:

$\$7.60 - \$9.20 = -\$1.60$

The rate of change is:

$$\frac{-\$1.60}{2 \text{ booths}} = -\$0.80/\text{booth}$$

The rate of change is negative, so the amount Kashala has in change decreases with time.

At every toll booth, Kashala pays \$0.80.

16. a) A person who works 0 h earns \$0.

So, substitute $x = 0$ into the equations:

Equation 1:

$$y = 500 + 40x$$

$$y = 500 + 40(0)$$

$$y = 500$$

Equation 2:

$$y = 35 - 0.06x$$

$$y = 35 - 0.06(0)$$

$$y = 35$$

Equation 3:

$$y = 20x$$

$$y = 20(0)$$

$$y = 0$$

So, Equation 3 seems to be the best match. It could represent an hourly wage of \$20/h.

So, after 1 h, the person earns \$20. Look for a set of ordered pairs that contains (1, 20).

Set B contains (1, 20), so it seems to be the best match.

So, Equation 3 and Set B match this situation.

b) The cost of a banquet is the sum of a fixed cost and a variable cost, so look for an equation that has a constant plus the product of a constant and a variable.

Equation 1 seems to be the best match. It could represent a flat fee of \$500 plus \$40/person.

So, if 100 people attend, the cost is $\$500 + \$4000 = \$4500$. Look for a set of ordered pairs that contains (100, 4500).

Set C contains (100, 4500), so it seems to be the best match.

So, Equation 1 and Set C match this situation.

c) The volume of gas in the tank decreases as the car is driven, so look for an equation with a negative rate of change.

Equation 2 seems to be the best match. It could represent a car with a tank size of 35 L that uses 0.06 L of gas per kilometre driven.

So, if the car is driven for 100 km, the volume of gas remaining in the tank is:

$$35 \text{ L} - 6 \text{ L} = 29 \text{ L}$$

So, look for a set with the ordered pair (100, 29).

Set A contains (100, 29), so it seems to be the best match.

So, Equation 2 and Set A match this situation.

17. a) i) A constant speed is a constant rate of change; that is, a measure of how the distance changes with time.

The change in altitude is 500 m in 10 min. So, the rate of change is:

$$\frac{-500 \text{ m}}{10 \text{ min}} = -50 \text{ m/min}$$

So, the relation is linear.

- ii) Create a table of values for the situation.

Time (h)	Number of bacteria
0	100
1	300
2	900
3	2700
4	8100

The change in time is constant, but the change in the number of bacteria is not constant. So, the situation does not represent a linear relation.

- iii) Create a table of values for the situation.

Distance (km)	Cost (\$)
0	5
1	7
2	9
3	11
4	13

Since the change in each variable is constant, the situation represents a linear relation.

- iv) Create a table of values for the situation.

Number of yearbooks	Cost (\$)
0	500
100	$(100)(5) + 500 = 1000$
200	$(200)(5) + 500 = 1500$
300	$(300)(5) + 500 = 2000$
400	$(400)(5) + 500 = 2500$

Since the change in each variable is constant, the situation represents a linear relation.

- v) Create a table of values for the situation.

Time (years)	Value of investment (\$)
0	1000.00
1	$1000(1.12) = 1120.00$
2	$1120(1.12) = 1254.40$
3	$1254.40(1.12) = 1404.93$
4	$1404.93(1.12) = 1573.52$

The change in time is constant, but the change in the value of the investment is not constant. So, the situation does not represent a linear relation.

- b) i)** The altitude of the hang glider depends on the time since she started her descent. So, altitude is the dependent variable and time is the independent variable.
From part a, the rate of change is -50 m/min
The rate of change is negative because the hang glider's altitude is decreasing over time.
Every minute, the altitude decreases by 50 m.
- iii)** The taxi cost depends on the distance travelled. So, the cost is the dependent variable and the distance travelled is the independent variable.
The rate of change is: $\frac{\$2}{1 \text{ km}} = \$2/\text{km}$
The rate of change is positive because the cost increases with the distance travelled.
Every kilometre, the cost increases by \$2.
- iv)** The printing cost depends on the number of yearbooks to be printed. So, the cost is the dependent variable and the number of yearbooks to be printed is the independent variable.
The rate of change is: $\$5/\text{yearbook}$
The rate of change is positive because the cost increases with the number of yearbooks printed.
For every yearbook to be printed, the cost increases by \$5.

C

- 18. a)** The formula represents a relation with P as the dependent variable and s as the independent variable. The variable s is raised to the exponent 1, so the relation is linear.
- b)** The formula represents a relation with A as the dependent variable and s as the independent variable. The variable s is raised to the exponent 2, so the relation is not linear.
- c)** The formula represents a relation with V as the dependent variable and r as the independent variable. The variable r is raised to the exponent 3, so the relation is not linear.
- d)** The formula represents a relation with C as the dependent variable and d as the independent variable. The variable d is raised to the exponent 1, so the relation is linear.
- e)** The formula represents a relation with A as the dependent variable and r as the independent variable. The variable r is raised to the exponent 2, so the relation is not linear.

- 19. a)** Each equation represents a relation with V as the dependent variable and n as the independent variable.
In the first equation, $V = 24\,000 - 2000n$, the variable n is raised to the exponent 1, so the relation is linear.
In the second equation, $V = 24\,000(0.2^n)$, the variable n is the exponent of a power, so the relation is not linear.
- b)** The rate of change for the first equation is $-\$2000/\text{year}$.
The rate of change is negative, so the value of the truck decreases with time.
Every year, the value of the truck depreciates by $\$2000$.
- 20.** The speed of sound is constant, and since speed is the rate of change of distance over time, the relation is linear.
- 21.** Suppose the pickers pick berries at the same rate and 1 picker could harvest the patch in 8 h.
So, 2 pickers could harvest the patch in 4 h.
4 pickers could harvest the patch in 2 h.
8 pickers could harvest the patch in 1 h.
So, a set of ordered pairs for the relation is: $\{(1, 8), (2, 4), (4, 2), (8, 1)\}$
The rate of change calculated from the first 2 ordered pairs is:
$$\frac{4-8}{2-1} = -4$$

The rate of change calculated from the 2nd and 3rd ordered pairs is:
$$\frac{2-4}{4-2} = \frac{-2}{2}, \text{ or } -1$$

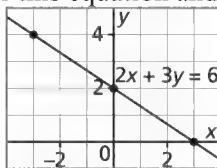
Since the rate of change is not constant, the relation is not linear.
- 22. a)** The change in each variable will be constant because there is only one pair of points to consider.
For example, this relation has exactly 2 ordered pairs: $\{(0, 1), (4, 22)\}$
- $\{(0, 1), (4, 22)\}$
- The change in each variable is constant.
So, the statement is true.

- b) The statement is true. An equation of the form $Ax + By = C$ represents a linear function because the terms in x and y are added, and the exponents of x and y are 1. The only linear relation that is not a function is a vertical line. A vertical line has the form $x = D$, for a constant D . Since B is a non-zero constant, it is impossible to write an equation that is equivalent to the equation $Ax + By = C$ and does not contain a term involving y . So, it does not represent a vertical line.

For example, this equation has the form $Ax + By = C$, where A , B , and C are non-zero constants: $2x + 3y = 6$

Create a table of values for this equation and graph them on a coordinate grid.

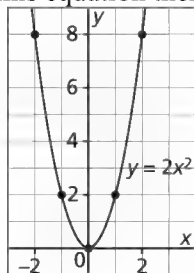
x	y
-3	4
0	2
3	0



This graph represents a linear function.

- c) This equation has the form $y = Cx^2$, where C is a non-zero constant: $y = 2x^2$
Create a table of values for this equation then graph it on a coordinate grid.

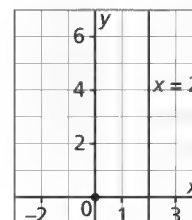
x	y
-2	8
-1	2
0	0
1	2
2	8



The graph is a curve.

So, the statement is false.

- d) The graph of any equation of the form $x = C$, where C is a constant, is a vertical line.
A line always represents a linear relation.
For example, $x = 2$ has the form $x = C$, where C is the constant 2.
The graph of $x = 2$ is a vertical line through $(2, 0)$.
So, the statement is true.



- e) A linear relation is only a linear function when the relation does not contain two different ordered pairs with the same first element. From part d, any vertical line, such as $x = 2$, is a linear relation. However, every ordered pair has the same first element, 2. So, the statement is false.

Lesson 5.7 Interpreting Graphs of Linear Functions

Exercises (pages 319–323)

A

4. a) i) On the vertical axis, the point of intersection has coordinates (0, 0).
The vertical intercept is 0.
On the horizontal axis, the point of intersection has coordinates (0, 0).
The horizontal intercept is 0.

- ii) Two points on the graph are: (0, 0) and (3, 120)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{120 \text{ km} - 0 \text{ km}}{3 \text{ h} - 0 \text{ h}} \\ &= \frac{120 \text{ km}}{3 \text{ h}} \\ &= 40 \text{ km/h}\end{aligned}$$

- iii) The domain is all possible t -values:

$$0 \leq t \leq 3$$

The range is all possible d -values:

$$0 \leq d \leq 120$$

- b) i) On the vertical axis, the point of intersection has coordinates (0, 100).
The vertical intercept is 100.
On the horizontal axis, the point of intersection has coordinates (4, 0).
The horizontal intercept is 4.

- ii) Use the coordinates in part i.

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{0 \text{ km} - 100 \text{ km}}{4 \text{ h} - 0 \text{ h}} \\ &= \frac{-100 \text{ km}}{4 \text{ h}} \\ &= -25 \text{ km/h}\end{aligned}$$

- iii) The domain is all possible t -values:

$$0 \leq t \leq 4$$

The range is all possible d -values:

$$0 \leq d \leq 100$$

5. a) i) On the vertical axis, the point of intersection has coordinates (0, 400).
The vertical intercept is 400.

- ii) Two points on the graph have coordinates: (0, 400) and (8, 1200)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } A}{\text{Change in } t} &= \frac{1200 \text{ ft.} - 400 \text{ ft.}}{8 \text{ min} - 0 \text{ min}} \\ &= \frac{800 \text{ ft.}}{8 \text{ min}} \\ &= 100 \text{ ft./min}\end{aligned}$$

- iii) The domain is all possible t -values:

$$0 \leq t \leq 8$$

The range is all possible A -values:

$$400 \leq A \leq 1200$$

- b) i) On the vertical axis, the point of intersection has coordinates (0, 1000).
The vertical intercept is 1000.

- ii) Two points on the graph have coordinates: (0, 1000) and (8, 600)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } A}{\text{Change in } t} &= \frac{600 \text{ ft.} - 1000 \text{ ft.}}{8 \text{ min} - 0 \text{ min}} \\ &= \frac{-400 \text{ ft.}}{8 \text{ min}} \\ &= -50 \text{ ft./min}\end{aligned}$$

- iii) The domain is all possible t -values:

$$0 \leq t \leq 8$$

The range is all possible A -values:

$$600 \leq A \leq 1000$$

B

6. Since each function is linear, its graph is a straight line.

a) $f(x) = 4x + 3$

Determine the y -intercept:

When $x = 0$,

$$f(0) = 4(0) + 3$$

$$f(0) = 3$$

Determine the x -intercept:

When $f(x) = 0$,

$$0 = 4x + 3$$

$$-3 = 4x$$

$$x = -\frac{3}{4}$$

Determine the coordinates of a third point on the graph.

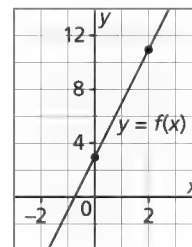
When $x = 2$,

$$f(2) = 4(2) + 3$$

$$f(2) = 11$$

Plot the points $(0, 3)$ and $(2, 11)$, then draw a line through them.

Check that the graph passes through $(-\frac{3}{4}, 0)$.



b) $g(x) = -3x + 5$

Determine the y -intercept:

When $x = 0$,

$$g(0) = -3(0) + 5$$

$$g(0) = 5$$

Determine the x -intercept:

When $g(x) = 0$,

$$0 = -3x + 5$$

$$3x = 5$$

$$x = \frac{5}{3}$$

Determine the coordinates of a third point on the graph.

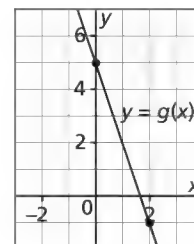
When $x = 2$,

$$g(2) = -3(2) + 5$$

$$g(2) = -1$$

Plot the points $(0, 5)$ and $(2, -1)$, then draw a line through them.

Check that the graph passes through $(\frac{5}{3}, 0)$.



c) $h(x) = 9x - 2$

Determine the y -intercept:

When $x = 0$,

$$h(0) = 9(0) - 2$$

$$h(0) = -2$$

Determine the x -intercept:

When $h(x) = 0$,

$$h(x) = 9x - 2$$

$$0 = 9x - 2$$

$$2 = 9x$$

$$x = \frac{2}{9}$$

Determine the coordinates of a third point on the graph.

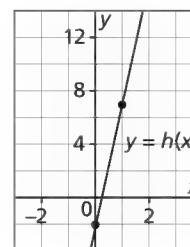
When $x = 1$,

$$h(1) = 9(1) - 2$$

$$h(1) = 7$$

Plot the points $(0, -2)$ and $(1, 7)$, then draw a line through them.

Check that the graph passes through $(\frac{2}{9}, 0)$.



d) $k(x) = -5x - 2$

Determine the y -intercept:

When $x = 0$,

$$k(0) = -5(0) - 2$$

$$k(0) = -2$$

Determine the x -intercept:

When $k(x) = 0$,

$$0 = -5x - 2$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

Determine the coordinates of a third point on the graph.

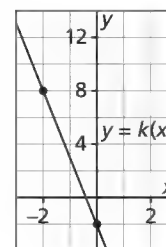
When $x = -2$,

$$k(-2) = -5(-2) - 2$$

$$k(-2) = 8$$

Plot the points $(0, -2)$ and $(-2, 8)$, then draw a line through them.

Check that the graph passes through $(-\frac{2}{5}, 0)$.



7. a) Choose two points on the graph: (0, 0) and (2, 18)

The rate of change is:

$$\frac{\text{Change in } A}{\text{Change in } V} = \frac{18 \text{ m}^2 - 0 \text{ m}^2}{2 \text{ L} - 0 \text{ L}}$$

$$= 9 \text{ m}^2/\text{L}$$

Every litre of paint covers an area of 9 m^2 .

- b) Since 1 L of paint covers 9 m^2 , then 6 L of paint will cover: $6(9 \text{ m}^2) = 54 \text{ m}^2$
Six litres of paint cover an area of 54 m^2 .

- c) Since 9 m^2 is covered by 1 L, then 45 m^2 will be covered by: $\frac{45}{9} \text{ L} = 5 \text{ L}$
Five litres of paint would cover 45 m^2 .

8. Determine the rate of change and vertical intercept for each graph.

- i) Two points on the graph are: (0, 20) and (10, 0)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - 20^{\circ}\text{C}}{10 \text{ h} - 0 \text{ h}} \\ &= \frac{-20^{\circ}\text{C}}{10 \text{ h}} \\ &= -2^{\circ}\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, 20).

The vertical intercept is 20.

- ii) Two points on the graph are: (2, 0) and (4, 10)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{10^{\circ}\text{C} - 0^{\circ}\text{C}}{4 \text{ h} - 2 \text{ h}} \\ &= \frac{10^{\circ}\text{C}}{2 \text{ h}} \\ &= 5^{\circ}\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, -10).

The vertical intercept is -10.

- iii) Two points on the graph are: (0, 20) and (2, 0)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - 20^{\circ}\text{C}}{2 \text{ h} - 0 \text{ h}} \\ &= \frac{-20^{\circ}\text{C}}{2 \text{ h}} \\ &= -10^{\circ}\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, 20).

The vertical intercept is 20.

- iv) Two points on the graph are: (0, -10) and (10, 0)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - (-10^{\circ}\text{C})}{10 \text{ h} - 0 \text{ h}} \\ &= \frac{10^{\circ}\text{C}}{10 \text{ h}} \\ &= 1^{\circ}\text{C/h}\end{aligned}$$

On the vertical axis, the point of intersection has coordinates (0, -10).

The vertical intercept is -10.

- a) The graph in part ii has a rate of change of 5°C/h and a vertical intercept of -10°C .

- b) The graph in part iii has a rate of change of $-10^{\circ}\text{C}/\text{h}$ and a vertical intercept of 20°C .
9. a) On the vertical axis, the point of intersection has coordinates $(0, 0)$. The vertical intercept is 0.
On the horizontal axis, the point of intersection has coordinates $(0, 0)$. The horizontal intercept is 0.
The point where the graph intersects the axes, $(0, 0)$, represents the cost of running the backhoe for 0 h: \$0
- b) Choose two points on the graph: $(0, 0)$ and $(5, 400)$
The rate of change is:

$$\frac{\text{Change in } C}{\text{Change in } t} = \frac{\$400 - \$0}{5 \text{ h} - 0 \text{ h}}$$

$$= \frac{\$400}{5 \text{ h}}$$

$$= \$80/\text{h}$$
 It costs \$80 to run the backhoe for 1 h.
- c) The domain is all possible t -values:
 $0 \leq t \leq 10$
 The range is all possible C -values:
 $0 \leq C \leq 800$
- d) From part a, it costs \$80 to run the backhoe for 1 h.
 So, the cost for 7 h is: $7(\$80) = \$560$
 It costs \$560 to run the backhoe for 7 h.
- e) From part a, it costs \$80 to run the backhoe for 1 h.
 So, the time for a cost of \$360 is:

$$\frac{360}{80} \text{ h} = 4.5 \text{ h}$$
 When the cost is \$360, the backhoe is run for 4.5 h.

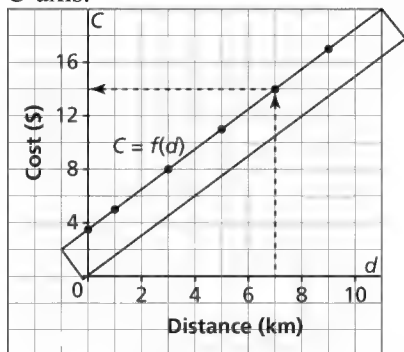
10. a) Choose two points on the graph: (3, 8) and (7, 14)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } C}{\text{Change in } d} &= \frac{\$14 - \$8}{7 \text{ km} - 3 \text{ km}} \\ &= \frac{\$6}{4 \text{ km}} \\ &= \$1.50/\text{km}\end{aligned}$$

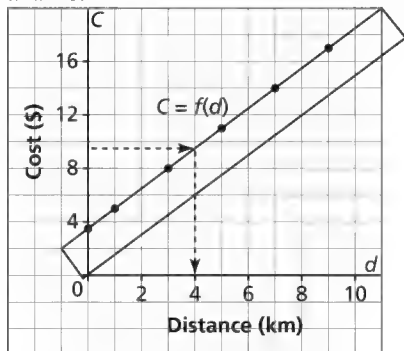
Every kilometre driven costs an additional \$1.50.

- b) To estimate the cost when the distance is 7 km, use the graph.
From 7 on the d -axis, draw a vertical line to the graph, then a horizontal line to the C -axis.



From the graph, when the distance is 7 km, the cost is approximately \$14.

- c) To estimate the distance when the cost is \$9.50, use the graph.
From 9.5 on the C -axis, draw a horizontal line to the graph, then a vertical line to the d -axis.



From the graph, when the cost is \$9.50, the distance is approximately 4 km.

11. Choose two points on the Smart car graph: (0, 35) and (500, 5)

The rate of change for the graph of the fuel consumption of a Smart car is:

$$\begin{aligned}\frac{\text{Change in } V}{\text{Change in } d} &= \frac{5 \text{ L} - 35 \text{ L}}{500 \text{ km} - 0 \text{ km}} \\ &= \frac{-30 \text{ L}}{500 \text{ km}} \\ &= -0.06 \text{ L/km}\end{aligned}$$

The Smart car uses 0.06 L of fuel to drive 1 km.

Choose two points on the SUV graph: (0, 80) and (550, 10)

The rate of change for the graph of the fuel consumption of an SUV is:

$$\begin{aligned}\frac{\text{Change in } V}{\text{Change in } d} &= \frac{10 \text{ L} - 80 \text{ L}}{550 \text{ km} - 0 \text{ km}} \\ &= \frac{-70 \text{ L}}{550 \text{ km}} \\ &= -0.127 \text{ L/km}\end{aligned}$$

An SUV uses 0.127 L of fuel to drive 1 km.

The SUV uses more than twice as much fuel to drive 1 km. So, a driver of an SUV will pay more than twice as much for gas as a driver of a Smart car.

12. a) The dogsled finished the race when its distance from the finish line was 0 km.
From the graph, the dogsled finished the race after 2.5 h.

- b) The average speed is the rate of change of distance over time.

Choose two points: (0, 60) and (2.5, 0)

The rate of change is:

$$\begin{aligned}\frac{\text{Change in } d}{\text{Change in } t} &= \frac{0 \text{ km} - 60 \text{ km}}{2.5 \text{ h} - 0 \text{ h}} \\ &= \frac{-60 \text{ km}}{2.5 \text{ h}} \\ &= -24 \text{ km/h}\end{aligned}$$

The rate of change is negative, which means that the distance to the finish line is decreasing with time.

The average speed of the dogsled was 24 km/h.

- c) The length of the race is the distance of the dogsled from the finish line at 0 h.
From the graph, the race was 60 km long.

- d) When the dogsled had completed $\frac{2}{3}$ of the race, it had travelled: $\frac{2}{3}(60 \text{ km}) = 40 \text{ km}$

So, its distance from the finish line was:

$$60 \text{ km} - 40 \text{ km} = 20 \text{ km}$$

From part b, the dogsled travels 24 km/h.

So, to travel 40 km, it takes: $\frac{40}{24} \text{ h} = 1.\overline{6} \text{ h}$, or $1\frac{2}{3} \text{ h}$

The dogsled completed $\frac{2}{3}$ of the race in $1\frac{2}{3} \text{ h}$, or 1 h 40 min.

- 13. a)** From Graph A, it takes 50 min to fill the empty tank to 100 m^3 .
From Graph B, it takes 25 min to empty the tank that starts with 100 m^3 of fuel.
So, it takes longer to fill the empty tank.
- b)** When the tank in Graph B was half empty, it contained 50 m^3 of fuel and 12.5 min had elapsed.
From Graph A, after 12.5 min, the tank that was being filled would contain 25 m^3 of fuel.
- 14. a)** The points on the graph are joined because the scale on the axes is so small that it would be impossible to distinguish every point on the graph.
- b) i)** The vertical intercept of the graph is 200, so the fixed cost is \$200.
Choose two points on the graph: (0, 200) and (40, 800)
The rate of change is:

$$\frac{\text{Change in } C}{\text{Change in } n} = \frac{\$800 - \$200}{40 \text{ sweatshirts} - 0 \text{ sweatshirts}}$$

$$= \$15/\text{sweatshirt}$$
 An equation that represents this situation is: $C = 200 + 15n$
 Substitute: $C = 700$
 $700 = 200 + 15n$ Solve for n .
 $500 = 15n$
 $\frac{500}{15} = n$
 $n = 33.\overline{3}$
 So, approximately 33 sweatshirts can be bought for \$700.
- ii)** From part a, the rate of change is \$15/sweatshirt.
So, one more sweatshirt would increase the cost by \$15.

15. Since each function is linear, its graph is a straight line.

a) $f(x) = 5 - 2.5x$

Determine the y-intercept:

When $x = 0$,

$$f(0) = 5 - 2.5(0)$$

$$f(0) = 5$$

Determine the x-intercept:

When $f(x) = 0$,

$$0 = 5 - 2.5x$$

$$2.5x = 5$$

$$x = \frac{5}{2.5}$$

$$x = 2$$

Determine the coordinates of a third point on the graph.

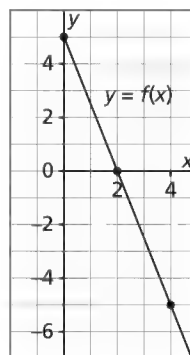
When $x = 4$,

$$f(x) = 5 - 2.5x$$

$$f(4) = 5 - 2.5(4)$$

$$f(4) = -5$$

Plot the points $(0, 5)$, $(2, 0)$, and $(4, -5)$, then draw a line through them.



b) $g(t) = 85t$

Determine the vertical intercept:

When $t = 0$,

$$g(0) = 85(0)$$

$$g(0) = 0$$

Determine the coordinates of a second point on the graph:

When $t = 2$,

$$g(2) = 85(2)$$

$$g(2) = 190$$

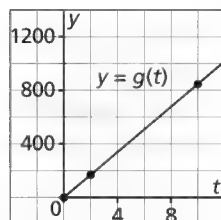
Determine the coordinates of a third point on the graph.

When $t = 10$,

$$g(10) = 85(10)$$

$$g(10) = 850$$

Plot the points $(0, 0)$, $(2, 190)$, and $(10, 850)$, then draw a line through them.



c) $h(n) = 750 + 55n$

Determine the vertical intercept:

When $n = 0$,

$$h(0) = 750 + 55(0)$$

$$h(0) = 750$$

Determine the horizontal intercept:

When $h(n) = 0$,

$$0 = 750 + 55n$$

$$-750 = 55n$$

$$-\frac{750}{55} = n$$

$$n = -\frac{150}{11}, \text{ or } -13.\overline{63}$$

Since the horizontal intercept is outside the domain, determine the coordinates of two other points on the graph.

When $n = 2$,

$$h(2) = 750 + 55(2)$$

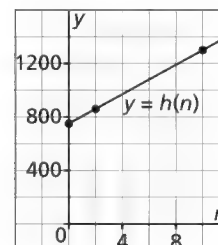
$$h(2) = 860$$

When $n = 10$,

$$h(10) = 750 + 55(10)$$

$$h(10) = 1300$$

Plot the points $(0, 750)$, $(2, 860)$, and $(10, 1300)$, then draw a line through them.



d) $V(d) = 55 - 0.08d$

Determine the vertical intercept:

When $d = 0$,

$$V(0) = 55 - 0.08(0)$$

$$V(0) = 55$$

Determine the d -intercept:

When $V(d) = 0$,

$$0 = 55 - 0.08d$$

$$0.08d = 55$$

$$d = \frac{55}{0.08}$$

$$d = 687.5$$

The horizontal intercept is not easy to graph, so determine the coordinates of two other points on the graph.

When $d = 300$,

$$V(300) = 55 - 0.08(300)$$

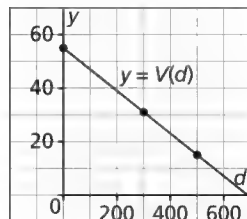
$$V(300) = 31$$

When $d = 500$,

$$V(500) = 55 - 0.08(500)$$

$$V(500) = 15$$

Plot the points $(0, 55)$, $(300, 31)$ and $(500, 15)$, then draw a line through them.



16. a) The profit on each bar sold is the rate of change of the graph.
Choose two points on the graph: $(0, -40)$ and $(300, 200)$

$$\begin{aligned}\frac{\text{Change in } P}{\text{Change in } n} &= \frac{\$200 - (-\$40)}{300 \text{ bars} - 0 \text{ bars}} \\ &= \frac{\$240}{300 \text{ bars}} \\ &= \$0.80/\text{bar}\end{aligned}$$

So, the profit is \$0.80 per bar sold.

- b) On the vertical axis, the point of intersection has coordinates $(0, -40)$. The vertical intercept is -40 . This point of intersection represents the loss when 0 bars are sold: \$40; that is, the cost price of the power bars.

On the horizontal axis, the point of intersection has coordinates $(50, 0)$. The horizontal intercept is 50. This point of intersection represents the number of bars that must be sold to reach the break-even point, when no profit is made and there is no loss: 50 bars

- c) The domain of the function is:
All whole numbers up to 300
The range of the function is:
All multiples of 0.80 between and including -40 to 200.
I wouldn't want to list all the values in the range because there are 301 of them.

17. a) There are no intercepts on the graph because it is unlikely that people less than 10 years old or more than 90 years old will be doing a stress test.

- b) Choose two points on the graph: (10, 175) and (90, 110)

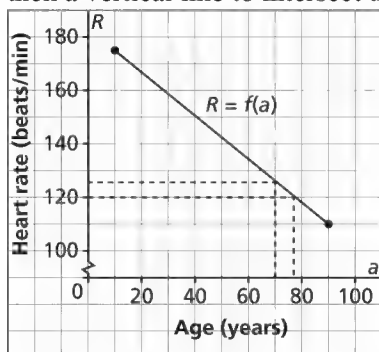
The rate of change is:

$$\begin{aligned}\frac{\text{Change in } R}{\text{Change in } a} &= \frac{110 \text{ beats/min} - 175 \text{ beats/min}}{90 \text{ years} - 10 \text{ years}} \\ &= \frac{-65 \text{ beats/min}}{80 \text{ years}} \\ &= -0.8125(\text{beats/min})/\text{year}\end{aligned}$$

The rate of change is negative, so the recommended heart rate decreases with age.

For every additional year of age, the recommended heart rate decreases by approximately 1 beat/min.

- c) Use the graph. From 120 on the vertical axis, draw a horizontal line to intersect the graph, then a vertical line to intersect the horizontal axis at approximately 77.



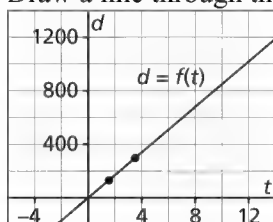
The recommended maximum heart rate is 120 beats/min for someone aged 77.

- d) Use the graph. From 70 on the horizontal axis, draw a vertical line to intersect the graph, then a horizontal line to intersect the vertical axis at approximately 125. A person aged 70 has a recommended heart rate of approximately 125 beats/min.

C

- 18. a) i)** The line intersects the x -axis at $(5, 0)$, so the x -intercept is 5.
The line intersects the y -axis at $(0, 5)$, so the y -intercept is 5.
- ii)** The coordinates of each point on the line have the sum of 5.
Since the intercepts are the same, the relation is described by this equation:
 $x + y = 5$
- b) i)** The line intersects the x -axis at $(5, 0)$, so the x -intercept is 5.
The line intersects the y -axis at $(0, -5)$, so the y -intercept is -5 .
- ii)** The coordinates of each point on the line have a difference of 5, with $x > y$.
Since the intercepts are opposites, the relation is described by this equation:
 $x - y = 5$

- 19. a)** Plot the points $(1.5, 127.5)$ and $(3.5, 297.5)$.
Draw a line through the points.



- b)** The line passes through these points: $(1.5, 127.5)$ and $(3.5, 297.5)$
The rate of change is:
$$\frac{\text{Change in } d}{\text{Change in } t} = \frac{297.5 - 127.5}{3.5 - 1.5} = 85$$

For each increase of $t = 1$, d increases by 85.

For each increase of $t = 1.5$, d increases by $85\left(\frac{3}{2}\right) = 127.5$

$$f(3.5) = 297.5$$

$$\text{So, } f(5) = 297.5 + 127.5 = 425$$

- c)** Let the coordinates of the point for which $f(t) = 212.5$ be $(t, 212.5)$.
Use this point and the point $(1.5, 127.5)$ to write an expression for the rate of change.
Equate this expression to 85.

$$\frac{212.5 - 127.5}{t - 1.5} = 85$$

$$\frac{85}{t - 1.5} = 85$$

$$\frac{85}{85} = t - 1.5$$

$$1 = t - 1.5$$

$$t = 2.5$$

Use the equation: $f(t) = 85t$

When $f(t) = 212.5$,

$$f(t) = 85t$$

$$212.5 = 85t$$

$$\frac{212.5}{85} = t$$

$$t = 2.5$$

- d)** For each decrease in $t = 1.5$, d decreases by 127.5. Since $f(1.5) = 127.5$, then $f(0) = 0$. The graph passes through the origin. So, a possible context is a car's distance from home as it travels away at a constant speed of 85 km/h.

20. a) The vertical intercept indicates the person's distance from Duke Point when starting the journey at Parksville.
The horizontal intercept indicates the person's distance from Parksville after completing the journey and reaching Duke Point.

The distance between the two locations doesn't change, so the intercepts have the same value.

- b) Use the points: (0, 50) and (50, 0)

The rate of change is:

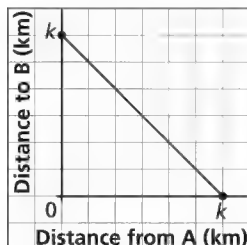
$$\frac{\text{Change in } d}{\text{Change in } p} = \frac{0 \text{ km} - 50 \text{ km}}{50 \text{ km} - 0 \text{ km}} \\ = -1$$

The rate of change has no units because it is the quotient of two measurements in kilometres.

For every 1 km the ferry moves away from Parksville, it moves 1 km closer to Duke Point.

- c) Interchanging the dependent and independent variables would interchange the labels on the axes, but the line on the graph would stay the same.
- d) The graph will go down to the right.
Both the horizontal and vertical intercepts will be k .
Both the domain and range will be all real numbers from 0 to k .
The rate of change will be -1 .

Distance from A to B



Review

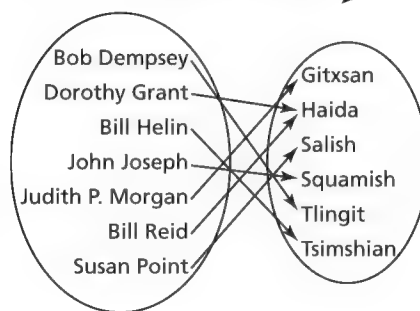
(pages 326–328)

5.1

1. a) The table shows a relation with the association “has this cultural heritage” from a set of artists to a set of First Nations heritages.

- b) i) The relation as a set of ordered pairs:
{(Bob Dempsey, Tlingit), (Dorothy Grant, Haida), (Bill Helin, Tsimshian), (John Joseph, Squamish), (Judith P. Morgan, Gitxsan), (Bill Reid, Haida), (Susan Point, Salish)}

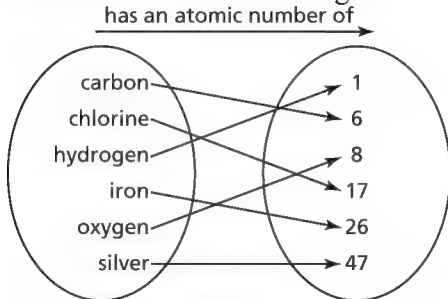
- ii) The relation as an arrow diagram:
has this cultural heritage



2. a) The relation in a table:

Element	Atomic Number
carbon	6
chlorine	17
hydrogen	1
iron	26
oxygen	8
silver	47

The relation as an arrow diagram:



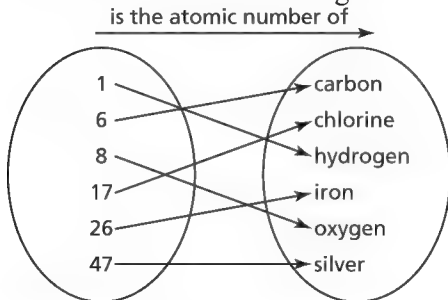
The relation as a set of ordered pairs:

$\{(\text{carbon}, 6), (\text{chlorine}, 17), (\text{hydrogen}, 1), (\text{iron}, 26), (\text{oxygen}, 8), (\text{silver}, 47)\}$

b) The relation in a table:

Atomic Number	Element
1	hydrogen
6	carbon
8	oxygen
17	chlorine
26	iron
47	silver

The relation as an arrow diagram:



The relation as a set of ordered pairs:

$\{(1, \text{hydrogen}), (6, \text{carbon}), (8, \text{oxygen}), (17, \text{chlorine}), (26, \text{iron}), (47, \text{silver})\}$

5.2

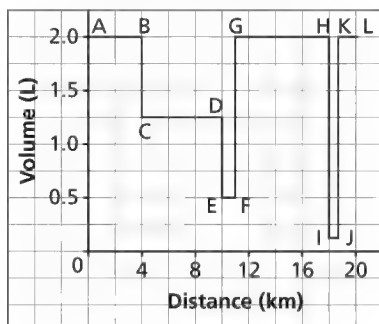
3. a) $\{(4, 3), (4, 2), (4, 1), (4, 0)\}$
This relation is not a function because the number 4 is a first element in 4 different ordered pairs.
- b) $\{(2, 4), (-2, 4), (3, 9), (-3, 9)\}$
This relation is a function because the ordered pairs have different first elements.
- c) $\{(2, 8), (3, 12), (4, 16), (5, 20)\}$
This relation is a function because the ordered pairs have different first elements.
- d) $\{(5, 5), (5, -5), (-5, 5), (-5, -5)\}$
This relation is not a function because each of the numbers 5 and -5 appears as the first element in 2 different ordered pairs.
4. a) $f(x) = -4x + 9$
- b) $C(n) = 12n + 75$
- c) $D(t) = -20t + 150$
- d) $P(s) = 4s$
5. a) As an equation in 2 variables, $P(n) = 5n - 300$ is:
 $P = 5n - 300$
- b) The profit depends on the number of students who attend. So, n is the independent variable and P is the dependent variable.
- c) To determine $P(150)$, use:
 $P(n) = 5n - 300$ Substitute: $n = 150$
 $P(150) = 5(150) - 300$
 $P(150) = 450$
 This means that if 150 students attend the dance, the profit will be \$450.
- d) To determine the value of n when $P(n) = 700$, use:
 $P(n) = 5n - 300$ Substitute: $P(n) = 700$
 $700 = 5n - 300$
 $1000 = 5n$
 $n = 200$
 This means that the profit is \$700 when 200 students attend the dance.

5.3

6. a) The distance to Laura's house from school is a positive number, so the vertical intercept of the graph should be a positive number. When Laura cycles home, the graph should show a steep decrease because her distance from home is decreasing. When Laura reaches home, her distance from home is 0. When she walks back to school, the graph should show a moderate increase because her walking speed is slower than her cycling speed and her distance from home is increasing. So, Graph A best matches the situation.

- b) Graph D could represent Laura's journey to school to pick up her bike. She walks to school, then picks up her bicycle and rides home.

7. a) Label the segments of the graph.
Water in Liam's Flask

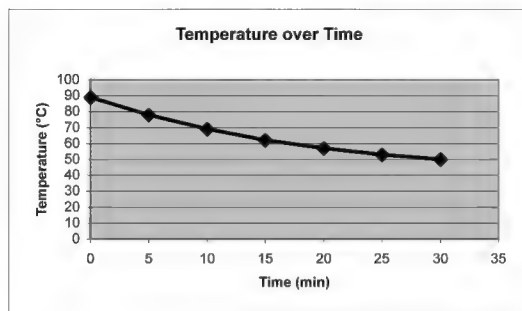


Segment	Graph	Journey
AB	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 4 km with 2.0 L of water in his flask.
BC	The graph is vertical below point B, so the volume of water in the flask decreases.	Liam drinks or spills 0.75 L of water.
CD	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 6 km with 1.25 L of water in his flask.
DE	The graph is vertical below point D, so the volume of water in the flask decreases.	Liam drinks or spills 0.75 L of water from his flask.
EF	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 1 km with 0.5 L of water in his flask.
FG	The graph is vertical above point F, so the volume of water in the flask increases.	Liam fills his flask so that it contains 2.0 L of water.
GH	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks 7 km with 2.0 L of water in his flask.
HI	The graph is vertical below point H, so the volume of water in the flask decreases.	Liam drinks or spills approximately 1.875 L of water from his flask.
IJ	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks approximately 0.7 km with 0.125 L of water in his flask.
JK	The graph is vertical above point J, so the volume of water in the flask increases.	Liam fills his flask so that it contains 2.0 L of water.
KL	The graph is horizontal, so as distance increases the volume of water in the flask stays the same.	Liam walks approximately 1.3 km with 2.0 L of water remaining in his flask.

- b) Liam filled his flask two times as shown on the graph by line segments FG and JK.
- c) The vertical intercept of the graph is 2.0, so Liam started the hike with 2.0 L of water in his flask.
- d) The independent variable is the distance Liam hikes. The dependent variable is the volume of water in his flask.

5.4

8. a) Graph the data using a computer spreadsheet program. Label the axes.



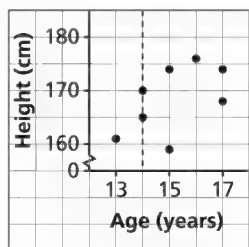
I joined the points because all times between 0 min and 30 min are permissible and all temperatures between 50°C and 89°C are permissible.

- b) The graph represents a function because a vertical line drawn on the graph passes through at most one point.

5.5

9. a) This graph is not a function. A vertical line can be drawn through the points (14, 165) and (14, 170) on the graph.

Heights and Ages of 8 Students



The domain is the set of ages: {13, 14, 15, 16, 17}

The domain is the set of heights: approximately {159, 161, 165, 168, 170, 174, 176}

- b) This graph is a function. Any vertical line drawn on the graph would pass through 0 points or 1 point.

The domain is the set of times:

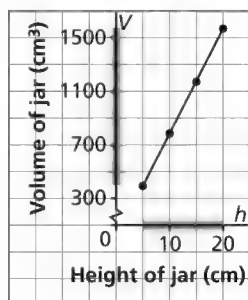
{08:00, 10:00, 12:00, 14:00, 16:00, 18:00}

The range is the set of the numbers of bicycles:

{2, 5, 10, 20, 25}

10. a) i) Graph A represents the volume of a jar, V cubic centimetres, as a function of its height, h centimetres.
- ii) Graph B represents the number of marbles in a jar, n , as a function of the jar's height, h centimetres.
- b) i) For Graph A, the independent variable is the height of the jar, h . The dependent variable is the volume of the jar, V .
- ii) For Graph B, the independent variable is the height of the jar, h . The dependent variable is the number of marbles in the jar, n .
- c) i) Visualize the shadow of the graph on the axes.

Graph A



The domain is the set of heights:

$$5 \leq h \leq 20$$

The range is the set of volumes:

$$\text{approximately } 400 \leq V \leq 1575$$

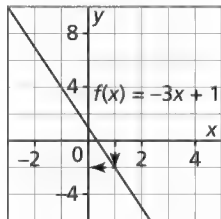
- ii) For Graph B: The domain is the set of heights.
 $\{5, 10, 15, 20\}$
 The range is the set of the numbers of marbles:
 $\{14, 28, 42, 56\}$
- d) The points are joined in Graph A because it is possible for a jar to have any height between 5 cm and 20 cm and any volume between 400 cm^3 and 1575 cm^3 .
 The points are not joined in Graph B because only whole numbers of marbles are permissible.

11. The domain value is a value of x . The range value is a value of $f(x)$.

a) To determine the value of $f(x)$ when $x = 1$:

Begin at $x = 1$ on the x -axis.

Draw a vertical line to the graph, then a horizontal line to the y -axis.



The line intersects the y -axis at -2 .

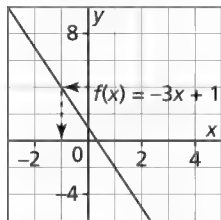
So, $f(1) = -2$

When the domain value is 1, the range value is -2 .

b) To determine the value of x when $f(x) = 4$:

Begin at $f(x) = 4$ on the y -axis.

Draw a horizontal line to the graph, then a vertical line to the x -axis.



The line intersects the x -axis at -1 .

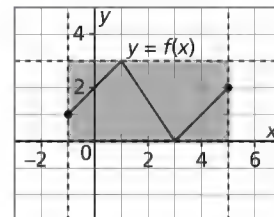
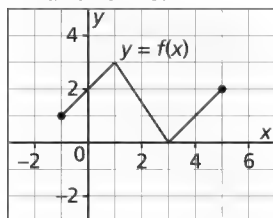
So, $f(-1) = 4$

When the range value is 4, the domain value is -1 .

12. a) On a grid, draw faint vertical lines through $x = -1$ and $x = 5$; and faint horizontal lines through $y = 0$ and $y = 3$.

Draw line segments inside the region enclosed by the faint lines so that there is exactly one point on the graph for every x -value between -1 and 5 , and there is at least one point on the graph for every y -value between 0 and 3 .

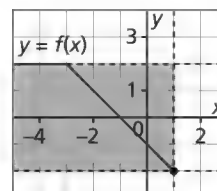
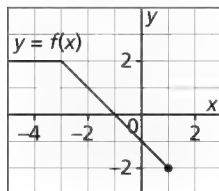
A function is:



- b) On a grid, draw a faint vertical line through $x = 1$; and faint horizontal lines through $y = -2$ and $y = 2$.

Draw line segments inside the region between the horizontal lines and to the left of the vertical line so that there is exactly one point on the graph for every x -value less than or equal to 1 , and there is at least one point on the graph for every y -value between -2 and 2 .

A function is:



5.6

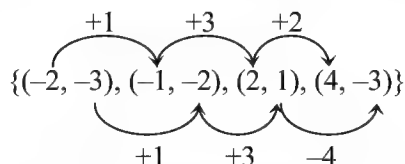
13. a) $\{(1, 5), (5, 5), (9, 5), (13, 5)\}$

Since every second element in the ordered pairs is 5, the points lie on a horizontal line. So, the set of ordered pairs represents a linear relation.

- b) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$

Since every first element in the ordered pairs is 1, the points lie on a vertical line. So, the set of ordered pairs represents a linear relation.

- c) The first elements are in numerical order. So, calculate the change in each variable.

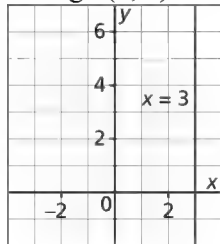


There is no constant change in the first elements or the second elements. For the last two

pairs, as the first element increases by 2, the second element decreases by 4. So, the set of ordered pairs does not represent a linear relation.

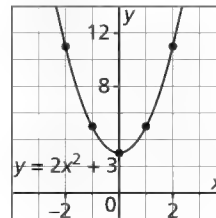
14. a) i) $x = 3$

This is a vertical line that passes through $(3, 0)$.



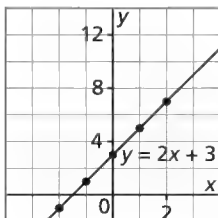
ii) $y = 2x^2 + 3$

x	y
-2	11
-1	5
0	3
1	5
2	11



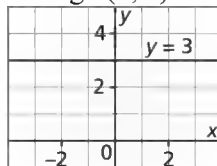
iii) $y = 2x + 3$

x	y
-2	-1
-1	1
0	3
1	5
2	7



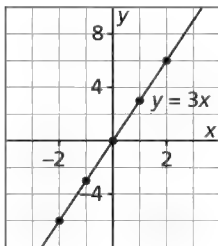
iv) $y = 3$

This is a horizontal line that passes through $(0, 3)$.



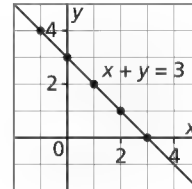
v) $y = 3x$

x	y
-2	-6
-1	-3
0	0
1	3
2	6



vi) $x + y = 3$

x	y
-1	4
0	3
1	2
2	1
3	0



- b) The equations in parts i, iii, iv, v, and vi represent linear relations because their graphs are straight lines.

15. a) The equation represents a linear relation because, when g changes by 1, N changes by $\frac{1}{15}$.
- b) The rate of change is $\frac{1}{15}$. For every 1 g of carbohydrates that Isabelle consumes, she gives herself $\frac{1}{15}$ of a unit of insulin.

5.7

16. a) Distance is shown on the vertical axis of the graph.
Jadan travelled 6000 m from Whitehorse to the Grey Mountain Road lookout, so at the start of her bicycle trip she was 6000 m, or 6 km, from the lookout.
- b) The domain is all possible values of the number of revolutions: $0 \leq n \leq 2800$
The range is all possible values of the distance: $0 \leq d \leq 6000$
- c) Choose two points on the graph: (0, 0) and (2800, 6000)
The rate of change is:

$$\frac{\text{Change in } d}{\text{Change in } n} = \frac{6000 \text{ m} - 0 \text{ m}}{2800 \text{ revolutions} - 0 \text{ revolutions}}$$

$$= 2.1428... \text{ m/revolution}$$
 In one revolution of the wheel, the bicycle covers a distance of approximately 2 m.
- d) The distance the bicycle travels in one revolution of its wheel is the circumference of the wheel.
An equation that relates the circumference, C , of a circle to its diameter, d , is:

$$C = \pi d$$
 Substitute: $C \doteq 2.1428$

$$2.1428 \doteq \pi d$$

$$\frac{2.1428}{\pi} \doteq d$$

$$d \doteq 0.6820$$
 So, the bicycle wheel has a diameter of approximately 0.68 m, or 68 cm.

17. Determine the vertical intercept and the rate of change for each graph.

- a) When $t = 0$, $T = 3$, so the vertical intercept is 3°C .

Choose 2 points on the graph: $(0, 3)$ and $(1, 0)$

Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - 3^{\circ}\text{C}}{1 \text{ h} - 0 \text{ h}} \\ &= -3^{\circ}\text{C/h}\end{aligned}$$

So, this graph matches part ii.

- b) When $t = 0$, $T = -3$, so the vertical intercept is -3°C .

Choose 2 points on the graph: $(0, -3)$ and $(1, 0)$

Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - (-3^{\circ}\text{C})}{1 \text{ h} - 0 \text{ h}} \\ &= 3^{\circ}\text{C/h}\end{aligned}$$

So, this graph matches part iii.

- c) When $t = 0$, $T = -3$, so the vertical intercept is -3°C .

Choose 2 points on the graph: $(0, -3)$ and $(9, 0)$

Calculate the rate of change:

$$\begin{aligned}\frac{\text{Change in } T}{\text{Change in } t} &= \frac{0^{\circ}\text{C} - (-3^{\circ}\text{C})}{9 \text{ h} - 0 \text{ h}} \\ &= \frac{3^{\circ}\text{C}}{9 \text{ h}} \\ &= \frac{1}{3}^{\circ}\text{C/h}\end{aligned}$$

So, this graph matches part i.

18. a) On the graph, the company makes a profit when the profit, P , is a positive number.
When $P = 0$, $n = 200$; this is the break-even point, when the company does not suffer a loss or make a profit.
The company begins to make a profit when 201 baseball caps have been sold.

- b) The profit on the sale of each baseball cap is the rate of change.

Choose two points on the graph: (200, 0) and (300, 400)

$$\begin{aligned}\frac{\text{Change in } P}{\text{Change in } n} &= \frac{\$400 - \$0}{300 \text{ caps} - 200 \text{ caps}} \\ &= \frac{\$400}{100 \text{ caps}} \\ &= \$4/\text{cap}\end{aligned}$$

So, the profit on each baseball cap is \$4.

- c) The vertical intercept is -800 .

The rate of change is 4.

So, an equation to represent the situation is: $P = 4n - 800$

- i) $P = 4n - 800$ Substitute: $P = 600$

$$600 = 4n - 800$$

$$1400 = 4n$$

$$n = 350$$

350 caps must be sold to make a profit of \$600.

- ii) $P = 4n - 800$ Substitute: $P = 1200$

$$1200 = 4n - 800$$

$$2000 = 4n$$

$$n = 500$$

500 caps must be sold to make a profit of \$1200.

- d) The profit doesn't double because it depends on the sale of caps and the initial cost of \$800 to buy or make the caps. So, doubling the number of caps does not double the profit.

Practice Test

(page 329)

1. To determine $f(-3)$, use:

$$f(x) = 3 - 6x$$

Substitute: $x = -3$

$$f(-3) = 3 - 6(-3)$$

$$f(-3) = 21$$

So, answer B is correct.

2. Equations A and D represent horizontal lines.

Equation B represents a line with a rate of change of 5 that passes through the origin.

Equation C has x raised to the exponent 2, so it is not a linear function.

So, answer C is correct.

3. a) i) $\{(2, 5), (-3, 6), (1, 5), (-1, 4), (0, 2)\}$

The relation is a function because each ordered pair has a different first element.

- ii) The domain is the set of first elements:

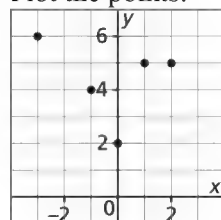
$$\{-3, -1, 0, 1, 2\}$$

The range is the set of second elements:

$$\{2, 4, 5, 6\}$$

Represent the function as a graph.

Plot the points:



The function is not linear because the points on the graph do not lie on a straight line.

- b) i) The relation is a function because no number appears more than once in the first column of the table of values.

- ii) The domain is the set of numbers in the first column of the table:

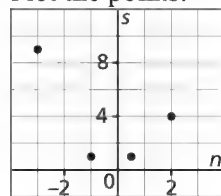
$$\{-3, -1, 1, 2, \dots\}$$

The range is the set of numbers in the second column of the table:

$$\{1, 4, 9, \dots\}$$

Represent the function as a graph.

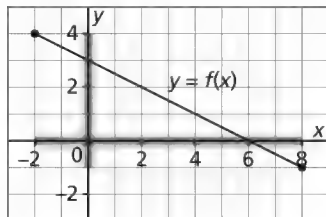
Plot the points:



The function is not linear because the points do not lie on a straight line.

- c) i) The relation is a function because any vertical line passes through at most 1 point on the graph.

- ii) Visualize the shadow of the graph on the x - and y -axes.



The domain is the set of x -values of the graph:

$$-2 \leq x \leq 8$$

The range is the set of y -values of the graph:

$$-1 \leq y \leq 4$$

Represent the relation as a table of values.

x	-2	0	2	4	6	8
y	4	3	2	1	0	-1

Choose two points on the graph: $(0, 3)$ and $(-1, 8)$

The rate of change is $-\frac{1}{2}$.

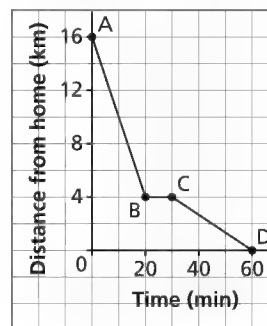
The function is linear because the graph is a non-vertical line.

- iii) The independent variable is x . The dependent variable is y .

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Chapter 5
Relations and Functions

Jamie's Journey Home



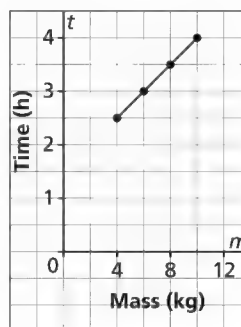
4. Situation: Jamie's school is 16 km from her house. Jamie rides her friend's bike from school to her friend's house, which is 4 km from her own house. She arrives at her friend's house 20 min after she left school. She talks to her friend for 10 min, then jogs the remaining 4 km home in 30 min.

Justification: Line segment AB goes down to the right, so the person's distance from home is decreasing with time. Segment BC is horizontal, so the person's distance from home remains constant. Segment CD goes down to the right, so the person's distance from home decreases with time.

5. a) The relation is a function because no number is repeated in the first row.
- b) The time, t , needed to cook the turkey depends on its mass, m . So, time is the dependent variable and mass is the independent variable.
- c) Graph the data.

Connect the points because it is possible for a turkey to have any mass between 4 kg and 10 kg, in which case, its cooking time is likely to be between 2.5 h and 4.0 h.

Time Needed to Cook a Turkey



- d) The domain is all possible values of the mass: $4 \leq m \leq 10$
 The range is all possible values of the cooking time: $2.5 \leq t \leq 4.0$
 The graph could be extended for turkeys with greater or lesser mass.
 The domain and range are restricted to positive real numbers because it is impossible for a turkey to have a negative mass or to cook a turkey for a negative length of time.
- e) Choose 2 points on the graph: (4, 2.5) and (6, 3.0)
 The rate of change is:

$$\frac{\text{Change in } t}{\text{Change in } m} = \frac{3.0 \text{ h} - 2.5 \text{ h}}{6 \text{ kg} - 4 \text{ kg}}$$

$$= \frac{0.5 \text{ h}}{2 \text{ kg}}$$

$$= 0.25 \text{ h/kg}$$
 For every additional 1 kg, the time needed to cook the turkey increases by 0.25 h.
- f) Use the rate of change:
 7 kg is 1 kg more than 6 kg, and a turkey with mass 6 kg should be cooked for 3.0 h.

So, a turkey with mass 7 kg should be cooked for:
 $3.0 \text{ h} + 0.25 \text{ h} = 3.25 \text{ h}$

Lesson 6.1 Slope of a Line

Exercises (pages 339–343)

A

4. Count squares to determine the rise and run.

- a) The rise is 2.
The run is 11.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{2}{11}$$

- b) The rise is 4.
The run is 14.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{4}{14} \quad \text{Simplify.}$$

$$\text{Slope} = \frac{2}{7}$$

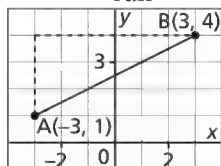
5. a) The segment goes down to the right, so its slope is negative.

- b) The segment goes up to the right, so its slope is positive.

- c) The segment is vertical, so its slope is not defined.

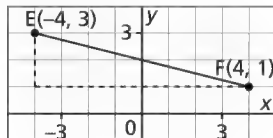
- d) The segment is horizontal, so its slope is 0.

6. a) $\text{Slope} = \frac{\text{rise}}{\text{run}}$



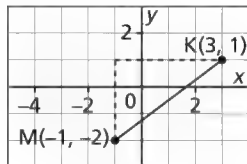
$$\text{Slope} = \frac{3}{6}, \text{ or } \frac{1}{2}$$

- b) $\text{Slope} = \frac{\text{rise}}{\text{run}}$



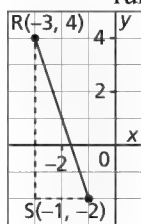
$$\text{Slope} = \frac{-2}{8}, \text{ or } -\frac{1}{4}$$

c) $\text{Slope} = \frac{\text{rise}}{\text{run}}$



$$\text{Slope} = \frac{3}{4}$$

d) $\text{Slope} = \frac{\text{rise}}{\text{run}}$



$$\text{Slope} = \frac{-6}{2}, \text{ or } -3$$

7. a) Since x increases by 1, the run is 1.
Since y increases by 3, the rise is 3.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{3}{1}, \text{ or } 3$$

- b) Since x increases by 2, the run is 2.
Since y decreases by 7, the rise is -7 .

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{-7}{2}, \text{ or } -\frac{7}{2}$$

- c) Since x decreases by 4, the run is -4 .
Since y decreases by 2, the rise is -2 .

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{-2}{-4}, \text{ or } \frac{1}{2}$$

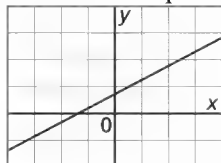
- d) Since x decreases by 2, the run is -2 .
Since y increases by 1, the rise is 1.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

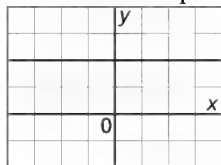
$$\text{Slope} = \frac{1}{-2}, \text{ or } -\frac{1}{2}$$

8. Sketches may vary. The lines may be in different positions on the grid but they should have the same orientations as those shown.

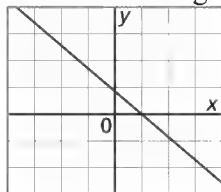
- a) A line with a positive slope goes up to the right.



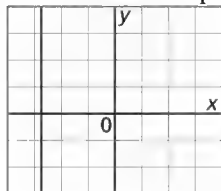
- b) A line with slope 0 is horizontal.



- c) A line with a negative slope goes down to the right.



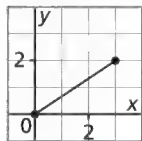
- d) A line with a slope that is not defined is vertical.



9. Diagrams may vary. The line segments may have different lengths and may have different endpoints at the origin but they should have the same orientations as those shown.

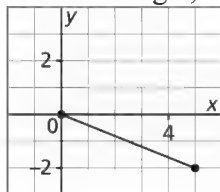
- a) Mark a point at the origin. For a slope of $\frac{2}{3}$, the rise is 2 and the run is 3.

From the origin, move 2 squares up and 3 squares right. Mark a point. Join the points.



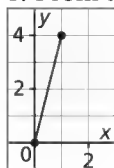
- b) Mark a point at the origin. Write the slope $-\frac{2}{5}$ as $\frac{-2}{5}$; then the rise is -2 and the run is 5 .

From the origin, move 2 squares down and 5 squares right. Mark a point. Join the points.



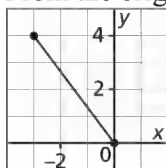
- c) Mark a point at the origin. For a slope of 4 , write 4 as $\frac{4}{1}$; then the rise is 4 and the run is 1 .

1. From the origin, move 4 squares up and 1 square right. Mark a point. Join the points.



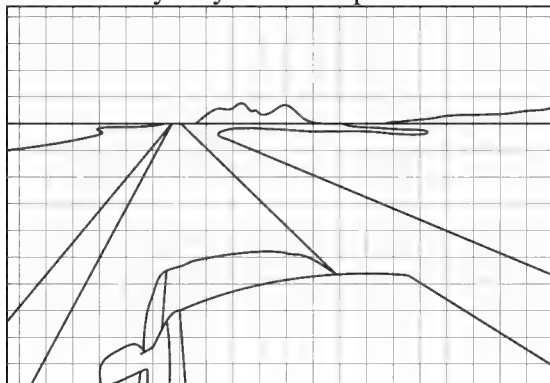
- d) Mark a point at the origin. Write the slope $-\frac{4}{3}$ as $\frac{4}{-3}$; then the rise is 4 and the run is -3 .

From the origin, move 4 squares up and 3 squares left. Mark a point. Join the points.



10. a) Answers may vary; for example, I can use the slopes of the lines to draw them at the correct angle when I copy them. The horizon, and the edges of the crops, road, and the vehicle are straight lines. So, I determine the rise and run of these lines, then draw these lines on a grid to represent the edges of each object in the copy of the picture.

- b) Sketches may vary. For example:



B

11. Chosen points may vary.

- a) I chose the points with coordinates (4, 1) and (−4, −3).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-3 - 1}{-4 - 4}$$

$$\text{Slope} = \frac{-4}{-8}, \text{ or } \frac{1}{2}$$

- b) I chose the points with coordinates (2, 0) and (−2, −2).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

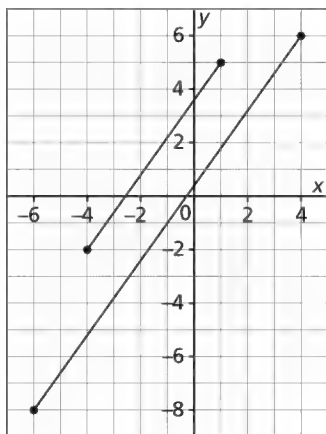
$$\text{Slope} = \frac{-2 - 0}{-2 - 2}$$

$$\text{Slope} = \frac{-2}{-4}, \text{ or } \frac{1}{2}$$

- c) The slopes in parts a and b are equal because the slopes of all segments of the same line are equal.

12. Diagrams may vary. The line segments may be different lengths and may have different endpoints but they should have the same orientation as those shown.

a)



- b) The line segments have the same slope; that is, both of them go up to the right with a rise of 7 and a run of 5. The line segments have different lengths and they are in different places on the grid.

13. a) $\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

i) $\text{Slope of PQ} = \frac{6 - 2}{3 - 1}$

$\text{Slope of PQ} = \frac{4}{2}, \text{ or } 2$

ii) $\text{Slope of ST} = \frac{5 - 1}{8 - 0}$

$\text{Slope of ST} = \frac{4}{8}, \text{ or } \frac{1}{2}$

iii) $\text{Slope of VR} = \frac{-8 - 4}{3 - (-1)}$

$\text{Slope of VR} = \frac{-12}{4}, \text{ or } -3$

iv) $\text{Slope of UW} = \frac{-5 - (-7)}{-6 - (-12)}$

$\text{Slope of UW} = \frac{2}{6}, \text{ or } \frac{1}{3}$

b) i) The slope of PQ is 2. It tells me that the line goes up to the right with a rise of 2 and a run of 1.

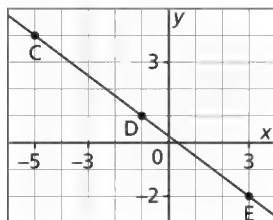
ii) The slope of ST is $\frac{1}{2}$. It tells me that the line goes up to the right with a rise of 1 and a run of 2. This line is less steep than the line in part a.

iii) The slope of VR is -3 . This tells me that the line goes down to the right with a rise of -3 and a run of 1. I can also think of this line as having a rise of 3 and a run of -1 .

iv) The slope of UW is $\frac{1}{3}$. This tells me that the line goes up to the right with a rise of 1 and a run of 3. It is less steep than both the lines PQ and ST.

14. Diagrams may vary.

a)



$$\text{b) Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{i) Slope of CD} = \frac{1 - 4}{-1 - (-5)}$$

$$\text{Slope of CD} = \frac{-3}{4}, \text{ or } -\frac{3}{4}$$

$$\text{ii) Slope of DE} = \frac{-2 - 1}{3 - (-1)}$$

$$\text{Slope of DE} = \frac{-3}{4}, \text{ or } -\frac{3}{4}$$

$$\text{iii) Slope of CE} = \frac{-2 - 4}{3 - (-5)}$$

$$\text{Slope of CE} = \frac{-6}{8}, \text{ or } -\frac{3}{4}$$

The slopes of the segments are equal because all segments on the same line have the same slope.

$$15. \text{ a) Slope} = \frac{\text{rise}}{\text{run}} \quad \text{Substitute the values for the rise and run.}$$

$$\text{Slope} = \frac{6}{90}$$

$$\text{Slope} = \frac{1}{15}$$

The treadmill has a slope of $\frac{1}{15}$.

$$\text{b) Slope} = \frac{\text{rise}}{\text{run}}$$

Substitute the values for the slope and the run. Let the rise be represented by r .

$$0.15 = \frac{r}{90} \quad \text{Solve for the rise. Multiply each side by 90.}$$

$$90(0.15) = \frac{r}{90}(90)$$

$$r = 90(0.15)$$

$$r = 13.5$$

At the maximum slope, the rise is 13.5 in.

16. a) $\text{Slope} = \frac{\text{rise}}{\text{run}}$ Substitute the values for the rise and run. Write 4 ft. as 48 in.

$$\text{Slope} = \frac{-1}{48}, \text{ or } -\frac{1}{48}$$

The trench has a slope of $-\frac{1}{48}$, or the trench has a slope of $\frac{1}{48}$ downward.

- b) The length of the trench is the run.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Substitute the values for the slope and the rise. Write $6\frac{1}{2}$ as 6.5. Let the run be represented by r .

$$\frac{1}{48} = \frac{6.5}{r} \quad \text{Solve for } r. \text{ Multiply each side by } 48r.$$

$$\frac{1}{48}(48r) = \frac{6.5}{r}(48r)$$

$$r = 312$$

A length of 312 in. is $\frac{312}{12}$ ft., or 26 ft.

The trench is 26 ft. measured horizontally.

- c) The length of the drop is the rise.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

Substitute the values for the slope and the run. Let the rise be represented by r .

$$\frac{1}{48} = \frac{r}{18}$$

Solve for r . Multiply each side by a common denominator of 48 and 18; that is, 144.

$$\frac{1}{48}(144) = \frac{r}{18}(144)$$

$$3 = 8r \quad \text{Divide each side by 8.}$$

$$r = \frac{3}{8}$$

A depth of $\frac{3}{8}$ ft. is $\frac{3}{8}(12 \text{ in.})$, or $4\frac{1}{2}$ in.

The trench drops $4\frac{1}{2}$ in. over a horizontal distance of 18 ft.

17. a) A line with a slope of -2 goes down to the right, so the line could be line ii or line iv.

From the graph, the slope of line ii is: $\frac{-1}{2}$, or $-\frac{1}{2}$.

From the graph, the slope of line iv is: $-\frac{2}{1}$, or -2

So, the line with a slope of -2 is line iv.

- b) A line with a slope of $\frac{1}{2}$ goes up to the right, so the line could be line i or line iii.

From the graph, the slope of line i is: $\frac{2}{1}$, or 2

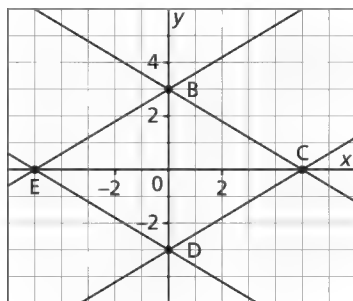
From the graph, the slope of line iii is: $\frac{1}{2}$

So, the line with a slope of $\frac{1}{2}$ is line iii.

- c) From part a, line ii has a slope of $-\frac{1}{2}$.

- d) From part b, line i has a slope of 2.

18. a)



From the graph:

i) Slope of BC = $-\frac{3}{5}$, or $-\frac{3}{5}$

ii) Slope of DC = $\frac{3}{5}$

iii) Slope of DE = $\frac{3}{-5}$, or $-\frac{3}{5}$

iv) Slope of BE = $\frac{3}{5}$

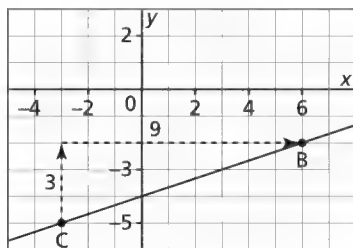
- b) The slopes of BC and ED are equal. The slopes of BE and CD are equal.
The two different slopes are opposites.

19. Slope = $\frac{\text{rise}}{\text{run}}$

- a) The slope of a horizontal line is 0 because its rise is 0, and the quotient of 0 and any number is zero.

- b) The slope of a vertical line is undefined because its run is 0, and the quotient of any number and 0 is undefined; that is, I cannot divide by 0.

20. a)



From the graph:

$$\text{Slope of BC} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope of BC} = \frac{3}{9}, \text{ or } \frac{1}{3}$$

The correct answer is $\frac{1}{3}$.

- b) The student who got the answer 3 may have written the expression for the slope as $\frac{\text{run}}{\text{rise}}$, and then written:

$$\text{Slope of BC} = \frac{9}{3}, \text{ or } 3$$

The student who got the answer -3 may have written the expression for the slope as $\frac{\text{run}}{\text{rise}}$, and then wrote the wrong sign for either the rise or the run:

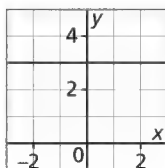
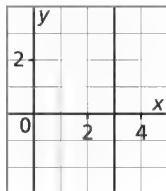
$$\text{Slope of BC} = \frac{-9}{3}, \text{ or } -3$$

The student who got the answer $-\frac{1}{3}$ may have subtracted one set of coordinates in the wrong order and written:

$$\begin{aligned} \text{Slope of BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - (-2)}{6 - (-3)} \\ &= \frac{-3}{9}, \text{ or } -\frac{1}{3} \end{aligned}$$

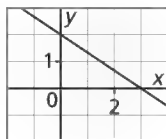
21. Positions of lines on the grid may vary.

a) i)



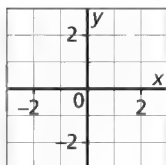
b) i) I could draw more lines than I can count, but each one would be vertical with a slope not defined, or horizontal with a slope equal to 0.

a) ii)



b) ii) I could draw more lines than I can count, but each one would be oblique with a slope equal to any real number except 0.

a) iii)



b) iii) A line that coincides with the x -axis or with the y -axis has more intercepts than I can count. The slope of the x -axis is 0 and the slope of the y -axis is not defined.

22. Slope = $\frac{\text{rise}}{\text{run}}$

The slope must be less than $\frac{1}{12}$.

So, $\frac{\text{rise}}{\text{run}} < \frac{1}{12}$

Let r represent the minimum horizontal distance.

Substitute: rise = 70 and run = r

$$\frac{70}{r} < \frac{1}{12} \quad \text{Multiply each side of the inequality by } 12r.$$

$$12r \left(\frac{70}{r} \right) < 12r \left(\frac{1}{12} \right)$$

$$12(70) < r$$

$$r > 840$$

The minimum horizontal distance must be 840 cm, or 8.4 m.

To justify the answer, determine the slope for a greater horizontal distance, such as 900 cm.

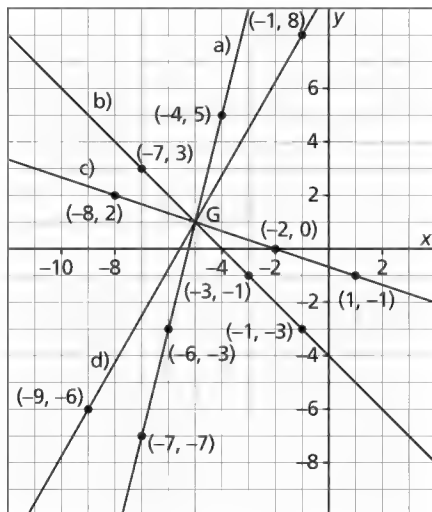
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{70}{900}, \text{ or } 0.0\bar{7}$$

$$\text{Compare this value with } \frac{1}{12} = 0.08\bar{3}$$

Since $0.0\bar{7} < 0.08\bar{3}$, a ramp with a run of 900 cm will be less steep than a ramp with a run of 840 cm.

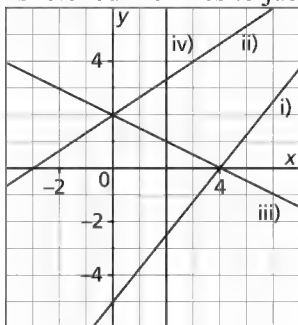
23.



- a) A line with slope 4 has a rise of 4 and a run of 1; so, from point G I moved 4 squares up and 1 square right to point $(-4, 5)$. A line with slope 4 also has a rise of -4 and a run of -1 : so from point G I moved 4 squares down and 1 square left to reach point $(-6, -3)$, then I moved 4 squares down and 1 square left again to reach point $(-7, -7)$.
- b) A line with slope -1 has a rise of 2 and a run of -2 ; so from point G I moved 2 squares up and 2 squares left to reach point $(-7, 3)$. A line with slope -1 also has a run of -2 and a rise of 2, so from point G I moved 2 squares down and 2 squares right to reach $(-3, -1)$, then I moved 2 squares down and 2 squares right again to reach $(-1, -3)$.
- c) A line with slope $-\frac{1}{3}$ has a rise of 1 and a run of -3 ; so from point G I moved 1 square up and 3 squares left to reach point $(-8, 2)$. A line with slope $-\frac{1}{3}$ also has a rise of -1 and a run of 3; so from point G I moved 1 square down and 3 squares right to reach point $(-2, 0)$, then I moved 1 square down and 3 squares right again to reach point $(1, -1)$.
- d) A line with slope $\frac{7}{4}$ has a rise of 7 and a run of 4; so from point G I moved 7 squares up and 4 squares right to reach point $(-1, 8)$. A line with slope $\frac{7}{4}$ also has a rise of -7 and a

run of -4 ; so from point G I moved 7 squares down and 4 squares left to reach point $(-9, -6)$. My grid wasn't big enough to plot another point, so I added -4 to -9 to get -13 ; and I added -7 to -6 to get -13 ; so my third point has coordinates $(-13, -13)$.

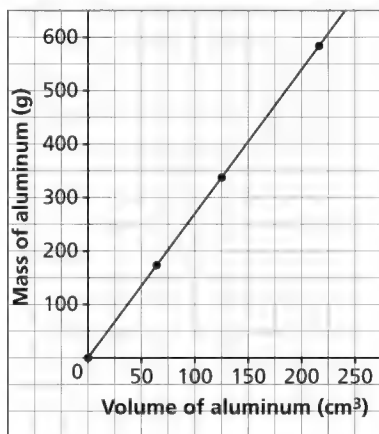
24. I sketched the lines to justify my answers.



- a), b) i) The slope of the line is positive because it goes up to the right.
 ii) The slope of the line is positive because it goes up to the right.
 iii) The slope of the line is negative because it goes down to the right.
 iv) The slope of the line is not defined because it is vertical.

25. a) Since the mass of aluminum depends on its volume, I plotted *Volume* horizontally and *Mass* vertically.

Mass and Volume of Aluminum



b) I checked that the points lie on the same straight line. I calculated the slope of the line through different pairs of points.

For the points $(64, 172.8)$ and $(125, 337.5)$:

$$\text{Slope} = \frac{337.5 - 172.8}{125 - 64}$$

$$\text{Slope} = \frac{164.7}{61}$$

$$\text{Slope} = 2.7$$

For the points $(125, 337.5)$ and $(216, 583.2)$:

$$\text{Slope} = \frac{583.2 - 337.5}{216 - 125}$$

$$\text{Slope} = \frac{245.7}{91}$$

$$\text{Slope} = 2.7$$

Since the slopes are equal, the points lie on the same line with slope 2.7.

- c) The slope is 2.7 g/cm^3 ; that is, it shows that the mass of 1 cm^3 of aluminum is 2.7 g; this is the density of aluminum.

- d) 1 cm^3 of aluminum has a mass of 2.7 g.

i) 50 cm^3 has a mass of $50(2.7 \text{ g}) = 135 \text{ g}$

ii) 275 cm^3 has a mass of $275(2.7 \text{ g}) = 742.5 \text{ g}$

- e) The volume of 1 g is $\frac{1}{2.7} \text{ cm}^3$.

i) The volume of 100 g is: $100\left(\frac{1}{2.7}\right) \text{ cm}^3 = 37.0370\dots \text{ cm}^3$

The volume of 100 g is approximately 37 cm^3 .

ii) The volume of 450 g is: $450\left(\frac{1}{2.7}\right) \text{ cm}^3 = 166.\bar{6} \text{ cm}^3$

The volume of 450 g is approximately 167 cm^3 .

26. a) The number of text messages is a whole number, so there are no permissible values between those represented by the plotted points. So, the points may not be joined.

- b) From the graph, the cost for 10 messages is \$1.50.

So, the cost for 1 message is: $\frac{\$1.50}{10} = \$0.15$, or 15¢

- c) The cost for 33 messages is: $33(\$0.15) = \$4.95$

- d) The number of messages that can be sent for \$7.20 is: $\frac{\$7.20}{\$0.15 / \text{message}} = 48 \text{ messages}$

- e) I assumed that all messages cost the same.

27. a) I subtracted the months: $5 - 2 = 3$

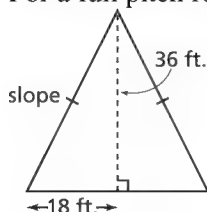
I subtracted the amounts saved: $\$280 - \$145 = \$135$

To calculate the amount saved each month, I divided: $\frac{\$135}{3} = \$45$

If I graphed the Account balance against the Months saved, the slope of the graph would be the amount saved per month.

- b) Charin saves \$45 a month.
So, in 10 months, Charin will have saved: $10(\$45) = \$450$
- c) Charin saves \$45 a month. So, in 2 months, he saves \$90. I subtract this amount from the account balance after 2 months to determine how much money was in the account at the start: $\$145 - \$90 = \$55$
- d) I assumed that the savings account did not earn any interest, or, if it did, then it was not paid into the account in the first 5 months.

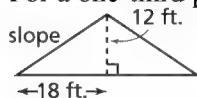
28. a) For a full pitch roof, the height and span are equal.



To calculate the slope, I divided: $\frac{36 \text{ ft.}}{18 \text{ ft.}} = 2$

So, the slope is 2.

- b) For a one-third pitch roof, the height is one-third the span.



To calculate the slope, I divided: $\frac{12 \text{ ft.}}{18 \text{ ft.}} = \frac{2}{3}$

So, the slope is $\frac{2}{3}$.

C

29. I calculated the slope of the glide path.

The rise is the change in altitude: $5500 \text{ m} - 7000 \text{ m} = -1500 \text{ m}$, or -1.5 km

The run is 18 km.

$$\text{Slope} = \frac{-1.5 \text{ km}}{18 \text{ km}}, \text{ or } -0.08\bar{3}$$

Determine the run, r , when the rise is -2600 m , or -2.6 km .

$$\text{Slope} = \frac{\text{rise}}{\text{run}} \quad \text{Substitute for slope, rise, and run.}$$

$$-0.08\bar{3} = \frac{-2.6 \text{ km}}{r} \quad \text{Multiply each side by } r.$$

$$-0.08\bar{3}r = -2.6 \text{ km} \quad \text{Divide each side by } -0.08\bar{3}.$$

$$r = \frac{-2.6 \text{ km}}{-0.08\bar{3}}$$

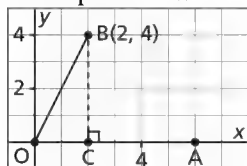
$$r = 31.2 \text{ km}$$

When the rise is -2.6 km , the run is 31.2 km. This means that when the plane was at an

altitude of 2600 m, it travelled a horizontal distance of 31.2 km before it landed. Since Winnipeg was 63 km away, the plane could not have reached Winnipeg.

30. Positions of point A may vary.

- a), b) Draw segment OB. Mark a point A anywhere on the positive x -axis, such as at (6, 0). Mark point C where the vertical line through B intersects the x -axis.



From the diagram,

$$\text{Slope of OB is: } \frac{4}{2} = 2$$

In right $\triangle OBC$

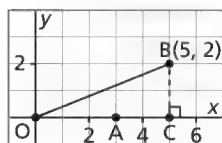
$$\tan \angle COB = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \angle COB = \frac{4}{2}$$

$$\tan \angle COB = 2$$

Since both C and A are on the same line, $\tan \angle COB = \tan \angle AOB = 2$

- c) Draw segment OB. Mark a point A anywhere on the positive x -axis, such as at (3, 0). Mark point C where the vertical line through B intersects the x -axis.



From the diagram,

$$\text{Slope of OB is: } \frac{2}{5}$$

In right $\triangle OBC$

$$\tan \angle COB = \frac{\text{opposite}}{\text{adjacent}}$$

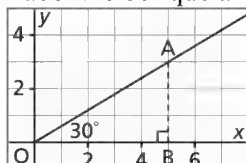
$$\tan \angle COB = \frac{2}{5}$$

Since both C and A are on the same line, $\tan \angle COB = \tan \angle AOB = \frac{2}{5}$

- d) The slope of a line segment is equal to the tangent of the angle formed by the segment and the positive x -axis. Both the slope and the tangent are equal to the quotient of the same two numbers.

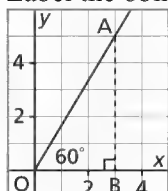
31. (Solution if Exercise 30 has been previously completed.)

- a) Label the oblique arm of the 30° angle as OA.



From exercise 30, the slope of OA is $\tan 30^\circ = 0.5773\dots$

- b) Label the oblique arm of the 60° angle as OB.

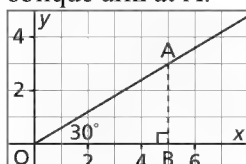


From exercise 30, the slope of OB is $\tan 60^\circ = 1.7320\dots$

- c) For an angle with one arm horizontal and one arm oblique:
Since $1.7320\dots$ is not double $0.5773\dots$; when an angle doubles, the slope of the oblique arm does not double.

31. (Solution if Exercise 30 has not been previously completed.)

- a) Draw the 30° angle. From B(5, 0), draw the perpendicular to the x-axis to intersect the oblique arm at A.



Determine the rise AB.

In right $\triangle AOB$:

$$\tan \angle AOB = \frac{AB}{OB}$$

Substitute: $\angle AOB = 30^\circ$ and $OB = 5$

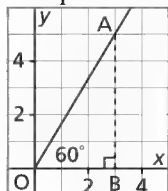
$$\tan 30^\circ = \frac{AB}{5}$$

Multiply each side by 5.

$$AB = 5 \tan 30^\circ$$

$$\text{So, the slope of OA is: } \frac{5 \tan 30^\circ}{5} = \tan 30^\circ$$

- b) Draw the 60° angle. From B(3, 0), draw the perpendicular to the x-axis to intersect the oblique arm at A.



Determine the rise AB.

In right $\triangle AOB$:

$$\tan \angle AOB = \frac{AB}{OB} \quad \text{Substitute: } \angle AOB = 60^\circ \text{ and } OB = 3$$

$$\tan 60^\circ = \frac{AB}{3} \quad \text{Multiply each side by 3.}$$

$$AB = 3 \tan 60^\circ$$

$$\text{So, the slope of OA is: } \frac{3 \tan 60^\circ}{3} = \tan 60^\circ$$

c) $\tan 30^\circ = 0.5773\dots$ and $\tan 60^\circ = 1.7320\dots$

For an angle with one arm horizontal and one arm oblique:

Since $1.7320\dots$ is not double $0.5773\dots$; when an angle doubles, the slope of the oblique arm does not double.

Lesson 6.2 Slopes of Parallel and Perpendicular Lines Exercises (pages 349–351)**A**

3. Parallel lines have the same slope.
 - a) A parallel line has slope $\frac{4}{5}$.
 - b) A parallel line has slope $-\frac{4}{3}$.
 - c) A parallel line has slope 3.
 - d) A parallel line has slope 0.
4. Perpendicular lines have slopes that are negative reciprocals. When the slope of a line is a fraction, to determine the slope of a perpendicular line, invert the fraction and change its sign. When the slope of a line is an integer, write it in fraction form with denominator 1; then invert and change its sign.
 - a) A perpendicular line has slope $-\frac{6}{7}$.
 - b) A perpendicular line has slope $\frac{8}{5}$.
 - c) Write the slope 9 as $\frac{9}{1}$. A perpendicular line has slope $-\frac{1}{9}$.
 - d) Write the slope -5 as $-\frac{5}{1}$. A perpendicular line has slope $\frac{1}{5}$.
5. Two lines are parallel when their slopes are equal. Two lines are perpendicular when the product of their slopes is -1 .
 - a) The slopes are equal, so the lines are parallel.
 - b) The slopes are not equal and their product is not -1 , so the lines are neither parallel nor perpendicular.
 - c) The slopes are not equal and their product is not -1 , so the lines are neither parallel nor perpendicular.
 - d) The product of the slopes is -1 , so the lines are perpendicular.
6. Two lines are parallel when their slopes are equal.
Perpendicular lines have slopes that are negative reciprocals. When the slope of a line is a fraction, to determine the slope of a perpendicular line, invert the fraction and change its sign.

When the slope of a line is an integer, write it in fraction form with denominator 1; then invert and change its sign.

- a) For a line with slope $-\frac{4}{9}$:
- i) A parallel line has slope $-\frac{4}{9}$.
 - ii) A perpendicular line has slope $\frac{9}{4}$.
- b) For a line with slope 5:
- i) A parallel line has slope 5.
 - ii) Write the slope as $\frac{5}{1}$. A perpendicular line has slope $-\frac{1}{5}$.
- c) For a line with slope $\frac{7}{3}$:
- i) A parallel line has slope $\frac{7}{3}$.
 - ii) A perpendicular line has slope $-\frac{3}{7}$.
- d) For a line with slope -4 :
- i) A parallel line has slope -4 .
 - ii) Write the slope as $\frac{-4}{1}$. A perpendicular line has slope $\frac{1}{4}$.

B

7. Calculate the slope of the club and the slope of the line through the tips of the golfer's shoes.
For the slope of the club:
From the diagram, the rise is approximately -1 and the run is approximately 6.

Slope of the club is approximately: $-\frac{1}{6}$

For the slope of the line through the tips of the golfer's shoes:
From the diagram, the rise is approximately -1 and the run is approximately 6.

Slope of the line through tips of shoes is approximately: $-\frac{1}{6}$

Since the slopes are the same, the club is parallel to the line through the golfer's shoes, and the golfer is set up correctly.

8. Lines are parallel if they have the same slope. Perpendicular lines have slopes that are negative reciprocals. So, calculate the slope of each line segment then check to see if they are equal or are negative reciprocals. Use the formula: $\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$

- a) i) A(-5, -2), B(1, 5); and C(-1, -4), D(4, 1)

ii) $\text{Slope of AB} = \frac{5 - (-2)}{1 - (-5)}$

$$\text{Slope of AB} = \frac{7}{6}$$

$$\text{Slope of CD} = \frac{1 - (-4)}{4 - (-1)}$$

$$\text{Slope of CD} = \frac{5}{5}, \text{ or } 1$$

The slopes of AB and CD are not equal, nor are they negative reciprocals, so the lines are neither parallel nor perpendicular.

- b) i) E(-3, 4), F(3, 2); and G(2, 5), H(0, -1)

ii) $\text{Slope of EF} = \frac{2 - 4}{3 - (-3)}$

$$\text{Slope of EF} = \frac{-2}{6}, \text{ or } -\frac{1}{3}$$

$$\text{Slope of GH} = \frac{5 - (-1)}{2 - 0}$$

$$\text{Slope of GH} = \frac{6}{2}, \text{ or } 3$$

Since $3\left(-\frac{1}{3}\right) = -1$; the slopes of EF and GH are negative reciprocals, so the lines are perpendicular.

- c) i) J(-2, 3), K(1, -3); and M(3, 1), N(-4, -2)

ii) $\text{Slope of JK} = \frac{-3 - 3}{1 - (-2)}$

$$\text{Slope of JK} = \frac{-6}{3}, \text{ or } -2$$

$$\text{Slope of MN} = \frac{-2 - 1}{-4 - 3}$$

$$\text{Slope of MN} = \frac{-3}{-7}, \text{ or } \frac{3}{7}$$

The slopes of JK and MN are not equal, nor are they negative reciprocals, so the lines are neither parallel nor perpendicular.

- d) i) P(0, 5), Q(6, 2); and R(-4, -1), S(0, -3)

ii) Slope of PQ = $\frac{2 - 5}{6 - 0}$

Slope of PQ = $\frac{-3}{6}$, or $-\frac{1}{2}$

Slope of RS = $\frac{-3 - (-1)}{0 - (-4)}$

Slope of RS = $\frac{-2}{4}$, or $-\frac{1}{2}$

The slopes of AB and CD are equal, so the lines are parallel.

9. Lines are parallel if they have the same slope. Perpendicular lines have slopes that are negative reciprocals. So, calculate the slope of each line segment then check to see if the slopes are equal or are negative reciprocals. Use the formula: Slope = $\frac{\text{change in } y}{\text{change in } x}$

a) Slope of ST = $\frac{5 - (-1)}{-1 - (-4)}$

Slope of ST = $\frac{6}{3}$, or 2

Slope of UV = $\frac{-1 - 1}{5 - 1}$

Slope of UV = $\frac{-2}{4}$, or $-\frac{1}{2}$

The slopes of ST and UV are negative reciprocals, so the line segments are perpendicular.

b) Slope of BC = $\frac{3 - (-2)}{-3 - (-6)}$

Slope of BC = $\frac{5}{3}$

Slope of DE = $\frac{5 - 0}{5 - 2}$

Slope of DE = $\frac{5}{3}$

The slopes of BC and DE are equal, so the line segments are parallel.

c) Slope of NP = $\frac{-4 - 2}{-3 - (-6)}$

Slope of NP = $\frac{-6}{3}$, or -2

$$\text{Slope of QR} = \frac{4 - (-3)}{3 - 1}$$

$$\text{Slope of QR} = \frac{7}{2}$$

The slopes of NP and QR are not equal, nor are they negative reciprocals, so the line segments are neither parallel nor perpendicular.

d) $\text{Slope of GH} = \frac{1 - 5}{4 - (-2)}$

$$\text{Slope of GH} = \frac{-4}{6}, \text{ or } -\frac{2}{3}$$

$$\text{Slope of JK} = \frac{0 - (-4)}{7 - 1}$$

$$\text{Slope of JK} = \frac{4}{6}, \text{ or } \frac{2}{3}$$

The slopes of GH and JK are not equal, nor are they negative reciprocals, so the line segments are neither parallel nor perpendicular.

- 10. a)** For DE, the coordinates of the points at the intercepts are: (4, 0) and (0, -6)

$$\text{Slope of DE} = \frac{-6 - 0}{0 - 4}$$

$$\text{Slope of DE} = \frac{-6}{-4}, \text{ or } \frac{3}{2}$$

For FG, the coordinates of the points at the intercepts are: (-6, 0) and (0, 4)

$$\text{Slope of FG} = \frac{4 - 0}{0 - (-6)}$$

$$\text{Slope of FG} = \frac{4}{6}, \text{ or } \frac{2}{3}$$

Both lines have positive slopes, which are reciprocals.

- b)** For HJ, the coordinates of the points at the intercepts are: (-2, 0) and (0, 3)

$$\text{Slope of HJ} = \frac{3 - 0}{0 - (-2)}$$

$$\text{Slope of HJ} = \frac{3}{2}$$

For KM, the coordinates of the points at the intercepts are: (-9, 0) and (0, 6)

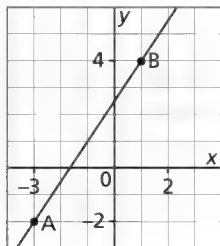
$$\text{Slope of KM} = \frac{6 - 0}{0 - (-9)}$$

$$\text{Slope of KM} = \frac{6}{9}, \text{ or } \frac{2}{3}$$

Both lines have positive slopes, which are reciprocals.

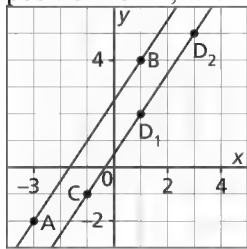
11. a) Slope of AB = $\frac{4 - (-2)}{1 - (-3)}$

Slope of AB = $\frac{6}{4}$, or $\frac{3}{2}$



b) Line CD is parallel to line AB, so line CD has slope $\frac{3}{2}$.

c) Since line CD has slope $\frac{3}{2}$, its rise is 3 and its run is 2. From point C, move 3 squares up and 2 squares right to get to one possible position for D, with coordinates (1, 2). From this point, move 3 squares up and 2 squares right again to get to a second possible position for D, with coordinates (3, 5).

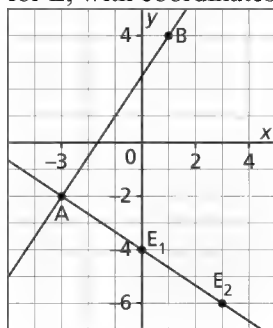


My answers may be different because a classmate may choose the coordinates of any other point on the line CD that I drew.

d) Line AE is perpendicular to line AB; so the slope of AE is the negative reciprocal of $\frac{3}{2}$, which is $-\frac{2}{3}$.

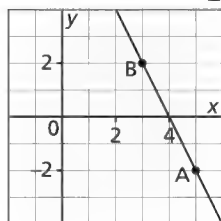
e) Since line AE has slope $-\frac{2}{3}$, its rise is -2 and its run is 3. From point A, move 2 squares down and 3 squares right to get to one possible for E, with coordinates (0, -4). From this point, move 2 squares down and 3 squares right again to get to a second possible position

for E, with coordinates $(3, -6)$.



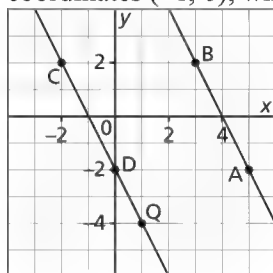
12. a) Slope of AB = $\frac{2 - (-2)}{3 - 5}$

Slope of AB = $\frac{4}{-2}$, or -2



b) Line CD is parallel to line AB, so line CD has slope -2 .

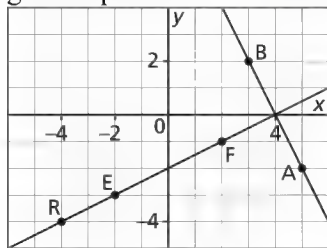
c) Since line CD has slope -2 , its rise is 2 and its run is -1 . From point Q, move 2 squares up and 1 square left to get to a point with coordinates $(0, -2)$, which is the y -intercept. From this point, move 2 squares up and 1 square left again to get to a point with coordinates $(-1, 0)$, which is the x -intercept.



d) Line EF is perpendicular to line AB; so the slope of EF is the negative reciprocal of -2 , which is $\frac{1}{2}$.

e) Since line EF has slope $\frac{1}{2}$, its rise is 1 and its run is 2. From point R, move 1 square up and 2 squares right to get to a point with coordinates $(-2, -3)$. From this point, move 1 square up and 2 squares right again to get to a point with coordinates $(0, -2)$, which is the y -intercept. From this point, move 1 square up and 2 squares right again to get to a point with coordinates $(2, -1)$. From this point, move 1 square up and 2 squares right again to

get to a point with coordinates (4, 0), which is the x -intercept.



13. a) Quadrilateral HJKM is a parallelogram if its opposite sides are parallel.
I counted squares to determine the rise and the run of each side.

$$\text{Slope of HJ} = \frac{2}{7}$$

$$\text{Slope of MK} = \frac{2}{7}$$

Since the slopes of HJ and MK are equal, those sides are parallel.

$$\text{Slope of HM} = \frac{-6}{2}, \text{ or } -3$$

$$\text{Slope of JK} = \frac{-6}{2}, \text{ or } -3$$

Since the slopes of HM and JK are equal, those sides are parallel.

Since both pairs of opposite sides are parallel, quadrilateral HJKM is a parallelogram.

- b) Quadrilateral HJKM is a rectangle if its angles are right angles; that is, if adjacent sides are perpendicular.

The slopes of adjacent sides are $\frac{2}{7}$ and -3 ; since these are not negative reciprocals, the sides are not perpendicular, and the parallelogram is not a rectangle.

14. From the diagram, I can see that sides GF and DE are not parallel.

I check to see if sides GD and FE are parallel.

I counted squares to determine the rise and the run of each side.

$$\text{Slope of GD} = \frac{3}{3}, \text{ or } 1$$

$$\text{Slope of FE} = \frac{4}{4}, \text{ or } 1$$

Since the slopes of GD and FE are equal, those sides are parallel and quadrilateral DEFG is a trapezoid.

15. When I know the coordinates of Q and R, I can calculate the slope of QR. This slope is equal to the slope of the opposite side of the rectangle, TS. The other pair of opposite sides, QT and RS, also have equal slopes. This slope is the negative reciprocal of the slope of QR because the sides are perpendicular to QR. But I do not know the lengths of sides RS and QT, so I cannot determine the coordinates of S and T.

16. I calculate the slopes of the 3 sides of $\triangle ABC$, then check to see which two of the slopes are negative reciprocals.

A(-3, 1), B(6, -2), and C(3, 4)

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{-2 - 1}{6 - (-3)}$$

$$\text{Slope of AB} = -\frac{3}{9}$$

$$\text{Slope of AB} = -\frac{1}{3}$$

$$\text{Slope of BC} = \frac{4 - (-2)}{3 - 6}$$

$$\text{Slope of BC} = \frac{6}{-3}, \text{ or } -2$$

$$\text{Slope of AC} = \frac{4 - 1}{3 - (-3)}$$

$$\text{Slope of AC} = \frac{3}{6}, \text{ or } \frac{1}{2}$$

Since -2 and $\frac{1}{2}$ are negative reciprocals, then BC is perpendicular to AC, and $\triangle ABC$ is a right triangle, with $\angle C = 90^\circ$.

17. I calculate the slopes of the 3 sides of $\triangle DEF$, then check to see if any two of the slopes are negative reciprocals.

D(-3, -2), E(1, 4), and F(4, 2)

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of DE} = \frac{4 - (-2)}{1 - (-3)}$$

$$\text{Slope of DE} = \frac{6}{4}, \text{ or } \frac{3}{2}$$

$$\text{Slope of DF} = \frac{2 - (-2)}{4 - (-3)}$$

$$\text{Slope of DF} = \frac{4}{7}$$

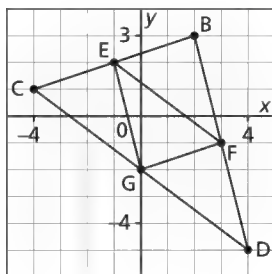
$$\text{Slope of EF} = \frac{2 - 4}{4 - 1}$$

$$\text{Slope of EF} = -\frac{2}{3}$$

Since $\frac{3}{2}$ and $-\frac{2}{3}$ are negative reciprocals, then DE is perpendicular to EF, and $\triangle DEF$ is a right triangle, with $\angle E = 90^\circ$.

18. Triangles may vary. I chose vertices so that the midpoints of the lines lie where grid lines intersect. Triangle BCD has vertices B(2, 3), C(-4, 1), and D(4, -5).

a)



$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of BC} = \frac{3 - 1}{2 - (-4)}$$

$$\text{Slope of BC} = \frac{2}{6}, \text{ or } \frac{1}{3}$$

$$\text{Slope of CD} = \frac{-5 - 1}{4 - (-4)}$$

$$\text{Slope of CD} = \frac{-6}{8}, \text{ or } -\frac{3}{4}$$

$$\text{Slope of BD} = \frac{-5 - 3}{4 - 2}$$

$$\text{Slope of BD} = \frac{-8}{2}, \text{ or } -4$$

b) The midpoints of the sides are: E(-1, 2), F(3, -1), and G(0, -2); they form $\triangle EFG$.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of EF} = \frac{-1 - 2}{3 - (-1)}$$

$$\text{Slope of EF} = -\frac{3}{4}$$

$$\text{Slope of FG} = \frac{-2 - (-1)}{0 - 3}$$

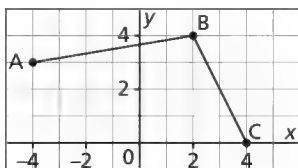
$$\text{Slope of FG} = \frac{-1}{-3}, \text{ or } \frac{1}{3}$$

$$\text{Slope of EG} = \frac{-2 - 2}{0 - (-1)}$$

$$\text{Slope of EG} = \frac{-4}{1}, \text{ or } -4$$

c) The slopes of the sides of $\triangle BCD$ are equal to the slopes of the sides of $\triangle EFG$; that is, BC is parallel to FG; BD is parallel to EG, and CD is parallel to EF. In each case, the segment that joins the midpoint of two sides of a triangle is parallel to the third side of the triangle.

19.



- a) If ABCD was a rectangle, then AB would be perpendicular to BC. From the diagram, they do not look perpendicular, but I shall determine the slopes of these sides to verify.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{4 - 3}{2 - (-4)}$$

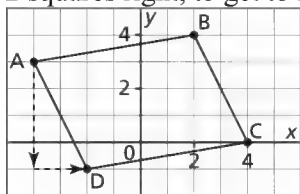
$$\text{Slope of AB} = \frac{1}{6}$$

$$\text{Slope of BC} = \frac{0 - 4}{4 - 2}$$

$$\text{Slope of BC} = \frac{-4}{2}, \text{ or } -2$$

Since $\frac{1}{6}$ and -2 are not negative reciprocals, then AB is not perpendicular to BC and ABCD is not a rectangle.

- b) Side AD is parallel to side BC, so they have the same rise and the same run. BC has a rise of -4 and a run of 2 . So, from point A, move 4 squares down, then 2 squares right, to get to D($-2, -1$).

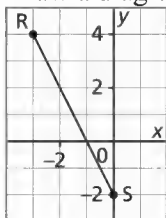


- c) I could have used the fact that AB is parallel to DC, and moved 1 square down from C, then 6 squares left; I would get to the same point D.

I could also have added coordinates:

Point C has coordinates $(4, 0)$. I could have added -1 to the y -coordinate, to represent a rise of -1 , then added -6 to the x -coordinate to represent a rise of -6 . Then the coordinates of D are: $(4 - 6, 0 - 1)$, or $(-2, -1)$

20. Draw a diagram.



For a right triangle, $\angle R$, $\angle S$, or $\angle T$ is a right angle.

Suppose $\angle R$ is the right angle.

Then, RT is perpendicular to RS at R .

The slope of RT is the negative reciprocal of the slope of RS .

From the diagram, the rise of RS is -6 and the run of RS is 3 .

So the slope of RS is $-\frac{6}{3}$.

The negative reciprocal of $-\frac{6}{3}$ can be written as $\frac{3}{6}$.

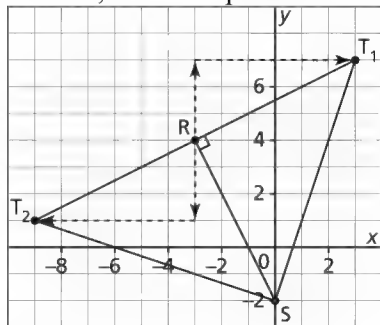
So, the rise of RT is 3 and the run of RT is 6 .

From R , move 3 squares up and 6 squares right to reach point $T_1(3, 7)$.

The negative reciprocal of $-\frac{6}{3}$ can also be written as $-\frac{3}{6}$.

So, the rise of RT is -3 and the run of RT is -6 .

From R , move 3 squares down and 6 squares left to reach $T_2(-9, 1)$.



Suppose $\angle S$ is the right angle.

Repeat a similar process as for $\angle R$.

Then, ST is perpendicular to RS at S .

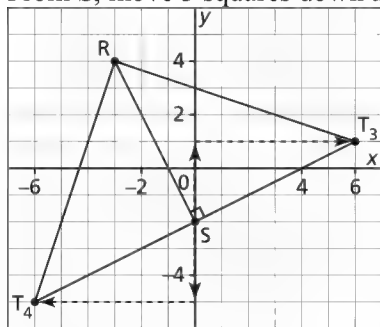
The slope of ST is the negative reciprocal of the slope of RS .

So, the rise of ST is 3 and the run of ST is 6 .

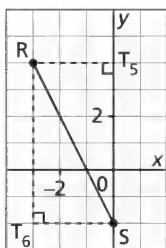
From S , move 3 squares up and 6 squares right to reach point $T_3(6, 1)$.

So, the rise of ST is -3 and the run of ST is -6 .

From S , move 3 squares down and 6 squares left to reach $T_4(-6, -5)$.



Suppose $\angle T$ is the right angle. From the Angle in a Semicircle Property, T is a point on the circumference of a circle with diameter RS .



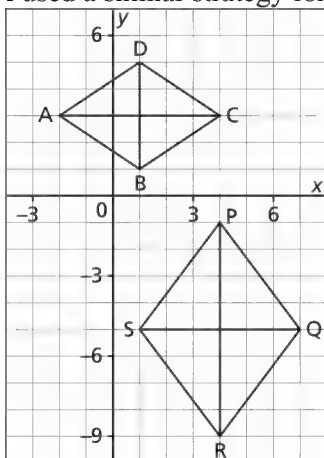
Possible coordinates for T are: (3, 7), (-9, 1), (6, 1), (-6, -5), (4, 0), and (-3, -2)

C

21. Rhombuses may vary.

For rhombus ABCD, to make sure all the sides were the same length and to make sure I didn't draw a square, I started at A and moved 2 squares down and 3 squares right to get to B. From point B, I moved 3 squares right and 2 squares up to reach C. From C I moved 2 squares up and 3 squares left to reach D. I then joined AD.

I used a similar strategy for rhombus PQRS.



Wherever I drew the rhombuses, the diagonals were always vertical and horizontal, so the diagonals intersect at right angles. To verify this, I calculated the slopes of the diagonals. For rhombus ABCD, the diagonals are AC and BD.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AC} = \frac{3 - 3}{4 - (-2)}$$

$$\text{Slope of AC} = \frac{0}{6}, \text{ or } 0; \text{ so AC is horizontal.}$$

$$\text{Slope of BD} = \frac{5 - 1}{1 - 1}$$

$$\text{Slope of BD} = \frac{4}{0}, \text{ which is undefined; so BD is vertical.}$$

Since AC is horizontal and BD is vertical, then the diagonals of the rhombus are perpendicular.

For rhombus PQRS, the diagonals are PR and SQ.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of SQ} = \frac{-5 - (-5)}{7 - 1}$$

$$\text{Slope of SQ} = \frac{0}{6}, \text{ or } 0; \text{ so SQ is horizontal.}$$

$$\text{Slope of PR} = \frac{-9 - (-1)}{4 - 4}$$

$$\text{Slope of PR} = \frac{-8}{0}, \text{ which is undefined; so PR is vertical.}$$

Since SQ is horizontal and PR is vertical, then the diagonals of the rhombus are perpendicular.

22. Method 1: Use grid paper.

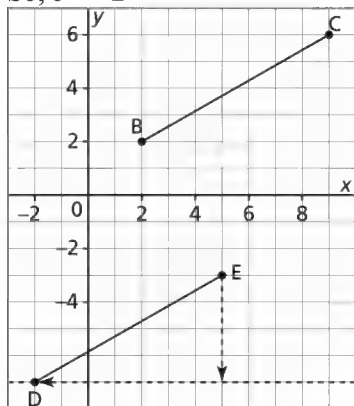
Plot points B(2, 2), C(9, 6), and E(5, -3). Draw a broken line through $y = -7$ because D lies on that line.

Since BC and DE are parallel, their slopes are equal.

From the graph, the rise from C to B is -4 and the run is -7.

So, from E, move 4 squares down and 7 squares left to reach D at (-2, -7).

So, $c = -2$



Method 2: Use algebra.

Determine the slope of each segment.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of BC} = \frac{6 - 2}{9 - 2}$$

$$\text{Slope of BC} = \frac{4}{7}$$

$$\text{Slope of DE} = \frac{-3 - (-7)}{5 - c}, \text{ or } \frac{4}{5 - c}$$

Since BC and DE are parallel, equate the two slopes:

$$\frac{4}{7} = \frac{4}{5 - c} \quad \text{Multiply each side by 7.}$$

$$7\left(\frac{4}{7}\right) = 7\left(\frac{4}{5-c}\right)$$

$$4 = \frac{28}{5-c}$$

Multiply each side by $(5-c)$.

$$4(5-c) = (5-c)\left(\frac{28}{5-c}\right)$$

$$20 - 4c = 28$$

$$-4c = 28 - 20$$

$$-4c = 8$$

Collect like terms.

Divide each side by -4 .

$$c = \frac{8}{-4}$$

$$c = -2$$

23. Determine the slope of each segment.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{10-5}{7-3}$$

$$\text{Slope of AB} = \frac{5}{4}$$

$$\text{Slope of CD} = \frac{a-2}{1-0}$$

$$\text{Slope of CD} = \frac{a-2}{1}, \text{ or } a-2$$

a) Since AB and CD are parallel, equate the two slopes:

$$\frac{5}{4} = a-2$$

Multiply each side by 4.

$$4\left(\frac{5}{4}\right) = 4(a-2)$$

$$5 = 4a - 8$$

$$13 = 4a$$

Collect like terms.

Divide each side by 4.

$$\frac{13}{4} = a$$

$$a = 3.25$$

b) Since AB and CD are perpendicular, then their slopes are negative reciprocals. That is, the product of the slopes is equal to -1 .

$$\left(\frac{5}{4}\right)(a-2) = -1$$

Multiply each side by 4.

$$4\left(\frac{5}{4}\right)(a-2) = 4(-1)$$

$$5(a-2) = -4$$

$$5a - 10 = -4$$

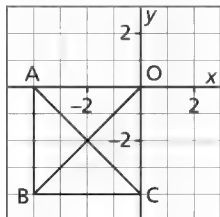
$$5a = 6$$

$$a = \frac{6}{5}$$

$$a = 1.2$$

24. Positions of squares may vary.

- a) The coordinates of the vertices are: O(0, 0), A(-4, 0), B(-4, -4), and C(0, -4)



The diagonals are AC and OB.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

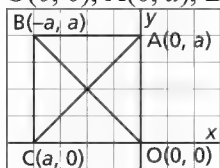
$$\text{Slope of AC} = \frac{-4 - 0}{0 - (-4)}$$

$$\text{Slope of AC} = \frac{-4}{4}, \text{ or } -1$$

$$\text{Slope of OB} = \frac{-4 - 0}{-4 - 0}, \text{ or } 1$$

Since the slopes of OB and AC are negative reciprocals, then OB and AC are perpendicular.

- b) Since each side of the square has length a units, the coordinates of the vertices are: O(0, 0), A(0, a), B(- a , a), and C(- a , 0)



The diagonals are AC and OB.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AC} = \frac{0 - a}{-a - 0}$$

$$\text{Slope of AC} = \frac{-a}{-a}, \text{ or } 1$$

$$\text{Slope of OB} = \frac{a - 0}{-a - 0}$$

$$\text{Slope of OB} = \frac{a}{-a}, \text{ or } -1$$

Since the slopes of OB and AC are negative reciprocals, then OB and AC are perpendicular.

Checkpoint 1

Assess Your Understanding (page 353)

6.1

1. $\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$

Point A has coordinates $(-6, 5)$. Point B has coordinates $(3, -1)$.

$$\text{Slope of AB} = \frac{-1 - 5}{3 - (-6)}$$

$$\text{Slope of AB} = \frac{-6}{9}$$

$$\text{Slope of AB} = -\frac{2}{3}$$

Point C has coordinates $(-5, -1)$. Point D has coordinates $(3, 1)$.

$$\text{Slope of CD} = \frac{1 - (-1)}{3 - (-5)}$$

$$\text{Slope of CD} = \frac{2}{8}$$

$$\text{Slope of CD} = \frac{1}{4}$$

2. $\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$

a) Point Q has coordinates $(-2, 5)$. Point R has coordinates $(2, -10)$.

$$\text{Slope of QR} = \frac{-10 - 5}{2 - (-2)}$$

$$\text{Slope of QR} = \frac{-15}{4}$$

b) The point at the x -intercept has coordinates $A(3, 0)$. The point at the y -intercept has coordinates $B(0, -5)$.

$$\text{Slope of AB} = \frac{-5 - 0}{0 - 3}$$

$$\text{Slope of AB} = \frac{-5}{-3}$$

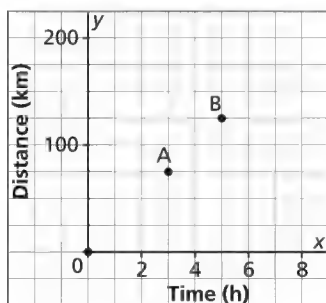
$$\text{Slope of AB} = \frac{5}{3}$$

The slope of the line through the intercepts is $\frac{5}{3}$.

3. The slope of a line is equal to the slope of any segment of the line, so we can use any two points that form that segment to determine the slope of the line.

4. a) Label two points on the line: $A(3, 75)$ and $B(5, 125)$
Label the axes x and y .

Jordan's Snowmobile Journey



$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{125 \text{ km} - 75 \text{ km}}{5 \text{ h} - 3 \text{ h}}$$

$$\text{Slope of AB} = \frac{50 \text{ km}}{2 \text{ h}}$$

$$\text{Slope of AB} = 25 \text{ km/h}$$

The slope is Jordan's average speed.

- b) Since Jordan travels at an average speed of 25 km/h, in $1\frac{1}{4}$ h, he will travel:

$$25 \text{ km/h} (1.25 \text{ h}) = 31.25 \text{ km}$$

In $1\frac{1}{4}$ h, Jordan will travel approximately 31 km.

- c) Jordan travels at an average speed of 25 km/h. To travel 65 km, it will take him:

$$\frac{65 \text{ km}}{25 \text{ km/h}} = 2.6 \text{ h}$$

There are 60 min in 1 h, so 0.6 h is $0.6(60 \text{ min}) = 36 \text{ min}$

It will take Jordan 2 h 36 min to travel 65 km.

6.2

5. The positions of the lines on the grids and their labels may vary.

- a) The lines are neither parallel nor perpendicular because the slopes are neither equal nor are they negative reciprocals.

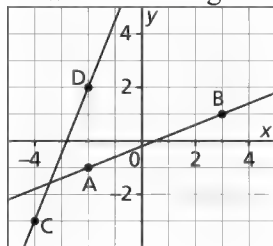
A line with slope $\frac{2}{5}$ has a rise of 2 and a run of 5. To draw line AB with slope $\frac{2}{5}$, I

began at point A, moved 2 squares up and 5 squares right to reach point B. I drew a line

through AB. A line with slope $\frac{5}{2}$ has a rise of 5 and a run of 2. To draw line CD with

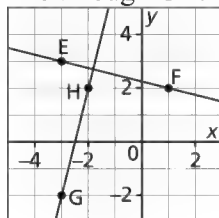
slope $\frac{5}{2}$, I began at point C, moved 5 squares up and 2 squares right to reach point D. I

drew a line through CD.

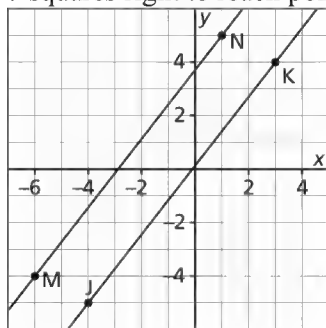


- b) The lines are perpendicular because their slopes are negative reciprocals.

A line with slope $-\frac{1}{4}$ has a rise of -1 and a run of 4 . To draw line EF with slope $-\frac{1}{4}$, I began at point E, moved 1 square down and 4 squares right to reach point F. I drew a line through EF. A line with slope 4 has a rise of 4 and a run of 1 . To draw line GH with slope 4 , I began at point G, moved 4 squares up and 1 square right to reach point H. I drew a line through GH.



- c) The lines are parallel because their slopes are equal; that is, the fraction $\frac{18}{14}$ simplifies to $\frac{9}{7}$. A line with slope $\frac{9}{7}$ has a rise of 9 and a run of 7 . To draw line JK with slope $\frac{9}{7}$, I began at point J, moved 9 squares up and 7 squares right to reach point K. I drew a line through JK. To draw line MN with slope $\frac{9}{7}$, I began at point M, moved 9 squares up and 7 squares right to reach point N. I drew a line through MN.



6. Coordinates may vary.
Draw a line through the points D(-6, -1) and E(2, 5).

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

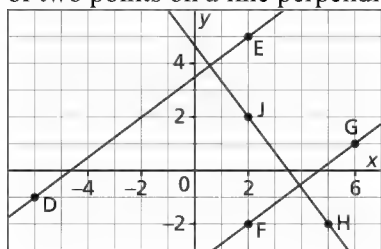
$$\text{Slope of DE} = \frac{5 - (-1)}{2 - (-6)}$$

$$\text{Slope of DE} = \frac{6}{8}, \text{ or } \frac{3}{4}$$

- a) A line parallel to DE has slope $\frac{3}{4}$. To draw a parallel line FG, choose any point such as F(2, -2). From F, move 3 squares up and 4 squares right to reach point G with coordinates (6, 1). So, the coordinates of two points on a line parallel to DE are (2, -2) and (6, 1).

- b) A line perpendicular to DE has a slope that is the negative reciprocal of $\frac{3}{4}$; that is, $-\frac{4}{3}$.

To draw a perpendicular line HJ, choose any point such as H(5, -2). From H, move 4 squares up and 3 squares left to reach point J with coordinates (2, 2). So, the coordinates of two points on a line perpendicular to DE are (5, -2) and (2, 2).



7. The triangle is a right triangle if two sides are perpendicular; that is, they form a right angle.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of AB} = \frac{-6 - 5}{-5 - (-1)}$$

$$\text{Slope of AB} = \frac{-11}{-4}, \text{ or } \frac{11}{4}$$

$$\text{Slope of BC} = \frac{1 - (-6)}{3 - (-5)}$$

$$\text{Slope of BC} = \frac{7}{8}$$

$$\text{Slope of AC} = \frac{1 - 5}{3 - (-1)}$$

$$\text{Slope of AC} = \frac{-4}{4}, \text{ or } -1$$

Since no two slopes are negative reciprocals, no two sides are perpendicular, and the triangle is not a right triangle.

8. Plot M and P on a grid.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope of MP} = \frac{-3 - 6}{3 - (-3)}$$

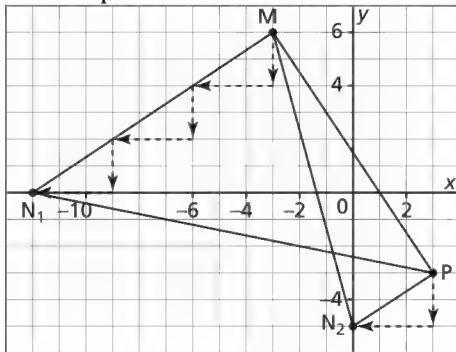
$$\text{Slope of MP} = \frac{-9}{6}, \text{ or } -\frac{3}{2}$$

So, if the right angle is at M, the slope of MN is the negative reciprocal of $-\frac{3}{2}$, which is $\frac{2}{3}$.

From point M, I move 2 squares down and 3 squares left. I continue to do this until I reach the x -axis at $(-12, 0)$. This is one position of N.

If the right angle is at P, the slope of PN is $\frac{2}{3}$.

From point P, I move 2 squares down and 3 squares left. I reach the y -axis at $(0, -5)$. This is another position of N.



Lesson 6.3 **Math Lab:** **Assess Your Understanding (page 356)**
Investigating Graphs of Linear Functions

1. Each line has an equation of the form $y = mx + b$, where m is the slope and b is the y -intercept.

- a) Since the slope of each line is $\frac{1}{2}$, then the equation of each line has the form $y = \frac{1}{2}x + b$

From the top of the screen to the bottom, the equations are:

$$y = \frac{1}{2}x + 4, y = \frac{1}{2}x + 2, y = \frac{1}{2}x - 1, y = \frac{1}{2}x - 2, y = \frac{1}{2}x - 3$$

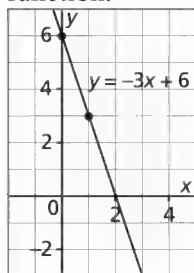
- b) Since the slope of each line is $-\frac{1}{3}$, then the equation of each line has the form

$$y = -\frac{1}{3}x + b$$

From the top of the screen to the bottom, the equations are:

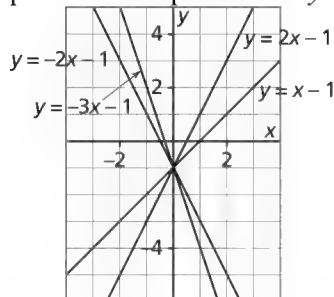
$$y = -\frac{1}{3}x + 4, y = -\frac{1}{3}x + 3, y = -\frac{1}{3}x + 1, y = -\frac{1}{3}x - 2, y = -\frac{1}{3}x - 3$$

2. The graph of a function with an equation of the form $y = mx + b$ has a slope of m and a y -intercept of b . To graph the function, I would mark a point at the y -intercept, then use the slope to identify the rise and run. From the y -intercept, I would move vertically a number of squares equal to the rise, then horizontally a number of squares equal to the run, then mark another point. When I draw a line through the points, I have drawn a graph of the function.
3. The graph of the linear function with equation $y = -3x + 6$ has a y -intercept of 6 and a slope of -3 . From 6 on the y -axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept. This line is the graph of the function.



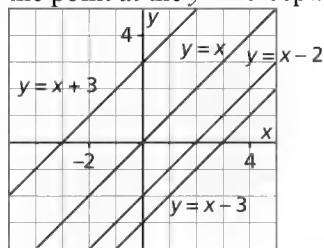
4. a) Predictions may vary. For example:
When m is positive, the graphs go up to the right. As m increases, the graphs get steeper.
When m is negative, the graphs go down to the right. As m decreases, the graphs get steeper.
- b) i) The graph of $y = x - 1$ has slope 1 and y -intercept -1 . From -1 on the y -axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.

- ii) The graph of $y = 2x - 1$ has slope 2 and y -intercept -1 . From -1 on the y -axis, I move 2 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
- iii) The graph of $y = -3x - 1$ has slope -3 and y -intercept -1 . From -1 on the y -axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
- iv) The graph of $y = -2x - 1$ has slope -2 and y -intercept -1 . From -1 on the y -axis, I move 2 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.



The graphs show that my predictions in part a were correct.

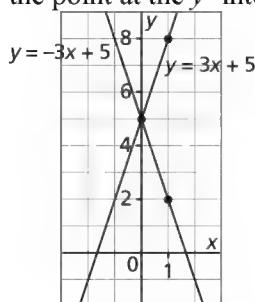
5. a) Predictions may vary. For example:
The graph moves up if b is increasing and the graph moves down if b is decreasing.
- b) i) The graph of $y = x - 3$ has slope 1 and y -intercept -3 . From -3 on the y -axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
 - ii) The graph of $y = x - 2$ has slope 1 and y -intercept -2 . From -2 on the y -axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
 - iii) The graph of $y = x$ has slope 1 and y -intercept 0. From the origin, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
 - iv) The graph of $y = x + 3$ has slope 1 and y -intercept 3. From 3 on the y -axis, I move 1 square up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.



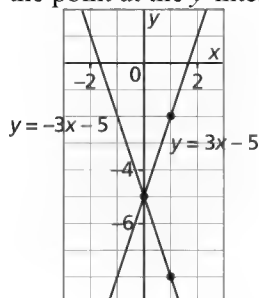
The graphs show that my prediction in part a was correct.

6. I used the same strategy as in question 3.

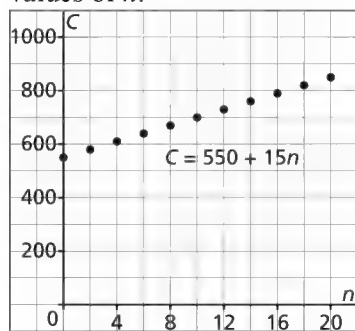
- a) The graph of $y = 3x + 5$ has slope 3 and y -intercept 5. From 5 on the y -axis, I move 3 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
- b) The graph of $y = -3x + 5$ has slope -3 and y -intercept 5. From 5 on the y -axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.



- c) The graph of $y = 3x - 5$ has slope 3 and y -intercept -5 . From -5 on the y -axis, I move 3 squares up, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.
- d) The graph of $y = -3x - 5$ has slope -3 and y -intercept -5 . From -5 on the y -axis, I move 3 squares down, then 1 square right and mark a point. I draw a line through this point and the point at the y -intercept.



7. a) The graph of $C = 550 + 15n$ has a C -intercept of 550 and a slope of 15. Since n is a whole number, the graph is a set of points. From 550 on the C -axis, I move 15 up and 1 right. Since it is difficult to move 15 up when the scale is 1 square represents \$100, I move 150 up and 10 right, then mark a point. From this point, I move 150 up and 10 right, then mark another point. I use a straightedge to mark more points along this line at different values of n .



- b) m represents the slope or rate of change; that is, \$15 per person. b represents the initial cost of \$550 to rent the hall. After this has been paid, each additional person costs \$15.

Lesson 6.4 **Slope-Intercept Form of
the Equation for a Linear Function**

Exercises (pages 362–364)

A

4. Compare each given equation with the equation of the form $y = mx + b$, where m is the slope and b is the y -intercept.

- a) The graph of $y = 4x - 7$ has slope 4 and y -intercept -7 .
- b) When a coefficient of a variable is not indicated, the coefficient is 1. The graph of $y = x + 12$ has slope 1 and y -intercept 12.
- c) The graph of $y = -\frac{4}{9}x + 7$ has slope $-\frac{4}{9}$ and y -intercept 7.
- d) The graph of $y = 11x - \frac{3}{8}$ has slope 11 and y -intercept $-\frac{3}{8}$.
- e) The equation $y = \frac{1}{5}x$ can be written as $y = \frac{1}{5}x + 0$. The slope is $\frac{1}{5}$ and the y -intercept is 0; the graph passes through the origin.
- f) The equation $y = 3$ can be written as $y = 0x + 3$. The slope is 0 and the y -intercept is 3.

5. Substitute the given values for the slope m and the y -intercept b into the equation $y = mx + b$.

- a) With slope 7 and y -intercept 16, the equation is: $y = 7x + 16$
- b) With slope $-\frac{3}{8}$ and y -intercept 5, the equation is: $y = -\frac{3}{8}x + 5$
- c) The point $H(0, -3)$ represents a y -intercept of -3 ; the slope is $\frac{7}{16}$; so the equation is:

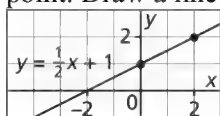
$$y = \frac{7}{16}x - 3$$

- d) With slope $-\frac{6}{5}$ and y -intercept -8 , the equation is: $y = -\frac{6}{5}x - 8$

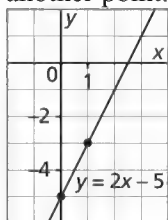
- e) The origin $O(0, 0)$ represents a y -intercept of 0; the slope is $-\frac{5}{12}$; so the equation is:

$$y = -\frac{5}{12}x$$

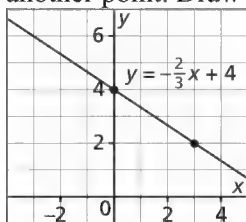
6. a) Mark a point at 1 on the y -axis, then move 1 square up and 2 squares right. Mark another point. Draw a line through the points.



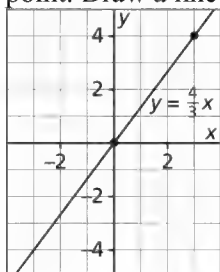
- b) Mark a point at -5 on the y -axis, then move 2 squares up and 1 square right. Mark another point. Draw a line through the points.



- c) Mark a point at 4 on the y -axis, then move 2 squares down and 3 squares right. Mark another point. Draw a line through the points.

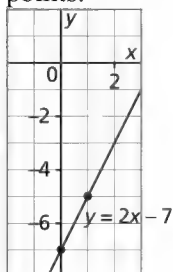


- d) Mark a point at the origin, then move 4 squares up and 3 squares right. Mark another point. Draw a line through the points.

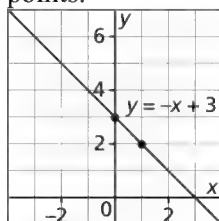


B

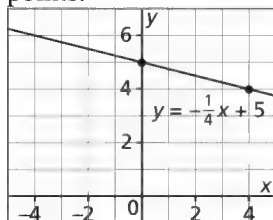
7. a) The slope of the line is 2 and its y -intercept is -7 . Mark a point at -7 on the y -axis, then move 2 squares up and 1 square right. Mark another point. Draw a line through the points.



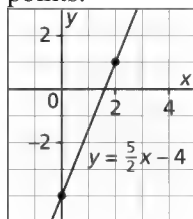
- b) The slope of the line is -1 and its y -intercept is 3. Mark a point at 3 on the y -axis, then move 1 square down and 1 square right. Mark another point. Draw a line through the points.



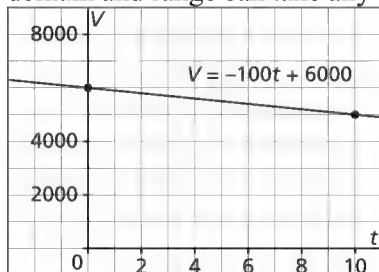
- c) The slope of the line is $-\frac{1}{4}$ and its y -intercept is 5. Mark a point at 5 on the y -axis, then move 1 square down and 4 squares right. Mark another point. Draw a line through the points.



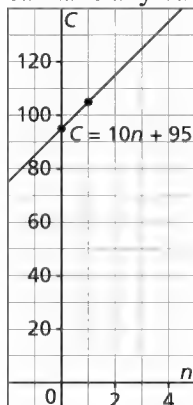
- d) The slope of the line is $\frac{5}{2}$ and its y -intercept is -4 . Mark a point at -4 on the y -axis, then move 5 squares up and 2 squares right. Mark another point. Draw a line through the points.



- e) The slope of the line is -100 and its V -intercept is 6000. Mark a point at 6000 on the V -axis, then move 1000 units down and 10 units right. Mark another point. I assume the domain and range can take any values so I draw a line through the points.



- f) The slope of the line is 10 and its C -intercept is 95. Mark a point at 95 on the C -axis, then move 10 units up and 1 unit right. Mark another point. I assume the domain and range can take any values so I draw a line through the points.



8. a) An equation of the line has the form $C = mt + b$.
Substitute $m = 50$ because the slope or rate of change is \$50/h.
Substitute $b = 80$ because the initial fee is \$80.
An equation is: $C = 50t + 80$
- b) Use the general form of the equation, $C = mt + b$.
Substitute $m = 40$ because the new slope or rate of change is \$40/h.
Substitute $b = 100$ because the new initial fee is \$100.
An equation is: $C = 40t + 100$
9. An equation of the line has the form $F = md + b$.
Substitute $m = 0.02$ because the slope or rate of change is 2% of the amount d dollars.
Substitute $b = 3.50$ because the withdrawal fee is \$3.50.
An equation is: $F = 0.02d + 3.50$
10. Screens may vary.
I used a graphing calculator.

- a) I pressed $\boxed{Y=}$, then I input the expression $Y1 = \left(\frac{-3}{13}\right)X + \frac{4}{11}$.

```

Plot1 Plot2 Plot3
Y1 (-3/13)X+4/11
1
Y2=
Y3=
Y4=
Y5=
Y6=

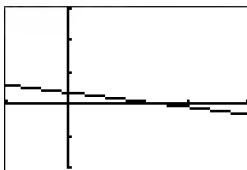
```

I pressed $\boxed{\text{GRAPH}}$. I changed the $\boxed{\text{WINDOW}}$ to $X_{\min} = -1$, $X_{\max} = 3$, $Y_{\min} = -2$, and $Y_{\max} = 3$ to get this screen:

```

WINDOW
Xmin=-1
Xmax=3
Xscl=1
Ymin=-2
Ymax=3
Yscl=1
Xres=1

```



- b) I pressed $\boxed{Y=}$, then I input the expression $Y1 = 3.75X - 2.95$.

```

Plot1 Plot2 Plot3
Y1=3.75X-2.95
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

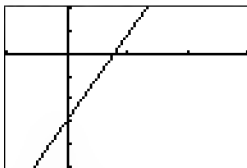
```

I pressed $\boxed{\text{GRAPH}}$. I changed the $\boxed{\text{WINDOW}}$ to $X_{\min} = -1$, $X_{\max} = 3$, $Y_{\min} = -5$, and $Y_{\max} = 2$ to get this screen:

```

WINDOW
Xmin=-1
Xmax=3
Xscl=1
Ymin=-5
Ymax=2
Yscl=1
Xres=1

```



- c) I pressed $\boxed{Y=}$, then I input the expression $Y1 = 0.45X + 25.50$.

```

Plot1 Plot2 Plot3
Y1=0.45X+25.50
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

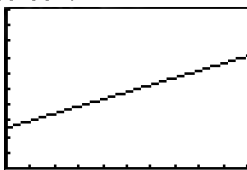
```

I pressed $\boxed{\text{GRAPH}}$. I changed the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 100$, $Y_{\min} = 0$, and $Y_{\max} = 100$ to get this screen:

```

WINDOW
Xmin=0
Xmax=100
Xscl=10
Ymin=0
Ymax=100
Yscl=10
Xres=1

```



- d) I pressed $\boxed{Y=}$, then I input the expression $Y1 = \left(\frac{9}{5}\right)X + 32$.

```

Plot1 Plot2 Plot3
Y1=(9/5)X+32
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

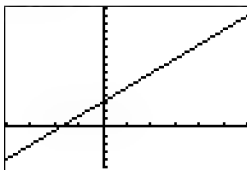
```

I pressed $\boxed{\text{GRAPH}}$. I changed the $\boxed{\text{WINDOW}}$ to $X_{\min} = -40$, $X_{\max} = 60$, $Y_{\min} = -50$, and $Y_{\max} = 150$ to get this screen:

```

WINDOW
Xmin=-40
Xmax=60
Xscl=10
Ymin=-50
Ymax=150
Yscl=10
Xres=1

```



11. a) The student may have confused the values of the slope and the y-intercept.

- b) From the graph:

Two points on the line have coordinates: (1, 1) and (0, -3)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\frac{1 - (-3)}{1 - 0} = 4$

The y -intercept is -3 .

Substitute $m = 4$ and $b = -3$ in the equation $y = mx + b$.

The correct equation is: $y = 4x - 3$

12. a) i) From the graph:

Two points on the line have coordinates: $(-2, 3)$ and $(2, 1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1 - 3}{2 - (-2)} = \frac{-2}{4}$, or $-\frac{1}{2}$

The y -intercept is 2.

ii) Substitute $m = -\frac{1}{2}$ and $b = 2$ in the equation $y = mx + b$.

The equation is: $y = -\frac{1}{2}x + 2$

To verify the equation, substitute the coordinates of another point on the line, such as $(4, 0)$.

Substitute $x = 4$ and $y = 0$ in $y = -\frac{1}{2}x + 2$.

L.S. = y R.S. = $-\frac{1}{2}x + 2$

L.S. = 0 R.S. = $-\frac{1}{2}(4) + 2$

R.S. = $-2 + 2$

R.S. = 0

= L.S.

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = -\frac{1}{2}x + 2$.

$y = -\frac{1}{2}(10) + 2$

$y = -5 + 2$

$y = -3$

When $x = 10$, $y = -3$

b) i) From the graph:

Two points on the line have coordinates: $(0, -6)$ and $(1, -2)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-2 - (-6)}{1 - 0} = \frac{4}{1}$, or 4

The y -intercept is -6 .

- ii) Substitute $m = 4$ and $b = -6$ in the equation $y = mx + b$.

The equation is: $y = 4x - 6$

To verify the equation, substitute the coordinates of another point on the line, such as $(2, 2)$.

Substitute $x = 2$ and $y = 2$ in $y = 4x - 6$.

$$\text{L.S.} = y$$

$$\text{R.S.} = 4x - 6$$

$$\text{L.S.} = 2$$

$$\text{R.S.} = 4(2) - 6$$

$$\text{R.S.} = 8 - 6$$

$$\text{R.S.} = 2$$

$$= \text{L.S.}$$

Since the left side is equal to the right side, the equation is correct.

- iii) Substitute $x = 10$ in $y = 4x - 6$.

$$y = 4(10) - 6$$

$$y = 40 - 6$$

$$y = 34$$

When $x = 10$, $y = 34$

- c) i) From the graph:

Two points on the line have coordinates: $(-4, -2)$ and $(0, 1)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{1 - (-2)}{0 - (-4)} = \frac{3}{4}$$

The y -intercept is 1.

- ii) Substitute $m = \frac{3}{4}$ and $b = 1$ in the equation $y = mx + b$.

$$\text{The equation is: } y = \frac{3}{4}x + 1$$

To verify the equation, substitute the coordinates of another point on the line, such as $(4, 4)$.

$$\text{Substitute } x = 4 \text{ and } y = 4 \text{ in } y = \frac{3}{4}x + 1.$$

$$\text{L.S.} = y$$

$$\text{R.S.} = \frac{3}{4}(4) + 1$$

$$\text{L.S.} = 4$$

$$\text{R.S.} = 3 + 1$$

$$\text{R.S.} = 4$$

$$= \text{L.S.}$$

Since the left side is equal to the right side, the equation is correct.

- iii) Substitute $x = 10$ in $y = \frac{3}{4}x + 1$.

$$y = \frac{3}{4}(10) + 1$$

$$y = \frac{30}{4} + 1$$

$$y = \frac{30}{4} + \frac{4}{4}$$

$$y = \frac{34}{4}, \text{ or } \frac{17}{2}$$

When $x = 10$, $y = 8.5$

d) i) From the graph:

Two points on the line have coordinates: $(0, -2)$ and $(3, -3)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{-3 - (-2)}{3 - 0} = \frac{-1}{3}, \text{ or } -\frac{1}{3}$$

The y -intercept is -2 .

ii) Substitute $m = -\frac{1}{3}$ and $b = -2$ in the equation $y = mx + b$.

$$\text{The equation is: } y = -\frac{1}{3}x - 2$$

To verify the equation, substitute the coordinates of another point on the line, such as $(6, -4)$.

$$\text{Substitute } x = 6 \text{ and } y = -4 \text{ in } y = -\frac{1}{3}x - 2.$$

$$\text{L.S.} = y \qquad \text{R.S.} = -\frac{1}{3}(6) - 2$$

$$\text{L.S.} = -4 \qquad \text{R.S.} = -2 - 2$$

$$\text{R.S.} = -4$$

$$= \text{L.S.}$$

Since the left side is equal to the right side, the equation is correct.

iii) Substitute $x = 10$ in $y = -\frac{1}{3}x - 2$.

$$y = -\frac{1}{3}(10) - 2$$

$$y = -\frac{10}{3} - \frac{6}{3}$$

$$y = -\frac{16}{3}$$

$$\text{When } x = 10, y = -\frac{16}{3}$$

13. a) From the graph:

Two points on the line have coordinates: $(0, 900)$ and $(10, 100)$

$$\text{Slope} = \frac{\text{change in } h\text{-coordinates}}{\text{change in } t\text{-coordinates}}$$

Slope of the line is: $\frac{100 - 900}{10 - 0} = \frac{-800}{10}$, or -80

The slope is the quotient of height in metres and time in minutes, so the slope is -80 m/min; this is the rate at which the plane is descending; every minute, it descends 80 m.

The h -intercept is 900; this is the height in metres of the plane before it began its descent.

- b) Rewrite the equation $y = mx + b$ as $h = mt + b$, to use the dependent and independent variables on the graph.

Substitute $m = -80$ and $b = 900$ in the equation $h = mt + b$.

The equation is: $h = -80t + 900$

To verify the equation, substitute the coordinates of another point on the line, such as $(5, 500)$.

Substitute $t = 5$ and $h = 500$ in $h = -80t + 900$.

L.S. = h	R.S. = $-80t + 900$
L.S. = 500	R.S. = $-80(5) + 900$
	R.S. = $-400 + 900$
	= 500
	= L.S.

Since the left side is equal to the right side, the equation is correct.

- c) Substitute $t = 5.5$ in $h = -80t + 900$.

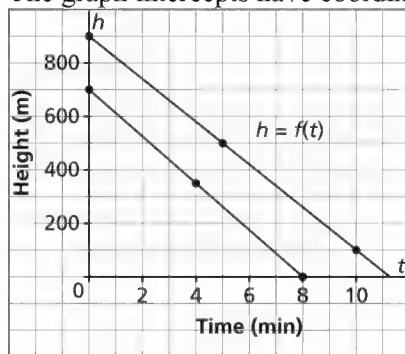
$$h = -80(5.5) + 900$$

$$h = -440 + 900$$

$$h = 460$$

When $t = 5.5$ min, $h = 460$ m; that is, 5.5 min after beginning its descent, the plane is 460 m high.

- d) i) The graph intercepts have coordinates: $(0, 700)$ and $(8, 0)$



The graph is steeper.

$$\text{Slope} = \frac{\text{change in } h\text{-coordinates}}{\text{change in } t\text{-coordinates}}$$

Slope of the line is: $\frac{0 - 700}{8 - 0} = \frac{-700}{8}$, or -87.5

The slope is -87.5 m/min; the plane is descending at a faster rate; every minute, it descends 87.5 m.

- ii) Substitute $m = -87.5$ and $b = 700$ in the equation $h = mt + b$.

The equation is: $h = -87.5t + 700$

To verify the equation, substitute the coordinates of another point on the line, such as (4, 350).

Substitute $t = 4$ and $h = 350$ in $h = -87.5t + 700$.

$$\text{L.S.} = h \qquad \text{R.S.} = -87.5t + 700$$

$$\text{L.S.} = 350 \qquad \text{R.S.} = -87.5(4) + 700$$

$$\text{R.S.} = -350 + 700$$

$$= 350$$

$$= \text{L.S.}$$

Since the left side is equal to the right side, the equation is correct.

14. a) From the given data:

Rewrite the equation $y = mx + b$ as $C = mn + b$, to use the dependent and independent variables given.

The C -intercept is 20, which is the initial cost in dollars.

The slope is 0.80, which is the cost per song downloaded.

Substitute $m = 0.80$ and $b = 20$ in the equation $C = mn + b$.

The equation is: $C = 0.80n + 20$

- b) Substitute $n = 109$ in the equation: $C = 0.80n + 20$

$$C = 0.80(109) + 20$$

$$C = 87.2 + 20$$

$$C = 107.2$$

The total cost to download 109 songs is \$107.20.

- c) Substitute $C = 120$ in the equation: $C = 0.80n + 20$

$$120 = 0.80n + 20 \qquad \text{Solve for } n.$$

$$100 = 0.80n \qquad \text{Divide each side by 0.80.}$$

$$125 = n$$

Michelle downloaded 125 songs.

15. a) A horizontal line has a slope of 0, so the equation

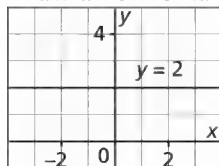
$$y = mx + b \text{ becomes}$$

$$y = 0x + b, \text{ or}$$

$$y = b$$

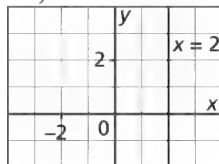
So, for the line $y = 2$, the slope is 0 and the y -intercept is 2.

Draw a horizontal line through 2 on the y -axis.



- b) I cannot use $y = mx + b$ to graph $x = 2$ because it has an infinite slope.

So, I use the fact that $x = 2$ is a vertical line through 2 on the x -axis.



16. a) From the given data:

Rewrite the equation $y = mx + b$ as $E = mt + b$, to use the dependent and independent variables given.

The E -intercept is 34, which is Alun's nightly earnings in dollars before tips are added.

The slope is 0.05, which is the percent of tips that Alun gets, written as a decimal.

Substitute $m = 0.05$ and $b = 34$ in the equation $E = mt + b$.

The equation is: $E = 0.05t + 34$

- b) Substitute $t = 400$ in the equation: $E = 0.05t + 34$

$$E = 0.05(400) + 34$$

$$E = 20 + 34$$

$$E = 54$$

Alun earns \$54 when the tips are \$400.

- c) Substitute $E = 64$ in the equation: $E = 0.05t + 34$

$$64 = 0.05t + 34$$

Solve for t .

$$30 = 0.05t$$

Divide each side by 0.05.

$$600 = t$$

When Alun earned \$64, the tips were \$600.

17. a) From the graph:

Two points on the line have coordinates: (0, 1) and (1, 5)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{5 - 1}{1 - 0} = 4$$

The y -intercept is 1.

For the equation of the line, substitute $m = 4$ and $b = 1$ in $y = mx + b$.

So, the equation is: $y = 4x + 1$

- b) From the graph:

Two points on the line have coordinates: (0, -1) and (3, 1)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{1 - (-1)}{3 - 0} = \frac{2}{3}$$

The y -intercept is -1.

For the equation of the line, substitute $m = \frac{2}{3}$ and $b = -1$ in $y = mx + b$.

So, the equation is: $y = \frac{2}{3}x - 1$

- c) From the graph:

Two points on the line have coordinates: (-3, -2) and (0, -7)

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

Slope of the line is: $\frac{-7 - (-2)}{0 - (-3)} = \frac{-5}{3}$, or $-\frac{5}{3}$

The y -intercept is -7 .

For the equation of the line, substitute $m = -\frac{5}{3}$ and $b = -7$ in $y = mx + b$.

So, the equation is: $y = -\frac{5}{3}x - 7$

18. From Graph A:

Two points on the line have coordinates: $(1, 2)$ and $(0, -1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-1 - 2}{0 - 1} = \frac{-3}{-1}$, or 3

The y -intercept is -1 .

For the equation of the line, substitute $m = 3$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = 3x - 1$, which is the equation in part b.

From Graph B:

Two points on the line have coordinates: $(3, 0)$ and $(0, -1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-1 - 0}{0 - 3} = \frac{-1}{-3}$, or $\frac{1}{3}$

The y -intercept is -1 .

For the equation of the line, substitute $m = \frac{1}{3}$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = \frac{1}{3}x - 1$, which is the equation in part d.

From Graph C:

Two points on the line have coordinates: $(2, 3)$ and $(1, 1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1 - 3}{1 - 2} = \frac{-2}{-1}$, or 2

The y -intercept is -1 .

For the equation of the line, substitute $m = 2$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = 2x - 1$, which is the equation in part a.

From Graph D:

Two points on the line have coordinates: $(-2, 1)$ and $(1, -2)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1 - (-2)}{-2 - 1} = \frac{3}{-3}$, or -1

The y -intercept is -1 .

For the equation of the line, substitute $m = -1$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = -x - 1$, which is the equation in part c.

19. From Graph A:

Two points on the line have coordinates: $(4, 3)$ and $(2, 1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1 - 3}{2 - 4} = \frac{-2}{-2}$, or 1

The y -intercept is -1 .

For the equation of the line, substitute $m = 1$ and $b = -1$ in $y = mx + b$.

So, the equation is $y = x - 1$, which is the equation in part d.

From Graph B:

Two points on the line have coordinates: $(2, 5)$ and $(-2, 1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{1 - 5}{-2 - 2} = \frac{-4}{-4}$, or 1

The y -intercept is 3 .

For the equation of the line, substitute $m = 1$ and $b = 3$ in $y = mx + b$.

So, the equation is $y = x + 3$, which is the equation in part c.

From Graph C:

Two points on the line have coordinates: $(-3, -1)$ and $(1, -5)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-5 - (-1)}{1 - (-3)} = \frac{-4}{4}$, or -1

The y -intercept is -4 .

For the equation of the line, substitute $m = -1$ and $b = -4$ in $y = mx + b$.

So, the equation is $y = -x - 4$, which is the equation in part a.

From Graph D:

Two points on the line have coordinates: $(-2, 3)$ and $(2, -1)$

Slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

Slope of the line is: $\frac{-1 - 3}{2 - (-2)} = \frac{-4}{4}$, or -1

The y -intercept is 1 .

For the equation of the line, substitute $m = -1$ and $b = 1$ in $y = mx + b$.

So, the equation is $y = -x + 1$, which is the equation in part b.

20. From Graph A:

Two points on the line have coordinates: $(-3, 3)$ and $(3, 1)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{1 - 3}{3 - (-3)} = \frac{-2}{6}, \text{ or } -\frac{1}{3}$$

The y -intercept is 2.

These match the slope and y -intercept in part d.

From Graph B:

Two points on the line have coordinates: $(1, 5)$ and $(-1, -1)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{-1 - 5}{-1 - 1} = \frac{-6}{-2}, \text{ or } 3$$

The y -intercept is 2.

These match the slope and y -intercept in part a.

From Graph C:

Two points on the line have coordinates: $(3, -1)$ and $(-3, -3)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{-3 - (-1)}{-3 - 3} = \frac{-2}{-6}, \text{ or } \frac{1}{3}$$

The y -intercept is -2 .

These match the slope and y -intercept in part b.

From Graph D:

Two points on the line have coordinates: $(-1, 1)$ and $(0, -2)$

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{-2 - 1}{0 - (-1)} = -3$$

The y -intercept is -2 .

These match the slope and y -intercept in part c.

21. Parallel lines have the same slope.

Look for equations with the same value of m .

The graphs of these equations have the same slope of -5 , so they represent parallel lines:

$$y = -5x - 7 \text{ and } y = -5x + 13$$

The graphs of these equations have the same slope of 5 , so they represent parallel lines:

$$y = 5x + 15 \text{ and } y = 5x + 24$$

The graphs of these equations have the same slope of $\frac{1}{5}$, so they represent parallel lines:

$$y = \frac{1}{5}x + 9 \text{ and } y = \frac{1}{5}x + 21$$

The graphs of these equations have the same slope of $-\frac{1}{5}$, so they represent parallel lines:

$$y = -\frac{1}{5}x + 15 \text{ and } y = -\frac{1}{5}x$$

Perpendicular lines have slopes that are negative reciprocals.

Look for equations whose values of m have the product of -1 .

The graphs of these equations have slopes with the product of -1 , so they represent perpendicular lines:

$$y = -5x - 7 \text{ and } y = \frac{1}{5}x + 9; y = -5x - 7 \text{ and } y = \frac{1}{5}x + 21;$$

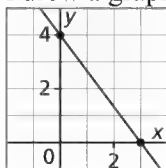
$$y = -5x + 13 \text{ and } y = \frac{1}{5}x + 9; y = -5x + 13 \text{ and } y = \frac{1}{5}x + 21;$$

$$y = 5x + 15 \text{ and } y = -\frac{1}{5}x + 15; y = 5x + 15 \text{ and } y = -\frac{1}{5}x;$$

$$y = 5x + 24 \text{ and } y = -\frac{1}{5}x + 15; y = 5x + 24 \text{ and } y = -\frac{1}{5}x$$

C

22. I drew a graph with y -intercept 4 and x -intercept 3.



$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of the line is: } \frac{0 - 4}{3 - 0} = \frac{-4}{3}, \text{ or } -\frac{4}{3}$$

The y -intercept is 4.

For the equation of the line, substitute $m = -\frac{4}{3}$ and $b = 4$ in $y = mx + b$.

$$\text{So, the equation is: } y = -\frac{4}{3}x + 4$$

23. Since $F(4, -6)$ lies on the line with equation $y = \frac{5}{3}x + c$, the coordinates of F must satisfy the equation.

$$\text{Substitute } x = 4 \text{ and } y = -6 \text{ in } y = \frac{5}{3}x + c.$$

$$-6 = \frac{5}{3}(4) + c \quad \text{Solve for } c.$$

$$-6 = \frac{20}{3} + c$$

$$-\frac{18}{3} - \frac{20}{3} = c$$

$$c = -\frac{38}{3}$$

24. Since $E(-3, 5)$ lies on the line with equation $y = mx - \frac{7}{8}$, the coordinates of E must satisfy the equation.

Substitute $x = -3$ and $y = 5$ in $y = mx - \frac{7}{8}$.

$$5 = m(-3) - \frac{7}{8}$$

Solve for m .

$$5 = -3m - \frac{7}{8}$$

$$\frac{40}{8} + \frac{7}{8} = -3m$$

$$\frac{47}{8} = -3m$$

Divide each side by -3 .

$$m = \frac{47}{-24}, \text{ or } -\frac{47}{24}$$

Lesson 6.5 **Slope-Point Form
of the Equation for a Linear Function**

Exercises (pages 372–374)

A

4. Compare each given equation with slope-point form: $y - y_1 = m(x - x_1)$, where m is the slope and the coordinates of a point on the line are (x_1, y_1) .

a) $y - 5 = -4(x - 1)$

The slope m is -4 .

The coordinates of a point on the line are $(1, 5)$.

b) $y + 7 = 3(x - 8)$; write this equation as:

$$y - (-7) = 3(x - 8)$$

The slope m is 3 .

The coordinates of a point on the line are $(8, -7)$.

c) $y + 11 = (x + 15)$; write this equation as:

$$y - (-11) = 1(x - (-15))$$

The slope m is 1 .

The coordinates of a point on the line are $(-15, -11)$.

d) $y = 5(x - 2)$; write this equation as:

$$y - 0 = 5(x - 2)$$

The slope m is 5 .

The coordinates of a point on the line are $(2, 0)$.

e) $y + 6 = \frac{4}{7}(x + 3)$; write this equation as:

$$y - (-6) = \frac{4}{7}(x - (-3))$$

The slope m is $\frac{4}{7}$.

The coordinates of a point are on the line $(-3, -6)$.

f) $y - 21 = -\frac{8}{5}(x + 16)$; write this equation as:

$$y - 21 = -\frac{8}{5}(x - (-16))$$

The slope m is $-\frac{8}{5}$.

The coordinates of a point are on the line $(-16, 21)$.

5. Substitute the given values of slope and coordinates into the equation in slope-point form: $y - y_1 = m(x - x_1)$, where m is the slope and the coordinates of a point on the line are (x_1, y_1) .

a) The graph has slope -5 and passes through $P(-4, 2)$.

Substitute: $y_1 = 2$, $m = -5$, and $x_1 = -4$

$$y - 2 = -5(x - (-4))$$

An equation is: $y - 2 = -5(x + 4)$

- b) The graph has slope 7 and passes through Q(6, -8).

Substitute: $y_1 = -8$, $m = 7$, and $x_1 = 6$

$$y - (-8) = 7(x - 6)$$

An equation is: $y + 8 = 7(x - 6)$

- c) The graph has slope $-\frac{3}{4}$ and passes through R(7, -5).

Substitute: $y_1 = -5$, $m = -\frac{3}{4}$, and $x_1 = 7$

$$y - (-5) = -\frac{3}{4}(x - 7)$$

An equation is: $y + 5 = -\frac{3}{4}(x - 7)$

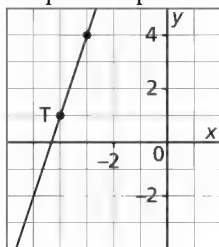
- d) The graph has slope 0 and passes through S(3, -8).

A line with slope 0 is horizontal and it intersects the y -axis.

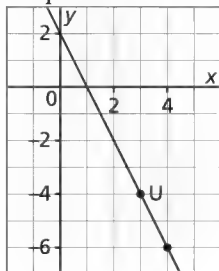
The y -coordinate of P is -8.

So, an equation is: $y = -8$

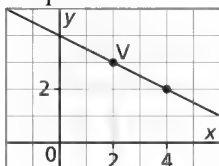
6. a) Mark a point at T(-4, 1). The slope is 3, so the rise is 3 and the run is 1. From T, move 3 squares up and 1 square right. Mark another point. Draw a line through the points.



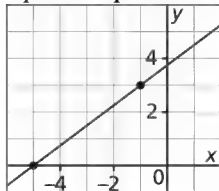
- b) Mark a point at U(3, -4). The slope is -2, so the rise is -2 and the run is 1. From U, move 2 squares down and 1 square right. Mark another point. Draw a line through the points.



- c) Mark a point at V(2, 3). The slope is $-\frac{1}{2}$, so the rise is -1 and the run is 2. From V, move 1 square down and 2 squares right. Mark another point. Draw a line through the points.



- d) The point at an x -intercept of -5 has coordinates $(-5, 0)$. Mark a point at $(-5, 0)$. The slope is $\frac{3}{4}$, so the rise is 3 and the run is 4. From the point at the intercept, move 3 squares up and 4 squares right. Mark another point. Draw a line through the points.



B

7. Compare each given equation with the equation $y - y_1 = m(x - x_1)$, to identify the slope m and the coordinates of a point on the line (x_1, y_1) .

- a) $y + 2 = -3(x - 4)$; write this equation as:

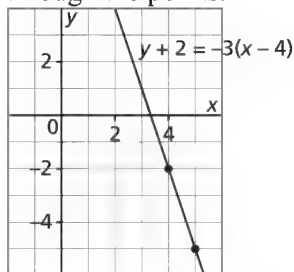
$$y - (-2) = -3(x - 4)$$

The slope m is -3 .

The coordinates of a point on the line are $(4, -2)$.

The graph has slope -3 and passes through the point $(4, -2)$.

Mark a point at $(4, -2)$. The slope is -3 , so the rise is -3 and the run is 1. From the marked point, move 3 squares down and 1 square right. Mark another point. Draw a line through the points.



- b) $y + 4 = 2(x + 3)$; write this equation as:

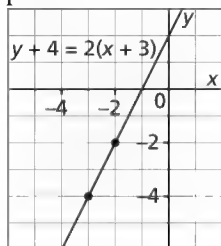
$$y - (-4) = 2(x - (-3))$$

The slope m is 2.

The coordinates of a point on the line are $(-3, -4)$.

The graph has slope 2 and passes through the point $(-3, -4)$.

Mark a point at $(-3, -4)$. The slope is 2, so the rise is 2 and the run is 1. From the marked point, move 2 squares up and 1 square right. Mark another point. Draw a line through the points.



- c) $y - 3 = (x + 5)$; write this equation as:

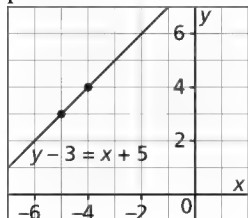
$$y - 3 = 1(x - (-5))$$

The slope m is 1.

The coordinates of a point on the line are $(-5, 3)$.

The graph has slope 1 and passes through the point $(-5, 3)$.

Mark a point at $(-5, 3)$. The slope is 1, so the rise is 1 and the run is 1. From the marked point, move 1 square up and 1 square right. Mark another point. Draw a line through the points.



- d) $y = -(x - 2)$; write this equation as:

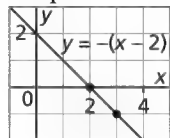
$$y - 0 = -1(x - 2)$$

The slope m is -1 .

The coordinates of a point on the line are $(2, 0)$.

The graph has slope -1 and passes through the point $(2, 0)$.

Mark a point at $(2, 0)$. The slope is -1 , so the rise is -1 and the run is 1. From the marked point, move 1 square down and 1 square right. Mark another point. Draw a line through the points.



8. a) To determine the equation of a line, I can use the slope-point of the equation:

$y - y_1 = m(x - x_1)$, and substitute the given values.

I substitute: $y_1 = 5$, $m = -4$, and $x_1 = -3$

$y - y_1 = m(x - x_1)$ becomes

$$y - 5 = -4(x - (-3))$$

$$y - 5 = -4(x + 3)$$

This is the same equation as that given in the question.

- b) I apply the distributive property to the equation in part a.

$$y - 5 = -4(x + 3)$$

$$y - 5 = -4x - 12 \quad \text{Collect like terms.}$$

$$y = -4x - 12 + 5$$

$$y = -4x - 7$$

This is the same equation as that given in the question.

9. a) i) From the graph:

A point on the line has coordinates $P(-2, 4)$. Another point on the line has coordinates $(1, 0)$. Use the coordinates of these points to calculate the slope of the line.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{0 - 4}{1 - (-2)}$$

$$\text{Slope} = \frac{-4}{3}, \text{ or } -\frac{4}{3}$$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

$$\text{Substitute: } y_1 = 4, m = -\frac{4}{3}, \text{ and } x_1 = -2$$

$$y - 4 = -\frac{4}{3}(x - (-2))$$

$$\text{An equation is: } y - 4 = -\frac{4}{3}(x + 2)$$

b) i) $y - 4 = -\frac{4}{3}(x + 2)$ Solve for y .

$$y - 4 = -\frac{4}{3}x - \frac{8}{3}$$

$$y = -\frac{4}{3}x - \frac{8}{3} + 4$$

$$y = -\frac{4}{3}x - \frac{8}{3} + \frac{12}{3}$$

$$y = -\frac{4}{3}x + \frac{4}{3}$$

From the equation, the y -intercept is $\frac{4}{3}$.

For the x -intercept, substitute $y = 0$ in the equation above.

$$y = -\frac{4}{3}x + \frac{4}{3}$$

$$0 = -\frac{4}{3}x + \frac{4}{3}$$

Solve for x .

$$\frac{4}{3}x = \frac{4}{3}$$

Divide each side by $\frac{4}{3}$.

$$x = 1$$

The x -intercept is 1.

a) ii) From the graph:

A point on the line has coordinates P(3, 3). Another point on the line has coordinates (-2, 1). Use the coordinates of these points to calculate the slope of the line.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{1 - 3}{-2 - 3}$$

$$\text{Slope} = \frac{-2}{-5}, \text{ or } \frac{2}{5}$$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

$$\text{Substitute: } y_1 = 3, m = \frac{2}{5}, \text{ and } x_1 = 3$$

$$y - 3 = \frac{2}{5}(x - 3)$$

An equation is: $y - 3 = \frac{2}{5}(x - 3)$

b) ii) $y - 3 = \frac{2}{5}(x - 3)$ Solve for y .

$$y - 3 = \frac{2}{5}x - \frac{6}{5}$$

$$y = \frac{2}{5}x - \frac{6}{5} + 3$$

$$y = \frac{2}{5}x - \frac{6}{5} + \frac{15}{5}$$

$$y = \frac{2}{5}x + \frac{9}{5}$$

From the equation, the y -intercept is $\frac{9}{5}$.

For the x -intercept, substitute $y = 0$ in the equation above.

$$y = \frac{2}{5}x + \frac{9}{5}$$

$$0 = \frac{2}{5}x + \frac{9}{5}$$

Solve for x .

$$\frac{2}{5}x = -\frac{9}{5}$$

Divide each side by $\frac{2}{5}$.

$$x = \left(-\frac{9}{5}\right)\left(\frac{5}{2}\right)$$

$$x = -\frac{9}{2}$$

The x -intercept is $-\frac{9}{2}$.

a) iii) From the graph:

A point on the line has coordinates $P(-4, -2)$. Another point on the line has coordinates $(-1, -1)$. Use the coordinates of these points to calculate the slope of the line.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - (-2)}{-1 - (-4)}$$

$$\text{Slope} = \frac{1}{3}$$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = \frac{1}{3}$, and $x_1 = -4$

$$y - (-2) = \frac{1}{3}(x - (-4))$$

$$\text{An equation is: } y + 2 = \frac{1}{3}(x + 4)$$

b) iii) $y + 2 = \frac{1}{3}(x + 4)$

Solve for y .

$$y + 2 = \frac{1}{3}x + \frac{4}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3} - 2$$

$$y = \frac{1}{3}x + \frac{4}{3} - \frac{6}{3}$$

$$y = \frac{1}{3}x - \frac{2}{3}$$

From the equation, the y -intercept is $-\frac{2}{3}$.

For the x -intercept, substitute $y = 0$ in the equation above.

$$y = \frac{1}{3}x - \frac{2}{3}$$

$$0 = \frac{1}{3}x - \frac{2}{3}$$

Solve for x .

$$\frac{1}{3}x = \frac{2}{3}$$

Multiply each side by 3.

$$x = 3\left(\frac{2}{3}\right)$$

$$x = 2$$

The x -intercept is 2.

a) iv) From the graph:

A point on the line has coordinates $P(1, -2)$. Another point on the line has coordinates $(-1, 3)$. Use the coordinates of these points to calculate the slope of the line.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{3 - (-2)}{-1 - 1}$$

$$\text{Slope} = \frac{5}{-2}, \text{ or } -\frac{5}{2}$$

Substitute the slope and the coordinates of point P in the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = -\frac{5}{2}$, and $x_1 = 1$

$$y - (-2) = -\frac{5}{2}(x - 1)$$

An equation is: $y + 2 = -\frac{5}{2}(x - 1)$

b) iv) $y + 2 = -\frac{5}{2}(x - 1)$ Solve for y .

$$y + 2 = -\frac{5}{2}x + \frac{5}{2}$$

$$y = -\frac{5}{2}x + \frac{5}{2} - 2$$

$$y = -\frac{5}{2}x + \frac{5}{2} - \frac{4}{2}$$

$$y = -\frac{5}{2}x + \frac{1}{2}$$

From the equation, the y -intercept is $\frac{1}{2}$.

For the x -intercept, substitute $y = 0$ in the equation above.

$$y = -\frac{5}{2}x + \frac{1}{2}$$

$$0 = -\frac{5}{2}x + \frac{1}{2}$$

Solve for x .

$$\frac{5}{2}x = \frac{1}{2}$$

Multiply each side by $\frac{2}{5}$.

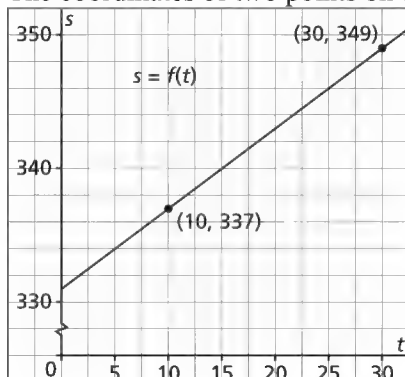
$$x = \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)$$

$$x = \frac{1}{5}$$

The x -intercept is $\frac{1}{5}$.

10. Different variables may be used.

- a)** Sketch a graph of the speed of sound as a function of the air temperature.
The coordinates of two points on the graph are: (10, 337) and (30, 349)



Use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{s - s_1}{t - t_1} = \frac{s_2 - s_1}{t_2 - t_1}$$

Substitute: $s_1 = 337$, $t_1 = 10$, $s_2 = 349$, $t_2 = 30$

$$\frac{s - 337}{t - 10} = \frac{349 - 337}{30 - 10}$$

$$\frac{s - 337}{t - 10} = \frac{12}{20}$$

$$\frac{s - 337}{t - 10} = 0.6 \quad \text{Multiply each side by } (t - 10).$$

$$(t - 10) \left(\frac{s - 337}{t - 10} \right) = 0.6(t - 10)$$

$$s - 337 = 0.6(t - 10)$$

The linear equation above represents the function.

- b)** Substitute $t = 0$ in the equation: $s - 337 = 0.6(t - 10)$

$$s - 337 = 0.6(0 - 10) \quad \text{Solve for } s.$$

$$s - 337 = -6$$

$$s = 337 - 6$$

$$s = 331$$

When the air temperature is 0°C , the speed of sound is 331 m/s.

- 11.** Use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

- a)** The two points are B(-2, -5) and C(1, 1).

Substitute: $y_1 = -5$, $x_1 = -2$, $y_2 = 1$, and $x_2 = 1$

$$\frac{y - (-5)}{x - (-2)} = \frac{1 - (-5)}{1 - (-2)}$$

$$\frac{y + 5}{x + 2} = \frac{6}{3}$$

$$\frac{y + 5}{x + 2} = 2$$

Multiply each side by $(x + 2)$.

$$(x + 2) \left(\frac{y + 5}{x + 2} \right) = 2(x + 2)$$

$$y + 5 = 2(x + 2)$$

The equation above is in slope-point form.

The slope is 2. Another form of the equation is: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 1$, $m = 2$, and $x_1 = 1$

$$y - 1 = 2(x - 1)$$

Apply the distributive property to one of the equations above.

$$y + 5 = 2(x + 2)$$

$$y + 5 = 2x + 4$$

$$y = 2x + 4 - 5$$

$$y = 2x - 1$$

The equation above is in slope-intercept form.

- b) The two points are Q(-4, 7) and R(5, -2).

Substitute: $y_1 = 7$, $x_1 = -4$, $y_2 = -2$, and $x_2 = 5$

$$\frac{y - 7}{x - (-4)} = \frac{-2 - 7}{5 - (-4)}$$

$$\frac{y - 7}{x + 4} = \frac{-9}{9}$$

$$\frac{y - 7}{x + 4} = -1 \quad \text{Multiply each side by } (x + 4).$$

$$(x + 4) \left(\frac{y - 7}{x + 4} \right) = -1(x + 4)$$

$$y - 7 = -(x + 4)$$

The equation above is in slope-point form.

The slope is -1. Another form of the equation is: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = -1$, and $x_1 = 5$

$$y + 2 = -(x - 5)$$

Apply the distributive property to one of the equations above.

$$y - 7 = -(x + 4)$$

$$y - 7 = -x - 4$$

$$y = -x - 4 + 7$$

$$y = -x + 3$$

The equation above is in slope-intercept form.

- c) The two points are U(-3, -7) and V(2, 8).

Substitute: $y_1 = -7$, $x_1 = -3$, $y_2 = 8$, and $x_2 = 2$

$$\frac{y - (-7)}{x - (-3)} = \frac{8 - (-7)}{2 - (-3)}$$

$$\frac{y - (-7)}{x - (-3)} = \frac{15}{5}$$

$$\frac{y + 7}{x + 3} = 3 \quad \text{Multiply each side by } (x + 3).$$

$$(x + 3) \left(\frac{y + 7}{x + 3} \right) = 3(x + 3)$$

$$y + 7 = 3(x + 3)$$

The equation above is in slope-point form.

The slope is 3. Another form of the equation is: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 8$, $m = 3$, and $x_1 = 2$

$$y - 8 = 3(x - 2)$$

Apply the distributive property to one of the equations above.

$$y + 7 = 3(x + 3)$$

$$y + 7 = 3x + 9$$

$$y = 3x + 9 - 7$$

$$y = 3x + 2$$

The equation above is in slope-intercept form.

- d) The two points are H(-7, -1) and J(-5, -5).

Substitute: $y_1 = -1$, $x_1 = -7$, $y_2 = -5$, and $x_2 = -5$

$$\frac{y - (-1)}{x - (-7)} = \frac{-5 - (-1)}{-5 - (-7)}$$

$$\frac{y - (-1)}{x - (-7)} = \frac{-4}{2}$$

$$\frac{y + 1}{x + 7} = -2 \quad \text{Multiply each side by } (x + 7).$$

$$(x + 7) \left(\frac{y + 1}{x + 7} \right) = -2(x + 7)$$

$$y + 1 = -2(x + 7)$$

The equation above is in slope-point form.

Apply the distributive property to the equation above.

$$y + 1 = -2(x + 7)$$

$$y + 1 = -2x - 14$$

$$y = -2x - 14 - 1$$

$$y = -2x - 15$$

The equation above is in slope-intercept form.

12. From Graph A:

Its slope is 1 and its y -intercept is 1.

The only given equation with slope 1 is the equation in part b: $y - 3 = (x - 2)$

Apply the distributive property to determine this equation in slope-intercept form.

$$y - 3 = (x - 2)$$

$$y - 3 = x - 2$$

$$y = x - 2 + 3$$

$$y = x + 1$$

The above equation represents a line with slope 1 and y -intercept 1.

So, the equation in part b represents Graph A.

From Graph B:

Its slope is 2 and its y -intercept is 5.

There are two given equations with slope 2.

They are the equation in part a: $y + 3 = 2(x - 1)$ and the equation in part c: $y - 3 = 2(x + 1)$

Apply the distributive property to determine each equation in slope-intercept form.

For the equation in part a:

$$y + 3 = 2(x - 1)$$

$$y + 3 = 2x - 2$$

$$y = 2x - 2 - 3$$

$$y = 2x - 5$$

The above equation represents a line with slope 2 and y -intercept -5 .

Graph C has slope 2 and y -intercept -5 .

So, the equation in part a represents Graph C.

For the equation in part c:

$$y - 3 = 2(x + 1)$$

$$y - 3 = 2x + 2$$

$$y = 2x + 2 + 3$$

$$y = 2x + 5$$

The above equation represents a line with slope 2 and y -intercept 5.

Graph B has slope 2 and y -intercept 5

So, the equation in part c represents Graph B.

From Graph D:

Its slope is -1 and its y -intercept is -5 .

The only given equation with slope -1 is the equation in part d: $y + 3 = -(x + 2)$

Apply the distributive property to determine this equation in slope-intercept form

$$y + 3 = -(x + 2)$$

$$y + 3 = -x - 2$$

$$y = -x - 2 - 3$$

$$y = -x - 5$$

The above equation represents a line with slope -1 and y -intercept -5 .

So, the equation in part d represents Graph D.

13. The graph of $y + y_1 = m(x + x_1)$ has slope m and passes through the point with coordinates $(-x_1, -y_1)$.

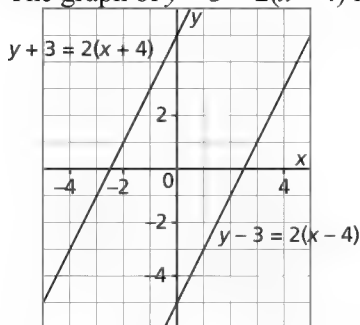
The graph of $y - y_1 = m(x - x_1)$ has slope m and passes through the point with coordinates (x_1, y_1) .

So, the lines are parallel.

For example:

The graph of $y + 3 = 2(x + 4)$ has slope 2 and passes through the point with coordinates $(-4, -3)$.

The graph of $y - 3 = 2(x - 4)$ has slope 2 and passes through the point with coordinates $(4, 3)$.



From the graph, it looks as though the y -intercepts are opposite numbers and the x -intercepts are opposite numbers. Determine the intercepts of each graph to check.

For $y + 3 = 2(x + 4)$:

Substitute $x = 0$ to determine the y -intercept.

$$y + 3 = 2(0 + 4)$$

$$y + 3 = 8$$

$$y = 5$$

Substitute $y = 0$ to determine the x -intercept.

$$0 + 3 = 2(x + 4)$$

$$3 = 2x + 8$$

$$-5 = 2x$$

$$x = -2.5$$

For $y - 3 = 2(x - 4)$:

Substitute $x = 0$ to determine the y -intercept.

$$y - 3 = 2(0 - 4)$$

$$y - 3 = -8$$

$$y = -5$$

Substitute $y = 0$ to determine the x -intercept.

$$0 - 3 = 2(x - 4)$$

$$-3 = 2x - 8$$

$$5 = 2x$$

$$x = 2.5$$

So, the graphs have opposite y -intercepts, opposite x -intercepts, and equal slopes.

- 14. a)** The graph has slope 2 and y -intercept 4.

Three of the given equations represent graphs with slope 2.

Write each equation in slope-intercept form.

$$y + 1 = 2(x - 2) \text{ becomes}$$

$$y + 1 = 2x - 4$$

$$y = 2x - 4 - 1$$

$$y = 2x - 5$$

The above equation represents a graph with y -intercept 5, so it is not the correct equation.

$$y + 2 = 2(x - 1) \text{ becomes}$$

$$y + 2 = 2x - 2$$

$$y = 2x - 2 - 2$$

$$y = 2x - 4$$

The above equation represents a graph with y -intercept -4 , so it is not the correct equation.

$$y - 2 = 2(x + 1) \text{ becomes}$$

$$y - 2 = 2x + 2$$

$$y = 2x + 2 + 2$$

$$y = 2x + 4$$

The above equation represents a graph with y -intercept 4, so it is the correct equation.

- b)** The graph has slope $\frac{1}{3}$ and y -intercept between 1 and 2.

Three of the given equations represent graphs with slope $\frac{1}{3}$.

Write each equation in slope-intercept form.

$$y - 1 = \frac{1}{3}(x - 2) \text{ becomes}$$

$$y - 1 = \frac{1}{3}x - \frac{2}{3}$$

$$y = \frac{1}{3}x - \frac{2}{3} + 1$$

$$y = \frac{1}{3}x + \frac{1}{3}$$

The above equation represents a graph with y -intercept $\frac{1}{3}$, so it is not the correct equation.

$$y + 2 = \frac{1}{3}(x + 1) \text{ becomes}$$

$$y + 2 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{1}{3} - 2$$

$$y = \frac{1}{3}x - \frac{5}{3}$$

The above equation represents a graph with y -intercept $-\frac{5}{3}$, so it is not the correct equation.

$$y - 2 = \frac{1}{3}(x - 1) \text{ becomes}$$

$$y - 2 = \frac{1}{3}x - \frac{1}{3}$$

$$y = \frac{1}{3}x - \frac{1}{3} + 2$$

$$y = \frac{1}{3}x + \frac{5}{3}$$

The above equation represents a graph with y -intercept $\frac{5}{3}$; since this number is between 1 and 2, this is the correct equation.

- c) The graph has slope $-\frac{2}{3}$ and y -intercept between 2 and 3.

Two of the given equations represent graphs with slope $-\frac{2}{3}$.

Write each equation in slope-intercept form.

$$y - 1 = -\frac{2}{3}(x - 2) \text{ becomes}$$

$$y - 1 = -\frac{2}{3}x + \frac{4}{3}$$

$$y = -\frac{2}{3}x + \frac{4}{3} + 1$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

The above equation represents a graph with y -intercept $\frac{7}{3}$, since this number is between 2 and 3, this might be the correct equation.

$$y - 2 = -\frac{2}{3}(x - 1) \text{ becomes}$$

$$y - 2 = -\frac{2}{3}x + \frac{2}{3}$$

$$y = -\frac{2}{3}x + \frac{2}{3} + 2$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

The above equation represents a graph with y -intercept $\frac{8}{3}$; since this number is between 2 and 3, this might be the correct equation.

The graph passes through (2, 1). Substitute $x = 2$ in both equations to check the y values.

$$y = -\frac{2}{3}x + \frac{7}{3}$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

$$y = -\frac{2}{3} \times 2 + \frac{7}{3}$$

$$y = -\frac{2}{3} \times 2 + \frac{8}{3}$$

$$y = -\frac{4}{3} + \frac{7}{3}$$

$$y = -\frac{4}{3} + \frac{8}{3}$$

$$y = \frac{3}{3}, \text{ or } 1$$

$$y = \frac{4}{3}$$

So, equation $y = -\frac{2}{3}x + \frac{7}{3}$ is the correct equation.

15. I used a graphing calculator. I wrote each equation in the form $y = f(x)$.

a) $y + \frac{2}{7} = \frac{3}{8}(x - 5)$ becomes $y = \frac{3}{8}(x - 5) - \frac{2}{7}$

On a TI-83 calculator, press $\boxed{Y=}$, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1=(3/8)(X-5)-(
2/7)
Y2=
Y3=
Y4=
Y5=
Y6=

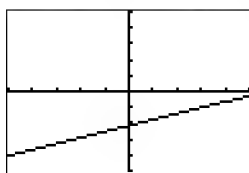
```

Press $\boxed{\text{GRAPH}}$. To get the screen below, press $\boxed{\text{WINDOW}}$ and set the window as follows:

```

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1

```



b) $y - \frac{10}{3} = -\frac{2}{9}(x + 11)$ becomes $y = -\frac{2}{9}(x + 11) + \frac{10}{3}$

On a TI-83 calculator, press $\boxed{Y=}$, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1 (-2/9)(X+11)
+(10/3)
Y2=
Y3=
Y4=
Y5=
Y6=

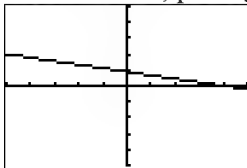
```

Press **GRAPH**. To get the screen below, press **WINDOW** and set the window as follows:

```

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Xres=1

```



- c) $y + 1.4 = 0.375(x + 4)$ becomes $y = 0.375(x + 4) - 1.4$

On a TI-83 calculator, press **Y=**, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1 0.375(X+4)-1.4
Y2=
Y3=
Y4=
Y5=
Y6=

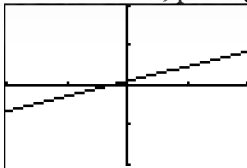
```

Press **GRAPH**. To get the screen below, press **WINDOW** and set the window as follows:

```

WINDOW
Xmin=-2
Xmax=2
Xscl=1
Ymin=-2
Ymax=2
Yscl=1
Xres=1

```



- d) $y - 2.35 = -0.5(x - 6.3)$ becomes $y = -0.5(x - 6.3) + 2.35$

On a TI-83 calculator, press **Y=**, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1 -0.5(X-6.3)+2.35
Y2=
Y3=
Y4=
Y5=
Y6=

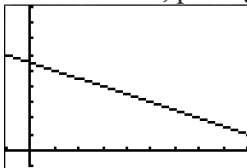
```

Press **GRAPH**. To get the screen below, press **WINDOW** and set the window as follows:

```

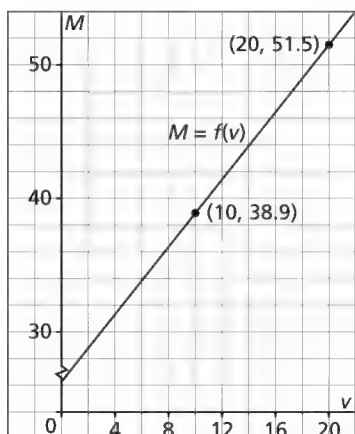
WINDOW
Xmin=-1
Xmax=9
Xscl=1
Ymin=-1
Ymax=9
Yscl=1
Xres=1

```



16. a) Sketch a graph of the mass, M grams, of the cylinder and liquid as a function of the volume, v millilitres, of liquid.

The coordinates of two points on the graph are: (10, 38.9) and (20, 51.5)



$$\text{Slope} = \frac{\text{change in } M\text{-coordinates}}{\text{change in } v\text{-coordinates}}$$

$$\text{Slope} = \frac{51.5 - 38.9}{20 - 10}$$

$$\text{Slope} = \frac{12.6}{10}$$

$$\text{Slope} = 1.26$$

The slope is the quotient of a mass in grams and a volume in millilitres, so the slope is 1.26 g/mL.

The slope is the mass in grams of 1 mL of liquid.

- b)** Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Replace y with M , and replace x with v .

$$M - M_1 = m(v - v_1)$$

Substitute: $M_1 = 51.5$, $m = 1.26$, and $v_1 = 20$

$$M - 51.5 = 1.26(v - 20)$$

The linear equation above represents the function.

- c)** Substitute $v = 30$ in the equation: $M - 51.5 = 1.26(v - 20)$

$$M - 51.5 = 1.26(30 - 20)$$

$$M - 51.5 = 1.26(10)$$

$$M - 51.5 = 12.6$$

$$M = 12.6 + 51.5$$

$$M = 64.1$$

When the volume of liquid is 30 mL, the mass of the cylinder and liquid is 64.1 g.

- d)** When the volume of the liquid is 0, the mass of the cylinder and liquid, M , will represent the mass of just the cylinder.

Substitute $v = 0$ in the equation: $M - 51.5 = 1.26(v - 20)$

$$M - 51.5 = 1.26(0 - 20)$$

$$M - 51.5 = 1.26(-20)$$

$$M - 51.5 = -25.2$$

$$M = -25.2 + 51.5$$

$$M = 26.3$$

The mass of the empty graduated cylinder is 26.3 g.

17. a) The mass of potash is a linear function of the time in years since 2005.

The year 2005 is 0 years since 2005.

The year 2007 is 2 years since 2005.

So, two ordered pairs that satisfy this function are: (0, 8.2) and (2, 9.4)

Use these ordered pairs to determine the slope of a graph of the function.

Let t represent the time in years since 2005.

Let p represent the mass of potash in millions of tonnes.

$$\text{Slope} = \frac{\text{change in } p\text{-coordinates}}{\text{change in } t\text{-coordinates}}$$

$$\text{Slope} = \frac{9.4 - 8.2}{2 - 0}$$

$$\text{Slope} = \frac{1.2}{2}$$

$$\text{Slope} = 0.6$$

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Replace y with p , and replace x with t .

$$p - p_1 = m(t - t_1)$$

Substitute: $p_1 = 8.2$, $m = 0.6$, and $t_1 = 0$

$$p - 8.2 = 0.6(t - 0)$$

$$p - 8.2 = 0.6t$$

$$p = 0.6t + 8.2$$

The linear equation above represents the relation.

- b) The year 2010 is 5 years since 2005.

Substitute $t = 5$ in the equation: $p = 0.6t + 8.2$

$$p = 0.6(5) + 8.2$$

$$p = 3 + 8.2$$

$$p = 11.2$$

In 2010, the sales of potash will be 11.2 million tonnes.

The year 2015 is 10 years since 2005.

Substitute $t = 10$ in the equation: $p = 0.6t + 8.2$

$$p = 0.6(10) + 8.2$$

$$p = 6 + 8.2$$

$$p = 14.2$$

In 2015, the sales of potash will be 14.2 million tonnes.

I assume that the relation continues for times beyond 2007 and remains linear.

18. a) The number of students is a linear function of the time in years since 2001.

The year 2003 is 2 years after 2001.

So, an ordered pair that satisfies this function is: (2, 3470)

The increase in student population is 198 students per year. This is the slope of a graph of this function.

Let t represent the time in years after 2001.

Let p represent the student population.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Replace y with p , and replace x with t .

$$p - p_1 = m(t - t_1)$$

Substitute: $p_1 = 3470$, $m = 198$, and $t_1 = 2$

$$p - 3470 = 198(t - 2)$$

The linear equation above represents the relation.

- b) The year 2005 is 4 years after 2001.

Substitute $t = 4$ in the equation: $p - 3470 = 198(t - 2)$

$$p - 3470 = 198(4 - 2)$$

$$p - 3470 = 198(2)$$

$$p = 396 + 3470$$

$$p = 3866$$

In January 2005, there were approximately 3866 students in francophone schools.

To check the answer, I know that the number of students increases by 198 a year, so, in the 2 years from January 2003 to January 2005, the number of students will increase by:
 $2(198) = 396$

I add this number to the number of students in January 2003: $3470 + 396 = 3866$

This agrees with the answer I obtained by substitution.

19. a) Given $G(-3, 11)$ and $H(4, -3)$

$$\text{Slope of GH} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope of GH} = \frac{-3 - 11}{4 - (-3)}$$

$$\text{Slope of GH} = \frac{-14}{7}$$

$$\text{Slope of GH} = -2$$

- b) Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of G and the slope; that is, substitute: $y_1 = 11$, $m = -2$, and $x_1 = -3$

$$y - 11 = -2(x - (-3))$$

$$y - 11 = -2(x + 3)$$

- c) Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of H and the slope; that is, substitute: $y_1 = -3$, $m = -2$, and $x_1 = 4$

$$y - (-3) = -2(x - 4)$$

$$y + 3 = -2(x - 4)$$

- d) Write each equation in slope-intercept form.

$$y - 11 = -2(x + 3) \text{ becomes}$$

$$y - 11 = -2x - 6$$

$$y = -2x - 6 + 11$$

$$y = -2x + 5$$

$$y + 3 = -2(x - 4) \text{ becomes}$$

$$y + 3 = -2x + 8$$

$$y = -2x + 8 - 3$$

$$y = -2x + 5$$

Since both equations have the same slope-intercept form, the equations are equivalent.

Another strategy I could have used is to graph both equations to see if the graphs are the same line.

20. a) i) A line that is parallel to the line with equation $y = -\frac{4}{3}x + 1$ has the same slope; that

is, its slope is $-\frac{4}{3}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of D(-5, -3) and the slope; that is, substitute: $y_1 = -3$,

$$m = -\frac{4}{3}, \text{ and } x_1 = -5$$

$$y - (-3) = -\frac{4}{3}(x - (-5))$$

$$y + 3 = -\frac{4}{3}(x + 5)$$

- ii) A line that is perpendicular to the line with equation $y = -\frac{4}{3}x + 1$ has a slope that is

the negative reciprocal of $-\frac{4}{3}$; that is, its slope is $\frac{3}{4}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of D(-5, -3) and the slope; that is, substitute: $y_1 = -3$, $m =$

$$\frac{3}{4}, \text{ and } x_1 = -5$$

$$y - (-3) = \frac{3}{4}(x - (-5))$$

$$y + 3 = \frac{3}{4}(x + 5)$$

- b) The two equations have the same constant terms on each side of the equation. The equations have different values for the coefficient of x .

21. a) i) A line that is parallel to the line with equation $y = 2x + 3$ has the same slope; that is, its slope is 2.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of C(1, -2) and the slope; that is, substitute: $y_1 = -2$, $m = 2$, and $x_1 = 1$

$$y - (-2) = 2(x - 1)$$

$$y + 2 = 2(x - 1)$$

- ii) A line that is perpendicular to the line with equation $y = 2x + 3$ has a slope that is the negative reciprocal of 2; that is, its slope is $-\frac{1}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of C(1, -2) and the slope; that is, substitute: $y_1 = -2$, $m =$

$$-\frac{1}{2}, \text{ and } x_1 = 1$$

$$y - (-2) = -\frac{1}{2}(x - 1)$$

$$y + 2 = -\frac{1}{2}(x - 1)$$

- 22. a)** A line that is parallel to the line with equation $y - 3 = -\frac{5}{2}(x + 2)$ has the same slope; that is, its slope is $-\frac{5}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of E(2, 6) and the slope; that is, substitute: $y_1 = 6$, $m = -\frac{5}{2}$, and

$$x_1 = 2$$

$$y - 6 = -\frac{5}{2}(x - 2)$$

- b)** A line that is perpendicular to the line with equation $y - 3 = -\frac{5}{2}(x + 2)$ has a slope that is the negative reciprocal of $-\frac{5}{2}$; that is, its slope is $\frac{2}{5}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of E(2, 6) and the slope; that is, substitute: $y_1 = 6$, $m = \frac{2}{5}$, and

$$x_1 = 2$$

$$y - 6 = \frac{2}{5}(x - 2)$$

I know that each equation is correct because I can identify the coordinates of the given point and the given slope from each equation.

- 23. a)** The point at an x -intercept of 4 has coordinates (4, 0). The given equation is $y = \frac{3}{5}x - 7$;

so, the equation of a parallel line has the same slope; that is, its slope is $\frac{3}{5}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates (4, 0) and the slope; that is, substitute: $y_1 = 0$, $m = \frac{3}{5}$, and

$$x_1 = 4$$

$$y - 0 = \frac{3}{5}(x - 4)$$

$$y = \frac{3}{5}(x - 4)$$

- b)** The line with x -intercept -3 and y -intercept 6 passes through the points with coordinates $(-3, 0)$ and $(0, 6)$. To determine the slope of this line, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{6 - 0}{0 - (-3)}$$

$$\text{Slope} = \frac{6}{3}, \text{ or } 2$$

A perpendicular line has a slope that is the negative reciprocal of 2; that is, the slope of a perpendicular line is $-\frac{1}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of F(4, -1) and the slope; that is, substitute: $y_1 = -1$, $m = -\frac{1}{2}$, and $x_1 = 4$

$$y - (-1) = -\frac{1}{2}(x - 4)$$

$$y + 1 = -\frac{1}{2}(x - 4)$$

24. Determine the y -intercept of the line with equation: $y - 3 = \frac{2}{9}(x + 5)$

Substitute $x = 0$.

$$y - 3 = \frac{2}{9}(0 + 5) \quad \text{Solve for } y.$$

$$y - 3 = \frac{2}{9}(5)$$

$$y - 3 = \frac{10}{9}$$

$$y = \frac{10}{9} + \frac{27}{9}$$

$$y = \frac{37}{9}$$

The other line has y -intercept $\frac{37}{9}$ and a slope that is the negative reciprocal of $\frac{2}{9}$; that is, its slope is $-\frac{9}{2}$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute the coordinates of the intercept $\left(0, \frac{37}{9}\right)$ and the slope; that is, substitute: $y_1 = \frac{37}{9}$,

$$m = -\frac{9}{2}, \text{ and } x_1 = 0$$

$$y - \frac{37}{9} = -\frac{9}{2}(x - 0)$$

$$y - \frac{37}{9} = -\frac{9}{2}x$$

$$y = -\frac{9}{2}x + \frac{37}{9}$$

25. The given equation is: $y = -\frac{5}{3}x - \frac{25}{3}$

The slope of a graph of the equation above is $-\frac{5}{3}$.

The slope of the graph of a perpendicular line is the negative reciprocal of $-\frac{5}{3}$; that is, the

slope of a perpendicular line is $\frac{3}{5}$.

The perpendicular line passes through K(-2, -5).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -5$, $m = \frac{3}{5}$, and $x_1 = -2$

$$y - (-5) = \frac{3}{5}(x - (-2))$$

$$y + 5 = \frac{3}{5}(x + 2)$$

C

26. If two perpendicular lines intersect at a point, there are many possible equations for the lines; too many equations to count. The slope of one line could be any positive or negative real number. The slope of the perpendicular line would be the negative reciprocal of the real number.

Suppose the slope of one line is m .

The line passes through the point M(3, 5).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 5$ and $x_1 = 3$

$$y - 5 = m(x - 3)$$

The slope of the perpendicular line is the negative reciprocal of m ; that is, the slope is $-\frac{1}{m}$.

The line passes through the point M(3, 5).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 5$, $m = -\frac{1}{m}$ and $x_1 = 3$

$$y - 5 = -\frac{1}{m}(x - 3)$$

The two equations have the form: $y - 5 = m(x - 3)$ and $y - 5 = -\frac{1}{m}(x - 3)$, for all real values of m , except 0. When $m = 0$, the two equations are $y = 5$ and $x = 3$.

27. Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

The coordinates of the point at the y -intercept b are: $(0, b)$

Substitute: $y_1 = b$ and $x_1 = 0$ in the equation above.

$$y - b = m(x - 0)$$

Solve for y .

$$y - b = mx$$

$$y = mx + b$$

Checkpoint 2

Assess Your Understanding (page 376)

6.3

1. a) On a TI-83 calculator, press $\boxed{Y=}$, then input the equation $y = \frac{3}{2}x - 4$ with no spaces

between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1=(3/2)X-4
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

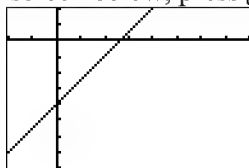
```

Press $\boxed{\text{GRAPH}}$. To get the screen below, press $\boxed{\text{WINDOW}}$ and set the window as follows:

```

WINDOW
Xmin=-2
Xmax=8
Xsc1=1
Ymin=-8
Ymax=2
Ysc1=1
Xres=1

```



- b) The line will have a greater slope when the x -coefficient increases. For example, use an x -coefficient of 4 and graph $y = 4x - 4$ on the same screen.

In the $\boxed{Y=}$ screen, below $Y1=(3/2)X-4$, input $Y2=4X-4$.

```

Plot1 Plot2 Plot3
Y1=(3/2)X-4
Y2=4X-4
Y3=
Y4=
Y5=
Y6=
Y7=

```

The line will have a lesser slope when the x -coefficient decreases. For example, use an x -coefficient of $\frac{2}{3}$ and graph the equation $y = \frac{2}{3}x - 4$ on the same screen.

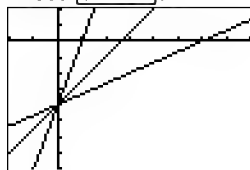
In the $\boxed{Y=}$ screen, below $Y2=4X-4$, input $Y3=(2/3)X-4$.

```

Plot1 Plot2 Plot3
Y1=(3/2)X-4
Y2=4X-4
Y3=(2/3)X-4
Y4=
Y5=
Y6=
Y7=

```

Press $\boxed{\text{GRAPH}}$.



- c) The line will have a greater y -intercept when the constant term increases. For example, use a constant term of -2 and graph $y = \frac{3}{2}x - 2$ on the same screen.

In the $\boxed{Y=}$ screen, below $Y1=(3/2)X-4$, input $Y2=(3/2)X-2$.

```

Plot1 Plot2 Plot3
Y1=(3/2)X-4
Y2=(3/2)X-2
Y3=
Y4=
Y5=
Y6=
Y7=

```

The line will have a lesser y -intercept when the constant term decreases. For example, use a constant term of -2 and graph $y = \frac{3}{2}x - 6$ on the same screen.

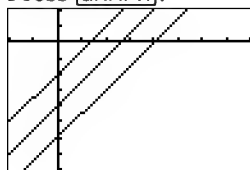
In the $[Y=]$ screen, below $Y2=(3/2)X-2$, input $Y3=(3/2)X-6$.

```

Plot1 Plot2 Plot3
Y1=(3/2)X-4
Y2=(3/2)X-2
Y3=(3/2)X-6
Y4=
Y5=
Y6=
Y7=

```

Press $[GRAPH]$.



6.4

2. a) From the graph, two points have coordinates $(0, 10)$ and $(2, 60)$.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{60 - 10}{2 - 0}$$

$$\text{Slope} = \frac{50}{2}$$

$$\text{Slope} = 25$$

Since the slope is the quotient of distance in kilometres and time in hours, the slope is 25 km/h, which is the average speed of Eric's snowmobile.

From the graph, the d -intercept is 10 km, which is how far Eric was from home when he began his snowmobile ride.

- b) An equation has the form $d = mt + b$, where m is the slope and b is the vertical intercept.

So, an equation is: $d = 25t + 10$

- c) i) Substitute $t = 2\frac{1}{4}$ in the equation, then solve for d .

Write $2\frac{1}{4}$ as 2.25.

$$d = 25t + 10$$

$$d = 25(2.25) + 10$$

$$d = 56.25 + 10$$

$$d = 66.25$$

Eric was 66.25 km from home after he had travelled $2\frac{1}{4}$ hours.

- ii) Substitute $d = 45$, then solve for t .

$$45 = 25t + 10$$

$$35 = 25t$$

$$\frac{35}{25} = t$$

$$t = 1.4$$

1.4 h is 1 h and 0.4×60 min, or 1 h 24 min.

Eric took 1 h 24 min to travel 45 km from home.

Subtract 10 from each side.

Divide each side by 25.

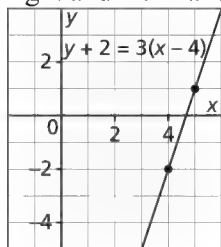
6.5

3. a) $y + 2 = 3(x - 4)$

The equation is in slope-point form, so I shall use the coordinates of a point and the slope to graph the equation.

From the equation, a point on the line has coordinates $(4, -2)$ and the slope of the line is 3.

On a grid, plot the point $(4, -2)$, then write the slope as $\frac{3}{1}$ and move 3 units up, 1 unit right and mark another point at $(5, 1)$. Draw a line through the points.

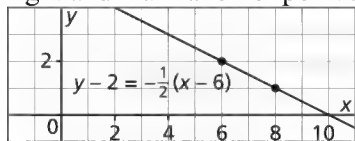


- b) $y - 2 = -\frac{1}{2}(x - 6)$

The equation is in slope-point form, so I will use the coordinates of a point and the slope to graph the equation.

From the equation, a point on the line has coordinates $(6, 2)$ and the slope of the line is $-\frac{1}{2}$.

On a grid, plot the point $(6, 2)$, then write the slope as $-\frac{1}{2}$ and move 1 unit down, 2 units right and mark another point at $(8, 1)$. Draw a line through the points.



- c) Determine the equation of the line that passes through $(-4, 7)$ and $(6, -1)$.

The slope of the line is: $\frac{\text{rise}}{\text{run}}$

$$\text{Slope} = \frac{-1 - 7}{6 - (-4)}$$

$$\text{Slope} = \frac{-8}{10}$$

$$\text{Slope} = \frac{-4}{5}$$

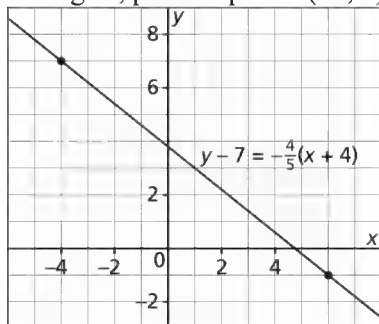
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 7$, $m = -\frac{4}{5}$, and $x_1 = -4$

$$y - 7 = -\frac{4}{5}(x - (-4))$$

$$y - 7 = -\frac{4}{5}(x + 4)$$

On a grid, plot the points $(-4, 7)$ and $(6, -1)$, then draw a line through them.



- d) The required line is perpendicular to the line with equation $y + 4 = 2(x + 2)$.
This line has slope 2.

The slope of a perpendicular line is the negative reciprocal of 2; that is, the slope is $-\frac{1}{2}$.

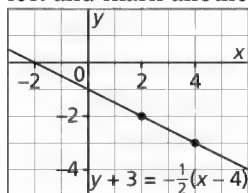
A line with this slope passes through the point with coordinates $(4, -3)$, so use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -3$, $m = -\frac{1}{2}$, and $x_1 = 4$

$$y - (-3) = -\frac{1}{2}(x - 4)$$

$$y + 3 = -\frac{1}{2}(x - 4)$$

On a grid, plot the point $(4, -3)$, then write the slope as $-\frac{1}{2}$ and move 1 unit up, 2 units left and mark another point at $(2, -2)$. Draw a line through the points.



- e) The point at an x -intercept of 5 has coordinates $(5, 0)$.
The point at a y -intercept of 3 has coordinates $(0, 3)$.

The slope of this line is: $\frac{\text{rise}}{\text{run}}$

$$\text{Slope} = \frac{0 - 3}{5 - 0}$$

$$\text{Slope} = \frac{-3}{5}, \text{ or } -\frac{3}{5}$$

A line parallel to this line has the same slope of $-\frac{3}{5}$.

The required line passes through $(-7, -2)$ and has slope $-\frac{3}{5}$.

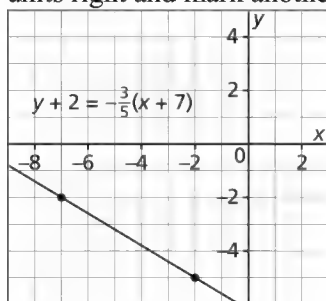
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -2$, $m = -\frac{3}{5}$, and $x_1 = -7$

$$y - (-2) = -\frac{3}{5}(x - (-7))$$

$$y + 2 = -\frac{3}{5}(x + 7)$$

On a grid, plot the point $(-7, -2)$, then write the slope as $-\frac{3}{5}$ and move 3 units down, 5 units right and mark another point at $(-2, -5)$. Draw a line through the points.



4. a) A line has slope 2 and y -intercept 3.
The slope-intercept form of an equation is: $y = mx + b$
Substitute $m = 2$ and $b = 3$.
The equation is: $y = 2x + 3$
- b) Determine the coordinates of a point on the line in part a.
Substitute $x = 1$.
 $y = 2(1) + 3$
 $y = 2 + 3$
 $y = 5$
A point on the line has coordinates $(1, 5)$.
Use the slope-point form of the equation of a line: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 5$, $m = 2$, and $x_1 = 1$
 $y - 5 = 2(x - 1)$
- c) Both equations indicate that the slope of the line is 2.
The slope-intercept form indicates that the y -intercept is 3.

The slope-point form indicates that the line passes through the point with coordinates $(1, 5)$.

**Lesson 6.6 General Form of the Equation for
a Linear Relation**

Exercises (pages 384–385)

A

4. Compare each given equation with each form given below:

Slope-intercept form: $y = mx + b$

Slope-point form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By = C$

General form: $Ax + By + C = 0$

- a) $8x - 3y = 52$ is in standard form.
- b) $9x + 4y + 21 = 0$ is in general form.
- c) $y = 4x + 7$ is in slope-intercept form.
- d) $y - 3 = 5(x + 7)$ is in slope-point form.

5. a) $8x - 3y = 24$

For the x -intercept, substitute $y = 0$.

$$8x - 3(0) = 24 \quad \text{Solve for } x.$$

$$8x = 24 \quad \text{Divide each side by 8.}$$

$$x = 3$$

For the y -intercept, substitute $x = 0$.

$$8(0) - 3y = 24 \quad \text{Solve for } y.$$

$$-3y = 24 \quad \text{Divide each side by } -3.$$

$$y = -8$$

The x -intercept is 3 and the y -intercept is -8 .

- b) $7x + 8y = 56$

For the x -intercept, substitute $y = 0$.

$$7x + 8(0) = 56 \quad \text{Solve for } x.$$

$$7x = 56 \quad \text{Divide each side by 7.}$$

$$x = 8$$

For the y -intercept, substitute $x = 0$.

$$7(0) + 8y = 56 \quad \text{Solve for } y.$$

$$8y = 56 \quad \text{Divide each side by 8.}$$

$$y = 7$$

The x -intercept is 8 and the y -intercept is 7.

- c) $4x - 11y = 88$

For the x -intercept, substitute $y = 0$.

$$4x - 11(0) = 88 \quad \text{Solve for } x.$$

$$4x = 88 \quad \text{Divide each side by 4.}$$

$$x = 22$$

For the y -intercept, substitute $x = 0$.

$$4(0) - 11y = 88 \quad \text{Solve for } y.$$

$$-11y = 88 \quad \text{Divide each side by } -11.$$

$$y = -8$$

The x -intercept is 22 and the y -intercept is -8 .

d) $2x - 9y = 27$

For the x -intercept, substitute $y = 0$.

$$2x - 9(0) = 27 \quad \text{Solve for } x.$$

$$2x = 27 \quad \text{Divide each side by 2.}$$

$$x = 13.5$$

For the y -intercept, substitute $x = 0$.

$$2(0) - 9y = 27 \quad \text{Solve for } y.$$

$$-9y = 27 \quad \text{Divide each side by } -9.$$

$$y = -3$$

The x -intercept is 13.5 and the y -intercept is -3 .

6. An equation is in general form when it is written as $Ax + By + C = 0$, where A is a whole number, and B and C are integers.

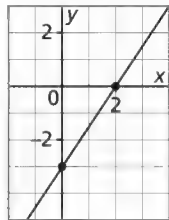
a) $4x + 3y = 36$ Subtract 36 from each side.
 $4x + 3y - 36 = 0$

b) $2x - y = 7$ Subtract 7 from each side.
 $2x - y - 7 = 0$

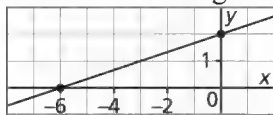
c) $y = -2x + 6$ Add $2x$ to each side and subtract 6 from each side.
 $y + 2x - 6 = 0$ Rearrange the terms on the left side so the x -term is first.
 $2x + y - 6 = 0$

d) $y = 5x - 1$ Subtract $5x$ from each side and add 1 to each side.
 $y - 5x + 1 = 0$ Rearrange the terms on the left side so the x -term is first.
 $-5x + y + 1 = 0$ Multiply each term by -1 so the x -term is positive.
 $5x - y - 1 = 0$

7. a) Mark a point at 2 on the x -axis, and mark a point at -3 on the y -axis.
 Draw a line through the points.



- b) Mark a point at -6 on the x -axis, and mark a point at 2 on the y -axis.
 Draw a line through the points.



B

8. a) i) The equation $-2x + 3y + 42 = 0$ is not in general form because the coefficient of x is not a whole number.
- ii) The equation $4y - 5x = 100$ is not in general form because all the terms are not on the left side and the first term is not an x -term with a whole number coefficient.

iii) The equation $\frac{1}{2}x - \frac{1}{2}y + 1 = 0$ is not in general form because the coefficient of x is not a whole number and the coefficient of y is not an integer.

iv) The equation $5y + 9x - 20 = 0$ is not in general form because the x -term is not first.

b) i) $-2x + 3y + 42 = 0$ Multiply each term by -1 .
 $2x - 3y - 42 = 0$

ii) $4y - 5x = 100$ Subtract 100 from each side.
 $4y - 5x - 100 = 0$ Rearrange the terms on the left side so the x -term is first.
 $-5x + 4y - 100 = 0$ Multiply each term by -1 so the x -term is positive.
 $5x - 4y + 100 = 0$

iii) $\frac{1}{2}x - \frac{1}{2}y + 1 = 0$ Multiply each term by 2.
 $x - y + 2 = 0$

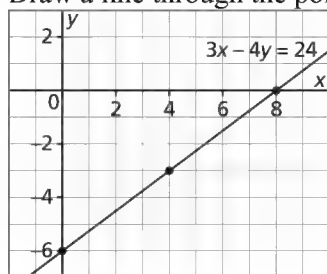
iv) $5y + 9x - 20 = 0$ Interchange the first two terms.
 $9x + 5y - 20 = 0$

9. a) i) $3x - 4y = 24$
 For the x -intercept, substitute $y = 0$.
 $3x - 4(0) = 24$ Solve for x .
 $3x = 24$ Divide each side by 3.
 $x = 8$

For the y -intercept, substitute $x = 0$.
 $3(0) - 4y = 24$ Solve for y .
 $-4y = 24$ Divide each side by -4 .
 $y = -6$

The x -intercept is 8 and the y -intercept is -6 .

ii) Mark a point at 8 on the x -axis, and mark a point at -6 on the y -axis.
 Draw a line through the points.



iii) A point on the graph appears to have coordinates $(4, -3)$. Substitute these coordinates into the equation of the line.

Substitute $x = 4$ and $y = -3$ in $3x - 4y = 24$.
 L.S. = $3x - 4y$ R.S. = 24
 $= 3(4) - 4(-3)$
 $= 12 + 12$

$$= 24$$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

b) i) $6x - 5y = -60$

For the x -intercept, substitute $y = 0$.

$$6x - 5(0) = -60$$

Solve for x .

$$6x = -60$$

Divide each side by 6.

$$x = -10$$

For the y -intercept, substitute $x = 0$.

$$6(0) - 5y = -60$$

Solve for y .

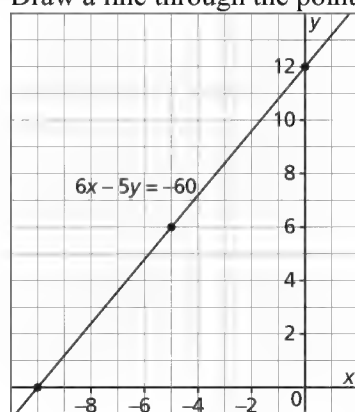
$$-5y = -60$$

Divide each side by -5 .

$$y = 12$$

The x -intercept is -10 and the y -intercept is 12 .

- ii) Mark a point at -10 on the x -axis, and mark a point at 12 on the y -axis.
Draw a line through the points.



- iii) A point on the graph appears to have coordinates $(-5, 6)$. Substitute these coordinates into the equation of the line.

Substitute $x = -5$ and $y = 6$ in $6x - 5y = -60$.

$$\text{L.S.} = 6x - 5y$$

$$\text{R.S.} = -60$$

$$= 6(-5) - 5(6)$$

$$= -30 - 30$$

$$= -60$$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

c) i) $3x - 2y = 24$

For the x -intercept, substitute $y = 0$.

$$3x - 2(0) = 24$$

Solve for x .

$$3x = 24$$

Divide each side by 3.

$$x = 8$$

For the y -intercept, substitute $x = 0$.

$$3(0) - 2y = 24$$

Solve for y .

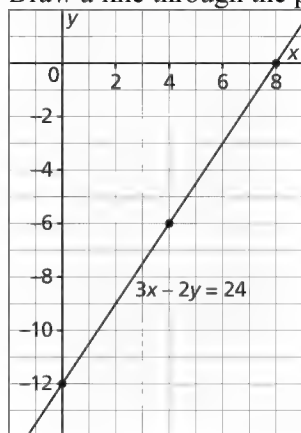
$$-2y = 24$$

Divide each side by -2 .

$$y = -12$$

The x -intercept is 8 and the y -intercept is -12 .

- ii) Mark a point at 8 on the x -axis, and mark a point at -12 on the y -axis.
Draw a line through the points.



- iii) A point on the graph appears to have coordinates $(4, -6)$. Substitute these coordinates into the equation of the line.

Substitute $x = 4$ and $y = -6$ in $3x - 2y = 24$.

$$\begin{aligned} \text{L.S.} &= 3x - 2y & \text{R.S.} &= 24 \\ &= 3(4) - 2(-6) \\ &= 12 + 12 \\ &= 24 \end{aligned}$$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

- d) i) $5x - y = 10$

For the x -intercept, substitute $y = 0$.

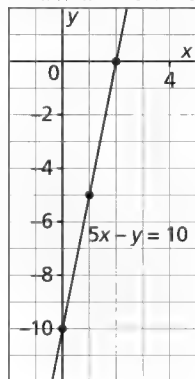
$$\begin{aligned} 5x - (0) &= 10 & \text{Solve for } x. \\ 5x &= 10 & \text{Divide each side by 5.} \\ x &= 2 \end{aligned}$$

For the y -intercept, substitute $x = 0$.

$$\begin{aligned} 5(0) - y &= 10 & \text{Solve for } y. \\ -y &= 10 & \text{Multiply each side by } -1. \\ y &= -10 \end{aligned}$$

The x -intercept is 2 and the y -intercept is -10 .

- ii) Mark a point at 2 on the x -axis, and mark a point at -10 on the y -axis.
Draw a line through the points.



- iii) A point on the graph appears to have coordinates $(1, -5)$. Substitute these coordinates into the equation of the line.

Substitute $x = 1$ and $y = -5$ in $5x - y = 10$.

$$\begin{aligned} \text{L.S.} &= 5x - y & \text{R.S.} &= 10 \\ &= 5(1) - (-5) \\ &= 5 + 5 \\ &= 10 \end{aligned}$$

Since the left side is equal to the right side, the point satisfies the equation, and the graph is correct.

10. a) Determine pairs of numbers that have a sum of 12.

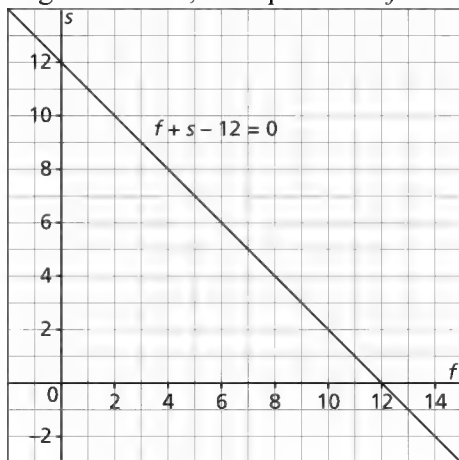
f	-2	0	4	10	12	15
s	14	12	8	2	0	-3

- b), c) Since the numbers are real numbers, I may join the points because all numbers with a sum of 12 are permissible.

The sum of the numbers is 12, so an equation is: $f + s = 12$

To write the equation in general form, subtract 12 from each side.

In general form, the equation is: $f + s - 12 = 0$



- d) From the graph:

When $f = -1$, $s = 13$

When $f = 1$, $s = 11$

When $f = 3$, $s = 9$

When $f = 7$, $s = 5$

When $f = 14$, $s = -2$

So, 6 pairs of integers with a sum of 12 are: $-1, 13$; $1, 11$; $3, 9$; $7, 5$; $14, -2$

11. a), b) When Rebecca uses only pans that hold 12 bars, she needs: $\frac{504}{12}$, or 42 pans

When Rebecca uses only pans that hold 36 bars, she needs: $\frac{504}{36}$, or 14 pans

When Rebecca uses 2 pans that hold 36 bars, she has $504 - 36 \times 2$, or 432 bars to put in

pans that hold 12 bars. So, she needs $\frac{432}{12}$, or 36 pans that hold 12 bars.

When Rebecca uses 6 pans that hold 36 bars, she has $504 - 36 \times 6$, or 288 bars to put in pans that hold 12 bars. So, she needs $\frac{288}{12}$, or 24 pans that hold 12 bars.

When Rebecca uses 10 pans that hold 36 bars, she has $504 - 36 \times 10$, or 144 bars to put in pans that hold 12 bars. So, she needs $\frac{144}{12}$, or 12 pans that hold 12 bars.

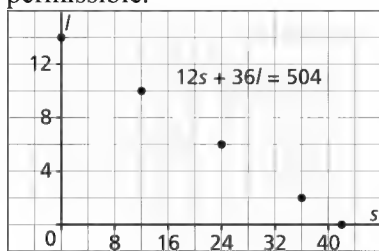
Let s represent the number of pans that hold 12 bars.

Let l represent the number of pans that hold 36 bars.

Write the data above in a table of values.

s	l
0	14
12	10
24	6
36	2
42	0

Plot the points on a grid. Do not join the points because only whole numbers of pans are permissible.



s pans hold $12s$ bars.

l pans hold $36l$ bars.

The total number of bars is 504.

So, an equation is: $12s + 36l = 504$

12. a) $4x + 3y - 24 = 0$ Solve for y . Subtract $4x$ from each side. Add 24 to each side.

$$3y = -4x + 24$$

Divide each side by 3.

$$y = -\frac{4}{3}x + 8$$

This equation is in slope-intercept form.

- b) $3x - 8y + 12 = 0$

$$-8y = -3x - 12$$

Solve for y . Subtract $3x$ and 12 from each side.

Divide each side by -8 .

$$y = \frac{-3}{-8}x - \frac{12}{-8}$$

Simplify.

$$y = \frac{3}{8}x - \frac{3}{2}$$

$$y = \frac{3}{8}x + \frac{3}{2}$$

This equation is in slope-intercept form.

c) $2x - 5y - 15 = 0$

Solve for y . Subtract $2x$ from each side.

Add 15 to each side.

Divide each side by -5 .

$$-5y = -2x + 15$$

$$y = \frac{-2}{-5}x + \frac{15}{-5}$$

Simplify.

$$y = \frac{2}{5}x + (-3)$$

$$y = \frac{2}{5}x - 3$$

This equation is in slope-intercept form.

d) $7x + 3y + 10 = 0$

Solve for y . Subtract $7x$ and 10 from each side.

Divide each side by 3.

$$3y = -7x - 10$$

$$y = -\frac{7}{3}x - \frac{10}{3}$$

This equation is in slope-intercept form.

13. I wrote each equation in slope-intercept form, then I identified the slope of the line from the equation. When an equation is in slope-intercept form, the coefficient of x is the slope of the line.

a) $4x + y - 10 = 0$

Solve for y . Subtract $4x$ from each side. Add 10 to each side.

$$y = -4x + 10$$

The coefficient of x is -4 , so the slope of the line is -4 .

b) $3x - y + 33 = 0$

Solve for y . Subtract $3x$ and 33 from each side.

$$-y = -3x - 33$$

Multiply each side by -1 .

$$y = 3x + 33$$

The coefficient of x is 3, so the slope of the line is 3.

c) $5x - y + 45 = 0$

Solve for y . Subtract $5x$ and 45 from each side.

$$-y = -5x - 45$$

Multiply each side by -1 .

$$y = 5x + 45$$

The coefficient of x is 5, so the slope of the line is 5.

d) $10x + 2y - 16 = 0$

Solve for y . Subtract $10x$ from each side. Add 16 to each side.

$$2y = -10x + 16$$

Divide each side by 2.

$$y = -5x + 8$$

The coefficient of x is -5 , so the slope of the line is -5 .

14. a) $x - 2y + 10 = 0$

Since the coefficients of x and y are factors of the constant term 10, I will use intercepts to graph the equation.

For the x -intercept, substitute $y = 0$.

$$x - 2(0) + 10 = 0$$

Solve for x . Subtract 10 from each side.

$$x = -10$$

For the y -intercept, substitute $x = 0$.

$$0 - 2y + 10 = 0$$

Solve for y . Subtract 10 from each side.

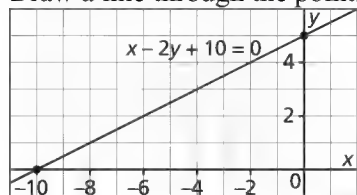
$$-2y = -10$$

Divide each side by -2 .

$$y = 5$$

On a grid, mark a point at -10 on the x -axis and mark a point at 5 on the y -axis.

Draw a line through the points.



b) $2x + 3y - 15 = 0$

Since the coefficient of y is a factor of the constant term 15, I will use the y -intercept as one point to graph the equation. I will determine the coordinates of another point by substituting a value for x that is a multiple of 3, so that the corresponding y -coordinate is a whole number.

For the y -intercept, substitute $x = 0$.

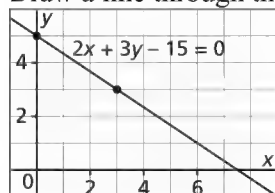
$$\begin{aligned} 2(0) + 3y - 15 &= 0 && \text{Solve for } y. \text{ Add 15 to each side.} \\ 3y &= 15 && \text{Divide each side by 3.} \\ y &= 5 \end{aligned}$$

Substitute $x = 3$.

$$\begin{aligned} 2(3) + 3y - 15 &= 0 && \text{Simplify.} \\ 6 + 3y - 15 &= 0 \\ 3y - 9 &= 0 && \text{Solve for } y. \text{ Add 9 to each side.} \\ 3y &= 9 && \text{Divide each side by 3.} \\ y &= 3 \end{aligned}$$

On a grid, mark a point at 5 on the y -axis and plot the point $(3, 3)$.

Draw a line through the points.



c) $7x + 4y + 4 = 0$

Since the coefficient of y is a factor of the constant term 4, I will use the y -intercept as one point to graph the equation. I will determine the coordinates of another point by substituting a value for x that is a multiple of 4, so that the corresponding y -coordinate is a whole number.

For the y -intercept, substitute $x = 0$.

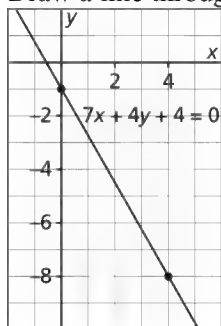
$$\begin{aligned} 7(0) + 4y + 4 &= 0 && \text{Solve for } y. \text{ Subtract 4 from each side.} \\ 4y &= -4 && \text{Divide each side by 4.} \\ y &= -1 \end{aligned}$$

Substitute $x = 4$.

$$\begin{aligned} 7(4) + 4y + 4 &= 0 && \text{Simplify.} \\ 28 + 4y + 4 &= 0 \\ 4y + 32 &= 0 && \text{Solve for } y. \text{ Subtract 32 from each side.} \\ 4y &= -32 && \text{Divide each side by 4.} \\ y &= -8 \end{aligned}$$

On a grid, mark a point at -1 on the y -axis and plot the point $(4, -8)$.

Draw a line through the points.



d) $6x - 10y + 15 = 0$

Since neither coefficient is a factor of the constant term, I will determine the coordinates of two points by substituting values for x . Whatever whole number values of x I choose, the y -coordinate will be a fraction.

Substitute $x = 0$.

$$\begin{aligned} 6(0) - 10y + 15 &= 0 \\ -10y &= -15 \\ y &= 1.5 \end{aligned}$$

Solve for y . Subtract 15 from each side.
Divide each side by -10 .

Substitute $x = 5$.

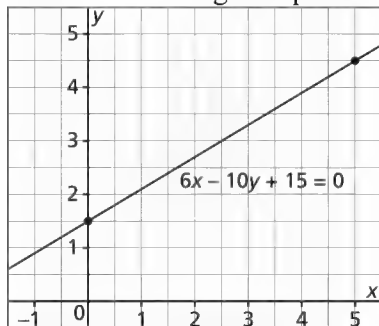
$$\begin{aligned} 6(5) - 10y + 15 &= 0 \\ 30 - 10y + 15 &= 0 \\ 45 - 10y &= 0 \\ -10y &= -45 \\ y &= 4.5 \end{aligned}$$

Simplify.

Solve for y . Subtract 45 from each side.
Divide each side by -10 .

On a grid, use a scale of 2 squares to 1 unit, so the points can be plotted accurately. Plot points at $(0, 1.5)$ and plot the point $(5, 4.5)$.

Draw a line through the points.



15. a) The pipe is 96 ft. long.

Four pieces of 6-ft. pipe are: $4 \times 6 \text{ ft.} = 24 \text{ ft.}$

So, $96 \text{ ft.} - 24 \text{ ft.} = 72 \text{ ft.}$ must be made from 8-ft. pieces.

The number of 8-ft. pieces is: $\frac{72}{8} = 9$

Nine 8-ft. pieces are needed.

- b) The pipe is 96 ft. long.

Three pieces of 8-ft. pipe are: $3 \times 8 \text{ ft.} = 24 \text{ ft.}$

So, $96 \text{ ft.} - 24 \text{ ft.} = 72 \text{ ft.}$ must be made from 6-ft. pieces.

The number of 6-ft. pieces is: $\frac{72}{6} = 12$

Twelve 6-ft. pieces are needed.

- c) The pipe is 96 ft. long.

Three pieces of 6-ft. pipe are: $3 \times 6 \text{ ft.} = 18 \text{ ft.}$

So, $96 \text{ ft.} - 18 \text{ ft.} = 78 \text{ ft.}$ must be made from 8-ft. pieces.

The number of 8-ft. pieces is: $\frac{78}{8} = 9.75$

Since this is not a whole number, 3 pieces of 6-ft. pipe could not be used.

- d) The pipe is 96 ft. long.

Four pieces of 8-ft. pipe are: $4 \times 8 \text{ ft.} = 32 \text{ ft.}$

So, $96 \text{ ft.} - 32 \text{ ft.} = 64 \text{ ft.}$ must be made from 6-ft. pieces.

The number of 6-ft. pieces is: $\frac{64}{6} = 10.\bar{6}$

Since this is not a whole number, 4 pieces of 8-ft. pipe could not be used.

16. a) When Pascal saves all toonies, he has $\frac{24}{2}$, or 12 toonies.

When Pascal saves all loonies, he has 24 loonies.

When Pascal saves 1 toonie, he has $\$24 - \$2$, or $\$22$ in loonies.

Some data are: 12 toonies, 0 loonies; 0 toonies, 24 loonies; and 1 toonie, 22 loonies

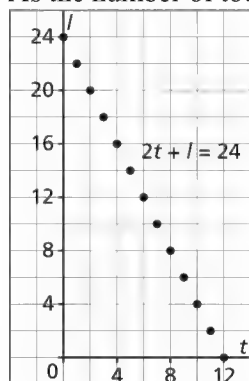
- b), c) Let t represent the number of toonies.

Let l represent the number of loonies.

Plot these points on a grid: (12, 0), (0, 24), (1, 22)

Do not join the points. Use a straightedge placed through the points to plot more points.

As the number of toonies increases by 1, the number of loonies decreases by 2.



The value of toonies in dollars + the value of loonies in dollars = $\$24$

Since 1 toonie is $\$2$, the value of toonies in dollars is: $2t$

Since 1 loonie is $\$1$, the value of loonies in dollars is: l

So, an equation is: $2t + l = 24$

- d) i) Using the graph:

If Pascal had 6 toonies and 8 loonies, then the point (6, 8) would lie on the line in the graph above.

From the graph, the point with t -coordinate 6 has l -coordinate 12.

So, the point (6, 8) does not lie on the line and Pascal cannot have 6 toonies and 8 loonies.

Using the equation:

If Pascal had 6 toonies and 8 loonies, then the point (6, 8) would satisfy the equation:

$$2t + l = 24$$

Substitute $t = 6$ and $l = 8$.

$$\text{L.S.} = 2t + l$$

$$\text{R.S.} = 24$$

$$\text{L.S.} = 2(6) + (8)$$

$$\text{L.S.} = 12 + 8$$

$$\text{L.S.} = 20$$

Since the left side does not equal the right side, the point (6, 8) does not satisfy the equation and Pascal cannot have 6 toonies and 8 loonies.

ii) Using the graph:

If Pascal had 8 toonies and 6 loonies, then the point (8, 6) would lie on the line in the graph above.

From the graph, the point with t -coordinate 8 has l -coordinate 8.

So, the point (8, 6) does not lie on the line and Pascal cannot have 8 toonies and 6 loonies.

Using the equation:

If Pascal had 8 toonies and 6 loonies, then the point (8, 6) would satisfy the equation:

$$2t + l = 24$$

Substitute $t = 8$ and $l = 6$.

$$\text{L.S.} = 2t + l$$

$$\text{R.S.} = 24$$

$$\text{L.S.} = 2(8) + (6)$$

$$\text{L.S.} = 16 + 6$$

$$\text{L.S.} = 22$$

Since the left side does not equal the right side, the point (8, 6) does not satisfy the equation and Pascal cannot have 8 toonies and 6 loonies.

17. I used a graphing TI-83 calculator. I wrote each equation in function form, so I could input the equation using the $\boxed{Y=}$ key.

a) $x - 22y - 15 = 0$

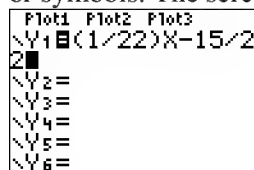
Solve for y . Subtract x from each side. Add 15 to each side.

$$-22y = -x + 15$$

Divide each side by -22 .

$$y = \frac{1}{22}x - \frac{15}{22}$$

On a TI-83 calculator, press $\boxed{Y=}$, then input the equation with no spaces between the terms or symbols. The screen should look like this:

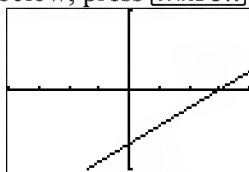


Press **[GRAPH]**. To get the screen below, press **[WINDOW]** and set the window as follows:

```

WINDOW
Xmin=-20
Xmax=20
Xscl=5
Ymin=-1
Ymax=1
Yscl=1
Xres=1

```



- b) $15x + 13y - 29 = 0$ Solve for y . Subtract $15x$ from each side. Add 29 to each side.
 $13y = -15x + 29$ Divide each side by 13.

$$y = \frac{-15}{13}x + \frac{29}{13}$$

On a TI-83 calculator, press **[Y=]**, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1=((-15/13)X+29
/13
Y2=
Y3=
Y4=
Y5=
Y6=

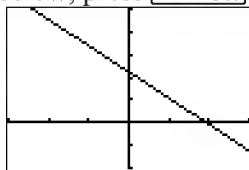
```

Press **[GRAPH]**. To get the screen below, press **[WINDOW]** and set the window as follows:

```

WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-2
Ymax=5
Yscl=1
Xres=1

```



- c) $33x + 2y + 18 = 0$ Solve for y . Subtract $33x$ and 18 from each side.
 $2y = -33x - 18$ Divide each side by 2.
 $y = \frac{-33}{2}x - 9$

On a TI-83 calculator, press **[Y=]**, then input the equation with no spaces between the terms or symbols. The screen should look like this:

```

Plot1 Plot2 Plot3
Y1=((-33/2)X-9
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```

Press **[GRAPH]**. To get the screen below, press **[WINDOW]** and set the window as follows:

```

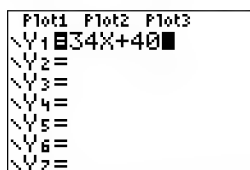
WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-20
Ymax=20
Yscl=5
Xres=1

```

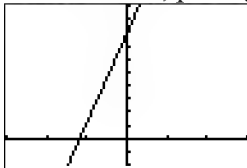
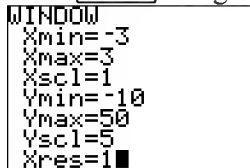


- d) $34x - y + 40 = 0$ Solve for y . Subtract $34x$ and 40 from each side.
 $-y = -34x - 40$ Multiply each side by -1 .
 $y = 34x + 40$

On a TI-83 calculator, press **[Y=]**, then input the equation with no spaces between the terms or symbols. The screen should look like this:



Press **GRAPH**. To get the screen below, press **WINDOW** and set the window as follows:



18. a) $y = \frac{1}{3}x - 4$ Multiply each side by 3 to remove the fraction.

$$3y = 3\left(\frac{1}{3}x\right) - 3(4) \quad \text{Simplify.}$$

$$3y = x - 12 \quad \text{Subtract } 3y \text{ from each side.}$$

$$0 = x - 3y - 12 \quad \text{Write the terms on the left side.}$$

$$x - 3y - 12 = 0 \quad \text{This equation is in general form.}$$

b) $y - 2 = \frac{1}{3}(x + 5)$ Multiply each side by 3 to remove the fraction.

$$3(y - 2) = 3\left(\frac{1}{3}\right)(x + 5) \quad \text{Simplify.}$$

$$3y - 6 = x + 5 \quad \text{Subtract } 3y \text{ from each side. Add 6 to each side.}$$

$$0 = x - 3y + 11 \quad \text{Write the terms on the left side.}$$

$$x - 3y + 11 = 0 \quad \text{This equation is in general form.}$$

c) $y + 3 = -\frac{1}{4}(x - 1)$ Multiply each side by 4 to remove the fraction.

$$4(y + 3) = 4\left(-\frac{1}{4}\right)(x - 1) \quad \text{Simplify.}$$

$$4y + 12 = -x + 1 \quad \text{Add } x \text{ to each side. Subtract 1 from each side.}$$

$$x + 4y + 11 = 0 \quad \text{This equation is in general form.}$$

d) $y = -\frac{3}{2}x + \frac{4}{3}$ Multiply each side by the common denominator: $2 \times 3 = 6$

$$6y = 6\left(-\frac{3}{2}x\right) + 6\left(\frac{4}{3}\right) \quad \text{Simplify.}$$

$$6y = -9x + 8 \quad \text{Add } 9x \text{ to each side. Subtract 8 from each side.}$$

$$9x + 6y - 8 = 0 \quad \text{This equation is in general form.}$$

19. a) In slope-intercept form, the equation is: $y = \frac{1}{3}x - 4$

To graph this line, mark a point at the y -intercept -4 , then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at $(3, -3)$.

From question 18a, in general form, the equation is: $x - 3y - 12 = 0$

Use intercepts to graph this line.

For the x -intercept, substitute $y = 0$.

$$x - 3(0) - 12 = 0 \quad \text{Solve for } x. \text{ Add 12 to each side.}$$

$$x = 12$$

For the y -intercept, substitute $x = 0$.

$$0 - 3y - 12 = 0 \quad \text{Solve for } y. \text{ Add 12 to each side.}$$

$$-3y = 12 \quad \text{Divide each side by } -3.$$

$$y = -4$$

To graph this line, mark a point at 12 on the x -axis and mark a point at -4 on the y -axis.

To write the equation in slope-point form, determine the coordinates of a point on the line.

Substitute $x = 3$ in $x - 3y - 12 = 0$.

$$3 - 3y - 12 = 0$$

$$-3y - 9 = 0 \quad \text{Add 9 to each side.}$$

$$-3y = 9 \quad \text{Divide each side by } -3.$$

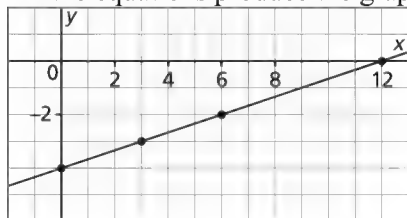
$$y = -3$$

A point on the line has coordinates $(3, -3)$. The slope of the line is $\frac{1}{3}$.

So, in slope-point form, the equation is: $y - (-3) = \frac{1}{3}(x - 3)$; or $y + 3 = \frac{1}{3}(x - 3)$

To graph this line, mark a point at $(3, -3)$, then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at $(6, -2)$.

All the equations produce the graph below.



b) In slope-point form, the equation is: $y - 2 = \frac{1}{3}(x + 5)$

From this equation, the slope of the line is $\frac{1}{3}$ and the coordinates of a point on the line are $(-5, 2)$.

To graph this line, mark a point at $(-5, 2)$, then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at $(-2, 3)$.

From question 18b, in general form, the equation is: $x - 3y + 11 = 0$

Use the x -intercept and one other point to graph this line.

For the x -intercept, substitute $y = 0$.

$$x - 3(0) + 11 = 0 \quad \text{Solve for } x. \text{ Subtract 11 from each side.}$$

$$x = -11$$

Substitute $x = 1$.

$$1 - 3y + 11 = 0$$

$$-3y + 12 = 0$$

Solve for y . Subtract 12 from each side.

$$-3y = -12$$

Divide each side by -3 .

$$y = 4$$

To graph this line, mark a point at -11 on the x -axis and plot the point $(1, 4)$.

To write the equation in slope-intercept form, solve this equation for y :

$$y - 2 = \frac{1}{3}(x + 5) \quad \text{Add 2 to each side.}$$

$$y = 2 + \frac{1}{3}(x + 5) \quad \text{Remove the brackets.}$$

$$y = 2 + \frac{1}{3}x + \frac{5}{3} \quad \text{Simplify.}$$

$$y = \frac{1}{3}x + \frac{6}{3} + \frac{5}{3}$$

$$y = \frac{1}{3}x + \frac{11}{3}$$

Since the y -intercept is not a whole number, determine the coordinates of another point on the line.

Substitute $x = 1$.

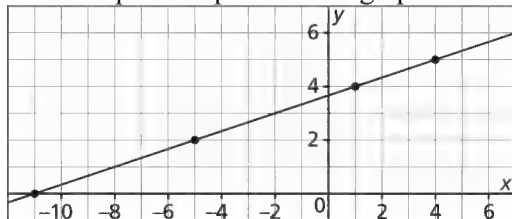
$$y = \frac{1}{3}(1) + \frac{11}{3}$$

$$y = \frac{12}{3}$$

$$y = 4$$

To graph this line, plot the point $(1, 4)$, then use the slope $\frac{1}{3}$ to move 1 unit up and 3 units right to mark a point at $(4, 5)$.

All the equations produce the graph below.



- c) In slope-point form, the equation is: $y + 3 = -\frac{1}{4}(x - 1)$

From this equation, the slope of the line is $-\frac{1}{4}$ and the coordinates of a point on the line are $(1, -3)$.

To graph this line, mark a point at $(1, -3)$, then use the slope $-\frac{1}{4}$ to move 1 unit down and 4 units right to mark a point at $(5, -4)$.

From question 18c, in general form, the equation is: $x + 4y + 11 = 0$

Use the x -intercept and one other point to graph this line.

For the x -intercept, substitute $y = 0$.

$$x + 4(0) + 11 = 0 \quad \text{Solve for } x. \text{ Subtract 11 from each side.}$$

$$x = -11$$

Substitute $x = 1$.

$$1 + 4y + 11 = 0$$

$$4y + 12 = 0$$

Solve for y . Subtract 12 from each side.

$$4y = -12$$

Divide each side by 4.

$$y = -3$$

To graph this line, mark a point at -11 on the x -axis and plot the point $(1, -3)$.

To write the equation in slope-intercept form, solve this equation for y :

$$y + 3 = -\frac{1}{4}(x - 1) \quad \text{Subtract 3 from each side.}$$

$$y = -3 - \frac{1}{4}(x - 1) \quad \text{Remove the brackets.}$$

$$y = -3 - \frac{1}{4}x + \frac{1}{4} \quad \text{Simplify.}$$

$$y = -\frac{1}{4}x - \frac{12}{4} + \frac{1}{4}$$

$$y = -\frac{1}{4}x - \frac{11}{4}$$

Since the y -intercept is not a whole number, determine the coordinates of another point on the line.

Substitute $x = 1$.

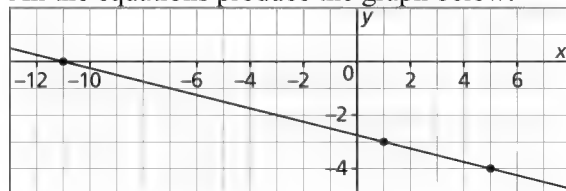
$$y = -\frac{1}{4}(1) - \frac{11}{4}$$

$$y = -\frac{12}{4}$$

$$y = -3$$

To graph this line, plot the point $(1, -3)$, then use the slope $-\frac{1}{4}$ to move 1 down up and 4 units right to mark a point at $(5, -4)$.

All the equations produce the graph below.



- d) In slope-intercept form, the equation is: $y = -\frac{3}{2}x + \frac{4}{3}$

Since the y -intercept is not a whole number, determine the coordinates of two other points on the line.

Substitute $x = 0$.

$$y = -\frac{3}{2}(0) + \frac{4}{3}$$

$$y = \frac{4}{3}$$

Substitute $x = 2$.

$$y = -\frac{3}{2}(2) + \frac{4}{3}$$

$$y = -3 + \frac{4}{3}$$

$$y = \frac{-9}{3} + \frac{4}{3}$$

$$y = \frac{-5}{3}$$

Since no points appear to have integer coordinates, use a scale on the y -axis of 3 squares to 1 unit.

To graph this line, plot points at $\left(0, \frac{4}{3}\right)$ and $\left(2, -\frac{5}{3}\right)$.

From question 18d, in general form, the equation is: $9x + 6y - 8 = 0$

Determine the coordinates of two points on this line.

Substitute $x = 0$.

$$9(0) + 6y - 8 = 0$$

Solve for y . Add 8 to each side.

$$6y = 8$$

Divide by 6.

$$y = \frac{8}{6}, \text{ or } \frac{4}{3}$$

Substitute $x = 2$.

$$9(2) + 6y - 8 = 0$$

Simplify.

$$10 + 6y = 0$$

Solve for y . Subtract 10 from each side.

$$6y = -10$$

Divide by 6.

$$y = -\frac{10}{6}, \text{ or } -\frac{5}{3}$$

To graph this line, plot the points at $\left(0, \frac{4}{3}\right)$ and $\left(2, -\frac{5}{3}\right)$.

To write the equation in slope-point form, use the slope $-\frac{3}{2}$ and the coordinates of a

point on the line, such as $\left(2, -\frac{5}{3}\right)$.

In slope-point form, the equation is: $y - \left(-\frac{5}{3}\right) = -\frac{3}{2}(x - 2)$, or $y + \frac{5}{3} = -\frac{3}{2}(x - 2)$

To graph this line, determine the coordinates of another point on the line.

Substitute $x = 0$.

$$y + \frac{5}{3} = -\frac{3}{2}(0 - 2)$$

$$y + \frac{5}{3} = 3 \quad \text{Subtract } \frac{5}{3} \text{ from each side.}$$

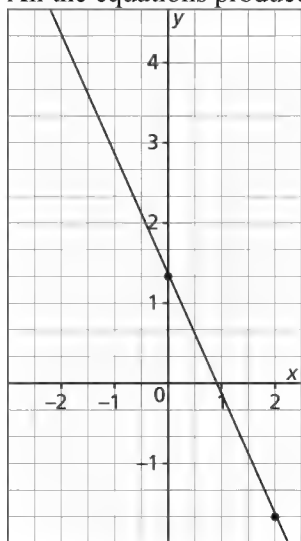
$$y = 3 - \frac{5}{3}$$

$$y = \frac{9}{3} - \frac{5}{3}$$

$$y = \frac{4}{3}$$

Plot the points $\left(2, -\frac{5}{3}\right)$ and $\left(0, \frac{4}{3}\right)$.

All the equations produce the graph below.



20. When $C = 0$, the equation $Ax + By + C = 0$ becomes $Ax + By = 0$.

Solve for y .

$$Ax + By = 0 \quad \text{Subtract } Ax \text{ from each side.}$$

$$By = -Ax \quad \text{Divide each side by } B.$$

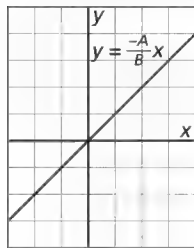
$$y = \frac{-A}{B}x$$

The slope of this line is $\frac{-A}{B}$ and its y -intercept is 0.

This line passes through the origin and has slope $\frac{-A}{B}$.

The slope is the opposite of the quotient of the coefficients of the x -term and the y -term.
When A and B have the same sign, the slope is negative.

When A and B have opposite signs, the slope is positive.



21. a) The x -intercept is 4 and the y -intercept is 6.

The coordinates of G are: (0, 6)

The coordinates of H are: (4, 0)

$$\text{Slope of GH} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope of GH} = \frac{6 - 0}{0 - 4}$$

$$\text{Slope of GH} = \frac{6}{-4}, \text{ or } -\frac{6}{4}$$

The slope of the line is the opposite of the quotient of the y -intercept and the x -intercept; that is,

$$\text{Slope} = -\frac{y\text{-intercept}}{x\text{-intercept}}$$

- b) This relationship is true for all lines. Suppose the x -intercept is a and the y -intercept is b . Then the coordinates of the point at the x -intercept is $(a, 0)$ and the coordinates of the point at the y -intercept is $(0, b)$.

The slope of the line through the points at the intercepts is:

$$\frac{b - 0}{0 - a} = -\frac{b}{a}$$

22. a) $2x + 3y - 6 = 0$

Determine the intercepts of the line with this equation.

For the x -intercept, substitute $y = 0$.

$$2x + 3(0) - 6 = 0$$

$$2x - 6 = 0 \quad \text{Add 6 to each side.}$$

$$2x = 6 \quad \text{Divide each side by 2.}$$

$$x = 3$$

For the y -intercept, substitute $x = 0$.

$$2(0) + 3y - 6 = 0$$

$$3y - 6 = 0 \quad \text{Add 6 to each side.}$$

$$3y = 6 \quad \text{Divide each side by 3.}$$

$$y = 2$$

The required graph has an x -intercept of 3 and a y -intercept of 2.

So, the required graph is Graph B.

- b) $2x - 3y + 6 = 0$

Determine the intercepts of the line with this equation.

For the x -intercept, substitute $y = 0$.

$$2x - 3(0) + 6 = 0$$

$$2x + 6 = 0$$

Subtract 6 from each side.

$$2x = -6$$

Divide each side by 2.

$$x = -3$$

For the y -intercept, substitute $x = 0$.

$$2(0) - 3y + 6 = 0$$

$$-3y + 6 = 0$$

Subtract 6 from each side.

$$-3y = -6$$

Divide each side by -3 .

$$y = 2$$

The required graph has an x -intercept of -3 and a y -intercept of 2 .

So, the required graph is Graph A.

23. a) Write $4x - y = 0$ in slope-intercept form.

$$4x - y = 0$$

Solve for y .

$$y = 4x$$

The graph of this line has y -intercept 0 , so it passes through the origin.

The x - and y -intercepts are at the same point so we cannot draw a line through them.

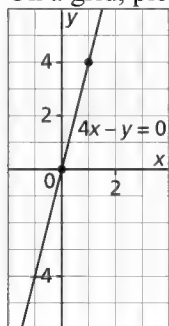
- b) Determine the coordinates of another point.

Substitute $x = 1$.

$$y = 4(1)$$

$$y = 4$$

On a grid, plot points at $(0, 0)$ and $(1, 4)$, then draw a line through the points.



24. Write each equation in slope-intercept form, then look for equations that are the same.

- a) $y = 3x + 6$ is in slope-intercept form.

- b) $2x - 3y - 3 = 0$

Solve for y . Subtract $2x$ from each side. Add 3 to each side.

$$-3y = -2x + 3$$

Divide each side by -3 .

$$y = \frac{2}{3}x - 1$$

This equation is in slope-intercept form.

- c) $y - 2 = \frac{2}{3}(x - 2)$

Solve for y . Add 2 to each side.

$$y = \frac{2}{3}(x - 2) + 2$$

Remove brackets.

$$y = \frac{2}{3}x - \frac{2}{3}(2) + 2$$

Simplify.

$$y = \frac{2}{3}x - \frac{4}{3} + 2$$

$$y = \frac{2}{3}x - \frac{4}{3} + \frac{6}{3}$$

$$y = \frac{2}{3}x + \frac{2}{3}$$

This equation is in slope-intercept form.

d) $3x - y - 6 = 0$

$$-y = -3x + 6$$

$$y = 3x - 6$$

Solve for y . Subtract $3x$ from each side. Add 6 to each side.

Multiply each side by -1 .

This equation is in slope-intercept form.

e) $y = \frac{2}{3}x - 1$

This equation is in slope-intercept form.

f) $y - 3 = 3(x - 3)$

$$y = 3(x - 3) + 3$$

$$y = 3x - 9 + 3$$

$$y = 3x - 6$$

Solve for y . Add 3 to each side.

Remove brackets.

Simplify.

g) $y - 1 = \frac{2}{3}(x - 3)$

Solve for y . Add 1 to each side.

$$y = \frac{2}{3}(x - 3) + 1$$

Remove brackets.

$$y = \frac{2}{3}x - \frac{2}{3}(3) + 1$$

Simplify.

$$y = \frac{2}{3}x - 2 + 1$$

$$y = \frac{2}{3}x - 1$$

h) $y + 3 = 3(x - 1)$

Solve for y . Subtract 3 from each side.

$$y = 3(x - 1) - 3$$

Remove brackets.

$$y = 3x - 3 - 3$$

Simplify.

$$y = 3x - 6$$

The equations in parts b, e, and g have the same slope-intercept form, so these equations

are equivalent: $2x - 3y - 3 = 0$; $y = \frac{2}{3}x - 1$; and $y - 1 = \frac{2}{3}(x - 3)$

The equations in parts d, f, and h have the same slope-intercept form, so these equations are equivalent: $3x - y - 6 = 0$; $y - 3 = 3(x - 3)$; and $y + 3 = 3(x - 1)$

- 25. a)** For an equation in general form to be difficult to graph using intercepts, the intercepts could be fractions; that is, the coefficients of the x - and y -terms are not factors of the constant term.

For example, $3x + 4y + 5 = 0$

- b)** To graph this equation, determine the coordinates of two points that satisfy the equation. Choose integer values of x so that when they are substituted in the equation, the values of y are also integers. Since the coefficient of the y -term is 4, find values of x so that when

the value is substituted, then the sum of the $3x$ term and 5 is a multiple of 4.

For example, substitute $x = 1$.

$$3(1) + 4y + 5 = 0$$

$$3 + 4y + 5 = 0$$

$$4y + 8 = 0$$

$$4y = -8$$

$$y = -2$$

Substitute $x = 5$.

$$3(5) + 4y + 5 = 0$$

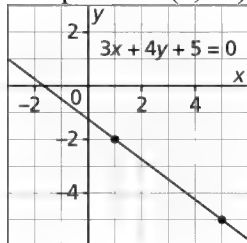
$$15 + 4y + 5 = 0$$

$$4y + 20 = 0$$

$$4y = -20$$

$$y = -5$$

Plot points at $(1, -2)$ and $(5, -5)$, then draw a line through the points.



C

26. a) $\frac{x}{4} + \frac{y}{3} = 1$ Multiply each side by the common denominator: $4 \times 3 = 12$

$$12\left(\frac{x}{4}\right) + 12\left(\frac{y}{3}\right) = 12(1)$$

$$3x + 4y = 12$$

$$3x + 4y - 12 = 0$$

Since this equation is in general form, it represents a linear function.

b) $y = \frac{10}{x}$ Multiply each side by x .

$$xy = x\left(\frac{10}{x}\right)$$

$$xy = 10$$

Since this equation cannot be written in general form, it does not represent a linear function.

c) $y = 2x(x + 4)$ Remove brackets.
 $y = 2x^2 + 8x$

Since this equation cannot be written in general form, it does not represent a linear function.

d) $y = \frac{x + y}{4} + 2$ Multiply each side by 4.

$$4y = 4\left(\frac{x + y}{4}\right) + 4(2) \quad \text{Simplify.}$$

$$4y = x + y + 8$$

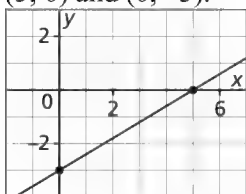
Collect like terms.

$$0 = x - 3y + 8$$

$$x - 3y + 8 = 0$$

Since this equation is in general form, it represents a linear function.

27. The x -intercept of a line is 5 and its y -intercept is -3 . The coordinates of these points are $(5, 0)$ and $(0, -3)$.



Use the formula for the equation of a line given two points on the line.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute: $y_1 = 0$, $x_1 = 5$, $y_2 = -3$, and $x_2 = 0$

$$\frac{y - 0}{x - 5} = \frac{-3 - 0}{0 - 5}$$

$$\frac{y}{x - 5} = \frac{3}{5} \quad \text{Multiply each side by } (x - 5).$$

$$y = \frac{3}{5}(x - 5)$$

28. a) $Ax + By + C = 0$

Write the equation in slope-intercept form.

$$Ax + By + C = 0$$

Solve for y . Subtract Ax and C from each side.

$$By = -Ax - C$$

Divide each side by B .

$$y = \frac{-A}{B}x - \frac{C}{B}$$

The slope of the line is the coefficient of x , which is $-\frac{A}{B}$.

- b) From part a, the y -intercept is the constant term in the equation in slope-intercept form;

that is, $-\frac{C}{B}$

Review

(pages 388–390)

6.1

1. a) From the graph, two points on the line have coordinates $(-2, 3)$ and $(4, -1)$.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - 3}{4 - (-2)}$$

$$\text{Slope} = \frac{-4}{6}$$

$$\text{Slope} = \frac{-2}{3}, \text{ or } -\frac{2}{3}$$

- b) From the graph, two points on the line have coordinates $(-5, 0)$ and $(0, 4)$.

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{4 - 0}{0 - (-5)}$$

$$\text{Slope} = \frac{4}{5}$$

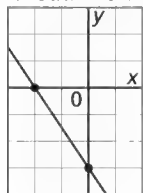
2. a) The change in x is positive, and the change in y is negative.

So, the slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$ is a negative number divided by a positive

number, which is a negative number.

So, the slope is negative.

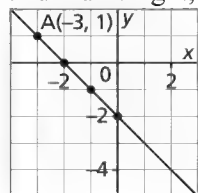
- b) Visualize the intercepts or sketch a diagram.



The line goes down to the right so its slope is negative.

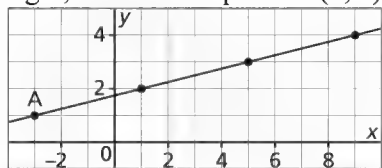
- c) If a line does not have an x -intercept, it is a horizontal line, which has a slope of 0.

3. a) i) Plot point $A(-3, 1)$. Write the slope of -1 as $\frac{-1}{1}$. From point A , move 1 unit down and 1 unit right, then mark a point at $(-2, 0)$. Draw a line through these points.



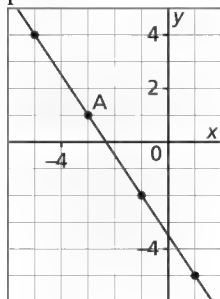
- ii) Identify the coordinates of three other points from the line: $(-2, 0)$, $(-1, -1)$, and $(0, -2)$

- b) i) Plot point $A(-3, 1)$. The slope is $\frac{1}{4}$, so from point A, move 1 unit up and 4 units right, then mark a point at $(1, 2)$. Draw a line through these points.



- ii) Identify the coordinates of three other points from the line: $(1, 2)$, $(5, 3)$, and $(9, 4)$

- c) i) Plot point $A(-3, 1)$. Write the slope of $-\frac{3}{2}$ as $\frac{-3}{2}$ and from point A, move 3 units down and 2 units right, then mark a point at $(-1, -2)$. Draw a line through these points.



- ii) Identify the coordinates of three other points from the line: $(-5, 4)$, $(-1, -2)$, and $(1, -5)$

4. Since I know the coordinates of two points on each line, I use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

- a) The two points have coordinates: $(-6, 8)$ and $(-1, -2)$

$$\text{Slope} = \frac{-2 - 8}{-1 - (-6)}$$

$$\text{Slope} = \frac{-10}{5}$$

$$\text{Slope} = -2$$

- b) The two points have coordinates: $(-3, 7)$ and $(5, -5)$

$$\text{Slope} = \frac{-5 - 7}{5 - (-3)}$$

$$\text{Slope} = \frac{-12}{8}$$

$$\text{Slope} = -\frac{3}{2}$$

5. a) Two points on the graph have coordinates: (2, 320) and (5, 800)

$$\text{Slope} = \frac{\text{change in vertical coordinates}}{\text{change in horizontal coordinates}}$$

$$\text{Slope} = \frac{800 - 320}{5 - 2}$$

$$\text{Slope} = \frac{480}{3}$$

$$\text{Slope} = 160$$

The vertical axis represents distance in metres and the horizontal axis represents time in minutes.

So the slope is 160 m/min, and this is Gabrielle's average speed.

- b) The slope is equal to the rate of change.

- c) i) From the graph, Gabrielle jogs 320 m in 2 min, so in 4 min, she will jog:
 $2 \times 320 \text{ m} = 640 \text{ m}$

- ii) Gabrielle jogs 160 m in 1 min.

$$\text{So, to jog 1000 m, she will take: } \frac{1000}{160} \text{ min} = 6.25 \text{ min}$$

6.2

6. Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals; that is, the product of their slopes is -1 .

- a) The slope of FG is 3.

- i) A parallel line also has slope 3.

- ii) A perpendicular line has slope $-\frac{1}{3}$.

- b) The slope of FG is $-\frac{6}{5}$.

- i) A parallel line also has slope $-\frac{6}{5}$.

- ii) A perpendicular line has slope $\frac{5}{6}$.

c) The slope of FG is $\frac{11}{8}$.

i) A parallel line also has slope $\frac{11}{8}$.

ii) A perpendicular line has slope $-\frac{8}{11}$.

d) The slope of FG is 1.

i) A parallel line also has slope 1.

ii) A perpendicular line has slope -1 .

7. Use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

to determine the slope of each line, then compare the slopes.

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals; that is, the product of their slopes is -1 .

a) The points are: H($-3, 3$) and J($-1, 7$)

$$\text{Slope of HJ} = \frac{7 - 3}{-1 - (-3)}$$

$$\text{Slope of HJ} = \frac{4}{2}$$

$$\text{Slope of HJ} = 2$$

The points are: K($-1, 2$) and M($5, -1$)

$$\text{Slope of KM} = \frac{-1 - 2}{5 - (-1)}$$

$$\text{Slope of KM} = \frac{-3}{6}$$

$$\text{Slope of KM} = -\frac{1}{2}$$

Since 2 and $-\frac{1}{2}$ are negative reciprocals, then HJ and KM are perpendicular.

b) The points are: N($-4, -2$) and P($-1, 7$)

$$\text{Slope of NP} = \frac{7 - (-2)}{-1 - (-4)}$$

$$\text{Slope of NP} = \frac{9}{3}$$

$$\text{Slope of NP} = 3$$

The points are: Q(2, 5) and R(4, -1)

$$\text{Slope of QR} = \frac{-1 - 5}{4 - 2}$$

$$\text{Slope of QR} = \frac{-6}{2}$$

$$\text{Slope of QR} = -3$$

Since the slopes of 3 and -3 are neither equal nor negative reciprocals, then the lines are neither parallel nor perpendicular.

8. From the diagram, I can see that STUV is not a parallelogram because the opposite sides are not parallel. To justify my answer, I determine the slopes of two opposite sides, SV and TU.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

From the diagram, count squares to determine the rise and run from V to S.

$$\text{Slope of SV} = \frac{5}{2}$$

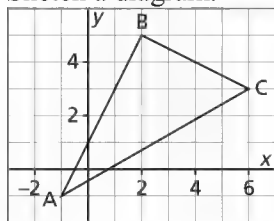
From the diagram, count squares to determine the rise and run from U to T.

$$\text{Slope of TU} = \frac{6}{2}$$

$$\text{Slope of TU} = 3$$

Since the slopes of SV and TU are not equal, these sides are not parallel, and the quadrilateral is not a parallelogram.

9. Sketch a diagram.



From the diagram, BC and AB might be perpendicular.

Determine their slopes to check.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

From the diagram, count squares to determine the rise and run from A to B.

$$\text{Slope of AB} = \frac{6}{3}, \text{ or } 2$$

From the diagram, count squares to determine the rise and run from C to B.

$$\text{Slope of BC} = \frac{2}{-4}, \text{ or } -\frac{1}{2}$$

Since the slopes of AB and BC are negative reciprocals, these sides are perpendicular, and $\triangle ABC$ is a right triangle, with $\angle B = 90^\circ$.

6.3

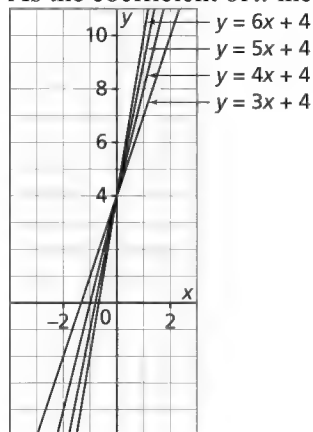
10. a) The graph of $y = 3x + 4$ has slope 3 and y -intercept 4.

When the coefficient of x increases by 1, the new equation is: $y = 4x + 4$; the slope of the graph is now 4 and the y -intercept is unchanged.

When the coefficient of x increases by 1 again, the new equation is: $y = 5x + 4$; the slope of the graph is now 5 and the y -intercept is unchanged.

When the coefficient of x increases by 1 again, the new equation is: $y = 6x + 4$; the slope of the graph is now 6 and the y -intercept is unchanged.

As the coefficient of x increases, the graphs are steeper.



- b) The graph of $y = 3x + 4$ has slope 3 and y -intercept 4.

When the constant term decreases by 1, the new equation is: $y = 3x + 3$; the slope of the graph is unchanged, and the y -intercept is now 3.

When the constant term decreases by 1 again, the new equation is: $y = 3x + 2$; the slope of the graph is unchanged, and the y -intercept is now 2.

When the constant term decreases by 1 again, the new equation is: $y = 3x + 1$; the slope of the graph is unchanged, and the y -intercept is now 1.

When the constant term decreases by 1 again, the new equation is: $y = 3x$; the slope of the graph is unchanged, and the y -intercept is now 0.

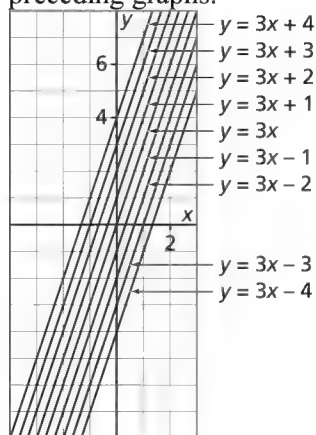
When the constant term decreases by 1 again, the new equation is: $y = 3x - 1$; the slope of the graph is unchanged, and the y -intercept is now -1 .

When the constant term decreases by 1 again, the new equation is: $y = 3x - 2$; the slope of the graph is unchanged, and the y -intercept is now -2 .

When the constant term decreases by 1 again, the new equation is: $y = 3x - 3$; the slope of the graph is unchanged, and the y -intercept is now -3 .

When the constant term decreases by 1 again, the new equation is: $y = 3x - 4$; the slope of the graph is unchanged, and the y -intercept is now -4 .

As the constant term decreases, the graph moves down the grid but remains parallel to the preceding graphs.



6.4

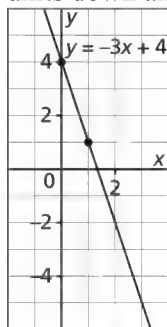
11. Compare each equation to the slope-intercept form: $y = mx + b$, where m is the slope of the graph of the equation and b is its y -intercept.

a) $y = -3x + 4$

$$y = mx + b$$

The slope, m , is -3 and the y -intercept, b , is 4 .

Mark a point at the y -intercept. Write the slope as $\frac{-3}{1}$, then from the y -intercept, move 3 units down and 1 unit right. Mark a point. Draw a line through the two points.

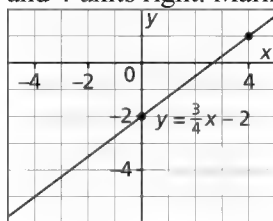


b) $y = \frac{3}{4}x - 2$

$$y = mx + b$$

The slope, m , is $\frac{3}{4}$ and the y -intercept, b , is -2 .

Mark a point at the y -intercept. Use the slope $\frac{3}{4}$; from the y -intercept, move 3 units up and 4 units right. Mark a point. Draw a line through the two points.



12. a) i) From the graph, two points on the graph have coordinates: $(0, 1)$ and $(3, 6)$

To determine the slope, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{6 - 1}{3 - 0}$$

$$\text{Slope} = \frac{5}{3}$$

From the graph, the y -intercept is 1 .

- ii) Use the slope-intercept form of the equation of a line: $y = mx + b$

Substitute: $m = \frac{5}{3}$ and $b = 1$

$$y = \frac{5}{3}x + 1$$

- iii) From the graph, another point on the graph has coordinates $(-3, -4)$

Substitute $x = -3$ and $y = -4$ in the equation $y = \frac{5}{3}x + 1$.

$$\text{L.S.} = y$$

$$= -4$$

$$\text{R. S.} = \frac{5}{3}x + 1$$

$$= \frac{5}{3}(-3) + 1$$

$$= -5 + 1$$

$$= -4$$

Since the left side is equal to the right side, the equation is correct.

- b) i) From the graph, two points on the graph have coordinates: $(-2, 2)$ and $(0, -1)$
To determine the slope, use:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - 2}{0 - (-2)}$$

$$\text{Slope} = \frac{-3}{2}, \text{ or } -\frac{3}{2}$$

From the graph, the y -intercept is -1 .

- ii) Use the slope-intercept form of the equation of a line: $y = mx + b$

Substitute: $m = -\frac{3}{2}$ and $b = -1$

$$y = -\frac{3}{2}x - 1$$

- iii) From the graph, another point on the graph has coordinates $(2, -4)$.

Substitute $x = 2$ and $y = -4$ in the equation $y = -\frac{3}{2}x - 1$.

$$\text{L.S.} = y$$

$$= -4$$

$$\text{R.S.} = -\frac{3}{2}x - 1$$

$$= -\frac{3}{2}(2) - 1$$

$$= -3 - 1$$

$$= -4$$

Since the left side is equal to the right side, the equation is correct.

13. Identify the slope and y -intercept of each graph.

To determine each slope, count squares then use the formula: $\text{Slope} = \frac{\text{rise}}{\text{run}}$

Graph A has slope $\frac{2}{1}$, or 2; and y -intercept 3.

Graph B has slope $\frac{-2}{1}$, or -2 ; and y -intercept 3.

Graph C has slope $\frac{1}{2}$; and y -intercept -3 .

Graph D has slope $\frac{-3}{1}$, or -3 ; and y -intercept -2 .

a) $y = \frac{1}{2}x - 3$

The graph of this equation has slope $\frac{1}{2}$ and y -intercept -3 , which is Graph C.

b) $y = -3x - 2$

The graph of this equation has slope -3 and y -intercept -2 , which is Graph D.

c) $y = 2x + 3$

The graph of this equation has slope 2 and y -intercept 3, which is Graph A.

d) $y = -2x + 3$

The graph of this equation has slope -2 and y -intercept 3, which is Graph B.

14. a) The total amount in the account, A dollars, is the sum of the amount saved in w weeks plus the initial amount, \$40.

Every week Mason saves \$15, so in w weeks he will have saved $15w$ dollars.

So, an equation is: $A = 15w + 40$

b) Substitute $A = 355$, then solve for w .

$$A = 15w + 40$$

$$355 = 15w + 40$$

Subtract 40 from each side.

$$315 = 15w$$

Divide each side by 15.

$$\frac{315}{15} = w$$

$$w = 21$$

Mason will have \$355 in his account after 21 weeks.

c) On a graph of $A = 15w + 40$:

The slope is 15, which is the rate of change of the amount in the account; that is, Mason saves \$15/week.

The vertical intercept is 40, which is the amount, \$40, that was in the account when Mason began saving regularly.

15. The graph of $y = \frac{4}{7}x - 5$ has slope $\frac{4}{7}$ and y -intercept -5 .

a) A line that is parallel to $y = \frac{4}{7}x - 5$ has the same slope of $\frac{4}{7}$ but a different y -intercept;

so its equation could be: $y = \frac{4}{7}x + 5$ or $y = \frac{4}{7}x - 3$

I know that all 3 lines are parallel because they have the same slope.

b) A line that is perpendicular to $y = \frac{4}{7}x - 5$ has a slope that is the negative reciprocal of $\frac{4}{7}$;

that is, its slope is $-\frac{7}{4}$. Its y -intercept may be the same as that of the original graph or it

may be different. The equation of a perpendicular line could be: $y = -\frac{7}{4}x - 5$ or

$$y = -\frac{7}{4}x + 6$$

I know that each of the two new lines is perpendicular to the original line because the product of their slopes, $\left(-\frac{7}{4}\right)\left(\frac{4}{7}\right)$, is equal to -1 .

6.5

16. The line with equation $y = 2x + 1$ has slope 2.

A perpendicular line DE has a slope that is the negative reciprocal of 2; that is, its slope is $-\frac{1}{2}$.

Line DE passes through $F(-2, 3)$.

To determine the equation of line DE, use the slope-point form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

Substitute: $y_1 = 3$, $m = -\frac{1}{2}$, and $x_1 = -2$

$$y - 3 = -\frac{1}{2}(x - (-2))$$

$$y - 3 = -\frac{1}{2}(x + 2)$$

An equation for line DE is: $y - 3 = -\frac{1}{2}(x + 2)$

17. a) i) $y + 4 = 2(x + 3)$

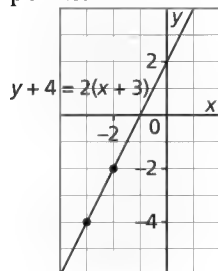
Write the equation as: $y - (-4) = 2(x - (-3))$

Compare the equation to the slope-point form: $y - y_1 = m(x - x_1)$

The slope of the graph is: $m = 2$

The coordinates of a point on the graph are: $(x_1, y_1) = (-3, -4)$

- ii) Plot the point $(-3, -4)$. From this point, use the slope, written as $\frac{2}{1}$, to move 2 squares up and 1 square right, then mark another point. Draw a line through the points.



- iii) From the graph, another point on the graph has coordinates $(-2, -2)$.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = -2$, $m = 2$, and $x_1 = -2$
 $y - (-2) = 2(x - (-2))$
 $y + 2 = 2(x + 2)$ This is a different way to write the equation.

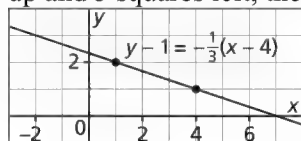
b) i) $y - 1 = -\frac{1}{3}(x - 4)$

Compare the equation to the slope-point form: $y - y_1 = m(x - x_1)$

The slope of the graph is: $m = -\frac{1}{3}$

The coordinates of a point on the graph are: $(x_1, y_1) = (4, 1)$

- ii) Plot the point $(4, 1)$. From this point, use the slope, written as $\frac{1}{-3}$, to move 1 square up and 3 squares left, then mark another point. Draw a line through the points.



- iii) From the graph, another point on the graph has coordinates $(1, 2)$.
Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$
Substitute: $y_1 = 2$, $m = -\frac{1}{3}$, and $x_1 = 1$
 $y - 2 = -\frac{1}{3}(x - 1)$ This is a different way to write the equation.

18. On each graph, I can identify the coordinates of two points, so I will write each equation in slope-point form, for which I need the slope and the coordinates of a point on the graph.

- a) Two points on the graph have coordinates $(2, 0)$ and $(-1, -2)$.
To determine the slope of the graph, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-2 - 0}{-1 - 2}$$

$$\text{Slope} = \frac{-2}{-3}, \text{ or } \frac{2}{3}$$

The slope is $\frac{2}{3}$, and one point on the line has coordinates (2, 0).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 0$, $m = \frac{2}{3}$, and $x_1 = 2$

$$y - 0 = \frac{2}{3}(x - 2)$$

$$y = \frac{2}{3}(x - 2)$$

To verify that this equation is correct, check that the coordinates of another point on the line satisfy the equation.

Another point on the line has coordinates (-1, -2).

Substitute $x = -1$ and $y = -2$ in the equation $y = \frac{2}{3}(x - 2)$.

$\begin{aligned} \text{L.S.} &= y \\ &= -2 \end{aligned}$	$\begin{aligned} \text{R.S.} &= \frac{2}{3}(x - 2) \\ &= \frac{2}{3}(-1 - 2) \\ &= \frac{2}{3}(-3) \\ &= -2 \end{aligned}$
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Since the left side is equal to the right side, the equation is correct.

- b)** Two points on the graph have coordinates (-3, 2) and (2, -1).

To determine the slope of the graph, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - 2}{2 - (-3)}$$

$$\text{Slope} = \frac{-3}{5}, \text{ or } -\frac{3}{5}$$

The slope is $-\frac{3}{5}$, and one point on the line has coordinates (-3, 2).

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 2$, $m = -\frac{3}{5}$, and $x_1 = -3$

$$y - 2 = -\frac{3}{5}(x - (-3))$$

$$y - 2 = -\frac{3}{5}(x + 3)$$

To verify that this equation is correct, check that the coordinates of another point on the

line satisfy the equation.

Another point on the line has coordinates $(2, -1)$.

Substitute $x = 2$ and $y = -1$ in the equation $y - 2 = -\frac{3}{5}(x + 3)$.

$$\begin{array}{ll} \text{L.S.} = y - 2 & \text{R.S.} = -\frac{3}{5}(x + 3) \\ & \\ = -1 - 2 & = -\frac{3}{5}(2 + 3) \\ & \\ = -3 & = -\frac{3}{5}(5) \\ & \\ & = -3 \end{array}$$

Since the left side is equal to the right side, the equation is correct.

- 19.** I know the coordinates of two points on a line, so I will write each equation in slope-point form, for which I need the slope and the coordinates of a point on the line.

- a) i)** Two points on the line have coordinates $(-3, -7)$ and $(1, 5)$.

To determine the slope of the line, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{5 - (-7)}{1 - (-3)}$$

$$\text{Slope} = \frac{12}{4}, \text{ or } 3$$

The slope is 3, and one point on the line has coordinates $(-3, -7)$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = -7$, $m = 3$, and $x_1 = -3$

$$y - (-7) = 3(x - (-3))$$

$$y + 7 = 3(x + 3)$$

- b) i)** To determine the coordinates of another point on the line, substitute a value for x , such as $x = 2$, in the equation, then solve for y .

$$y + 7 = 3(x + 3)$$

$$y + 7 = 3(2 + 3)$$

$$y + 7 = 3(5)$$

$$y + 7 = 15$$

$$y = 8$$

Another point on the line has coordinates $(2, 8)$.

- a) ii)** Two points on the line have coordinates $(-3, 3)$ and $(5, -1)$.

To determine the slope of the line, use the formula:

$$\text{Slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$\text{Slope} = \frac{-1 - 3}{5 - (-3)}$$

$$\text{Slope} = \frac{-4}{8}, \text{ or } -\frac{1}{2}$$

The slope is $-\frac{1}{2}$, and one point on the line has coordinates $(-3, 3)$.

Use the slope-point form of the equation: $y - y_1 = m(x - x_1)$

Substitute: $y_1 = 3$, $m = -\frac{1}{2}$, and $x_1 = -3$

$$y - 3 = -\frac{1}{2}(x - (-3))$$

$$y - 3 = -\frac{1}{2}(x + 3)$$

- b) ii)** To determine the coordinates of another point on the line, substitute a value for x , such as $x = 1$, in the equation, then solve for y .

$$y - 3 = -\frac{1}{2}(x + 3)$$

$$y - 3 = -\frac{1}{2}(1 + 3)$$

$$y - 3 = -\frac{1}{2}(4)$$

$$y - 3 = -2$$

$$y = 1$$

Another point on the line has coordinates $(1, 1)$.

- 20. a)** Let p represent the number of people on a trip, and let C represent the cost of the trip in dollars. If a graph was drawn of C as a function of p , then two points on the graph would have coordinates $(5, 220)$ and $(3, 132)$.

Determine the slope of the graph.

$$\text{Slope} = \frac{\text{change in } C\text{-coordinates}}{\text{change in } p\text{-coordinates}}$$

$$\text{Slope} = \frac{220 - 132}{5 - 3}$$

$$\text{Slope} = \frac{88}{2}, \text{ or } 44$$

The equation of the graph has the form:

$$C = mp + b$$

Substitute $C = 220$, $m = 44$, and $p = 5$, then solve for b .

$$220 = 44 \times 5 + b$$

$$220 = 220 + b$$

$$b = 0$$

An equation for the function is: $C = 44p$

- b)** The cost per person is \$44; this is the coefficient of p in the equation.
- c)** Substitute $C = 264$ in the equation, then solve for p .

$$C = 44p$$

$$264 = 44p$$

Divide each side by 44.

$$\frac{264}{44} = p$$

$$p = 6$$

Six people went on the trip.

6.6

21. a) i) $4y - 5x - 40 = 0$

This equation is not in general form because the first term on the left side is not the x -term.

ii) $\frac{1}{3}x + y = 4$

This equation is not in general form because the coefficient of the x -term is not a whole number and the constant term is not on the left side of the equation.

iii) $y - 2 = \frac{1}{3}(x + 4)$

This equation is in slope-point form.

iv) $y = \frac{1}{5}x + 3$

This equation is in slope-intercept form.

b) i) $4y - 5x - 40 = 0$

Multiply each side by -1 because the x -term must have a positive coefficient.

$$-4y + 5x + 40 = 0$$

Rearrange the first two terms so the x -term is first.

$$\text{In general form: } 5x - 4y + 40 = 0$$

ii) $\frac{1}{3}x + y = 4$

Multiply each side by 3 to remove the fraction.

$$3\left(\frac{1}{3}x\right) + 3y = 12$$

$$x + 3y = 12$$

Subtract 12 from each side.

$$\text{In general form: } x + 3y - 12 = 0$$

iii) $y - 2 = \frac{1}{3}(x + 4)$

Multiply each side by 3.

$$3(y - 2) = 3\left(\frac{1}{3}\right)(x + 4)$$

$$3y - 6 = x + 4$$

Subtract $3y$ from each side. Add 6 to each side.

$$0 = x - 3y + 10$$

$$\text{In general form: } x - 3y + 10 = 0$$

iv) $y = \frac{1}{5}x + 3$

Multiply each side by 5.

$$5y = 5\left(\frac{1}{5}\right)x + 5(3)$$

$$5y = x + 15$$

Subtract $5y$ from each side.

$$0 = x - 5y + 15$$

$$\text{In general form: } x - 5y + 15 = 0$$

22. a) i) $3x - 4y - 24 = 0$

Since the coefficients of x and y are factors of the constant term, I will use intercepts to graph the equation.

For the x -intercept, substitute $y = 0$.

$$3x - 4(0) - 24 = 0$$

$$3x - 24 = 0$$

$$3x = 24$$

$$x = 8$$

Solve for x .

Add 24 to each side.

Divide each side by 3.

For the y -intercept, substitute $x = 0$.

$$3(0) - 4y - 24 = 0$$

$$-4y - 24 = 0$$

$$-4y = 24$$

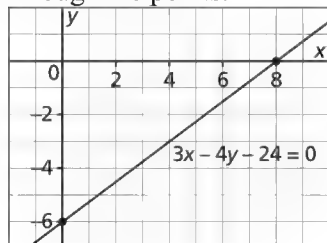
$$y = -6$$

Solve for y .

Add 24 to each side.

Divide each side by -4 .

On a grid, mark a point at 8 on the x -axis and at -6 on the y -axis, then draw a line through the points.



ii) $x - 3y + 12 = 0$

Since the coefficients of x and y are factors of the constant term, I will use intercepts to graph the equation.

For the x -intercept, substitute $y = 0$.

$$x - 3(0) + 12 = 0$$

$$x + 12 = 0$$

$$x = -12$$

Solve for x .

Subtract 12 from each side.

For the y -intercept, substitute $x = 0$.

$$(0) - 3y + 12 = 0$$

$$-3y + 12 = 0$$

$$-3y = -12$$

$$y = 4$$

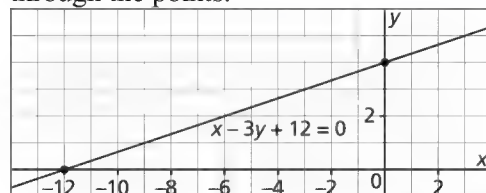
Solve for y .

Subtract 12 from each side.

Divide each side by -3 .

On a grid, mark a point at -12 on the x -axis and at 4 on the y -axis, then draw a line

through the points.



b) Use slope = $\frac{\text{rise}}{\text{run}}$

On each graph, count squares from the y -intercept to the origin to determine the rise, and count squares from the origin to the x -intercept to determine the run.

i) For the graph of $3x - 4y - 24 = 0$:

$$\text{Slope} = \frac{6}{8}, \text{ or } \frac{3}{4}$$

ii) For the graph of $x - 3y + 12 = 0$:

$$\text{Slope} = \frac{-4}{-12}, \text{ or } \frac{1}{3}$$

23. I cannot easily graph a line from its equation in general form when the coefficients of the x -term and y -term are not factors of the constant term; for example, $6x - 4y + 3 = 0$.

I write this equation in slope-intercept to determine the slope and y -intercept of the graph.

$$6x - 4y + 3 = 0$$

Subtract $6x$ and 3 from each side.

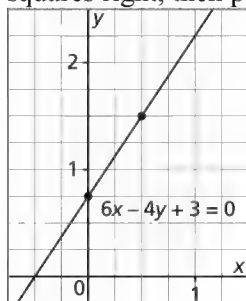
$$-4y = -6x - 3$$

Divide each side by -4 .

$$y = \frac{-6}{-4}x - \frac{3}{-4}$$

$$y = \frac{3}{2}x + \frac{3}{4}$$

To graph this equation, I use a scale of 4 squares to 1 unit so I can accurately plot a point at the y -intercept, which is $\frac{3}{4}$. From this point, I use the slope $\frac{3}{2}$ to move 3 squares up and 2 squares right, then plot another point. I draw a line through the points.



24. a) Choose a number, then subtract 6 from it; these two numbers differ by 6.

For example:

8, then $8 - 6 = 2$; the numbers are 8 and 2

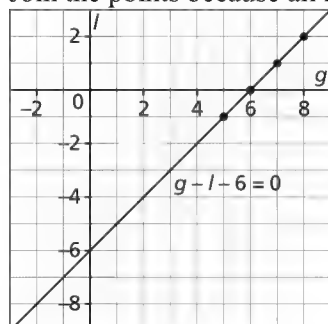
7, then $7 - 6 = 1$; the numbers are 7 and 1

6, then $6 - 0 = 0$; the numbers are 6 and 0

5, then $5 - 6 = -1$; the numbers are 5 and -1

Plot these numbers as ordered pairs, with the greater number first and g plotted on the horizontal axis: $(8, 2)$, $(7, 1)$, $(6, 0)$, $(5, -1)$

Join the points because all real numbers are possible values.



- b) Assume that g represents the greater number and l represents the lesser number.

Then an equation is: $g - l = 6$

Subtract 6 from each side:

$$g - l - 6 = 0 \quad \text{This equation is in general form.}$$

- c) From the graph, 5 other pairs of numbers are the coordinates of 5 points on the graph; for example, 4 and -2 ; 3 and -3 , 2 and -4 , 1 and -5 , 0 and -6

25. Write each equation in slope-intercept form, then compare the equations to determine which have the same slopes and the same y -intercepts.

a) $y = \frac{2}{5}x + 1$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is 1.

b) $y - 3 = \frac{2}{5}(x - 4)$ Add 3 to each side.

$$y = \frac{2}{5}(x - 4) + 3 \quad \text{Remove brackets.}$$

$$y = \frac{2}{5}x - \frac{2}{5}(4) + 3 \quad \text{Simplify.}$$

$$y = \frac{2}{5}x - \frac{8}{5} + 3$$

$$y = \frac{2}{5}x - \frac{8}{5} + \frac{15}{5}$$

$$y = \frac{2}{5}x + \frac{7}{5}$$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is

$$\frac{7}{5}.$$

c) $y - 1 = \frac{2}{5}(x - 1)$ Add 1 to each side.

$y = \frac{2}{5}(x - 1) + 1$ Remove brackets.

$y = \frac{2}{5}x - \frac{2}{5}(1) + 1$ Simplify.

$y = \frac{2}{5}x - \frac{2}{5} + 1$

$y = \frac{2}{5}x - \frac{2}{5} + \frac{5}{5}$

$y = \frac{2}{5}x + \frac{3}{5}$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is $\frac{3}{5}$.

d) $y - 3 = \frac{2}{5}(x - 5)$ Add 3 to each side.

$y = \frac{2}{5}(x - 5) + 3$ Remove brackets.

$y = \frac{2}{5}x - \frac{2}{5}(5) + 3$ Simplify.

$y = \frac{2}{5}x - 2 + 3$

$y = \frac{2}{5}x + 1$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is 1.

e) $2x - 5y + 7 = 0$ Subtract $2x$ and 7 from each side.
 $-5y = -2x - 7$ Divide each side by -5 .

$y = \frac{-2}{-5}x - \frac{7}{-5}$

$y = \frac{2}{5}x + \frac{7}{5}$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is $\frac{7}{5}$.

f) $2x - 5y - 5 = 0$ Subtract $2x$ from each side. Add 5 to each side.
 $-5y = -2x + 5$ Divide each side by -5 .

$$y = \frac{-2}{-5}x + \frac{5}{-5}$$

$$y = \frac{2}{5}x - 1$$

This equation is in slope-intercept form; the slope of its graph is $\frac{2}{5}$ and its y -intercept is -1 .

Equations $y = \frac{2}{5}x + 1$ and $y - 3 = \frac{2}{5}(x - 5)$ are equivalent.

Equations $y - 3 = \frac{2}{5}(x - 4)$ and $2x - 5y + 7 = 0$ are equivalent.

26. Determine the slope of each graph and identify its y -intercept where possible.

For Graph A:

Two points have coordinates $(-1, 5)$ and $(3, 0)$.

Use slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{0 - 5}{3 - (-1)}$$

$$\text{Slope} = \frac{-5}{4}, \text{ or } -\frac{5}{4}$$

For Graph B:

Its y -intercept is 3.

Two points have coordinates $(5, -1)$ and $(0, 3)$.

Use slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{3 - (-1)}{0 - 5}$$

$$\text{Slope} = \frac{4}{-5}, \text{ or } -\frac{4}{5}$$

For Graph C:

Two points have coordinates $(-3, 3)$ and $(2, -1)$.

Use slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{-1 - 3}{2 - (-3)}$$

$$\text{Slope} = \frac{-4}{5}, \text{ or } -\frac{4}{5}$$

Write each equation in slope-intercept form.

a) $y = -\frac{4}{5}x + 3$

This equation is in slope-intercept form; its graph has slope $-\frac{4}{5}$ and y -intercept 3. This matches Graph B.

b) $y - 3 = -\frac{4}{5}(x + 3)$ Add 3 to each side.

$$y = -\frac{4}{5}(x + 3) + 3$$
 Remove brackets.

$$y = -\frac{4}{5}x - \frac{4}{5}(3) + 3$$
 Simplify.

$$y = -\frac{4}{5}x - \frac{12}{5} + \frac{15}{5}$$

$$y = -\frac{4}{5}x + \frac{3}{5}$$

This equation is in slope-intercept form; its graph has slope $-\frac{4}{5}$ and y -intercept $\frac{3}{5}$.

This slope matches Graph C. Check that a point on Graph C satisfies this equation. Use the point (2, -1).

Substitute $x = 2$ and $y = -1$ in the equation $y - 3 = -\frac{4}{5}(x + 3)$.

$\begin{aligned} \text{L.S.} &= y - 3 \\ &= -1 - 3 \\ &= -4 \end{aligned}$	$\begin{aligned} \text{R.S.} &= -\frac{4}{5}(x + 3) \\ &= -\frac{4}{5}(2 + 3) \\ &= -\frac{4}{5}(5) \\ &= -4 \end{aligned}$
--	---

Since the left side is equal to the right side, the equation is correct and matches Graph C.

c) $5x + 4y - 15 = 0$ Subtract $5x$ from each side. Add 15 to each side.
 $4y = -5x + 15$ Divide each side by 4.
 $y = \frac{-5}{4}x + \frac{15}{4}$

This equation is in slope-intercept form; its graph has slope $-\frac{5}{4}$ and y -intercept $\frac{15}{4}$.

This slope matches Graph A. Check that a point on Graph A satisfies this equation. Use the point (3, 0).

Substitute $x = 3$ and $y = 0$ in the equation $5x + 4y - 15 = 0$.

$\begin{aligned} \text{L.S.} &= 5x + 4y - 15 \\ &= 5(3) + 4(0) - 15 \\ &= 15 - 15 \\ &= 0 \end{aligned}$	$\begin{aligned} \text{R.S.} &= 0 \end{aligned}$
--	--

Since the left side is equal to the right side, the equation is correct and matches Graph A.

27. a) Let a represent the time in hours Max works for family A who pays \$5/h.
Let b represent the time in hours Max works for family B who pays \$4/h.

When Max works only for family A, the time in hours he works is: $\frac{60}{5} = 12$

When Max works only for family B, the time in hours he works is: $\frac{60}{4} = 15$

When Max works for 4 h for family A, he earns: $4 \times \$5 = \$20$

So, he earns $\$60 - \$20 = \$40$ from family B.

So, Max works $\frac{40}{4}$ h, or 10 h for family B.

When Max works for 8 h for family A, he earns: $8 \times \$5 = \$40$

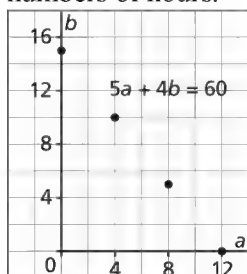
So, he earns $\$60 - \$40 = \$20$ from family B.

So, Max works $\frac{20}{4}$ h, or 5 h for family B.

Make a table for these data.

a hours	b hours
12	0
0	15
4	10
8	5

Plot these data on a grid. Do not join the points because we assume Max works for whole numbers of hours.



- b) When Max works a hours at \$5/h, he earns: $5a$ dollars
When Max works b hours at \$4/h, he earns: $4b$ dollars
His total earnings are: \$60
So, an equation is: $5a + 4b = 60$

28. a) Let n represent the number of new releases Kylie rents, at \$5 each.
Let m represent the number of older movies Kylie rents, at \$3 each.

When Kylie rents only new releases, the number she rents is: $\frac{45}{5} = 9$

When Kylie rents only older movies, the number she rents is: $\frac{45}{3} = 15$

When Kylie rents 6 new releases, it costs: $6 \times \$5 = \$30$

So, Kylie spends $\$45 - \$30 = \$15$ on older movies.

So, Kylie rents $\frac{15}{3}$, or 5 older movies.

When Kylie rents 3 new releases, it costs: $3 \times \$5 = \$15$

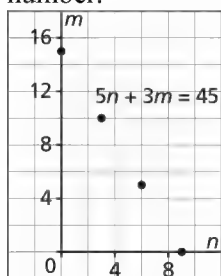
So, Kylie spends $\$45 - \$15 = \$30$ on older movies.

So, Kylie rents $\frac{30}{3}$, or 10 older movies.

Make a table for these data.

n	m
9	0
0	15
6	5
3	10

Plot these data on a grid. Do not join the points because the number of movies is a whole number.



When Kylie rents n movies at \$5 each, she pays: $5n$ dollars

When Kylie rents m movies at \$3 each, she pays: $3m$ dollars

The total Kylie spent is: \$45

So, an equation is: $5n + 3m = 45$

- b) i)** Five new releases cost: $5 \times \$5 = \$25$
 Six older movies cost: $6 \times \$3 = \$18$
 Total cost is: $\$25 + \$18 = \$43$
 Since this amount is not \$45, Kylie could not have rented these numbers of movies.
- ii)** Six new releases cost: $6 \times \$5 = \$30$
 Five older movies cost: $5 \times \$3 = \$15$
 Total cost is: $\$30 + \$15 = \$45$
 Since this amount is \$45, Kylie could have rented these numbers of movies.

Practice Test

(page 391)

1. For each line, count squares on the grid to determine the rise and the run.

$$\text{Use slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope of AB} = \frac{-4}{6}, \text{ or } -\frac{2}{3}$$

$$\text{Slope of CD} = \frac{6}{9}, \text{ or } \frac{2}{3}$$

$$\text{Slope of EF} = \frac{-6}{4}, \text{ or } -\frac{3}{2}$$

$$\text{Slope of GH} = \frac{6}{4}, \text{ or } \frac{3}{2}$$

The line with slope $-\frac{3}{2}$ is EF, which is choice C.

2. Write the equation $2x - 3y + 2 = 0$ in slope-intercept form.

$$2x - 3y + 2 = 0$$

Subtract $2x$ and 2 from each side.

$$-3y = -2x - 2$$

Divide each side by -3 .

$$y = \frac{-2}{-3}x - \frac{2}{-3}$$

$$y = \frac{2}{3}x + \frac{2}{3}$$

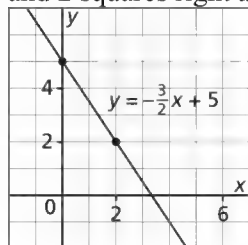
The graph of this equation has slope $\frac{2}{3}$.

From question 1, the line with slope $\frac{2}{3}$ is CD, which is choice B.

3 a) i) $y = -\frac{3}{2}x + 5$

Since the equation is in slope-intercept form, I use the slope $-\frac{3}{2}$ and the y -intercept 5 to graph the line.

I mark a point at 5 on the y -axis. I write the slope as $-\frac{3}{2}$, then move 3 squares down and 2 squares right and mark a point. I draw a line through the points.



ii) $y - 3 = \frac{1}{3}(x + 2)$

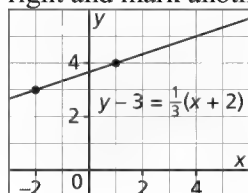
This equation is in slope-point form.

To identify the coordinates of the point, I write the equation as:

$$y - 3 = \frac{1}{3}(x - (-2))$$

A point on the line has coordinates $(-2, 3)$, and the slope of the line is $\frac{1}{3}$.

I mark a point at $(-2, 3)$, then use the slope $\frac{1}{3}$ to move 1 square up and 3 squares right and mark another point. I draw a line through the points.



iii) $3x - 4y - 12 = 0$

Since the coefficients of x and y in this equation are factors of 12, I can use intercepts to graph the equation.

For the x -intercept, substitute $y = 0$.

$$3x - 4(0) - 12 = 0$$

$$3x - 12 = 0$$

$$3x = 12$$

$$x = 4$$

Solve for x . Add 12 to each side.

Divide each side by 3.

For the y -intercept, substitute $x = 0$.

$$3(0) - 4y - 12 = 0$$

$$-4y - 12 = 0$$

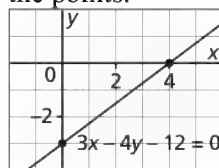
$$-4y = 12$$

$$y = -3$$

Solve for y . Add 12 to each side.

Divide each side by -4 .

I mark a point at 4 on the x -axis and a point at -3 on the y -axis. I draw a line through the points.



b) The graph of the equation $y = -\frac{3}{2}x + 5$ has slope $-\frac{3}{2}$, so any line parallel to this line

will also have slope $-\frac{3}{2}$.

The required line has slope $-\frac{3}{2}$ and passes through the point with coordinates $(6, 2)$.

Use the slope-point form of the equation:

$$y - y_1 = m(x - x_1)$$

Substitute: $y_1 = 2$, $m = -\frac{3}{2}$, and $x_1 = 6$

$$y - 2 = -\frac{3}{2}(x - 6)$$

I know my equation is correct because the slope of the graph of this equation is equal to the slope of the given parallel line, and the coordinates of the point on the graph given by this equation are equal to the given coordinates.

- c) The graph of the equation $y - 3 = \frac{1}{3}(x + 2)$ has slope $\frac{1}{3}$. The perpendicular line has a

slope that is the negative reciprocal of $\frac{1}{3}$; that is, its slope is -3 .

The required line has slope -3 and passes through the point with coordinates $(-1, 2)$.

Use the slope-point form of the equation:

$$y - y_1 = m(x - x_1)$$

Substitute: $y_1 = 2$, $m = -3$, and $x_1 = -1$

$$y - 2 = -3(x - (-1))$$

$$y - 2 = -3(x + 1)$$

$$y - 2 = -3x - 3$$

$$3x + y + 1 = 0$$

Write this equation in general form. Remove brackets.

Add $3x$ and 3 to each side.

This equation is in general form.

- d) To determine the coordinates of a point P on the graph of $3x - 4y - 12 = 0$, substitute a value for x , then solve the equation for y .
Since the coefficient of y is 4 and the constant term, 12 , is a multiple of 4 , choose a value of x that is a multiple of 4 so the value of y is an integer.

Substitute $x = 8$ in the equation:

$$3x - 4y - 12 = 0$$

$$3(8) - 4y - 12 = 0$$

$$24 - 4y - 12 = 0$$

$$12 - 4y = 0$$

$$-4y = -12$$

$$y = 3$$

Simplify.

Collect like terms.

Subtract 12 from each side of the equation.

Divide each side by -4 .

The coordinates of P are $(8, 3)$.

The line passes through P and Q(1, 5).

Determine the slope of PQ.

Use slope = $\frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$

$$\text{Slope} = \frac{5 - 3}{1 - 8}$$

$$\text{Slope} = \frac{2}{-7}, \text{ or } -\frac{2}{7}$$

Substitute the coordinates of Q and the slope in the slope-point form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

Substitute: $y_1 = 5$, $m = -\frac{2}{7}$, and $x_1 = 1$

$$y - 5 = -\frac{2}{7}(x - 1)$$

Solve for y . Add 5 to each side.

$$y = -\frac{2}{7}(x - 1) + 5$$

Remove brackets.

$$y = -\frac{2}{7}x + \frac{2}{7} + 5$$

$$y = -\frac{2}{7}x + \frac{2}{7} + \frac{35}{7}$$

$$y = -\frac{2}{7}x + \frac{37}{7}$$

This equation is in slope-intercept form.

4. a) Since I can identify the slope of the line and its y -intercept from the graph, I will write the equation in slope-intercept form: $y = mx + b$

I count squares on the grid to determine the rise and the run.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{-4}{2}, \text{ or } -2$$

From the graph, the y -intercept is -2 .

So, an equation is: $y = -2x - 2$

- b) The line is horizontal, so its equation is: $y = -1$

In general form, this is: $y + 1 = 0$

- c) Since I can calculate the slope and identify the coordinates of a point on the line, I shall use the slope-point form: $y - y_1 = m(x - x_1)$

The coordinates of two points on the line are $(-1, -2)$ and $(3, 1)$. I count squares on the grid between these points to determine the rise and the run.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

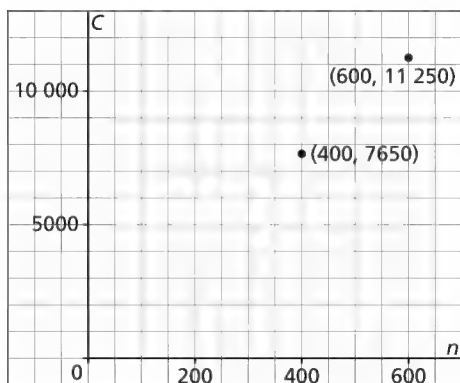
$$\text{Slope} = \frac{3}{4}$$

Substitute the coordinates of the point $(3, 1)$ and the slope $\frac{3}{4}$ in the equation

$$y - y_1 = m(x - x_1)$$

$$\text{So, an equation is: } y - 1 = \frac{3}{4}(x - 3)$$

5. The cost, C dollars, is a function of the number of people, n , who attend.
Use the given information as the coordinates of two points on a sketch of a graph of the function.
Plot these points: $(600, 11\,250)$ and $(400, 7650)$



The cost per person is the slope of the line through the points on the graph.

$$\text{Slope} = \frac{\text{change in } C\text{-coordinates}}{\text{change in } n\text{-coordinates}}$$

$$\text{Slope} = \frac{11\,250 - 7650}{600 - 400}$$

$$\text{Slope} = \frac{3600}{200}$$

$$\text{Slope} = 18$$

The cost per person is \$18.

Use the slope-point form of the equation:

$$C - C_1 = m(n - n_1)$$

Substitute: $C_1 = 7650$, $m = 18$, and $n_1 = 400$

$$C - 7650 = 18(n - 400) \quad \text{This is an equation of the function.}$$

a), c) To determine the cost for 340 people to attend, substitute $n = 340$ in the equation:

$$C - 7650 = 18(n - 400)$$

$$C - 7650 = 18(340 - 400) \quad \text{Solve for } C.$$

$$C - 7650 = 18(-60)$$

$$C - 7650 = -1080$$

$$C = 7650 - 1080$$

$$C = 6570$$

The cost for 340 people to attend is \$6570.

b), c) To determine the number of people who can attend for a cost of \$9810, substitute

$C = 9810$ in the equation:

$$C - 7650 = 18(n - 400)$$

$$9810 - 7650 = 18(n - 400) \quad \text{Solve for } n. \text{ Remove brackets and simplify.}$$

$$2160 = 18n - 7200$$

$$9360 = 18n$$

$$n = 520$$

For a cost of \$9810, 520 people can attend.

Lesson 7.1 Developing Systems of Linear Equations

Exercises (pages 401-402)

A

4. a) $2x + y = 11$

$x = 13 + y$

This is a linear system because both equations are linear.

b) $2x = 11 - y$

$4x - y = 13$

This is a linear system because both equations are linear.

c) $-\frac{1}{2}x - y = \frac{3}{4}$

$\frac{3}{2}x + 2 = -\frac{7}{8}$

This is a linear system because both equations are linear.

d) $-x^2 + y = 10$

$x + y = 5$

This is not a linear system because the first equation above is not linear; it contains an x^2 -term.

5. Substitute $x = -1$ and $y = 2$ into each equation to see if the values satisfy the equation.

a) $3x + 2y = -1$ ①

$2x - y = 1$ ②

Consider equation ①: $3x + 2y = -1$

Substitute: $x = -1$ and $y = 2$

$$\begin{aligned} \text{L.S.} &= 3x + 2y & \text{R.S.} &= -1 \\ &= 3(-1) + 2(2) \\ &= -3 + 4 \\ &= 1 \end{aligned}$$

Since the left side is not equal to the right side, $x = -1$ and $y = 2$ are not a solution of equation ①, so cannot be a solution of the linear system.

b) $3x - y = -1$ ①

$-x - y = -1$ ②

Consider equation ①: $3x - y = -1$

Substitute: $x = -1$ and $y = 2$

$$\begin{aligned} \text{L.S.} &= 3x - y & \text{R.S.} &= -1 \\ &= 3(-1) - (2) \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

Since the left side is not equal to the right side, $x = -1$ and $y = 2$ are not a solution of equation ①, so cannot be a solution of the linear system.

c) $-3x + 5y = 13$ ①

$4x - 3y = -10$ ②

Consider equation ①: $-3x + 5y = 13$

Substitute: $x = -1$ and $y = 2$

$$\begin{aligned}\text{L.S.} &= -3x + 5y & \text{R.S.} &= 13 \\ &= -3(-1) + 5(2) \\ &= 3 + 10 \\ &= 13\end{aligned}$$

Since the left side is equal to the right side, $x = -1$ and $y = 2$ are a solution of equation ①.

Consider equation ②: $4x - 3y = -10$

Substitute: $x = -1$ and $y = 2$

$$\begin{aligned}\text{L.S.} &= 4x - 3y & \text{R.S.} &= -10 \\ &= 4(-1) - 3(2) \\ &= -4 - 6 \\ &= -10\end{aligned}$$

Since the left side is equal to the right side, $x = -1$ and $y = 2$ are a solution of equation ②.

Since $x = -1$ and $y = 2$ satisfy both equations in the linear system, these values are a solution of the linear system.

B

6. Each linear system has the equation $2x + 2y = 228$. Look at the second equation in each system.

- a) A jacket costs \$44 more than a sweater; that is, the difference in costs is \$44.
Suppose x dollars represents the cost of a jacket and y dollars represents the cost of a sweater.

An equation to represent these costs is: $x - y = 44$

This matches the second equation in part iii, so the system in part iii matches this situation.

- b) The width of the tennis court is 42 ft. less than the length, or the length is 42 ft. more than the width.

Suppose x feet represents the length of the tennis court and y feet represents the width of the court.

An equation to represent these dimensions is: $x - y = 42$

This matches the second equation in part i, so the system in part i matches this situation.

- c) Forty more chapatti breads than naan breads were sold; that is, the difference in numbers of breads sold is 40.

Suppose x represents the number of chapatti breads that were sold and y represents the number of naan breads that were sold.

An equation to represent these numbers is: $x - y = 40$

This matches the second equation in part ii, so the system in part ii matches this situation.

7. a) Let s feet represent the length of a shorter piece of pipe.
Let l feet represent the length of a longer piece of pipe.

From the first diagram:

2 shorter pieces + 2 longer pieces = 20 ft.

So, one equation is: $2s + 2l = 20$

From the second diagram:

1 shorter piece + 3 longer pieces = 22 ft.

So, another equation is: $s + 3l = 22$

A linear system is:

$$2s + 2l = 20$$

$$s + 3l = 22$$

- b) From the first diagram:

The length of 2 shorter pieces is: $2 \times 4 \text{ ft.} = 8 \text{ ft.}$

The length of 2 longer pieces is: $2 \times 6 \text{ ft.} = 12 \text{ ft.}$

The sum of these lengths is: $8 \text{ ft.} + 12 \text{ ft.} = 20 \text{ ft.}$

This length is the same as the length on the diagram.

From the second diagram:

The length of 1 shorter piece is: 4 ft.

The length of 3 longer pieces is: $3 \times 6 \text{ ft.} = 18 \text{ ft.}$

The sum of these lengths is: $4 \text{ ft.} + 18 \text{ ft.} = 22 \text{ ft.}$

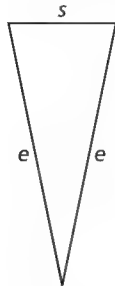
This length is the same as the length on the diagram.

Since both lengths agree with the diagrams given, the length of each piece of pipe is correct.

8. a) Sketch a diagram.

Let s centimetres represent the length of the shorter side.

Let e centimetres represent the length of each equal side.



The perimeter, 24 cm, is the sum of the lengths of the sides.

So, one equation is: $e + e + s = 24$

Or, $2e + s = 24$

An equal side is 6 cm longer than the shorter side.

So, another equation is: $e = 6 + s$

A linear system is:

$$2e + s = 24$$

$$e = 6 + s$$

- b) If the side lengths are 10 cm, 10 cm, and 4 cm, then the perimeter is:

$$10 \text{ cm} + 10 \text{ cm} + 4 \text{ cm} = 24 \text{ cm}$$

This agrees with the given information.

The difference in lengths between an equal side and a shorter side is:

$$10 \text{ cm} - 4 \text{ cm} = 6 \text{ cm}$$

This agrees with the given information.

Since all 3 lengths agree with the given information, the lengths are correct.

9. a) The sum of the masses on the left pan of the balance scales is equal to the sum of the masses on the right pan.

From the first diagram:

$$x + x + x + y = 10 + 5 + 2$$

So, one equation is: $3x + y = 17$

From the second diagram:

$$x = y + 1 + 1 + 1$$

So, another equation is: $x = y + 3$

A linear system is:

$$3x + y = 17$$

$$x = y + 3$$

- b) If a small bag of rice has a mass of 2 kg and a large bag of rice has a mass of 5 kg, then:
From the first balance scales, the mass in the left pan is:

$$5 \text{ kg} + 5 \text{ kg} + 5 \text{ kg} + 2 \text{ kg} = 17 \text{ kg}$$

The mass in the right pan is: $10 \text{ kg} + 5 \text{ kg} + 2 \text{ kg}$

Since these masses are equal, the scales balance and the individual masses are correct.

From the second balance scales, the mass in the left pan is: 5 kg

The mass in the right pan is: $2 \text{ kg} + 1 \text{ kg} + 1 \text{ kg} + 1 \text{ kg} = 5 \text{ kg}$

Since these masses are equal, the scales balance and the individual masses are correct.

- c) Check that the values of x and y satisfy each equation.

Substitute $x = 5$ and $y = 2$ in $3x + y = 17$

$$\text{L.S.} = 3x + y$$

$$= 3(5) + 2$$

$$= 15 + 2$$

$$= 17$$

$$= \text{R.S.}$$

Since the left side is equal to the right side, the values of x and y satisfy the equation.

Substitute $x = 5$ and $y = 2$ in $x = y + 3$

$$\text{L.S.} = x$$

$$= 5$$

$$\text{R.S.} = y + 3$$

$$= 2 + 3$$

$$= 5$$

Since the left side is equal to the right side, the values of x and y satisfy the equation.

10. The total distance is 25 km.

From the diagram the total distance is: $x + y + y + 5$, or $x + 2y + 5$

So, one equation is: $x + 2y + 5 = 25$, or $x + 2y = 20$

The distance from home to cabin 2 is 13 km.

From the diagram, this distance is: $x + y$

So, another equation is: $x + y = 13$

A linear system is:

$$x + 2y = 20$$

$$x + y = 13$$

The distance from home to cabin 1 is x , so substitute $x = 7$.

The distance from cabin 1 to cabin 2 is y , so substitute $y = 6$.

Check solution A.

Substitute $x = 7$ and $y = 6$ in $x + 2y = 20$

$$\text{L.S.} = x + 2y$$

$$= 7 + 2(6)$$

$$= 7 + 12$$

$$= 19$$

$$\neq \text{R.S.}$$

Since the left side is not equal to the right side, the solution is not correct.

Check solution B.

Substitute $x = 6$ and $y = 7$ in $x + 2y = 20$.

$$\text{L.S.} = x + 2y$$

$$= 6 + 2(7)$$

$$= 6 + 14$$

$$= 20$$

$$= \text{R.S.}$$

Since the left side is equal to the right side, $x = 6$ and $y = 7$ satisfy this equation.

Substitute $x = 6$ and $y = 7$ in $x + y = 13$.

$$\text{L.S.} = x + y$$

$$= 6 + 7$$

$$= 13$$

$$= \text{R.S.}$$

Since the left side is equal to the right side, $x = 6$ and $y = 7$ satisfy this equation.

Since $x = 6$ and $y = 7$ satisfy both equations, this solution is correct.

11. Let w minutes represent the time that Padma walked.

Let j minutes represent the time that Padma jogged.

Padma jogged for 1 h, which is 60 min.

So, one equation is: $w + j = 60$

Padma walked for 10 min more than she jogged.

So, another equation is: $w = j + 10$

A linear system is:

$$w + j = 60$$

$$w = j + 10$$

Check solution A.

Substitute $w = 35$ and $j = 25$ in $w + j = 60$.

$$\text{L.S.} = w + j$$

$$= 35 + 25$$

$$= 60$$

$$= \text{R.S.}$$

Since the left side is equal to the right side, $w = 35$ and $j = 25$ satisfy this equation.

Substitute $w = 35$ and $j = 25$ in $w = j + 10$.

$$\text{L.S.} = w$$

$$= 35$$

$$\text{R.S.} = j + 10$$

$$= 25 + 10$$

$$= 35$$

Since the left side is equal to the right side, $w = 35$ and $j = 25$ satisfy this equation.

Since $w = 35$ and $j = 25$ satisfy both equations, this solution is correct.

Check solution B.

Substitute $w = 25$ and $j = 35$ in $w + j = 60$.

$$\text{L.S.} = w + j$$

$$= 25 + 35$$

$$= 60$$

$$= \text{R.S.}$$

Since the left side is equal to the right side, $w = 25$ and $j = 35$ satisfy this equation.

Substitute $w = 25$ and $j = 35$ in $w = j + 10$.

$$\text{L.S.} = w$$

$$= 25$$

$$\text{R.S.} = j + 10$$

$$= 35 + 10$$

$$= 45$$

Since the left side is not equal to the right side, this solution is not correct.

12. a), b) The problem is about a collection of toonies and loonies. So, the variables could represent the number of toonies and the number of loonies in the collection.

Use the letters in the equations.

Let t represent the number of toonies and let l represent the number of loonies.

Then the first equation indicates that the total value for t toonies and l loonies is \$160.
The second equation indicates that the sum of the number of toonies and the number of loonies is 110.

Shen might have written this problem:

There are 110 coins in a collection of loonies and toonies.

The value of the collection of coins is \$160.

How many loonies and how many toonies are in the collection?

13. a), b) The problem is about the costs of tickets for a child and an adult, and about the total cost of tickets for a group of children and adults. So, the variables could represent the cost of a ticket for an adult and the cost of a ticket for a child.

Use the letters in the equations.

Let a dollars represent the cost of an adult ticket and let c dollars represent the cost of a ticket for a child.

Then the first equation indicates that the total cost for 5 adults and 2 children is \$38.

The second equation indicates that the difference in costs for an adult's ticket and a child's ticket is \$2.

Jacqui might have written this problem:

Five adults and 2 children went to a local fair.

The total cost for the tickets was \$38.

An adult's ticket cost \$2 more than a child's ticket.

What was the cost of each ticket?

14. Two variables are added and subtracted. The situation could involve the sum and the difference of two numbers.

From the first equation, the sum of two numbers is 100.

From the second equation, the difference of two numbers is 10.

Let x represent the greater number and let y represent the lesser number.

Then a problem could be:

The sum of two numbers is 100.

The difference of the two numbers is 10.

What are the two numbers?

C

15. $Ax + By = C$
 $Dx + Ey = F$

- a) Since the solution of the linear system is $(0, y)$, substitute $x = 0$ in each equation.

$Ax + By = C$ becomes

$$By = C \quad \text{Divide each side by } B.$$

$$y = \frac{C}{B}$$

$Dx + Ey = F$ becomes

$$Ey = F \quad \text{Divide each side by } E.$$

$$y = \frac{F}{E}$$

Since y is a solution of both equations, the two values of y are equal, so

$$\frac{C}{B} = \frac{F}{E}$$

- b) Since the solution of the linear system is $(x, 0)$, substitute $y = 0$ in each equation.

$Ax + By = C$ becomes

$$Ax = C \quad \text{Divide each side by } A.$$

$$x = \frac{C}{A}$$

$Dx + Ey = F$ becomes

$$Dx = F \quad \text{Divide each side by } D.$$

$$x = \frac{F}{D}$$

Since x is a solution to both equations, the two values of x are equal, so $\frac{C}{A} = \frac{F}{D}$.

$$16. \quad \frac{-3x + 24}{x + y} = -6 \quad \text{①}$$

$$-\frac{x}{5} - \frac{y}{3} = x + y \quad \text{②}$$

Consider equation ①:

$$\frac{-3x + 24}{x + y} = -6$$

Multiply each side by $(x + y)$ to remove the fraction.

$$(x + y) \left(\frac{-3x + 24}{x + y} \right) = -6(x + y)$$

Simplify. Remove brackets.

$$-3x + 24 = -6x - 6y$$

Collect like terms. Add $6x$ and $6y$ to each side.

$$6x + 6y - 3x + 24 = 0$$

$$3x + 6y + 24 = 0$$

$$x + 2y + 8 = 0$$

Simplify. Divide each side by 3.

This is a linear equation in x and y .

Consider equation ②:

$$-\frac{x}{5} - \frac{y}{3} = x + y$$

Multiply each side by the common denominator 15.

$$15 \left(-\frac{x}{5} \right) - 15 \left(\frac{y}{3} \right) = 15(x + y)$$

Simplify.

$$-3x - 5y = 15x + 15y$$

Collect like terms. Subtract $15x$ and $15y$ from each side.

$$-3x - 5y - 15x - 15y = 0$$

$$-18x - 20y = 0$$

$$9x + 10y = 0$$

Divide each side by -2 .

The linear system is:

$$x + 2y + 8 = 0$$

$$9x + 10y = 0$$

17. a) I wrote two equations in x and y that have the solution $x = 1$ and $y = 1$.

I wrote this solution as the ordered pair $(1, 1)$.

For the first equation, I chose another ordered pair $(3, 5)$.

I used the formula for writing an equation when the coordinates of two points that satisfy the equation are known:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

I substituted $y_1 = 1$, $x_1 = 1$, $y_2 = 5$, and $x_2 = 3$.

$$\frac{y-1}{x-1} = \frac{5-1}{3-1}$$

$$\frac{y-1}{x-1} = \frac{4}{2}$$

$$\frac{y-1}{x-1} = 2$$

Multiply each side by $(x-1)$.

$$(x-1)\left(\frac{y-1}{x-1}\right) = 2(x-1)$$

$$y-1 = 2x-2$$

Add 1 to each side.

$$y = 2x-1$$

For the second equation, I chose another ordered pair $(4, 2)$.

I used the formula:

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

I substituted $y_1 = 1$, $x_1 = 1$, $y_2 = 2$, and $x_2 = 4$.

$$\frac{y-1}{x-1} = \frac{2-1}{4-1}$$

$$\frac{y-1}{x-1} = \frac{1}{3}$$

Multiply each side by $(x-1)$.

$$(x-1)\left(\frac{y-1}{x-1}\right) = \frac{1}{3}(x-1)$$

$$y-1 = \frac{1}{3}x - \frac{1}{3}$$

Add 1 to each side.

$$y = \frac{1}{3}x + \frac{2}{3}$$

A linear system is:

$$y = 2x - 1$$

$$y = \frac{1}{3}x + \frac{2}{3}$$

- b)** There is more than one linear system with the same solution because I can write many different pairs of equations that have the solution $x = 1$ and $y = 1$. I can use the formula in part a and substitute different pairs of values for x_2 and y_2 .

- 18. a)** Answers may vary. For example:

I examined the two equations of the linear system:

$$x + 2y = 7 \quad \text{①}$$

$$x + 3y = 9 \quad \text{②}$$

I added y to each side of equation ①.

$$x + 2y + y = 7 + y$$

$$x + 3y = 7 + y \quad \text{③}$$

From ②, $x + 3y = 9$

So, comparing ① and ③, $7 + y = 9$, or $y = 2$

So, $y = 2$ is part of the solution of the linear system.

b) $x + 2y = 7$ ①
 $x + 3y = 9$ ②

I know $y = 2$ is part of the solution.

I substitute $y = 2$ in each equation, then solve for x .

For equation ①:

$$x + 2y = 7$$

$$x + 2(2) = 7$$

$$x + 4 = 7$$

$$x = 3$$

For equation ②:

$$x + 3y = 9$$

$$x + 3(2) = 9$$

$$x + 6 = 9$$

$$x = 3$$

The solution is $x = 3$ and $y = 2$.

Lesson 7.2 Solving a System of Linear Equations Graphically Exercises (pages 409-410)**A**

3. The solution is the coordinates of the point of intersection of the two lines on each graph.

- a) The solution is: $x = -4$ and $y = 2$
- b) The solution is: $x = 2$ and $y = 3$
- c) The solution is: $x = 1$ and $y = -3$
- d) The solution is: $x = -2$ and $y = -1$

B

4. a) The solution appears to be: $x = 9$ and $y = -2$

Substitute these values for x and y into each equation to check.

$$2x + 3y = 12$$

$$x - y = 11$$

$$\text{L.S.} = 2x + 3y$$

$$\text{L.S.} = x - y$$

$$= 2(9) + 3(-2)$$

$$= 9 - (-2)$$

$$= 18 - 6$$

$$= 9 + 2$$

$$= 12$$

$$= 11$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side.

So, $x = 9$ and $y = -2$ is a solution of the linear system. This solution is exact.

b) The solution appears to be: $x = -1\frac{3}{4}$ and $y = 2\frac{3}{4}$

Substitute these values for x and y into each equation to check.

$$-3x + y = \frac{31}{4}$$

$$3x - 4y = -16$$

$$\text{L.S.} = -3x + y$$

$$\text{L.S.} = 3x - 4y$$

$$= -3\left(-1\frac{3}{4}\right) + 2\frac{3}{4}$$

$$= 3\left(-1\frac{3}{4}\right) - 4\left(2\frac{3}{4}\right)$$

$$= (-3)\left(-\frac{7}{4}\right) + \frac{11}{4}$$

$$= 3\left(-\frac{7}{4}\right) - 4\left(\frac{11}{4}\right)$$

$$= \frac{21}{4} + \frac{11}{4}$$

$$= -\frac{21}{4} - \frac{44}{4}$$

$$= \frac{32}{4}$$

$$= -\frac{65}{4}, \text{ or } -16\frac{1}{4}$$

For each equation, the left and right sides are not equal, but they are close in value.

So, $x = -1\frac{3}{4}$ and $y = 2\frac{3}{4}$ is an approximate solution of the linear system.

5. a) i) $x + y = 7$ ①

$$3x + 4y = 24$$
 ②

Determine the x - and y -intercepts of the graph of each equation.

For equation ①:

$$x + y = 7$$

$$\begin{aligned}\text{When } x = 0, \\ 0 + y = 7 \\ y = 7\end{aligned}$$

$$\begin{aligned}\text{When } y = 0, \\ x + 0 = 7 \\ x = 7\end{aligned}$$

On a grid, mark a point at 7 on each axis, then draw a line through the points.

For equation ②:

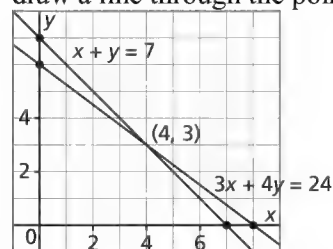
$$3x + 4y = 24$$

$$\begin{aligned}\text{When } x = 0, \\ 3(0) + 4y = 24 \\ 4y = 24 \\ y = 6\end{aligned}$$

When $y = 0$,

$$\begin{aligned}3x + 4(0) = 24 \\ 3x = 24 \\ x = 8\end{aligned}$$

On the grid, mark a point at 6 on the y -axis and mark a point at 8 on the x -axis, then draw a line through the points.



The point of intersection appears to be: (4, 3)

Verify the solution.

Substitute $x = 4$ and $y = 3$ into each equation.

$$\begin{aligned}x + y &= 7 \\ \text{L.S.} &= x + y \\ &= 4 + 3 \\ &= 7 \\ &= \text{R.S.}\end{aligned}$$

$$\begin{aligned}3x + 4y &= 24 \\ \text{L.S.} &= 3x + 4y \\ &= 3(4) + 4(3) \\ &= 12 + 12 \\ &= 24 \\ &= \text{R.S.}\end{aligned}$$

Since the left side is equal to the right side in each equation, then $x = 4$ and $y = 3$ is the solution of the linear system.

ii) $x - y = -1$ ①
 $3x + 2y = 12$ ②

Determine the x - and y -intercepts of the graph of each equation.

For equation ①:

$$x - y = -1$$

$$\begin{aligned}\text{When } x = 0, \\ 0 - y = -1 \\ y = 1\end{aligned}$$

$$\begin{aligned}\text{When } y = 0, \\ x - 0 = -1 \\ x = -1\end{aligned}$$

On a grid, mark a point at 1 on the y -axis and mark a point at -1 on the x -axis, then draw a line through the points.

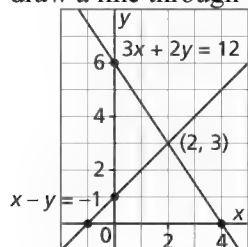
For equation ②:

$$3x + 2y = 12$$

$$\begin{aligned}\text{When } x = 0, \\ 3(0) + 2y = 12 \\ 2y = 12 \\ y = 6\end{aligned}$$

$$\begin{aligned}\text{When } y = 0, \\ 3x + 2(0) = 12 \\ 3x = 12 \\ x = 4\end{aligned}$$

On the grid, mark a point at 6 on the y -axis and mark a point at 4 on the x -axis, then draw a line through the points.



The point of intersection appears to be: $(2, 3)$

Verify the solution.

Substitute $x = 2$ and $y = 3$ into each equation.

$$x - y = -1$$

$$\text{L.S.} = x - y$$

$$= 2 - 3$$

$$= -1$$

$$= \text{R.S.}$$

$$3x + 2y = 12$$

$$\text{L.S.} = 3x + 2y$$

$$= 3(2) + 2(3)$$

$$= 6 + 6$$

$$= 12$$

$$= \text{R.S.}$$

Since the left side is equal to the right side in each equation, then $x = 2$ and $y = 3$ is the solution of the linear system.

iii) $5x + 4y = 10$ ①

$5x + 6y = 0$ ②

Determine the x - and y -intercepts of the graph of each equation.

For equation ①:

$$5x + 4y = 10$$

When $x = 0$,

$$5(0) + 4y = 10$$

$$4y = 10$$

$$y = \frac{10}{4}, \text{ or } 2\frac{1}{2}$$

When $y = 0$,

$$5x + 4(0) = 10$$

$$5x = 10$$

$$x = 2$$

Since the y -intercept is not where grid lines intersect, determine the coordinates of another point on the line.

Substitute: $x = 6$

$$5(6) + 4y = 10$$

$$30 + 4y = 10$$

$$4y = -20$$

$$y = -5$$

On a grid, mark a point at $(6, -5)$ and mark a point at 2 on the x -axis, then draw a line through the points.

For equation ②:

$$5x + 6y = 0$$

When $x = 0$,

$$5(0) + 6y = 0$$

$$6y = 0$$

$$y = 0$$

When $y = 0$,

$$5x + 6(0) = 0$$

$$5x = 0$$

$$x = 0$$

Since the x - and y -intercepts are equal, determine the coordinates of another point on the line.

Choose a value for x that is a multiple of 6, so the corresponding y -value is an integer.

Substitute: $x = 6$

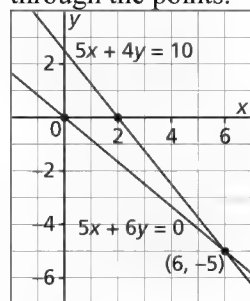
$$5(6) + 6y = 0$$

$$30 + 6y = 0$$

$$6y = -30$$

$$y = -5$$

On the grid, mark a point at $(6, -5)$ and mark a point at the origin, then draw a line through the points.



The point of intersection appears to be: $(6, -5)$

Since the coordinates of this point were determined using each equation, these coordinates satisfy each equation and $x = 6$ and $y = -5$ is the solution of the linear system.

iv) $x + 2y = -1$ ①
 $2x + y = -5$ ②

Determine the x - and y -intercepts of the graph of each equation.

For equation ①:

$$x + 2y = -1$$

When $x = 0$,

$$0 + 2y = -1$$

$$2y = -1$$

$$y = -\frac{1}{2}$$

When $y = 0$,

$$x + 2(0) = -1$$

$$x = -1$$

Since the y -intercept is not where grid lines intersect, determine the coordinates of another point on the line.

Substitute: $x = 1$

$$1 + 2y = -1$$

$$2y = -2$$

$$y = -1$$

On a grid, mark a point at $(1, -1)$ and mark a point at -1 on the x -axis, then draw a line through the points.

For equation ②:

$$2x + y = -5$$

When $x = 0$,

$$2(0) + y = -5$$

$$y = -5$$

When $y = 0$,

$$2x + 0 = -5$$

$$2x = -5$$

$$x = -2\frac{1}{2}$$

Since the x -intercept is not where grid lines intersect, determine the coordinates of another point on the line.

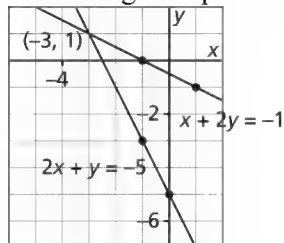
Substitute: $x = -1$

$$2(-1) + y = -5$$

$$-2 + y = -5$$

$$y = -3$$

On the grid, mark a point at $(-1, -3)$ and mark a point at -5 on the y -axis, then draw a line through the points.



The point of intersection appears to be: $(-3, 1)$

Verify the solution.

Substitute $x = -3$ and $y = 1$ into each equation.

$$x + 2y = -1$$

$$\text{L.S.} = -3 + 2(1)$$

$$= -3 + 2$$

$$= -1$$

$$= \text{R.S.}$$

$$2x + y = -5$$

$$\text{L.S.} = 2x + y$$

$$= 2(-3) + 1$$

$$= -6 + 1$$

$$= -5$$

$$= \text{R.S.}$$

Since the left side is equal to the right side in each equation, then $x = -3$ and $y = 1$ is the solution of the linear system.

b) For the linear system in part a, iv:

$$x + 2y = -1 \quad \textcircled{1}$$

$$2x + y = -5 \quad \textcircled{2}$$

The point of intersection of the graphs is the point with coordinates $(-3, 1)$.

This is the solution of the linear system. This point lies on both lines of the system, and its coordinates satisfy both equations of the system.

6. The linear system is:

$$3x - y = 1149$$

$$-x + 2y = 142$$

Substitute the solution $(500, 300)$ to check whether it is exact or approximate.

$$3x - y = 1149$$

$$\text{L.S.} = 3x - y$$

$$= 3(500) - 300$$

$$= 1500 - 300$$

$$= 1200$$

$$-x + 2y = 142$$

$$\text{L.S.} = -x + 2y$$

$$= -500 + 2(300)$$

$$= -500 + 600$$

$$= 100$$

For each equation, the left and right sides are not equal, but are close in value.

So, the solution $x = 500$ and $y = 300$ is an approximate solution.

7. a) $2x + 4y = -1 \quad \textcircled{1}$

$$3x - y = 9 \quad \textcircled{2}$$

Graph each equation.

For equation $\textcircled{1}$:

$$2x + 4y = -1$$

Since the coefficients of x and y are even and the constant term is odd, there are no

integer coordinates that satisfy the equation.

Determine the x - and y -intercepts, then graph the equation on a grid with a larger scale.

$$2x + 4y = -1$$

When $x = 0$,

$$2(0) + 4y = -1$$

$$4y = -1$$

$$y = -\frac{1}{4}$$

When $y = 0$,

$$2x + 4(0) = -1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

On a grid, use a scale of 4 squares represents 1 unit, mark a point at $-\frac{1}{4}$ on the y -axis

and mark a point at $-\frac{1}{2}$ on the x -axis, then draw a line through the points.

For equation ②:

$$3x - y = 9$$

When $x = 0$,

$$3(0) - y = 9$$

$$-y = 9$$

$$y = -9$$

When $y = 0$,

$$3x - 0 = 9$$

$$3x = 9$$

$$x = 3$$

For a y -intercept of -9 , the y -axis would be very long on a graph with a scale of 4 squares to 1 unit, so choose a different point to plot.

Substitute $x = 2$ in $3x - y = 9$.

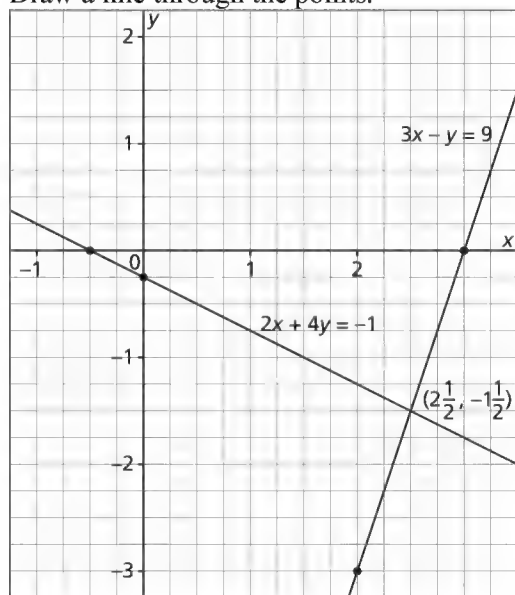
$$3(2) - y = 9$$

$$6 - y = 9$$

$$y = -3$$

On the grid, mark a point at $(2, -3)$ and mark a point at 3 on the x -axis.

Draw a line through the points.



The solution appears to be: $x = 2\frac{1}{2}$ and $y = -1\frac{1}{2}$

Write these numbers as decimals: $x = 2.5$ and $y = -1.5$

Substitute these values for x and y into each equation to check.

$$2x + 4y = -1$$

$$\text{L.S.} = 2x + 4y$$

$$= 2(2.5) + 4(-1.5)$$

$$= 5 - 6$$

$$= -1$$

$$= \text{R.S.}$$

$$3x - y = 9$$

$$\text{L.S.} = 3x - y$$

$$= 3(2.5) - (-1.5)$$

$$= 7.5 + 1.5$$

$$= 9$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 2.5$ and $y = -1.5$.

b) $5x + 5y = 17$ ①

$x - y = -1$ ②

Graph each equation.

For equation ①:

$$5x + 5y = 17$$

Since the coefficients of x and y are even and the constant term is odd, there are no integer coordinates that satisfy the equation.

Determine the x - and y -intercepts, then graph the equation on a grid with a larger scale.

$$5x + 5y = 17$$

When $x = 0$,

$$5(0) + 5y = 17$$

$$5y = 17$$

$$y = \frac{17}{5}, \text{ or } 3\frac{2}{5}$$

When $y = 0$,

$$5x + 5(0) = 17$$

$$5x = 17$$

$$x = \frac{17}{5}, \text{ or } 3\frac{2}{5}$$

On a grid, use a scale of 5 squares represents 1 unit, mark a point at $3\frac{2}{5}$ on the y -axis and

mark a point at $3\frac{2}{5}$ on the x -axis, then draw a line through the points.

For equation ②:

$$x - y = -1$$

When $x = 0$,

$$0 - y = -1$$

$$y = 1$$

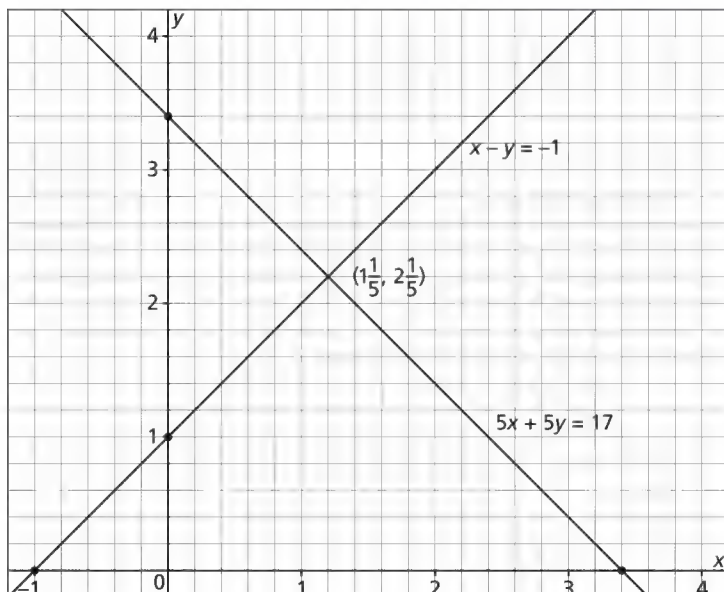
When $y = 0$,

$$x - 0 = -1$$

$$x = -1$$

On the grid, mark a point at 1 on the y -axis and mark a point at -1 on the x -axis.

Draw a line through the points.



The solution appears to be: $x = 1\frac{1}{5}$ and $y = 2\frac{1}{5}$

Write these numbers as decimals: $x = 1.2$ and $y = 2.2$

Substitute these values for x and y into each equation to check.

$$5x + 5y = 17$$

$$\text{L.S.} = 5x + 5y$$

$$= 5(1.2) + 5(2.2)$$

$$= 6 + 11$$

$$= 17$$

$$= \text{R.S.}$$

$$x - y = -1$$

$$\text{L.S.} = x - y$$

$$= 1.2 - 2.2$$

$$= -1$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution of the linear system is: $x = 1.2$ and $y = 2.2$

c) $x + y = \frac{23}{4}$ ①

$x - y = \frac{3}{4}$ ②

Graph each equation.

For equation ①:

$$x + y = \frac{23}{4}$$

Since the constant term is a rational number, there are no integer coordinates that satisfy the equation.

Determine the x - and y -intercepts, then graph the equation on a grid with a larger scale.

$$x + y = \frac{23}{4}$$

When $x = 0$,

$$0 + y = \frac{23}{4}$$

$$y = \frac{23}{4}, \text{ or } 5\frac{3}{4}$$

When $y = 0$,

$$x + 0 = \frac{23}{4}$$

$$x = \frac{23}{4}, \text{ or } 5\frac{3}{4}$$

On a grid, use a scale of 4 squares represents 1 unit, mark a point at $5\frac{3}{4}$ on the y -axis and

mark a point at $5\frac{3}{4}$ on the x -axis, then draw a line through the points.

For equation ②:

$$x - y = \frac{3}{4}$$

When $x = 0$,

$$0 - y = \frac{3}{4}$$

$$y = -\frac{3}{4}$$

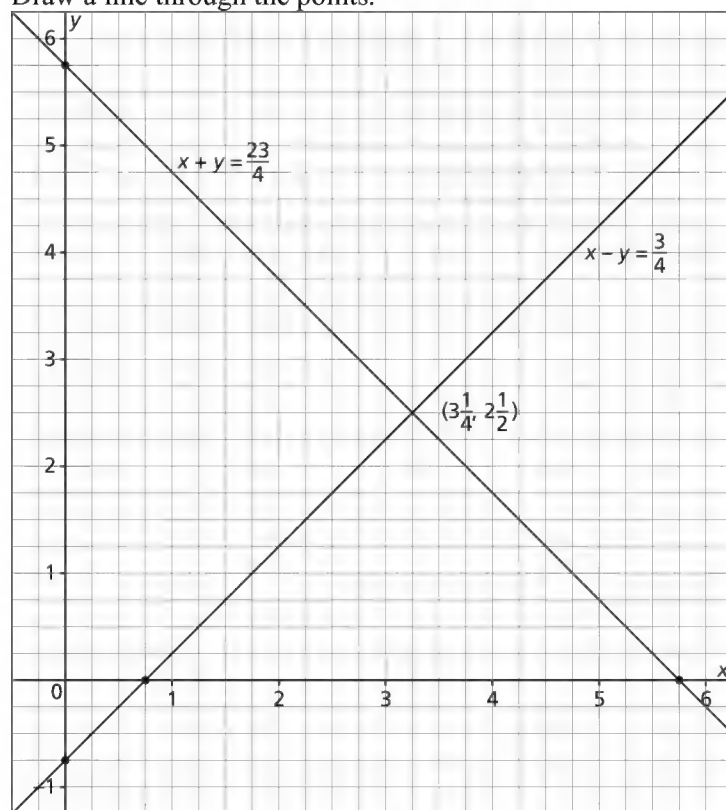
When $y = 0$,

$$x - 0 = \frac{3}{4}$$

$$x = \frac{3}{4}$$

On the grid, mark a point at $-\frac{3}{4}$ on the y -axis and mark a point at $\frac{3}{4}$ on the x -axis.

Draw a line through the points.



The solution appears to be: $x = 3\frac{1}{4}$ and $y = 2\frac{1}{2}$

Write these numbers as decimals: $x = 3.25$ and $y = 2.5$

Substitute these values for x and y into each equation to check. Write the rational numbers in the equations as decimals.

$$x - y = \frac{3}{4}$$

$$x - y = 0.75$$

$$\begin{aligned} \text{L.S.} &= x - y \\ &= 3.25 - 2.5 \end{aligned}$$

$$x + y = \frac{23}{4}$$

$$x + y = 5.75$$

$$\begin{aligned} \text{L.S.} &= 3.25 + 2.5 \\ &= 5.75 \end{aligned}$$

$$= 0.75$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 3.25$ and $y = 2.5$.

d) $3x + y = 6$ ①

$x + y = -\frac{4}{3}$ ②

Graph each equation.

For equation ①:

$$3x + y = 6$$

Determine the x - and y -intercepts.

$$3x + y = 6$$

$$\text{When } x = 0,$$

$$3(0) + y = 6$$

$$y = 6$$

$$\text{When } y = 0,$$

$$3x + 0 = 6$$

$$3x = 6$$

$$x = 2$$

On a grid, mark a point at 6 on the y -axis and mark a point at 2 on the x -axis, then draw a line through the points.

For equation ②:

$$x + y = -\frac{4}{3}$$

$$\text{When } x = 0,$$

$$0 + y = -\frac{4}{3}$$

$$y = -\frac{4}{3}$$

$$\text{When } y = 0,$$

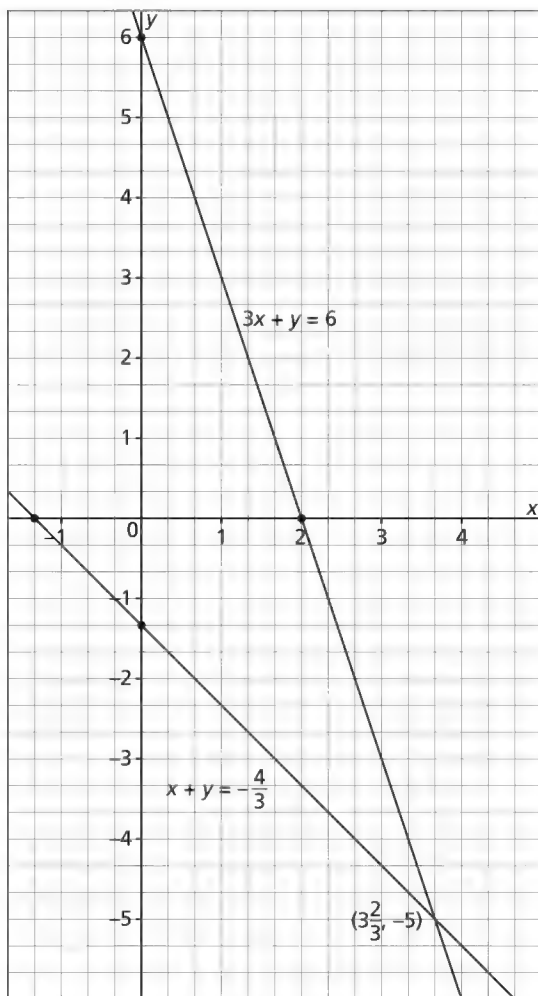
$$x - 0 = -\frac{4}{3}$$

$$x = -\frac{4}{3}$$

On the grid, use a scale of 3 squares to 1 unit. Mark a point at $x + y = -\frac{4}{3}$ on the y -axis

and mark a point at $-\frac{4}{3}$ on the x -axis.

Draw a line through the points.



The solution appears to be: $x = 3\frac{2}{3}$ and $y = -5$

Substitute these values for x and y into each equation to check.

$$x + y = -\frac{4}{3}$$

$$\text{L.S.} = x + y$$

$$= 3\frac{2}{3} - 5$$

$$= \frac{11}{3} - \frac{15}{3}$$

$$= -\frac{4}{3}$$

$$= \text{R.S.}$$

$$3x + y = 6$$

$$\text{L.S.} = 3x + y$$

$$= 3\left(3\frac{2}{3}\right) - 5$$

$$= (3)\left(\frac{11}{3}\right) - 5$$

$$= \frac{33}{3} - 5$$

$$= 11 - 5$$

$$= 6$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution of the linear system is $x = 3\frac{2}{3}$ and $y = -5$.

8. a) $C = 175 + 0.10n$ ①
 $C = 250 + 0.07n$ ②

For each equation, determine the C -intercept and the coordinates of another point on the line.

For equation ①:

$$C = 175 + 0.10n$$

Substitute: $n = 0$

$$C = 175$$

Substitute: $n = 1000$

$$C = 175 + 0.10 \times 1000$$

$$C = 175 + 100$$

$$C = 275$$

On a grid, use a scale of 2 squares to 100 units on the C -axis, and a scale of 2 squares to 500 units on the n -axis. Mark a point at 175 on the C -axis and mark a point at (1000, 275). Draw a line through the points. The data are discrete, but the scale is so small that if a point were plotted for each brochure, these points would form a line.

For equation ②:

$$C = 250 + 0.07n$$

Substitute: $n = 0$

$$C = 250$$

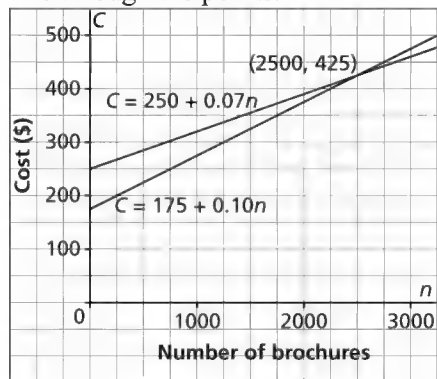
Substitute: $n = 1000$

$$C = 250 + 0.07 \times 1000$$

$$C = 250 + 70$$

$$C = 320$$

On the grid, mark a point at 250 on the C -axis and mark a point at (1000, 320). Draw a line through the points.



- b) i) From the graph, the cost will be the same at both companies at the point of intersection of the lines; that is, when 2500 brochures are printed for \$425. Check that this solution satisfies both equations.

Substitute $C = 425$ and $n = 2500$ in each equation.

For equation ①:

$$C = 175 + 0.10n$$

$$\text{L.S.} = C$$

$$= 425$$

$$\text{R.S.} = 175 + 0.10n$$

$$= 175 + 0.10(2500)$$

$$= 175 + 250$$

$$= 425$$

For equation ②:

$$C = 250 + 0.07n$$

$$\text{L.S.} = C$$

$$= 425$$

$$\text{R.S.} = 250 + 0.07n$$

$$= 250 + 0.07(2500)$$

$$= 250 + 175$$

$$= 425$$

Since the left side is equal to the right side for each equation, the solution is correct.

- ii) The equation that represents Company A is $C = 175 + 0.10n$. It is cheaper to use Company A when its line is beneath the line for Company B; that is, for any number of brochures less than 2500.

9. a) $P = 700 + 0.03s$ ①

$P = 1000 + 0.02s$ ②

For each equation, determine the P -intercept and the coordinates of another point on the line.

For equation ①:

$$P = 700 + 0.03s$$

Substitute: $s = 0$

$$P = 700$$

Substitute: $s = 10\,000$

$$P = 700 + 0.03 \times 10\,000$$

$$P = 700 + 300$$

$$P = 1000$$

On a grid, use a scale of 1 square to 200 units on the P -axis, and a scale of 2 squares to 10 000 units on the s -axis. Mark a point at 700 on the P -axis and mark a point at (10 000, 1000). Draw a line through the points.

For equation ②:

$$P = 1000 + 0.02s$$

Substitute: $s = 0$

$$P = 1000$$

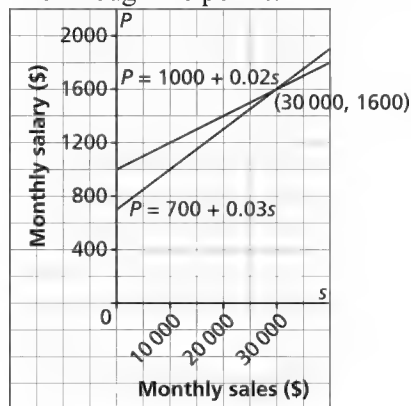
Substitute: $s = 10\,000$

$$P = 1000 + 0.02 \times 10\,000$$

$$P = 1000 + 200$$

$$P = 1200$$

On the grid, mark a point at 1000 on the P -axis and mark a point at (1000, 1200). Draw a line through the points.



- b) i) From the graph, a clerk will receive the same salary with both plans at the point of intersection of the lines; that is, when \$30 000 in sales earns a salary of \$1600. Check that this solution satisfies both equations.

Substitute $P = 1600$ and $s = 30\,000$ in each equation.

For equation ①:

$$P = 700 + 0.03s$$

$$\begin{aligned}\text{L.S.} &= P \\ &= 1600\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= 700 + 0.03s \\ &= 700 + 0.03(30\,000) \\ &= 700 + 900 \\ &= 1600\end{aligned}$$

For equation ②:

$$P = 1000 + 0.02s$$

$$\begin{aligned}\text{L.S.} &= P \\ &= 1600\end{aligned}$$

$$\begin{aligned}\text{R.S.} &= 1000 + 0.02s \\ &= 1000 + 0.02(30\,000) \\ &= 1000 + 600 \\ &= 1600\end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct.

- ii) The equation that represents Plan B is $P = 1000 + 0.02s$. A person on Plan B is earning more than he or she would on Plan A when the line for Plan B is above the line for Plan A; that is, for any value of sales up to \$30 000.

10. Let the area of the forested part be represented by f hectares.

Let the unforested area be represented by r hectares.

The area of Stanley park is 391 hectares. This is the sum of f hectares and r hectares.

So, one equation is: $f + r = 391$

The forested area is 141 hectares more than the unforested area.

So, another equation is: $f = r + 141$

A linear system is:

$$f + r = 391 \quad \text{①}$$

$$f = r + 141 \quad \text{②}$$

Graph the equations.

Determine the f - and r -intercepts.

For equation ①:

$$f + r = 391$$

$$\text{Substitute: } r = 0$$

$$f + 0 = 391$$

$$f = 391$$

$$\text{Substitute: } f = 0$$

$$0 + r = 391$$

$$r = 391$$

On a grid, plot r as a function of f ; that is, plot r on the vertical axis and f on the horizontal axis. Use a scale of 1 square to 50 units on each axis. Mark a point at 391 on each axis. Draw a line through the points. Since the point at 391 is an approximate position, the graph is approximate.

For equation ②:

$$f = r + 141$$

$$\text{Substitute: } r = 0$$

$$f = 0 + 141$$

$$f = 141$$

$$\text{Substitute: } f = 0$$

$$0 = r + 141$$

$$r = -141$$

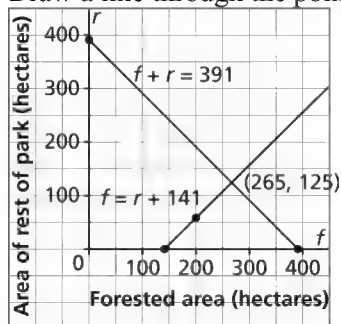
A negative value of r makes no sense when we are considering areas. This value of r is not in the range. Substitute a different value of f , such as $f = 200$.

$$200 = r + 141$$

$$r = 59$$

On the grid, mark a point at 141 on the f -axis and mark a point at (200, 59).

Draw a line through the points.



From the graph, the coordinates of the point of intersection indicate the two areas.

The f -coordinate is approximately 265 and the r -coordinate is approximately 125.

So, the forested area is about 265 hectares and the rest of the park is about 125 hectares.

Use the given information to check this solution.

The sum of the areas, in hectares, is: $265 + 125 = 390$; this is close to the given sum of 391.

The difference of the areas is: $265 - 125 = 140$; this is close to the given difference of 141.

The solution is approximate.

11. Let the number of wins be represented by w .

Let the number of overtime losses be represented by l .

The team had a total of 107 points.

The team gets 2 points for a win and 1 point for an overtime loss.

So, one equation is: $2w + l = 107$

The team had 43 more wins than overtime losses.

So, another equation is: $w - l = 43$

A linear system is:

$$2w + l = 107 \quad \text{①}$$

$$w - l = 43 \quad \text{②}$$

Graph the equations.

Determine the w - and l -intercepts.

For equation ①:

$$2w + l = 107 \quad \text{①}$$

Substitute: $l = 0$

$$2w + 0 = 107$$

$$2w = 107$$

$$w = 53.5$$

Substitute: $w = 0$

$$2(0) + l = 107$$

$$l = 107$$

Since the w -intercept is not a whole number, substitute a different value of l , and determine the corresponding w -coordinate.

Substitute: $l = 17$

$$2w + 17 = 107$$

$$2w = 90$$

$$w = 45$$

On a grid, plot l as a function of w ; that is, plot l on the vertical axis and w on the horizontal axis. Use a scale of 1 square to 10 units on each axis. Mark a point at (45, 17) and mark a point at 107 on the l -axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of w and l .

For equation ②:

$$w - l = 43 \quad \text{②}$$

Substitute: $l = 0$

$$w - 0 = 43$$

$$w = 43$$

Substitute: $w = 0$

$$0 - l = 43$$

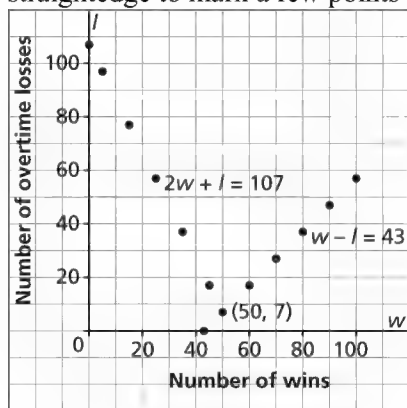
$$l = -43$$

A negative value of l makes no sense when we are considering numbers of wins and overtime losses. This value of l is not in the range. Substitute a different value of w , such as $w = 80$.

$$80 - l = 43$$

$$l = 37$$

On the grid, mark a point at 43 on the w -axis and mark a point at $(80, 37)$. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of w and l .



From the graph, the coordinates of the point of intersection are $(50, 7)$.

The w -coordinate is approximately 50 and the l -coordinate is approximately 7.

So, the number of wins is about 50 and the number of overtime losses is about 7.

Use the given information to check this solution.

The number of points is: $2(50) + 7 = 107$; this is equal to the given number of points.

The difference in wins and overtime losses is: $50 - 7 = 43$; this is equal to the given difference.

The solution is correct and the numbers are exact.

12. Let the number of \$5 gift cards sold be represented by f .

Let the number of \$10 gift cards be represented by t .

The class raised \$800.

So, one equation is: $5f + 10t = 800$

The class sold a total of 115 gift cards.

So, another equation is: $f + t = 115$

A linear system is:

$$5f + 10t = 800 \quad \text{①}$$

$$f + t = 115 \quad \text{②}$$

Graph the equations.

Determine the f - and t -intercepts.

For equation ①:

$$5f + 10t = 800 \quad \text{①}$$

Substitute: $t = 0$

$$5f + 10(0) = 800$$

$$5f = 800$$

$$f = 160$$

Substitute: $f = 0$

$$5(0) + 10t = 800$$

$$10t = 800$$

$$t = 80$$

On a grid, plot t as a function of f ; that is, plot t on the vertical axis and f on the horizontal

axis. Use a scale of 1 square to 10 units on each axis. Mark a point at 160 on the f -axis and mark a point at 80 on the t -axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of f and t .

For equation ②:

$$f + t = 115$$

②

Substitute: $t = 0$

$$f + 0 = 115$$

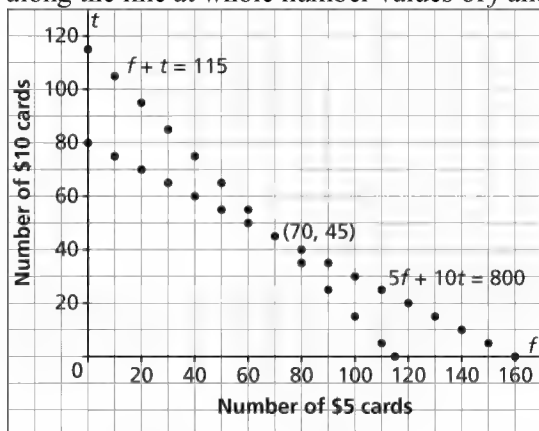
$$f = 115$$

Substitute: $f = 0$

$$0 + t = 115$$

$$t = 115$$

On the grid, mark a point at 115 on each axis. Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of f and t .



From the graph, the coordinates of the point of intersection appear to be (70, 45).

The f -coordinate is approximately 70 and the t -coordinate is approximately 45.

So, the number of \$5 gift cards sold is about 70 and the number of \$10 gift cards sold is about 45.

Use the given information to check this solution.

The amount of money raised, in dollars, is: $5(70) + 10(45) = 350 + 450$, or 800; this is equal to the given amount.

The total number of gift cards sold is: $70 + 45 = 115$; this is equal to the total number of cards sold.

The solution is correct and the numbers are exact.

13. Let the number of student tickets sold be represented by s .

Let the number of adult tickets sold be represented by a .

The total admission fee was \$152.

So, one equation is: $4.80s + 8a = 152$

There were 13 more students than adults.

So, another equation is: $s - a = 13$

A linear system is:

$$4.80s + 8a = 152 \quad \text{①}$$

$$s - a = 13 \quad \text{②}$$

Graph the equations.

Determine the s - and a -intercepts.

For equation ①:

$$4.80s + 8a = 152$$

Substitute: $a = 0$

$$4.80s + 8(0) = 152$$

$$4.80s = 152$$

$$s = 31.\bar{6}$$

Substitute: $s = 0$

$$4.80(0) + 8a = 152$$

$$8a = 152$$

$$a = 19$$

Since the s -intercept is not a whole number, substitute a different value for a , then solve for s .

Substitute: $a = 10$

$$4.80s + 8(10) = 152$$

$$4.80s + 80 = 152$$

$$4.80s = 72$$

$$s = 15$$

On a grid, plot a as a function of s ; that is, plot a on the vertical axis and s on the horizontal axis. Use a scale of 1 square to 2 units on each axis. Mark a point at 19 on the a -axis and mark a point at (15, 10). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of a and s .

For equation ②:

$$s - a = 13$$

②

Substitute: $a = 0$

$$s - 0 = 13$$

$$s = 13$$

Substitute: $s = 0$

$$0 - a = 13$$

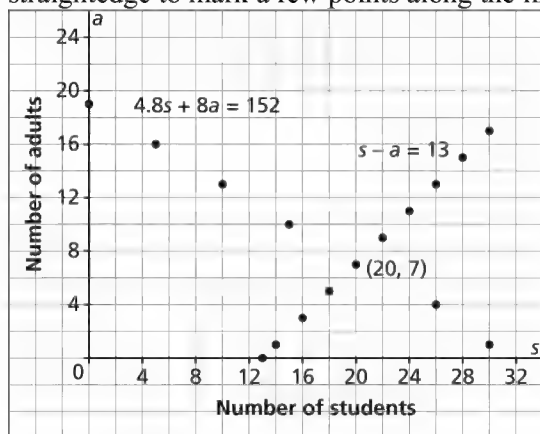
$$a = -13$$

A negative value of a makes no sense when we are considering numbers of adults. This value of a is not in the range. Substitute a different value of s , such as $s = 30$.

$$30 - a = 13$$

$$a = 17$$

On the grid, mark a point at 13 on the s -axis and mark a point at (30, 17). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of s and a .



From the graph, the coordinates of the point of intersection appear to be (20, 7).

The s -coordinate is approximately 20 and the a -coordinate is approximately 7.

So, the number of student tickets sold is about 20 and the number of adult tickets sold is about 7.

Use the given information to check this solution.

The total admission fee, in dollars, is: $4.8(20) + 7(8) = 96 + 56$, or 152; this is equal to the given admission fee.

The difference in the numbers of students and adults is: $20 - 7 = 13$; this is equal to the given difference.

The solution is correct and the numbers are exact.

14. a) Let the mass of the box be represented by b grams.
 Let the mass of a golf ball be represented by g grams.
 The box and 36 golf balls have a mass of 1806 g.
 So, one equation is: $b + 36g = 1806$
 When 12 balls are removed, 24 balls remain, so the mass of the box and 24 balls is 1254 g.
 $b + 24g = 1254$
 A linear system is:
 $b + 36g = 1806$ ①
 $b + 24g = 1254$ ②

- b) To graph the equations, determine the b - and g -intercepts.

For equation ①:

$$b + 36g = 1806$$

Substitute: $g = 0$

$$b + 36(0) = 1806$$

$$b = 1806$$

Substitute: $b = 0$

$$(0) + 36g = 1806$$

$$36g = 1806$$

$$g = 50.\overline{16}$$

The g -intercept is not a whole number, and it would be difficult to determine a whole number value, so the graph and solution will be approximate.

On a grid, plot g as a function of b ; that is, plot g on the vertical axis and b on the horizontal axis. Use a scale of 1 square to 100 units on the b -axis and 1 square to 5 units on the g -axis. Mark a point at 50 on the g -axis and mark a point at (1806). Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of g .

For equation ②:

$$b + 24g = 1254$$

②

Substitute: $g = 0$

$$b + 24(0) = 1254$$

$$b = 1254$$

Substitute: $b = 0$

$$0 + 24g = 1254$$

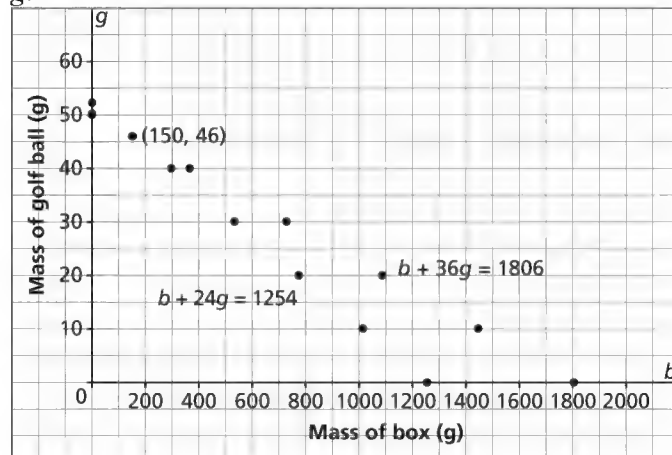
$$24g = 1254$$

$$g = 52.25$$

As with equation ①, the g -intercept is not a whole number, so the graph and solution will be approximate.

On the grid, mark a point at 1254 on the b -axis and mark a point at 52 on the g -axis.

Since the data are discrete, the graph is a set of points that lie on a line through the plotted points. Use a straightedge to mark a few points along the line at whole number values of g .



From the graph, the coordinates of the point of intersection appear to be (150, 45).

The b -coordinate is approximately 150 and the g -coordinate is approximately 45.

So, the mass of the box is about 150 g and the mass of a golf ball is about 45 g.

Use the given information to check this solution.

The mass of the box and 36 balls, in grams, is: $150 + 36(45) = 1770$; this is close to the given mass of 1806 g.

The mass of the box and 24 balls, in grams, is: $150 + 24(45) = 1230$; this is close to the given mass of 1254 g.

The mass of the box is about 150 g and the mass of one golf ball is about 45 g.

- c) It was difficult to determine the exact solution because I could not plot the intercepts accurately using the scale on the graph paper I had. When the intercepts are not accurate, the solution of the linear system will not be accurate.

15. The pentagon has perimeter 58 in.

So, $x + y + y + x + 17 = 58$ Collect like terms.

$$2x + 2y = 41$$

So, one equation is: $2x + 2y = 41$ ①

y is greater than x , and the difference between y and x is $3\frac{1}{2}$.

So, another equation is: $y - x = 3\frac{1}{2}$

A linear system is:

$$2x + 2y = 41 \quad \text{①}$$

$$y - x = 3\frac{1}{2} \quad \text{②}$$

Graph the equations.

Determine the x - and y -intercepts.

For equation ①:

$$2x + 2y = 41 \quad \text{①}$$

Substitute: $y = 0$

$$2x + 2(0) = 41$$

$$2x = 41$$

$$x = 20\frac{1}{2}$$

Substitute: $x = 0$

$$2(0) + 2y = 41$$

$$2y = 41$$

$$y = 20\frac{1}{2}$$

On a grid, plot y as a function of x . Mark a point at $20\frac{1}{2}$ on each axis. Draw a line through the points.

For equation ②:

$$y - x = 3\frac{1}{2} \quad \text{②}$$

Substitute: $y = 0$

$$0 - x = 3\frac{1}{2}$$

$$x = -3\frac{1}{2}$$

Substitute: $x = 0$

$$y - 0 = 3\frac{1}{2}$$

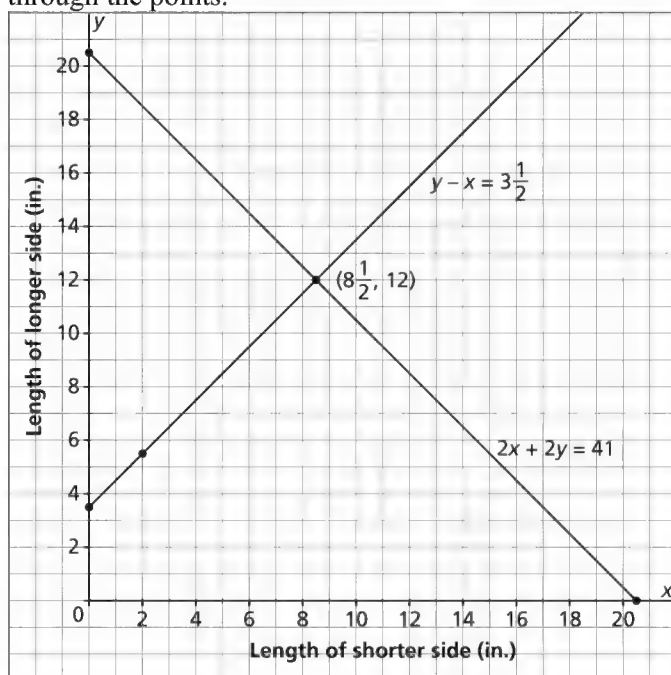
$$y = 3\frac{1}{2}$$

A negative value of x makes no sense when we are considering the length of the side of a pentagon. This value of x is not in the domain. Substitute a different value of x , such as $x = 2$.

$$y - 2 = 3\frac{1}{2}$$

$$y = 5\frac{1}{2}$$

On the grid, mark a point at $3\frac{1}{2}$ on the y -axis and mark a point at $(2, 5\frac{1}{2})$. Draw a line through the points.



From the graph, the coordinates of the point of intersection appear to be $(8\frac{1}{2}, 12)$.

The x -coordinate is approximately $8\frac{1}{2}$ and the y -coordinate is 12.

So, the value of x is about $8\frac{1}{2}$ in. and the value of y is about 12 in.

Use the given information to check this solution.

The perimeter, in inches, is: $17 + 2\left(8\frac{1}{2}\right) + 2(12) = 17 + 17 + 24$, or 58 in.; this is equal to the given perimeter.

The difference between y and x is: $12 - 8\frac{1}{2} = 3\frac{1}{2}$; this is equal to the given difference.

The solution is correct and the numbers are exact; that is, x is $8\frac{1}{2}$ in. and y is 12 in.

16. a) $2x + 7y = 3$ ①
 $4x + 3y = 7$ ②

To graph this linear system, determine the coordinates of two points on each line.

By looking at the equations, I can tell that the intercepts are fractions, so I choose other points.

For equation ①:

Since it is easier to divide by 2 than by 7, I choose values of y to substitute, then solve for x .

$$2x + 7y = 3$$

Substitute: $y = 1$

$$2x + 7(1) = 3$$

$$2x = -4$$

$$x = -2$$

Substitute: $y = -1$

$$2x + 7(-1) = 3$$

$$2x = 10$$

$$x = 5$$

Two points on equation ① have coordinates: $(-2, 1)$ and $(5, -1)$

For equation ②:

Since it is easier to divide by 4 than by 3, I choose values of y to substitute, then solve for x .

$$4x + 3y = 7$$

Substitute: $y = 1$

$$4x + 3(1) = 7$$

$$4x = 4$$

$$x = 1$$

Substitute: $y = -1$

$$4x + 3(-1) = 7$$

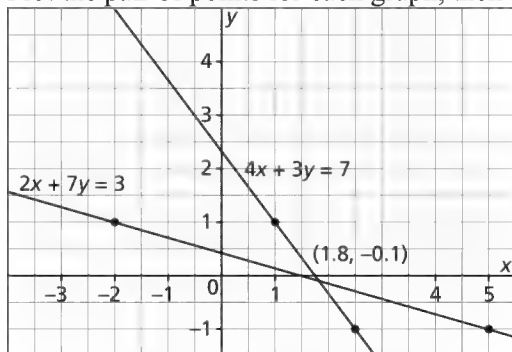
$$4x = 10$$

$$x = 2.5$$

Two points on equation ② have coordinates: $(1, 1)$ and $(2.5, -1)$

To plot the decimal values of the coordinates, use a scale of 2 squares to 1 unit on each axis.

Plot the pair of points for each graph, then draw a line through the points.



From the graph, the solution appears to be: $x = 1.8$ and $y = -0.1$

Check whether this solution satisfies both equations.

Substitute $x = 1.8$ and $y = -0.1$ in each equation.

For equation ①:

$$2x + 7y = 3$$

$$\text{L.S.} = 2x + 7y$$

$$\text{R.S.} = 3$$

$$= 2(1.8) + 7(-0.1)$$

$$= 3.6 - 0.7$$

$$= 2.9$$

For equation ②:

$$4x + 3y = 7$$

$$\text{L.S.} = 4x + 3y$$

$$\text{R.S.} = 7$$

$$= 4(1.8) + 3(-0.1)$$

$$= 7.2 - 0.3$$

$$= 6.9$$

For each equation, the left side is not equal to the right side, but the values are close.

So, the solution $x = 1.8$ and $y = -0.1$ is approximate.

- b) The solution is approximate because I cannot read the exact fraction or decimal value from the graph.

C

17. a) Equation ① has x -intercept 5 and y -intercept 5.

So, the graph of the equation passes through the points with coordinates (5, 0) and (0, 5).
Use the formula for the equation of a line when the coordinates of two points on the line

are known: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute: $y_1 = 0$, $x_1 = 5$, $y_2 = 5$, and $x_2 = 0$

$$\frac{y - 0}{x - 5} = \frac{5 - 0}{0 - 5}$$

$$\frac{y}{x - 5} = -1$$

Multiply each side by $(x - 5)$.

$$y = -1(x - 5)$$

$$y = -x + 5$$

This is equation ①.

Equation ② has x -intercept 4 and y -intercept 6.

So, the graph of the equation passes through the points with coordinates (4, 0) and (0, 6).

Use this formula: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute: $y_1 = 0$, $x_1 = 4$, $y_2 = 6$, and $x_2 = 0$

$$\frac{y - 0}{x - 4} = \frac{6 - 0}{0 - 4}$$

$$\frac{y}{x - 4} = -\frac{3}{2}$$

Multiply each side by $(x - 4)$.

$$y = -\frac{3}{2}(x - 4)$$

Remove brackets.

$$y = -\frac{3}{2}x + 6$$

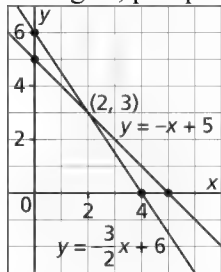
This is equation ②.

The linear system is:

$$y = -x + 5 \quad \text{①}$$

$$y = -\frac{3}{2}x + 6 \quad \text{②}$$

- b) On a grid, plot points at the intercepts for each graph, then draw a line through the points.



The solution appears to be: $x = 2$ and $y = 3$

Check that this solution satisfies both equations.

Substitute $x = 2$ and $y = 3$ in each equation.

For equation ①:

$$y = -x + 5$$

$$\text{L.S.} = y$$

$$= 3$$

$$\text{R.S.} = -x + 5$$

$$= -2 + 5$$

$$= 3$$

For equation ②:

$$y = -\frac{3}{2}x + 6$$

$$\text{L.S.} = y$$

$$= 3$$

$$\text{R.S.} = -\frac{3}{2}x + 6$$

$$= -\frac{3}{2}(2) + 6$$

$$= -3 + 6$$

$$= 3$$

Since the left side is equal to the right side for each equation, the solution is correct.

18. Graph the equation $y = 2x + 1$. Substitute two values for x .

When $x = 1$:

$$y = 2(1) + 1$$

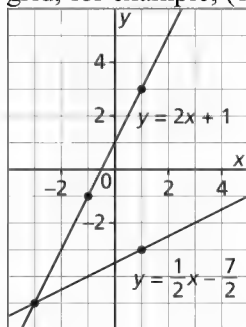
$$y = 3$$

When $x = -1$:

$$y = 2(-1) + 1$$

$$y = -1$$

On a grid, plot points at $(1, 3)$ and $(-1, -1)$, then draw a line through them. Extend the line to the third quadrant. Choose a point on the line in the third quadrant; for example, $(-3, -5)$. Let this be the coordinates of the point that is the solution of the linear system. To determine a second equation in the linear system, first choose the coordinates of any other point on the grid; for example, $(1, -3)$.



Use the formula for the equation of a line when the coordinates of two points on the line are

$$\text{known: } \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute: $y_1 = -5$, $x_1 = -3$, $y_2 = -3$, and $x_2 = 1$

$$\frac{y - (-5)}{x - (-3)} = \frac{-3 - (-5)}{1 - (-3)}$$

$$\frac{y + 5}{x + 3} = \frac{2}{4}$$

$$\frac{y + 5}{x + 3} = \frac{1}{2}$$

Multiply each side by $(x + 3)$.

$$y + 5 = \frac{1}{2}(x + 3)$$

$$y + 5 = \frac{1}{2}x + \frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{7}{2}$$

The second equation could be: $y = \frac{1}{2}x - \frac{7}{2}$

19. a) $2x + 3y = -5$ ①

$$\frac{x}{2} - \frac{y}{3} = 2$$
 ②

Write each equation in slope-intercept form.

For equation ①:

$$2x + 3y = -5 \quad \text{Subtract } 2x \text{ from each side.}$$

$$3y = -2x - 5 \quad \text{Divide each side by 3.}$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

The slope of the graph of equation ① is $-\frac{2}{3}$.

For equation ②:

$$\frac{x}{2} - \frac{y}{3} = 2 \quad \text{Solve for } y. \text{ Subtract } \frac{x}{2} \text{ from each side.}$$

$$-\frac{y}{3} = -\frac{x}{2} + 2 \quad \text{Multiply each side by } -3.$$

$$-3\left(-\frac{y}{3}\right) = -3\left(-\frac{x}{2}\right) + (-3)(2) \quad \text{Simplify.}$$

$$y = \frac{3}{2}x - 6$$

The slope of the graph of the equation ② is $\frac{3}{2}$.

The slopes $\frac{3}{2}$ and $-\frac{2}{3}$ are negative reciprocals, so the lines are perpendicular.

b) I chose two numbers that are negative reciprocals; such as $\frac{5}{3}$ and $-\frac{3}{5}$. I used these

numbers as the slopes of two lines.

I chose two y -intercepts, such as 4 and -2 .

To get two equations of a linear system, I substituted a slope and y -intercept into this form of the equation of a line: $y = mx + b$

A linear system is:

$$y = \frac{5}{3}x + 4$$

$$y = -\frac{3}{5}x - 2$$

Lesson 7.3 **Math Lab:** **Assess Your Understanding, pages 412-413**
Using Graphing Technology to Solve a System of Linear Equations

1. a) In the table:
The X-values increase by 1 each time. The values go from $X = 0$ to $X = 6$, with corresponding Y-values for each equation.
Gerard should use the table to find the X-value for which $Y_1 = Y_2$.
When $X = 4$, $Y_1 = 2$ and $Y_2 = 2$, so the solution of the linear system is approximately:
 $X = 4$ and $Y = 2$
- b) To solve this linear system, Gerard could graph each line and determine the coordinates of their point of intersection.

2. a) $3x - 6y = 14$ ①

$x + y = \frac{7}{6}$ ②

To solve the linear system above, I would use a graphing calculator.

I first have to write each equation in the form $y = mx + b$.

For equation ①:

$$3x - 6y = 14$$

Solve for y . Subtract $3x$ from each side.

$$-6y = -3x + 14$$

Divide each side by -6 .

$$y = \frac{-3}{-6}x + \frac{14}{-6}$$

$$y = \frac{1}{2}x - \frac{7}{3}$$

For equation ②:

$$x + y = \frac{7}{6}$$

Solve for y . Subtract x from each side.

$$y = -x + \frac{7}{6}$$

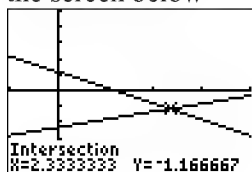
The linear system is:

$$y = \frac{1}{2}x - \frac{7}{3}$$

$$y = -x + \frac{7}{6}$$

On a TI-83 graphing calculator, I pressed $\boxed{Y=}$, then next to $Y1 =$ I input the expression: $(1/2)X - 7/3$. I moved the cursor down to $Y2 =$ and input the expression $-X + 7/6$.

I pressed $\boxed{\text{GRAPH}}$. To see the point of intersection, I set the $\boxed{\text{WINDOW}}$ to $X_{\min} = -1$, $X_{\max} = 4$, $Y_{\min} = -5$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, I pressed $\boxed{2\text{nd}} \boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. I pressed $\boxed{\text{ENTER}}$ 3 times to get the screen below



- b) From the calculator screen, the solution is: $x = 2.\overline{3}$ and $y = -1.\overline{16}$

In fraction form, the solution is: $x = \frac{7}{3}$ and $y = -\frac{7}{6}$

Verify the solution.

Substitute $x = \frac{7}{3}$ and $y = -\frac{7}{6}$ into each equation.

$$y = \frac{1}{2}x - \frac{7}{3}$$

$$y = -x + \frac{7}{6}$$

$$\text{L.S.} = y$$

$$\text{R.S.} = \frac{1}{2}x - \frac{7}{3}$$

$$\text{L.S.} = y$$

$$\text{R.S.} = -x + \frac{7}{6}$$

$$= -\frac{7}{6}$$

$$= \frac{1}{2}\left(\frac{7}{3}\right) - \frac{7}{3}$$

$$= -\frac{7}{6}$$

$$= -\frac{7}{3} + \frac{7}{6}$$

$$= \frac{7}{6} - \frac{7}{3}$$

$$= -\frac{14}{6} + \frac{7}{6}$$

$$= \frac{7}{6} - \frac{14}{6}$$

$$= -\frac{7}{6}$$

$$= -\frac{7}{6}$$

$$= \text{L.S.}$$

$$= \text{L.S.}$$

Since the left side is equal to the right side in each equation, then $x = \frac{7}{3}$ and $y = -\frac{7}{6}$ is the solution of the linear system.

3. Let c represent the number of cedar tree seedlings planted.
Let s represent the number of spruce tree seedlings planted.

A total of 72 seedlings was purchased.

So, one equation is: $c + s = 72$

There were twice as many cedar trees as spruce trees.

So, another equation is: $c = 2s$

A linear system is:

$$c + s = 72 \quad \text{①}$$

$$c = 2s \quad \text{②}$$

Write equation ① in $y = mx + b$ form.

$$c + s = 72 \quad \text{Subtract } s \text{ from each side.}$$

$$c = -s + 72$$

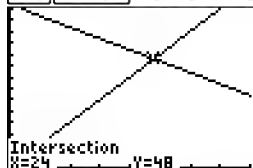
On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $-X+72$.

Move the cursor down to $Y2=$ and input the expression $2X$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 40$,

$Y_{\min} = 0$, and $Y_{\max} = 70$. To show the coordinates of the point of intersection, press

$\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for $\boxed{\text{CALC}}$, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below:



From the calculator screen, the solution is: $x = 24$ and $y = 48$

The value of x is the value of s , so the number of spruce tree seedlings is 24.

The value of y is the value of c , so the number of cedar tree seedlings is 48.

Verify the solution.

The total number of tree seedlings is: $24 + 48 = 72$; this is the same as the given information.

The number of cedar tree seedlings is 48, which is twice the number of spruce tree seedlings, 24; this is the same as the given information.

The solution is correct.

4. a) i) $x + 2y = 3$ ①
 $2x - y = 1$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

$$x + 2y = 3$$

Subtract x from each side.

$$2y = -x + 3$$

Divide each side by 2.

$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation ②:

$$2x - y = 1$$

Subtract $2x$ from each side.

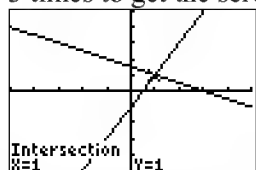
$$-y = -2x + 1$$

Multiply each side by -1 .

$$y = 2x - 1$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-1/2)X+3/2$. Move the cursor down to $Y2=$ and input the expression $2X-1$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $Xmin = -5$, $Xmax = 5$, $Ymin = -5$, and $Ymax = 5$. To show the coordinates of the point of intersection, press $\boxed{2nd} \boxed{\text{TRACE}}$ for CALC , then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 1$ and $y = 1$

Verify the solution.

Substitute $x = 1$ and $y = 1$ into each equation.

$$x + 2y = 3$$

$$2x - y = 1$$

$$\text{L.S.} = x + 2y$$

$$\text{R.S.} = 3$$

$$\text{L.S.} = 2x - y$$

$$\text{R.S.} = 1$$

$$= 1 + 2(1)$$

$$= 2(1) - 1$$

$$= 3$$

$$= 1$$

Since the left side is equal to the right side in each equation, then $x = 1$ and $y = 1$ is the solution of the linear system.

ii) $x + 2y = 3$ ①
 $2x - y = 6$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

From part i:

$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation ②:

$$2x - y = 6$$

Subtract $2x$ from each side.

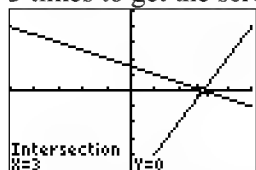
$$-y = -2x + 6$$

Multiply each side by -1 .

$$y = 2x - 6$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-1/2)X+3/2$. Move the cursor down to $Y2=$ and input the expression $2X-6$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $Xmin = -5$, $Xmax = 5$, $Ymin = -5$, and $Ymax = 5$. To show the coordinates of the point of intersection, press $\boxed{2nd} \boxed{\text{TRACE}}$ for CALC , then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 3$ and $y = 0$

Verify the solution.

Substitute $x = 3$ and $y = 0$ into each equation.

$$x + 2y = 3$$

$$2x - y = 6$$

$$\text{L.S.} = x + 2y$$

$$\text{R.S.} = 3$$

$$\text{L.S.} = 2x - y$$

$$\text{R.S.} = 6$$

$$= 3 + 2(0)$$

$$= 2(3) - 0$$

$$= 3$$

$$= 6$$

Since the left side is equal to the right side in each equation, then $x = 3$ and $y = 0$ is the solution of the linear system.

iii) $x + 2y = 3$ ①

$$2x - y = 11$$
 ②

Write each equation in $y = mx + b$ form.

For equation ①:

From part i:

$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation ②:

$$2x - y = 11$$

Subtract $2x$ from each side.

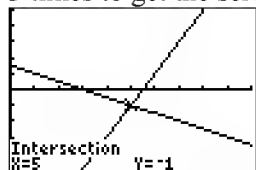
$$-y = -2x + 11$$

Multiply each side by -1 .

$$y = 2x - 11$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-1/2)X+3/2$. Move the cursor down to $Y2=$ and input the expression $2X-11$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $Xmin = 0$, $Xmax = 10$, $Ymin = -5$, and $Ymax = 5$. To show the coordinates of the point of intersection, press $\boxed{2nd} \boxed{\text{TRACE}}$ for CALC , then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 5$ and $y = -1$

Verify the solution.

Substitute $x = 5$ and $y = -1$ into each equation.

$$x + 2y = 3$$

$$2x - y = 11$$

$$\begin{aligned}\text{L.S.} &= x + 2y \\ &= 5 + 2(-1) \\ &= 5 - 2 \\ &= 3\end{aligned}$$

$$\text{R.S.} = 3$$

$$\begin{aligned}\text{L.S.} &= 2x - y \\ &= 2(5) - (-1) \\ &= 10 + 1 \\ &= 11\end{aligned}$$

$$\text{R.S.} = 11$$

Since the left side is equal to the right side in each equation, then $x = 5$ and $y = -1$ is the solution of the linear system.

iv) $x + 2y = 3$ ①
 $2x - y = 16$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

From part i:

$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation ②:

$$2x - y = 16$$

Subtract $2x$ from each side.

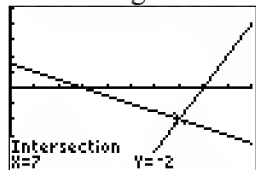
$$-y = -2x + 16$$

Multiply each side by -1 .

$$y = 2x - 16$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-1/2)X + 3/2$. Move the cursor down to $Y2 =$ and input the expression $2X - 16$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 10$, $Y_{\min} = -5$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, press $\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 7$ and $y = -2$

Verify the solution.

Substitute $x = 7$ and $y = -2$ into each equation.

$$x + 2y = 3$$

$$2x - y = 16$$

$$\begin{aligned}\text{L.S.} &= x + 2y \\ &= 7 + 2(-2) \\ &= 7 - 4 \\ &= 3\end{aligned}$$

$$\text{R.S.} = 3$$

$$\begin{aligned}\text{L.S.} &= 2x - y \\ &= 2(7) - (-2) \\ &= 14 + 2 \\ &= 16\end{aligned}$$

$$\text{R.S.} = 16$$

Since the left side is equal to the right side in each equation, then $x = 7$ and $y = -2$ is the solution of the linear system.

b) The first equation in each linear system is the same.

In the second equations, the x -coefficients are equal, the y -coefficients are equal, and the constant term increases by 5 each time.

So, the next linear system in the pattern will be:

$$x + 2y = 3 \quad \textcircled{1}$$

$$2x - y = 21 \quad \textcircled{2}$$

The solutions are:

$$x = 1 \text{ and } y = 1$$

$$x = 3 \text{ and } y = 0$$

$$x = 5 \text{ and } y = -1$$

$$x = 7 \text{ and } y = -2$$

The x -coordinate increases by 2 each time and the y -coordinate decreases by 1 each time.

So, the solution of the next linear system in the pattern is: $x = 9$ and $y = -3$

- c) Write each equation in part b in $y = mx + b$ form.

For equation $\textcircled{1}$:

From part a) i:

$$y = -\frac{1}{2}x + \frac{3}{2}$$

For equation $\textcircled{2}$:

$$2x - y = 21$$

Subtract $2x$ from each side.

$$-y = -2x + 21 \quad \text{Multiply each side by } -1.$$

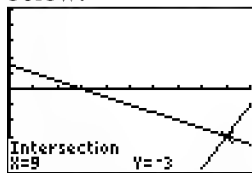
$$y = 2x - 21$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression

$(-1/2)X + 3/2$. Move the cursor down to $Y2 =$ and input the expression $2X - 21$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 10$, $Y_{\min} = -5$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, press

$\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 9$ and $y = -3$

Verify the solution.

Substitute $x = 9$ and $y = -3$ into each equation.

$$x + 2y = 3$$

$$\text{L.S.} = x + 2y$$

$$\text{R.S.} = 3$$

$$= 9 + 2(-3)$$

$$= 9 - 6$$

$$= 3$$

$$2x - y = 21$$

$$\text{L.S.} = 2x - y$$

$$\text{R.S.} = 21$$

$$= 2(9) - (-3)$$

$$= 18 + 3$$

$$= 21$$

Since the left side is equal to the right side in each equation, then $x = 9$ and $y = -3$ is the solution of the linear system.

5. For System A:

$$\frac{1}{2}x + y = 3 \quad \textcircled{1}$$

$$x + \frac{1}{2}y = 3 \quad \textcircled{2}$$

Write each equation in $y = mx + b$ form.

For equation ①:

$$\frac{1}{2}x + y = 3 \quad \text{Subtract } \frac{1}{2}x \text{ from each side.}$$

$$y = -\frac{1}{2}x + 3$$

For equation ②:

$$x + \frac{1}{2}y = 3 \quad \text{Subtract } x \text{ from each side.}$$

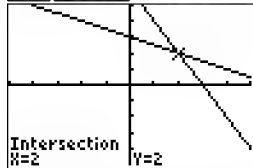
$$\frac{1}{2}y = -x + 3 \quad \text{Multiply each side by 2.}$$

$$y = -2x + 6$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-1/2)X+3$. Move the cursor down to $Y2=$ and input the expression $-2X+6$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = -5$, $X_{\max} = 5$, $Y_{\min} = -5$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, press

$\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then select 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 2$ and $y = 2$

Verify the solution.

Substitute $x = 2$ and $y = 2$ into each equation.

$$\frac{1}{2}x + y = 3$$

$$x + \frac{1}{2}y = 3$$

$$\text{L.S.} = \frac{1}{2}x + y$$

$$\text{L.S.} = x + \frac{1}{2}y$$

$$= \frac{1}{2}(2) + 2$$

$$= 2 + \frac{1}{2}(2)$$

$$= 1 + 2$$

$$= 2 + 1$$

$$= 3$$

$$= 3$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side in each equation, then $x = 2$ and $y = 2$ is the solution of the linear system.

So, a linear system with fractional coefficients does not always have an approximate solution.

For System B:

$$2x + y = \frac{23}{6} \quad \text{①}$$

$$\frac{x}{3} + \frac{y}{2} = \frac{55}{36} \quad \text{②}$$

Write each equation in $y = mx + b$ form.

For equation ①:

$$2x + y = \frac{23}{6}$$

Subtract $2x$ from each side.

$$y = -2x + \frac{23}{6}$$

For equation ②:

$$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$$

Subtract $\frac{x}{3}$ from each side.

$$\frac{y}{2} = -\frac{x}{3} + \frac{55}{36}$$

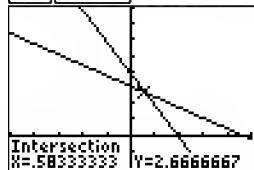
Multiply each side by 2.

$$y = -\frac{2x}{3} + \frac{55}{18}$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $-2X+23/6$. Move the cursor down to $Y2=$ and input the expression $-2X/3+55/18$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = -5$, $X_{\max} = 5$, $Y_{\min} = -2$, and $Y_{\max} = 8$. To show the coordinates of the point of intersection, press

$\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for $\boxed{\text{CALC}}$, then select 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 0.58\bar{3}$ and $y = 2.\bar{6}$

Verify the solution.

Substitute $x = 0.5833333$ and $y = 2.6666667$ into each equation.

$$2x + y = \frac{23}{6}$$

$$\begin{aligned} \text{L.S.} &= 2x + y \\ &= 2(0.5833333) + 2.6666667 \\ &= 3.83333360 \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{R.S.} &= \frac{23}{6} \\ &= 3.8\bar{3} \end{aligned}$$

$$\frac{x}{3} + \frac{y}{2} = \frac{55}{36}$$

$$\begin{aligned} \text{L.S.} &= \frac{x}{3} + \frac{y}{2} \\ &= \frac{0.5833333}{3} + \frac{2.6666667}{2} \\ &= 1.52777793 \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{R.S.} &= \frac{55}{36} \\ &= 1.52\bar{7} \end{aligned}$$

Since the left side is equal to the right side in each equation, then $x = 0.58\bar{3}$ and $y = 2.\bar{6}$ is the solution of the linear system.

These numbers are probably exact, and can be written as $x = \frac{7}{12}$ and $y = \frac{8}{3}$.

So, a linear system with fractional coefficients does not always have an approximate solution.

Checkpoint 1

Assess Your Understanding (page 415)

7.1

1. a) Let the length of the JumboTron be represented by l feet.

Let the width of the JumboTron be represented by w feet.

The perimeter of the JumboTron is 128 ft.

So, $l + l + w + w = 128$

One equation is: $2l + 2w = 128$

The width of the JumboTron is 16 ft. less than its length.

So, another equation is: $w = l - 16$

A linear system is:

$$2l + 2w = 128 \quad \textcircled{1}$$

$$w = l - 16 \quad \textcircled{2}$$

- b) i) Substitute $l = 40$ and $w = 24$ into each equation of the linear system.

For equation $\textcircled{1}$:

$$2l + 2w = 128$$

$$\text{L.S.} = 2l + 2w$$

$$= 2(40) + 2(24)$$

$$= 80 + 48$$

$$= 128$$

$$= \text{R.S.}$$

For equation $\textcircled{2}$:

$$w = l - 16$$

$$\text{L.S.} = w$$

$$= 24$$

$$\text{R.S.} = l - 16$$

$$= 40 - 16$$

$$= 24$$

For each equation, the left side is equal to the right side, so the solution is correct.

- ii) Use the fact that the length is 40 ft. and the width is 24 ft.

Then, the perimeter is: 40 ft. + 40 ft. + 24 ft. + 24 ft. = 128 ft.

This is the same as the given perimeter.

The difference between the length and the width is: 40 ft. - 24 ft. = 16 ft.

This is the same as the given information that the width is 16 ft. less than the length.

So, the solution is correct.

2. $10x + 5y = 850 \quad \textcircled{1}$

$$x - y = 10 \quad \textcircled{2}$$

The coefficients in the first equation are 5 and 10, which are the values of a nickel and a dime, so use these coins and their values to create a situation.

Let x represent the number of dimes and let y represent the number of nickels.

The value of x dimes is $10x$ cents and the value of y nickels is $5y$ cents.

From equation $\textcircled{1}$, the value of x dimes and y nickels is 850¢.

From equation $\textcircled{2}$, the difference in the numbers of dimes and nickels is 10.

A related problem is:

A collection of nickels and dimes has a value of \$8.50.

There are 10 more dimes than nickels.

How many dimes are there? How many nickels are there?

7.2

3. $2x + y = 1 \quad \textcircled{1}$

$$x + 2y = -1 \quad \textcircled{2}$$

Determine the coordinates of two points on each line.

For equation $\textcircled{1}$:

Substitute: $x = 0$

$$2x + y = 1$$

$$2(0) + y = 1$$

$$y = 1$$

Substitute: $x = 2$

$$2x + y = 1$$

$$2(2) + y = 1$$

$$4 + y = 1$$

$$y = -3$$

For equation ①, two points have coordinates: (0, 1) and (2, -3)

For equation ②:

Substitute: $x = 1$

$$x + 2y = -1$$

$$1 + 2y = -1$$

$$2y = -2$$

$$y = -1$$

Substitute: $x = 3$

$$x + 2y = -1$$

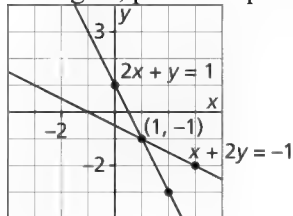
$$3 + 2y = -1$$

$$2y = -4$$

$$y = -2$$

For equation ②, two points have coordinates: (1, -1) and (3, -2)

On a grid, plot each pair of points, then draw a line through them.



From the graph, the solution appears to be: $x = 1$ and $y = -1$

Verify the solution.

The solution satisfies equation ② because the coordinates were determined to graph the line.

Substitute $x = 1$ and $y = -1$ in equation ①.

$$2x + y = 1$$

$$\text{L.S.} = 2x + y$$

$$\text{R.S.} = 1$$

$$= 2(1) + (-1)$$

$$= 2 - 1$$

$$= 1$$

Since the left side is equal to the right side, the solution is correct.

4. a) $F = 75 + 5v$ ①

$F = 10v$ ②

Determine the coordinates of two points on each line.

For equation ①:

Substitute: $v = 0$

$$F = 75 + 5v$$

$$F = 75 + 5(0)$$

$$F = 75$$

Substitute: $v = 10$

$$F = 75 + 5v$$

$$F = 75 + 5(10)$$

$$F = 75 + 50$$

$$F = 125$$

For equation ①, two points have coordinates: (0, 75) and (10, 125)

For equation ②:

Substitute: $v = 0$

$$F = 10v$$

$$F = 10(0)$$

$$F = 0$$

Substitute: $v = 10$

$$F = 10v$$

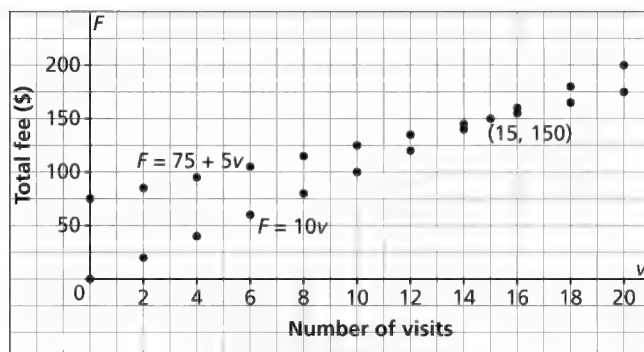
$$F = 10(10)$$

$$F = 100$$

For equation ②, two points have coordinates: (0, 0) and (10, 100)

The data are discrete. On a grid, plot each pair of points, then use a straightedge to mark more points on each line, with whole number values of v . Use a scale of 1 square to 1 unit

on the v -axis and 1 square to 25 units on the F -axis.



From the graph, the solution appears to be: $v = 15$ and $F = 150$

Verify the solution.

Substitute $v = 15$ and $F = 150$ into each equation.

For $F = 75 + 5v$ ①

$$\begin{aligned} \text{L.S.} &= F \\ &= 150 \end{aligned}$$

$$\begin{aligned} \text{R.S.} &= 75 + 5v \\ &= 75 + 5(15) \\ &= 75 + 75 \\ &= 150 \end{aligned}$$

For $F = 10v$

$$\begin{aligned} \text{L.S.} &= F \\ &= 150 \end{aligned}$$

②

$$\begin{aligned} \text{R.S.} &= 10v \\ &= 10(15) \\ &= 150 \end{aligned}$$

In each equation, the left side is equal to the right side, so the solution is correct.

- b) Plan A is represented by the equation $F = 75 + 5v$. Since the vertical axis represents the total fee, Plan A is cheaper when the graph of the line $F = 75 + 5v$ is below the other line on the graph. This occurs for values of v greater than 15. So, Plan A is cheaper for 16 or more visits.

5. a) Let the number of students who went to the aquarium be represented by s .
Let the number of adults who went to the aquarium be represented by a .
The cost for a student is \$21 and the cost for an adult is \$27, with a total cost of \$396.
So, one equation is: $21s + 27a = 396$
Eighteen people went to the aquarium.
So, another equation is: $s + a = 18$
A linear system is:

$$\begin{array}{ll} 21s + 27a = 396 & \text{①} \\ s + a = 18 & \text{②} \end{array}$$

- b) Determine the coordinates of two points on each line.

For equation ①:

Use guess and test to determine the whole-number coordinates of two points on the line.

Substitute: $s = 6$

$$21s + 27a = 396$$

$$21(6) + 27a = 396$$

$$126 + 27a = 396$$

$$27a = 396 - 126$$

$$27a = 270$$

$$a = 10$$

Substitute: $s = 15$

$$21s + 27a = 396$$

$$21(15) + 27a = 396$$

$$315 + 27a = 396$$

$$27a = 396 - 315$$

$$27a = 81$$

$$a = 3$$

For equation ①, two points have coordinates: (6, 10) and (15, 3)

For equation ②, use intercepts.

Substitute: $a = 0$

$$s + a = 18$$

$$s + 0 = 18$$

$$s = 18$$

Substitute: $s = 0$

$$s + a = 18$$

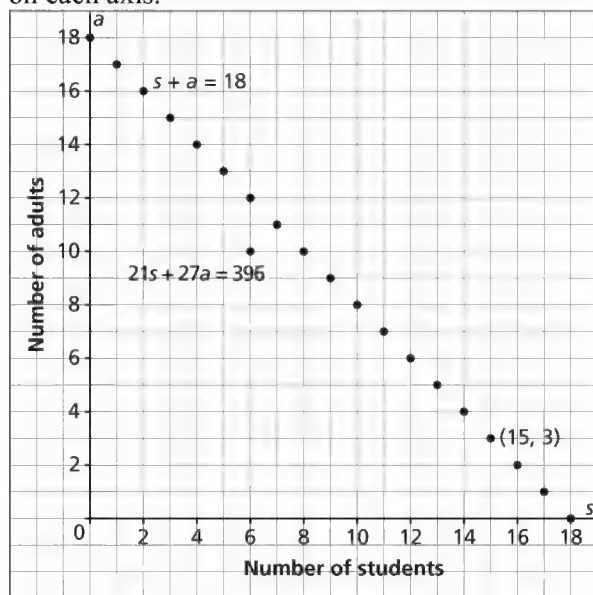
$$0 + a = 18$$

$$a = 18$$

For equation ②, two points have coordinates: $(18, 0)$ and $(0, 18)$

Graph a as a function of s ; that is, plot a on the vertical axis and s on the horizontal axis.

The data are discrete. On a grid, plot each pair of points, then use a straightedge to mark more points on each line, with whole number values of v . Use a scale of 1 square to 1 unit on each axis.



From the graph, the solution appears to be: $s = 15$ and $a = 3$; that is, 15 students and 3 adults went to the aquarium.

Verify the solution.

The total cost, in dollars, is: $15(21) + 3(27) = 315 + 81$, or 396; this agrees with the given information.

The total number of people who went to the aquarium is: $15 + 3 = 18$; this agrees with the given information.

So, the solution is correct.

7.3

6. a) Let l represent the number of large trees.

Let s represent the number of small trees.

A large tree removes 1.4 kg of pollution and a small tree removes 0.02 kg of pollution, with a total of 7200 kg of pollution being removed.

So, one equation is: $1.4l + 0.02s = 7200$

There is a total of 15 000 trees.

So, another equation is: $l + s = 15\,000$

A linear system is:

$$1.4l + 0.02s = 7200 \quad \text{①}$$

$$l + s = 15\,000 \quad \text{②}$$

- b) Write each equation in $y = mx + b$ form.

For equation ①:

$$1.4l + 0.02s = 7200$$

Subtract $1.4l$ from each side.

$$0.02s = -1.4l + 7200 \quad \text{Divide each side by 0.02.}$$

$$s = \frac{-1.4}{0.02}l + \frac{7200}{0.02} \quad \text{Simplify.}$$

$$s = -70l + 360\,000$$

For equation ②:

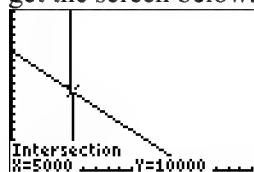
$$l + s = 15\,000$$

Subtract l from each side.

$$s = -l + 15\,000$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $-70X+360000$. Move the cursor down to $Y2=$ and input the expression $-X+15000$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 20000$, $Y_{\min} = 0$, and $Y_{\max} = 20000$. To show the coordinates of the point of intersection, press $\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 5000$ and $y = 10\,000$

The value of x is the value of l , so the number of large trees is 5000.

The value of y is the value of s , so the number of small trees is 10 000.

Verify the solution.

The total number of trees is: $5000 + 10\,000 = 15\,000$; this is the same as the given information.

The total mass of pollution removed, in kilograms, is:

$1.4(5000) + 0.02(10\,000) = 7000 + 200$, or 7200; this is the same as the given information.

The solution is correct.

Lesson 7.4 **Using a Substitution Strategy to Solve a System of Linear Equations**

Exercises (pages 425-427)

A

4. a) $y = 9 - x$ ①
 $2x + 3y = 11$ ②

Equation ① is solved for y , so substitute $y = 9 - x$ in equation ②.

$$\begin{aligned} 2x + 3y &= 11 && \text{②} \\ 2x + 3(9 - x) &= 11 && \text{Remove brackets.} \\ 2x + 27 - 3x &= 11 && \text{Collect like terms.} \end{aligned}$$

$$\begin{aligned} -x &= 11 - 27 \\ -x &= -16 && \text{Multiply each side by } -1. \\ x &= 16 \end{aligned}$$

Substitute $x = 16$ in equation ①.

$$\begin{aligned} y &= 9 - x && \text{①} \\ y &= 9 - 16 \\ y &= -7 \end{aligned}$$

Verify the solution.

In each equation, substitute: $x = 16$ and $y = -7$

$y = 9 - x$ ①	$2x + 3y = 11$ ②
L.S. = y	L.S. = $2x + 3y$
$= -7$	$= 2(16) + 3(-7)$
	$= 32 - 21$
	$= 11$
	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 16 \text{ and } y = -7$$

b) $x = y - 1$ ①
 $3x - y = 11$ ②

Equation ① is solved for x , so substitute $x = y - 1$ in equation ②.

$$\begin{aligned} 3x - y &= 11 && \text{②} \\ 3(y - 1) - y &= 11 && \text{Remove brackets.} \\ 3y - 3 - y &= 11 && \text{Collect like terms.} \end{aligned}$$

$$\begin{aligned} 2y &= 11 + 3 \\ 2y &= 14 && \text{Divide each side by 2.} \\ y &= 7 \end{aligned}$$

Substitute $y = 7$ in equation ①.

$$\begin{aligned} x &= y - 1 && \text{①} \\ x &= 7 - 1 \\ x &= 6 \end{aligned}$$

Verify the solution.

In each equation, substitute: $x = 6$ and $y = 7$

$x = y - 1$ ①	$3x - y = 11$ ②
L.S. = x	L.S. = $3x - y$
$= 6$	$= 3(6) - 7$
	$= 18 - 7$
	$= 11$
	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is: $x = 6$ and $y = 7$

c) $x = 7 + y$ ①
 $2x + y = -10$ ②
 Equation ① is solved for x , so substitute $x = 7 + y$ in equation ②.
 $2x + y = -10$ ②
 $2(7 + y) + y = -10$ Remove brackets.
 $14 + 2y + y = -10$ Collect like terms.
 $3y = -10 - 14$
 $3y = -24$ Divide each side by 3.
 $y = -8$
 Substitute $y = -8$ in equation ①.
 $x = 7 + y$ ①
 $x = 7 - 8$
 $x = -1$

Verify the solution.

In each equation, substitute: $x = -1$ and $y = -8$

$x = 7 + y$ ①	$2x + y = -10$ ②
L.S. = x	L.S. = $2x + y$
$= -1$	$= 2(-1) + (-8)$
R.S. = $7 + y$	$= -2 - 8$
$= 7 + (-8)$	$= -10$
$= -1$	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:

$x = -1$ and $y = -8$

d) $3x + y = 7$ ①
 $y = x + 3$ ②
 Equation ② is solved for y , so substitute $y = x + 3$ in equation ①.
 $3x + y = 7$ ①
 $3x + (x + 3) = 7$ Remove brackets.
 $3x + x + 3 = 7$ Collect like terms.
 $4x = 7 - 3$
 $4x = 4$ Divide each side by 4.
 $x = 1$
 Substitute $x = 1$ in equation ②.
 $y = x + 3$ ②
 $y = 1 + 3$
 $y = 4$

Verify the solution.

In each equation, substitute: $x = 1$ and $y = 4$

$3x + y = 7$ ①	$y = x + 3$ ②
L.S. = $3x + y$	L.S. = y
$= 3(1) + 4$	$= 4$
$= 7$	R.S. = $x + 3$
	$= 1 + 3$
	$= 4$

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 4$

5. a) $2x + 3y = 11$ ①

$4x - y = -13$ ②

Solve equation ② for y .

$4x - y = -13$ Subtract $4x$ from each side.

$-y = -4x - 13$ Multiply each side by -1 .

$y = 4x + 13$

Substitute $y = 4x + 13$ in equation ①.

$2x + 3y = 11$ ①

$2x + 3(4x + 13) = 11$ Remove brackets.

$2x + 12x + 39 = 11$ Collect like terms.

$14x = 11 - 39$

$14x = -28$ Divide each side by 14.

$x = -2$

Substitute $x = -2$ in equation ②.

$4x - y = -13$ ②

$4(-2) - y = -13$ Solve for y .

$-8 - y = -13$

$-y = -13 + 8$

$-y = -5$

$y = 5$

Verify the solution.

In each equation, substitute: $x = -2$ and $y = 5$

$2x + 3y = 11$ ①

L.S. $= 2x + 3y$

$= 2(-2) + 3(5)$

$= -4 + 15$

$= 11$

$= \text{R.S.}$

$4x - y = -13$ ②

L.S. $= 4x - y$

$= 4(-2) - 5$

$= -8 - 5$

$= -13$

$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:

$x = -2$ and $y = 5$

b) $4x + y = -5$ ①

$2x + 3y = 5$ ②

Solve equation ① for y .

$4x + y = -5$ Subtract $4x$ from each side.

$y = -4x - 5$

Substitute $y = -4x - 5$ in equation ②.

$2x + 3y = 5$ ②

$2x + 3(-4x - 5) = 5$ Remove brackets.

$2x - 12x - 15 = 5$ Collect like terms.

$-10x = 5 + 15$

$-10x = 20$

$x = -2$

Divide each side by -2 .

Substitute $x = -2$ in equation ①.

$4x + y = -5$ ①

$4(-2) + y = -5$ Solve for y .

$-8 + y = -5$

$y = -5 + 8$

$y = 3$

Verify the solution.

In each equation, substitute: $x = -2$ and $y = 3$

$$4x + y = -5 \quad \text{①}$$

$$\begin{aligned} \text{L.S.} &= 4x + y \\ &= 4(-2) + 3 \\ &= -8 + 3 \\ &= -5 \\ &= \text{R.S.} \end{aligned}$$

$$2x + 3y = 5 \quad \text{②}$$

$$\begin{aligned} \text{L.S.} &= 2x + 3y \\ &= 2(-2) + 3(3) \\ &= -4 + 9 \\ &= 5 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -2 \text{ and } y = 3$$

c) $x + 2y = 13 \quad \text{①}$

$$2x - 3y = -9 \quad \text{②}$$

Solve equation ① for x .

$$x + 2y = 13 \quad \text{Subtract } 2y \text{ from each side.}$$

$$x = -2y + 13$$

Substitute $x = -2y + 13$ in equation ②.

$$2x - 3y = -9 \quad \text{②}$$

$$2(-2y + 13) - 3y = -9 \quad \text{Remove brackets.}$$

$$-4y + 26 - 3y = -9 \quad \text{Collect like terms.}$$

$$-7y = -9 - 26$$

$$-7y = -35$$

Divide each side by -7 .

$$y = 5$$

Substitute $y = 5$ in equation ①.

$$x + 2y = 13 \quad \text{①}$$

$$x + 2(5) = 13$$

$$x + 10 = 13 \quad \text{Solve for } x.$$

$$x = 3$$

Verify the solution.

In each equation, substitute: $x = 3$ and $y = 5$

$$x + 2y = 13 \quad \text{①}$$

$$\begin{aligned} \text{L.S.} &= x + 2y \\ &= 3 + 2(5) \\ &= 3 + 10 \\ &= 13 \\ &= \text{R.S.} \end{aligned}$$

$$2x - 3y = -9 \quad \text{②}$$

$$\begin{aligned} \text{L.S.} &= 2x - 3y \\ &= 2(3) - 3(5) \\ &= 6 - 15 \\ &= -9 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is: $x = 3$ and $y = 5$

d) $3x + y = 7 \quad \text{①}$

$$5x + 2y = 13 \quad \text{②}$$

Solve equation ① for y .

$$3x + y = 7 \quad \text{Subtract } 3x \text{ from each side.}$$

$$y = -3x + 7$$

Substitute $y = -3x + 7$ in equation ②.

$$5x + 2y = 13 \quad \text{②}$$

$$5x + 2(-3x + 7) = 13 \quad \text{Remove brackets.}$$

$$5x - 6x + 14 = 13 \quad \text{Collect like terms.}$$

$$-x = 13 - 14$$

$$-x = -1$$

$$x = 1$$

Substitute $x = 1$ in equation ①.

$$3x + y = 7 \quad \text{①}$$

$$3(1) + y = 7$$

$$3 + y = 7$$

$$y = 4$$

Verify the solution.

In each equation, substitute: $x = 1$ and $y = 4$

$$3x + y = 7 \quad \text{①}$$

$$\text{L.S.} = 3x + y \quad \text{R.S.} = 7$$

$$= 3(1) + 4$$

$$= 7$$

$$5x + 2y = 13 \quad \text{②}$$

$$\text{L.S.} = 5x + 2y$$

$$= 5(1) + 2(4)$$

$$= 5 + 8$$

$$= 13$$

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 4$

B

6. a) i) $2x - 3y = 2 \quad \text{①}$

$$4x - 4y = 2 \quad \text{②}$$

The term $4x$ in equation ② is 2 times the term $2x$ in equation ①.

ii) $40x + 10y = 10 \quad \text{①}$

$$3x + 5y = 5 \quad \text{②}$$

The term $10y$ in equation ① is 2 times the term $5y$ in equation ②.

iii) $-3x + 6y = 9 \quad \text{①}$

$$5x - 2y = -7 \quad \text{②}$$

The term $6y$ in equation ① is 3 times the term $2y$ in equation ②.

iv) $-3x + 4y = 6 \quad \text{①}$

$$9x + 3y = 27 \quad \text{②}$$

The term $9x$ in equation ② is 3 times the term $3x$ in equation ①.

b) i) $2x - 3y = 2 \quad \text{①}$

$$4x - 4y = 2 \quad \text{②}$$

In equation ②, the term $4x$ can be written as $2(2x)$:

$$2(2x) - 4y = 2 \quad \text{③}$$

Solve equation ① for $2x$.

$$2x - 3y = 2 \quad \text{①}$$

$$2x = 3y + 2$$

Substitute for $2x$ in equation ③.

$$2(2x) - 4y = 2 \quad \text{③}$$

$$2(3y + 2) - 4y = 2 \quad \text{Simplify, then solve for } y.$$

$$6y + 4 - 4y = 2$$

$$2y = -2$$

$$y = -1$$

Substitute $y = -1$ in equation ①.

$$2x - 3y = 2 \quad \text{①}$$

$$2x - 3(-1) = 2 \quad \text{Simplify, then solve for } x.$$

$$2x + 3 = 2$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Verify the solution.

In each equation, substitute: $x = -\frac{1}{2}$ and $y = -1$

$$2x - 3y = 2 \quad \textcircled{1}$$

$$\text{L.S.} = 2x - 3y$$

$$= 2\left(-\frac{1}{2}\right) - 3(-1)$$

$$= -1 + 3$$

$$= 2$$

$$= \text{R.S.}$$

$$4x - 4y = 2 \quad \textcircled{2}$$

$$\text{L.S.} = 4x - 4y$$

$$= 4\left(-\frac{1}{2}\right) - 4(-1)$$

$$= -2 + 4$$

$$= 2$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is: $x = -\frac{1}{2}$
and $y = -1$

ii) $40x + 10y = 10 \quad \textcircled{1}$

$$3x + 5y = 5 \quad \textcircled{2}$$

In equation $\textcircled{1}$, the term $10y$ can be written as $2(5y)$:

$$40x + 2(5y) = 10 \quad \textcircled{3}$$

Solve equation $\textcircled{2}$ for $5y$.

$$3x + 5y = 5 \quad \textcircled{2}$$

$$5y = -3x + 5$$

Substitute for $5y$ in equation $\textcircled{3}$.

$$40x + 2(5y) = 10 \quad \textcircled{3}$$

$$40x + 2(-3x + 5) = 10 \quad \text{Simplify, then solve for } x.$$

$$40x - 6x + 10 = 10$$

$$34x = 0$$

$$x = 0$$

Substitute $x = 0$ in equation $\textcircled{1}$.

$$40x + 10y = 10 \quad \textcircled{1}$$

$$40(0) + 10y = 10 \quad \text{Simplify, then solve for } y.$$

$$10y = 10$$

$$y = 1$$

Verify the solution.

In each equation, substitute: $x = 0$ and $y = 1$

$$40x + 10y = 10 \quad \textcircled{1}$$

$$\text{L.S.} = 40x + 10y$$

$$= 40(0) + 10(1)$$

$$= 10$$

$$= \text{R.S.}$$

$$3x + 5y = 5 \quad \textcircled{2}$$

$$\text{L.S.} = 3x + 5y$$

$$= 3(0) + 5(1)$$

$$= 5$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:
 $x = 0$ and $y = 1$

iii) $-3x + 6y = 9 \quad \textcircled{1}$

$$5x - 2y = -7 \quad \textcircled{2}$$

In equation $\textcircled{1}$, the term $6y$ can be written as $3(2y)$:

$$-3x + 3(2y) = 9 \quad \textcircled{3}$$

Solve equation $\textcircled{2}$ for $2y$.

$$5x - 2y = -7 \quad \textcircled{2}$$

$$-2y = -5x - 7$$

$$2y = 5x + 7$$

Substitute for $2y$ in equation $\textcircled{3}$.

$$-3x + 3(2y) = 9 \quad \textcircled{3}$$

$$-3x + 3(5x + 7) = 9$$

Simplify, then solve for x .

$$-3x + 15x + 21 = 9$$

$$12x = -12$$

$$x = -1$$

Substitute $x = -1$ in equation $\textcircled{1}$.

$$-3x + 6y = 9 \quad \textcircled{1}$$

$$-3(-1) + 6y = 9$$

$$3 + 6y = 9$$

Simplify, then solve for y .

$$6y = 6$$

$$y = 1$$

Verify the solution.

In each equation, substitute: $x = -1$ and $y = 1$

$$-3x + 6y = 9 \quad \textcircled{1}$$

$$5x - 2y = -7 \quad \textcircled{2}$$

$$\text{L.S.} = -3x + 6y$$

$$\text{L.S.} = 5x - 2y$$

$$= -3(-1) + 6(1)$$

$$= 5(-1) - 2(1)$$

$$= 3 + 6$$

$$= -5 - 2$$

$$= 9$$

$$= -7$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -1 \text{ and } y = 1$$

iv) $-3x + 4y = 6 \quad \textcircled{1}$

$$9x + 3y = 27 \quad \textcircled{2}$$

In equation $\textcircled{2}$, the term $9x$ can be written as $3(3x)$:

$$3(3x) + 3y = 27 \quad \textcircled{3}$$

Solve equation $\textcircled{1}$ for $3x$.

$$-3x + 4y = 6 \quad \textcircled{1}$$

$$-3x = -4y + 6$$

$$3x = 4y - 6$$

Substitute for $3x$ in equation $\textcircled{3}$.

$$3(3x) + 3y = 27 \quad \textcircled{3}$$

$$3(4y - 6) + 3y = 27$$

Simplify, then solve for y .

$$12y - 18 + 3y = 27$$

$$15y = 45$$

$$y = 3$$

Substitute $y = 3$ in equation $\textcircled{1}$.

$$-3x + 4y = 6 \quad \textcircled{1}$$

$$-3x + 4(3) = 6$$

Simplify, then solve for x .

$$-3x + 12 = 6$$

$$-3x = -6$$

$$x = 2$$

Verify the solution.

In each equation, substitute: $x = 2$ and $y = 3$

$$-3x + 4y = 6 \quad \textcircled{1}$$

$$9x + 3y = 27 \quad \textcircled{2}$$

$$\text{L.S.} = -3x + 4y$$

$$\text{L.S.} = 9x + 3y$$

$$= -3(2) + 4(3)$$

$$= 9(2) + 3(3)$$

$$= -6 + 12$$

$$= 18 + 9$$

$$= 6$$

$$= 27$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 2 \text{ and } y = 3$$

7. a) I would choose the linear system in part i because one of the equations is already solved for one of the variables.

b) i) $x - y = -5 \quad \textcircled{1}$

$$x = -1 \quad \textcircled{2}$$

Equation $\textcircled{2}$ is solved for x , so substitute $x = -1$ in equation $\textcircled{1}$.

$$x - y = -5 \quad \textcircled{1}$$

$$-1 - y = -5$$

$$-y = -4$$

$$y = 4$$

Verify the solution.

In each equation, substitute: $x = -1$ and $y = 4$

$$x - y = -5 \quad \textcircled{1}$$

$$x = -1 \quad \textcircled{2}$$

$$\text{L.S.} = x - y$$

$$\text{L.S.} = x$$

$$= -1 - 4$$

$$= -1$$

$$= -5$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -1 \text{ and } y = 4$$

ii) $x - y = -5 \quad \textcircled{1}$

$$-x - y = 3 \quad \textcircled{2}$$

Solve equation $\textcircled{1}$ for x .

$$x - y = -5 \quad \textcircled{1}$$

$$x = y - 5$$

Substitute $x = y - 5$ in equation $\textcircled{2}$.

$$-x - y = 3 \quad \textcircled{2}$$

$$-(y - 5) - y = 3$$

Remove brackets.

$$-y + 5 - y = 3$$

Collect like terms.

$$-2y = -2$$

Divide each side by -2 .

$$y = 1$$

Substitute $y = 1$ in equation $\textcircled{1}$.

$$x - y = -5 \quad \textcircled{1}$$

$$x - 1 = -5$$

Solve for x .

$$x = -4$$

Verify the solution.

In each equation, substitute: $x = -4$ and $y = 1$

$$x - y = -5 \quad \text{①} \qquad -x - y = 3 \quad \text{②}$$

$$\begin{aligned} \text{L.S.} &= x - y \\ &= -4 - 1 \\ &= -5 \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{L.S.} &= -x - y \\ &= -(-4) - 1 \\ &= 4 - 1 \\ &= 3 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -4 \text{ and } y = 1$$

iii) $2x - 3y = 7 \quad \text{①}$

$$x - 2y = 3 \quad \text{②}$$

Solve equation ② for x .

$$x - 2y = 3 \quad \text{②}$$

$$x = 2y + 3$$

Substitute $x = 2y + 3$ in equation ①.

$$2x - 3y = 7 \quad \text{①}$$

$$2(2y + 3) - 3y = 7 \quad \text{Remove brackets.}$$

$$4y + 6 - 3y = 7 \quad \text{Collect like terms.}$$

$$y = 1$$

Substitute $y = 1$ in equation ②.

$$x - 2y = 3 \quad \text{②}$$

$$x - 2(1) = 3 \quad \text{Solve for } x.$$

$$x - 2 = 3$$

$$x = 5$$

Verify the solution.

In each equation, substitute: $x = 5$ and $y = 1$

$$2x - 3y = 7 \quad \text{①} \qquad x - 2y = 3 \quad \text{②}$$

$$\begin{aligned} \text{L.S.} &= 2x - 3y & \text{L.S.} &= x - 2y \\ &= 2(5) - 3(1) & &= 5 - 2(1) \\ &= 10 - 3 & &= 5 - 2 \\ &= 7 & &= 3 \\ &= \text{R.S.} & &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 5 \text{ and } y = 1$$

8. a) $\frac{x}{3} - \frac{y}{2} = 2 \quad \text{①}$

$$\frac{5x}{6} + \frac{3y}{4} = 1 \quad \text{②}$$

I multiply each term in equation ① by 6 because that is the lowest common denominator of the fractions; that is, 6 is the lowest common multiple of 2 and 3.

$$6\left(\frac{x}{3}\right) - 6\left(\frac{y}{2}\right) = 6(2) \quad \text{Simplify.}$$

$$2x - 3y = 12$$

I multiply each term in equation ② by 12 because that is the lowest common denominator of the fractions; that is, 12 is the lowest common multiple of 6 and 4.

$$12\left(\frac{5x}{6}\right) + 12\left(\frac{3y}{4}\right) = 12 \quad \text{Simplify.}$$

$$10x + 9y = 12$$

An equivalent linear system is:

$$2x - 3y = 12 \quad \textcircled{3}$$

$$10x + 9y = 12 \quad \textcircled{4}$$

b) Solve this linear system:

$$2x - 3y = 12 \quad \textcircled{3}$$

$$10x + 9y = 12 \quad \textcircled{4}$$

In equation $\textcircled{4}$, the term $10x$ can be written as $5(2x)$:

$$5(2x) + 9y = 12 \quad \textcircled{5}$$

Solve equation $\textcircled{3}$ for $2x$.

$$2x - 3y = 12 \quad \textcircled{3}$$

$$2x = 3y + 12$$

Substitute for $2x$ in equation $\textcircled{5}$.

$$5(2x) + 9y = 12 \quad \textcircled{5}$$

$$5(3y + 12) + 9y = 12 \quad \text{Simplify, then solve for } y.$$

$$15y + 60 + 9y = 12$$

$$24y = -48$$

$$y = -2$$

Substitute $y = -2$ in equation $\textcircled{3}$.

$$2x - 3y = 12 \quad \textcircled{3}$$

$$2x - 3(-2) = 12 \quad \text{Simplify, then solve for } x.$$

$$2x + 6 = 12$$

$$2x = 6$$

$$x = 3$$

Verify the solution.

In each equation, substitute: $x = 3$ and $y = -2$

$$2x - 3y = 12 \quad \textcircled{3}$$

$$\text{L.S.} = 2x - 3y$$

$$= 2(3) - 3(-2)$$

$$= 6 + 6$$

$$= 12$$

$$= \text{R.S.}$$

$$10x + 9y = 12 \quad \textcircled{4}$$

$$\text{L.S.} = 10x + 9y$$

$$= 10(3) + 9(-2)$$

$$= 30 - 18$$

$$= 12$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 3 \text{ and } y = -2$$

Verify the solution for the linear system in part a.

$$\frac{x}{3} - \frac{y}{2} = 2 \quad \textcircled{1}$$

$$\frac{5x}{6} + \frac{3y}{4} = 1 \quad \textcircled{2}$$

In each equation, substitute: $x = 3$ and $y = -2$

$$\frac{x}{3} - \frac{y}{2} = 2 \quad \textcircled{1}$$

$$\begin{aligned} \text{L.S.} &= \frac{x}{3} - \frac{y}{2} \\ &= \frac{3}{3} - \frac{-2}{2} \\ &= 1 - (-1) \\ &= 2 \\ &= \text{R.S.} \end{aligned}$$

$$\frac{5x}{6} + \frac{3y}{4} = 1 \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= \frac{5(3)}{6} + \frac{3(-2)}{4} \\ &= \frac{15}{6} + \frac{-6}{4} \\ &= \frac{5}{2} - \frac{3}{2} \\ &= \frac{2}{2} \\ &= 1 \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:
 $x = 3$ and $y = -2$

9. a) $2x + 2y = -4 \quad \textcircled{1}$

$-12x + 4y = -24 \quad \textcircled{2}$

I divide each term in equation $\textcircled{1}$ by 2 because 2 is the greatest common factor of the coefficients and constant term.

$2x + 2y = -4$ Divide each term by 2.

$x + y = -2$

I divide each term in equation $\textcircled{2}$ by 4 because 4 is the greatest common factor of the coefficients and constant term.

$-12x + 4y = -24$ Divide each term by 4.

$-3x + y = -6$

An equivalent linear system is:

$x + y = -2 \quad \textcircled{3}$

$-3x + y = -6 \quad \textcircled{4}$

b) Solve this linear system:

$x + y = -2 \quad \textcircled{3}$

$-3x + y = -6 \quad \textcircled{4}$

Solve equation $\textcircled{3}$ for y .

$x + y = -2$

$y = -x - 2 \quad \textcircled{5}$

Substitute $y = -x - 2$ in equation $\textcircled{4}$.

$-3x + y = -6 \quad \textcircled{4}$

$-3x + (-x - 2) = -6$ Simplify, then solve for x .

$-3x - x - 2 = -6$

$-4x = -4$

$x = 1$

Substitute $x = 1$ in equation $\textcircled{3}$.

$x + y = -2 \quad \textcircled{3}$

$1 + y = -2$ Simplify, then solve for y .

$y = -3$

Verify the solution.

In each equation, substitute: $x = 1$ and $y = -3$

$$x + y = -2 \quad \textcircled{3}$$

$$\text{L.S.} = x + y$$

$$= 1 + (-3)$$

$$= -2$$

$$= \text{R.S.}$$

$$-3x + y = -6 \quad \textcircled{4}$$

$$\text{L.S.} = -3x + y$$

$$= -3(1) + (-3)$$

$$= -3 - 3$$

$$= -6$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 1 \text{ and } y = -3$$

Verify the solution for the linear system in part a.

$$2x + 2y = -4 \quad \textcircled{1}$$

$$-12x + 4y = -24 \quad \textcircled{2}$$

In each equation, substitute: $x = 1$ and $y = -3$

$$2x + 2y = -4 \quad \textcircled{1}$$

$$\text{L.S.} = 2x + 2y$$

$$= 2(1) + 2(-3)$$

$$= 2 - 6$$

$$= -4$$

$$= \text{R.S.}$$

$$-12x + 4y = -24 \quad \textcircled{2}$$

$$\text{L.S.} = -12x + 4y$$

$$= -12(1) + 4(-3)$$

$$= -12 - 12$$

$$= -24$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 1 \text{ and } y = -3$$

10. Let r represent the number of bears that did appear to respond.

Let n represent the number of bears that did not appear to respond.

A total of 186 bears were investigated.

So, one equation is: $r + n = 186$

There were 94 more bears that did not respond than did respond. So, the difference in bears that did not respond and bears that did respond is 94.

Another equation is: $n - r = 94$

A linear system is:

$$r + n = 186 \quad \textcircled{1}$$

$$n - r = 94 \quad \textcircled{2}$$

Solve equation $\textcircled{1}$ for r .

$$r + n = 186 \quad \textcircled{1}$$

$$r = 186 - n$$

Substitute $r = 186 - n$ in equation $\textcircled{2}$.

$$n - r = 94 \quad \textcircled{2}$$

$$n - (186 - n) = 94 \quad \text{Remove brackets.}$$

$$n - 186 + n = 94 \quad \text{Collect like terms.}$$

$$2n = 94 + 186$$

$$2n = 280 \quad \text{Divide each side by 2.}$$

$$n = 140$$

Substitute $n = 140$ in equation $\textcircled{1}$.

$$r + n = 186 \quad \textcircled{1}$$

$$r + 140 = 186 \quad \text{Solve for } r.$$

$$r = 186 - 140$$

$$r = 46$$

Verify the solution.

The total number of bears is: $140 + 46 = 186$; this is the same as the given information.

The difference between the numbers of bears that did not respond and that did respond is:
 $140 - 46 = 94$; this is the same as the given information.

The solution is correct; that is, 46 bears responded and 140 bears did not appear to respond.

11. Let l centimetres represent the length of the flag.

Let w centimetres represent the width of the flag.

The length is 90 cm longer than the width.

So, one equation is: $l = 90 + w$

The perimeter of the flag is 540 cm.

So, $l + l + w + w = 540$ Simplify.

Another equation is: $2l + 2w = 540$

A linear system is:

$$l = 90 + w \quad \text{①}$$

$$2l + 2w = 540 \quad \text{②}$$

To solve this system:

From equation ①, substitute for $l = 90 + w$ in equation ②.

$$2l + 2w = 540 \quad \text{②}$$

$$2(90 + w) + 2w = 540 \quad \text{Remove brackets.}$$

$$180 + 2w + 2w = 540 \quad \text{Simplify, then solve for } w.$$

$$4w = 360$$

$$w = 90$$

Substitute $w = 90$ in equation ①.

$$l = 90 + w \quad \text{①}$$

$$l = 90 + 90$$

$$l = 180$$

The length of the flag is 180 cm and the width is 90 cm.

Verify the solution.

The difference between the length and width is: $180 \text{ cm} - 90 \text{ cm} = 90 \text{ cm}$; this is the same as the given information.

The perimeter of the flag is: $180 \text{ cm} + 180 \text{ cm} + 90 \text{ cm} + 90 \text{ cm} = 540 \text{ cm}$; this is the same as the given information.

So, the solution is correct.

12. Let s represent the number of students in the study.

Let a represent the number of adults in the study.

A total of 45 people were in the study.

So, one equation is: $s + a = 45$

80% of the students is: $0.8s$

60% of the adults is: $0.6a$

80% of the students and 60% of the adults is a total of 31 people.

So, another equation is: $0.8s + 0.6a = 31$

A linear system is:

$$s + a = 45 \quad \text{①}$$

$$0.8s + 0.6a = 31 \quad \text{②}$$

To solve this system:

Solve equation ① for s .

$$s + a = 45 \quad \text{①}$$

$$s = 45 - a$$

Substitute $s = 45 - a$ in equation ②.

$$0.8s + 0.6a = 31 \quad \text{②}$$

$$0.8(45 - a) + 0.6a = 31 \quad \text{Simplify, then solve for } a.$$

$$36 - 0.8a + 0.6a = 31$$

$$-0.2a = -5 \quad \text{Divide each side by } -0.2.$$

$$a = 25$$

Substitute $a = 25$ in equation ①.

$$s + a = 45 \quad \text{①}$$

$$s + 25 = 45$$

$$s = 20$$

There were 20 students and 25 adults in the study.

Verify the solution.

The total number of people in the study is: $20 + 25 = 45$; this is the same as the given information.

80% of the students is: $0.8 \times 20 = 16$

60% of the adults is: $0.6 \times 25 = 15$

The number of people who reported a heavy use of the internet is: $16 + 15 = 31$; this is the same as the given information.

So, the solution is correct.

13. Let the number of groups of 4 students be represented by x .

Let the number of groups of 5 students be represented by y .

A total of 47 students were in the groups.

So, one equation is: $4x + 5y = 47$

There were 11 groups of students.

So, another equation is: $x + y = 11$

A linear system is:

$$4x + 5y = 47 \quad \text{①}$$

$$x + y = 11 \quad \text{②}$$

To solve this system:

Solve equation ② for x .

$$x + y = 11 \quad \text{②}$$

$$x = 11 - y$$

Substitute $x = 11 - y$ in equation ①.

$$4x + 5y = 47 \quad \text{①}$$

$$4(11 - y) + 5y = 47 \quad \text{Simplify, then solve for } y.$$

$$44 - 4y + 5y = 47$$

$$y = 3$$

Substitute $y = 3$ in equation ②.

$$x + y = 11 \quad \text{②}$$

$$x + 3 = 11$$

$$x = 8$$

There were 8 groups of 4 students and 3 groups of 5 students.

Verify the solution.

8 groups of 4 is 32 students.

3 groups of 5 is 15 students.

So, the total number of students is: $32 + 15 = 47$; this is the same as the given information.

The total number of groups is: $8 + 3 = 11$; this is the same as the given information.

So, the solution is correct.

14. Let p represent the number of people masks.
 Let a represent the number of animal masks.
 The total number of masks is 85.
 So, one equation is: $p + a = 85$
 60% of the people masks is: $0.6p$
 40% of the animal masks is: $0.4a$
 60% of the people masks and 40% of the animal masks is a total of 38 masks made from yellow cedar.
 So, another equation is: $0.6p + 0.4a = 38$
 A linear system is:
 $p + a = 85$ ①
 $0.6p + 0.4a = 38$ ②
 To solve this system:
 Solve equation ① for p .
 $p + a = 85$ ①
 $p = 85 - a$
 Substitute $p = 85 - a$ in equation ②.
 $0.6p + 0.4a = 38$ ②
 $0.6(85 - a) + 0.4a = 38$ Simplify, then solve for a .
 $51 - 0.6a + 0.4a = 38$
 $-0.2a = -13$ Divide each side by -0.2 .
 $a = 65$
 Substitute $a = 65$ in equation ①.
 $p + a = 85$ ①
 $p + 65 = 85$
 $p = 20$
 There were 20 people masks and 65 animal masks.
- Verify the solution.
 The total number of masks is: $20 + 65 = 85$; this is the same as the given information.
 60% of 20 is: $0.6 \times 20 = 12$
 40% of 65 is: $0.4 \times 65 = 26$
 The total number of yellow cedar masks: $12 + 26 = 38$; this is the same as the given information.
 So, the solution is correct.

15. Let A represent the number of marks for part A of the test.
 Let B represent the number of marks for part B of the test.
 The total possible mark is 75.
 So, one equation is: $A + B = 75$
 80% of the marks for part A is: $0.8A$
 92% of the marks for part B is: $0.92B$
 80% of the marks for part A and 92% of the marks for part B is 63.
 So, another equation is: $0.8A + 0.92B = 63$
 A linear system is:
 $A + B = 75$ ①
 $0.8A + 0.92B = 63$ ②

To solve this system:

Solve equation ① for A .

$$A + B = 75 \quad \text{①}$$

$$A = 75 - B$$

Substitute $A = 75 - B$ in equation ②.

$$0.8A + 0.92B = 63 \quad \text{②}$$

$$0.8(75 - B) + 0.92B = 63$$

$$60 - 0.8B + 0.92B = 63 \quad \text{Simplify, then solve for } B.$$

$$0.12B = 3 \quad \text{Divide each side by 0.12.}$$

$$B = 25$$

Substitute $B = 25$ in equation ①.

$$A + B = 75 \quad \text{①}$$

$$A + 25 = 75$$

$$A = 50$$

There 50 marks for part A and 25 marks for part B of the test.

Verify the solution.

The total possible mark is: $50 + 25 = 75$; this is the same as the given information.

80% of 50 is: $0.8 \times 50 = 40$

92% of 25 is: $0.92 \times 25 = 23$

The total number of marks that Sam scored is: $40 + 23 = 63$; this is the same as the given information.

So, the solution is correct.

16. Let x dollars represent the amount invested at 2.5%.

Let y dollars represent the amount invested at 3.75%.

The total amount invested is \$5000.

So, one equation is: $x + y = 5000$

x dollars invested at 2.5% earns $0.025x$ dollars interest.

y dollars invested at 3.75% earns $0.0375y$ dollars.

The total interest earned is \$162.50.

So, another equation is: $0.025x + 0.0375y = 162.5$

A linear system is:

$$x + y = 5000 \quad \text{①}$$

$$0.025x + 0.0375y = 162.5 \quad \text{②}$$

To solve this system:

Solve equation ① for x .

$$x + y = 5000 \quad \text{①}$$

$$x = 5000 - y$$

Substitute $x = 5000 - y$ in equation ②.

$$0.025x + 0.0375y = 162.5 \quad \text{②}$$

$$0.025(5000 - y) + 0.0375y = 162.5$$

$$125 - 0.025y + 0.0375y = 162.5$$

$$0.0125y = 162.5 - 125$$

$$0.0125y = 37.5$$

$$y = 3000$$

Substitute $y = 3000$ in equation ①.

$$x + 3000 = 5000 \quad \text{①}$$

$$x = 2000$$

\$2000 was invested at 2.5% and \$3000 was invested at 3.75%.

Verify the solution.

The total amount invested is: $\$2000 + \$3000 = \$5000$; this is the same as the given information.

$\$2000$ invested at 2.5% earns: $\$2000 \times 0.025 = \$50$ interest

$\$3000$ invested at 3.75% earns: $\$3000 \times 0.0375 = \$112.50$ interest

The total interest is: $\$50 + \$112.50 = \$162.50$; this is the same as the given information.

So, the solution is correct.

17. Let s dollars represent the cost of a single-scoop cone.

Let d dollars represent the cost of a double-scoop cone.

76 single-scoop cones and 49 double-scoop cones cost \$474.25.

So, one equation is: $76s + 49d = 474.25$

54 single-scoop cones and 37 double-scoop cones cost \$346.25.

So, another equation is: $54s + 37d = 346.25$

A linear system is:

$$76s + 49d = 474.25 \quad \textcircled{1}$$

$$54s + 37d = 346.25 \quad \textcircled{2}$$

To solve this system, use a graphing calculator.

Write each equation in the form $y = mx + b$.

For equation $\textcircled{1}$:

$$76s + 49d = 474.25$$

$$49d = -76s + 474.25$$

$$d = \frac{-76}{49}s + \frac{474.25}{49}$$

Solve for d . Subtract $76s$ from each side.

Divide each side by 49.

For equation $\textcircled{2}$:

$$54s + 37d = 346.25$$

$$37d = -54s + 346.25$$

$$d = \frac{-54}{37}s + \frac{346.25}{37}$$

Solve for d . Subtract $54s$ from each side.

Divide each side by 37.

An equivalent linear system is:

$$d = \frac{-76}{49}s + \frac{474.25}{49}$$

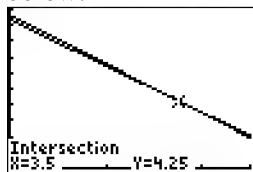
$$d = \frac{-54}{37}s + \frac{346.25}{37}$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression

$(-76/49)X + 474.25/49$. Move the cursor down to $Y2 =$ and input the expression

$(-54/37)X + 346.25/37$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, I set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 5$, $Y_{\min} = 0$, and $Y_{\max} = 10$. To show the coordinates of the point of intersection, I pressed $\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. I pressed $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $s = 3.5$ and $d = 4.25$

A single-scoop cone costs \$3.50 and a double-scoop cone costs \$4.25.

Verify the solution.

76 single-scoop cones and 49 double-scoop cones cost: $76(\$3.50) + 49(\$4.25) = \$474.25$;
this is the same as the given information.

54 single-scoop cones and 37 double-scoop cones cost: $54(\$3.50) + 37(\$4.25) = \$346.25$;
this is the same as the given information.

So, the solution is correct.

18. Let p dollars represent the cost for each person to work w weekends.

Joel is paid \$40 per weekend.

So, for w weekends, Joel's earnings in dollars are: $40w$

One equation is: $p = 40w$

Sue is paid \$150 plus \$30 each weekend.

So, for w weekends, Sue's earnings in dollars are: $150 + 30w$

Another equation is: $p = 150 + 30w$

A linear system is:

$$p = 40w \quad \text{①}$$

$$p = 150 + 30w \quad \text{②}$$

To solve this system:

Both equations are solved for p .

From equation ①, substitute $p = 40w$ in equation ②.

$$p = 150 + 30w \quad \text{②}$$

$$40w = 150 + 30w \quad \text{Solve for } w.$$

$$10w = 150$$

$$w = 15$$

Joel has to work 15 weekends before he earns the same as Sue.

Check the solution.

When Joel works for 15 weekends, he earns: $15(\$40) = \$600$

When Sue works for 15 weekends, she earns: $\$150 + 15(\$30) = \$600$

Since both people earn \$600 after working 15 weekends, the solution is correct.

19. a) $\frac{1}{2}x + \frac{2}{3}y = 1 \quad \text{①}$

$$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2} \quad \text{②}$$

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 2 and 3, which is 6:

$$\frac{1}{2}x + \frac{2}{3}y = 1$$

Multiply each term by 6.

$$6\left(\frac{1}{2}x\right) + 6\left(\frac{2}{3}y\right) = 6(1)$$

Simplify.

$$3x + 4y = 6 \quad \text{③}$$

For equation ②, the common denominator is the lowest common multiple of 4 and 3, which is 12:

$$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2}$$

Multiply each term by 12.

$$12\left(\frac{1}{4}x\right) - 12\left(\frac{1}{3}y\right) = 12\left(\frac{5}{2}\right)$$

Simplify.

$$3x - 4y = 30 \quad \text{④}$$

Solve equation ④ for $3x$.

$$3x - 4y = 30 \quad \text{④}$$

$$3x = 4y + 30$$

Substitute for $3x$ in equation ③.

$$3x + 4y = 6 \quad \text{③}$$

$$(4y + 30) + 4y = 6$$

Simplify, then solve for y .

$$8y = -24$$

$$y = -3$$

Substitute $y = -3$ into equation ④.

$$3x - 4y = 30 \quad \text{④}$$

$$3x - 4(-3) = 30$$

Simplify, then solve for x .

$$3x + 12 = 30$$

$$3x = 18$$

$$x = 6$$

Verify the solution.

In each original equation, substitute: $x = 6$ and $y = -3$

$$\frac{1}{2}x + \frac{2}{3}y = 1 \quad \text{①}$$

$$\frac{1}{4}x - \frac{1}{3}y = \frac{5}{2} \quad \text{②}$$

$$\text{L.S.} = \frac{1}{2}x + \frac{2}{3}y$$

$$\text{L.S.} = \frac{1}{4}x - \frac{1}{3}y$$

$$= \frac{1}{2}(6) + \frac{2}{3}(-3)$$

$$= \frac{1}{4}(6) - \frac{1}{3}(-3)$$

$$= 3 - 2$$

$$= \frac{3}{2} + 1$$

$$= 1$$

$$= \frac{5}{2}$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 6 \text{ and } y = -3$$

b) $\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12} \quad \text{①}$

$$x - y = -\frac{4}{3} \quad \text{②}$$

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 4, 2, and 12, which is 12:

$$\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$$

Multiply each term by 12.

$$12\left(\frac{3}{4}x\right) + 12\left(\frac{1}{2}y\right) = 12\left(-\frac{7}{12}\right)$$

Simplify.

$$9x + 6y = -7 \quad \text{③}$$

For equation ②, the common denominator is 3:

$$x - y = -\frac{4}{3} \quad \text{Multiply each term by 3.}$$

$$3x - 3y = 3\left(-\frac{4}{3}\right) \quad \text{Simplify.}$$

$$3x - 3y = -4 \quad \text{④}$$

Solve equation ④ for $3x$.

$$3x - 3y = -4 \quad \text{④}$$

$$3x = 3y - 4$$

Substitute for $3x$ in equation ③.

$$9x + 6y = -7 \quad \text{③}$$

$$3(3y - 4) + 6y = -7 \quad \text{Simplify, then solve for } y.$$

$$9y - 12 + 6y = -7$$

$$15y = 5$$

$$y = \frac{5}{15}, \text{ or } \frac{1}{3}$$

Substitute $y = \frac{1}{3}$ into equation ③.

$$9x + 6y = -7 \quad \text{③}$$

$$9x + 6\left(\frac{1}{3}\right) = -7 \quad \text{Simplify, then solve for } x.$$

$$9x + 2 = -7$$

$$9x = -9$$

$$x = -1$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = \frac{1}{3}$

$$\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12} \quad \text{①}$$

$$x - y = -\frac{4}{3} \quad \text{②}$$

$$\text{L.S.} = \frac{3}{4}x + \frac{1}{2}y$$

$$\text{L.S.} = x - y$$

$$= \frac{3}{4}(-1) + \frac{1}{2}\left(\frac{1}{3}\right)$$

$$= -1 - \frac{1}{3}$$

$$= -\frac{3}{4} + \frac{1}{6}$$

$$= -\frac{3}{3} - \frac{1}{3}$$

$$= -\frac{9}{12} + \frac{2}{12}$$

$$= -\frac{4}{3}$$

$$= -\frac{7}{12}$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -1 \text{ and } y = \frac{1}{3}$$

$$\text{c) } \frac{1}{3}x - \frac{3}{8}y = 1 \quad \textcircled{1}$$

$$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2} \quad \textcircled{2}$$

Write an equivalent system with integer coefficients.

For equation $\textcircled{1}$, the common denominator is the lowest common multiple of 3 and 8, which is 24:

$$\frac{1}{3}x - \frac{3}{8}y = 1 \quad \text{Multiply each term by 24.}$$

$$24\left(\frac{1}{3}x\right) - 24\left(\frac{3}{8}y\right) = 24(1) \quad \text{Simplify.}$$

$$8x - 9y = 24 \quad \textcircled{3}$$

For equation $\textcircled{2}$, the common denominator is the lowest common multiple of 4, 8, and 2, which is 8:

$$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2} \quad \text{Multiply each term by 8.}$$

$$8\left(-\frac{1}{4}x\right) - 8\left(\frac{1}{8}y\right) = 8\left(\frac{3}{2}\right) \quad \text{Simplify.}$$

$$-2x - y = 12 \quad \textcircled{4}$$

Solve equation $\textcircled{4}$ for y .

$$-2x - y = 12 \quad \textcircled{4}$$

$$-y = 2x + 12$$

$$y = -2x - 12$$

Substitute for y in equation $\textcircled{3}$.

$$8x - 9y = 24 \quad \textcircled{3}$$

$$8x - 9(-2x - 12) = 24 \quad \text{Simplify, then solve for } x.$$

$$8x + 18x + 108 = 24$$

$$26x = 24 - 108$$

$$26x = -84$$

$$x = \frac{-84}{26}, \text{ or } -\frac{42}{13}$$

To determine the value of y , solve equation $\textcircled{4}$ for $2x$.

$$-2x - y = 12 \quad \textcircled{4}$$

$$-2x = y + 12$$

$$2x = -y - 12$$

Substitute for $2x$ in equation $\textcircled{3}$.

$$8x - 9y = 24 \quad \textcircled{3}$$

$$4(-y - 12) - 9y = 24 \quad \text{Simplify, then solve for } y.$$

$$-4y - 48 - 9y = 24$$

$$-13y = 24 + 48$$

$$-13y = 72$$

$$y = \frac{72}{-13}, \text{ or } -\frac{72}{13}$$

Verify the solution.

In each original equation, substitute: $x = -\frac{42}{13}$ and $y = -\frac{72}{13}$

$$\frac{1}{3}x - \frac{3}{8}y = 1 \quad \textcircled{1}$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{3}x - \frac{3}{8}y \\ &= \frac{1}{3}\left(-\frac{42}{13}\right) - \frac{3}{8}\left(-\frac{72}{13}\right) \\ &= -\frac{14}{13} + \frac{27}{13} \\ &= \frac{13}{13} \\ &= 1 \\ &= \text{R.S.} \end{aligned}$$

$$-\frac{1}{4}x - \frac{1}{8}y = \frac{3}{2} \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= -\frac{1}{4}x - \frac{1}{8}y \\ &= -\frac{1}{4}\left(-\frac{42}{13}\right) - \frac{1}{8}\left(-\frac{72}{13}\right) \\ &= \frac{21}{26} + \frac{18}{26} \\ &= \frac{39}{26} \\ &= \frac{3}{2} \\ &= \text{R.S.} \end{aligned}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -\frac{42}{13} \text{ and } y = -\frac{72}{13}$$

d) $\frac{7}{4}x + \frac{4}{3}y = 3 \quad \textcircled{1}$

$$\frac{1}{2}x - \frac{5}{6}y = 2 \quad \textcircled{2}$$

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 4 and 3, which is 12:

$$\frac{7}{4}x + \frac{4}{3}y = 3 \quad \text{Multiply each term by 12.}$$

$$12\left(\frac{7}{4}x\right) + 12\left(\frac{4}{3}y\right) = 12(3) \quad \text{Simplify.}$$

$$21x + 16y = 36 \quad \textcircled{3}$$

For equation ②, the common denominator is the lowest common multiple of 2 and 6, which is 6:

$$\frac{1}{2}x - \frac{5}{6}y = 2 \quad \text{Multiply each term by 6.}$$

$$6\left(\frac{1}{2}x\right) - 6\left(\frac{5}{6}y\right) = 6(2) \quad \text{Simplify.}$$

$$3x - 5y = 12 \quad \textcircled{4}$$

Solve equation ④ for 3x.

$$3x - 5y = 12 \quad \textcircled{4}$$

$$3x = 5y + 12$$

Substitute for 3x in equation ③.

$$21x + 16y = 36 \quad \textcircled{3}$$

$$7(5y + 12) + 16y = 36 \quad \text{Simplify, then solve for } y.$$

$$35y + 84 + 16y = 36$$

$$51y = 36 - 84$$

$$51y = -48$$

$$y = \frac{-48}{51}, \text{ or } -\frac{16}{17}$$

Substitute $y = -\frac{16}{17}$ into equation ④.

$$3x - 5y = 12 \quad \text{④}$$

$$3x - 5\left(-\frac{16}{17}\right) = 12 \quad \text{Simplify, then solve for } x.$$

$$3x + \frac{80}{17} = 12$$

$$3x = 12 - \frac{80}{17}$$

$$3x = \frac{204}{17} - \frac{80}{17}$$

$$3x = \frac{124}{17}$$

$$x = \frac{124}{51}$$

Verify the solution.

In each original equation, substitute: $x = \frac{124}{51}$ and $y = -\frac{16}{17}$

$$\frac{7}{4}x + \frac{4}{3}y = 3 \quad \text{①}$$

$$\frac{1}{2}x - \frac{5}{6}y = 2 \quad \text{②}$$

$$\text{L.S.} = \frac{7}{4}x + \frac{4}{3}y$$

$$\text{L.S.} = \frac{1}{2}x - \frac{5}{6}y$$

$$= \frac{7}{4}\left(\frac{124}{51}\right) + \frac{4}{3}\left(-\frac{16}{17}\right)$$

$$= \frac{1}{2}\left(\frac{124}{51}\right) - \frac{5}{6}\left(-\frac{16}{17}\right)$$

$$= \frac{217}{51} - \frac{64}{51}$$

$$= \frac{62}{51} + \frac{40}{51}$$

$$= \frac{153}{51}$$

$$= \frac{102}{51}$$

$$= 3$$

$$= 2$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = \frac{124}{51} \text{ and } y = -\frac{16}{17}$$

20. a) $7.50r + 45c = 375 \quad \text{①}$

$r - c = 15 \quad \text{②}$

An ink cartridge is more expensive than a ream of paper, so assume an ink cartridge costs \$45.00 and a ream of paper costs \$7.50.

Let r represent the number of reams of paper bought.

Let c represent the number of ink cartridges bought.

Then equation ① represents the sum of the cost of r reams of paper at \$7.50/ream and the cost of c ink cartridges at \$45.00/cartridge. This sum is \$375.00.

Equation ② represents the difference in numbers of reams of paper and ink cartridges

bought. This difference is 15.

A related problem is:

An ink cartridge costs \$45.00.

A ream of paper costs \$7.50.

The cost of buying some ink cartridges and some reams of paper was \$375.00.

Fifteen more reams of paper were bought than ink cartridges.

How many reams of paper and how many ink cartridges were bought?

b) $7.50r + 45c = 375$ ①

$r - c = 15$ ②

Solve equation ② for r .

$r - c = 15$ ②

$r = c + 15$

Substitute $r = c + 15$ in equation ①.

$7.50r + 45c = 375$ ①

$7.50(c + 15) + 45c = 375$ Simplify, then solve for c .

$7.50c + 112.5 + 45c = 375$

$52.50c = 375 - 112.5$

$52.50c = 262.5$ Divide each side by 52.50.

$c = 5$

Substitute $c = 5$ into equation ②.

$r - c = 15$ ②

$r - 5 = 15$

Solve for r .

$r = 20$

20 reams of paper and 5 ink cartridges were bought.

Verify the solution.

In each equation, substitute: $r = 20$ and $c = 5$

$7.50r + 45c = 375$ ①

$r - c = 15$ ②

L.S. = $7.50r + 45c$

L.S. = $r - c$

$= 7.50(20) + 45(5)$

$= 20 - 5$

$= 150 + 225$

$= 15$

$= 375$

$= \text{R.S.}$

$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is correct.

21. $2x + 4y = 98$ ①

$x + y = 27$ ②

Each variable term in equation ① is the product of a constant that is not 1 and a variable.

This suggests a problem about two groups of people who pay different amounts.

Equation ② is the sum of two variable terms with coefficient 1, which suggests that the numbers of people in equation ① are added.

A situation that could be modelled by this linear system is students and adults attending a fall fair.

A related problem is:

The cost for a student to attend a local fall fair is \$2.

The cost for an adult to attend the fair is \$4.

In the first 5 min after the fair opened, 27 people paid a total of \$98 to go to the fair.

How many students and how many adults went to the fair in the first 5 min?

In this problem, x represents the number of students and y represents the number of adults.

$$2x + 4y = 98 \quad \textcircled{1}$$

$$x + y = 27 \quad \textcircled{2}$$

To solve the system:

Solve equation $\textcircled{2}$ for x .

$$x + y = 27 \quad \textcircled{2}$$

$$x = 27 - y$$

Substitute $x = 27 - y$ in equation $\textcircled{1}$.

$$2x + 4y = 98 \quad \textcircled{1}$$

$$2(27 - y) + 4y = 98 \quad \text{Simplify, then solve for } y.$$

$$54 - 2y + 4y = 98$$

$$2y = 98 - 54$$

$$2y = 44$$

$$y = 22$$

Substitute $y = 22$ into equation $\textcircled{2}$.

$$x + y = 27 \quad \textcircled{2}$$

$$x + 22 = 27$$

$$x = 5$$

5 students and 22 adults went to the fair in the first 5 min.

Verify the solution.

Substitute $x = 5$ and $y = 22$ into each equation.

$$2x + 4y = 98 \quad \textcircled{1}$$

$$\text{L.S.} = 2x + 4y$$

$$= 2(5) + 4(22)$$

$$= 10 + 88$$

$$= 98$$

$$= \text{R.S.}$$

$$x + y = 27 \quad \textcircled{2}$$

$$\text{L.S.} = x + y$$

$$= 5 + 22$$

$$= 27$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is correct.

22. a) $2x - y = -4 \quad \textcircled{1}$

$$3x + 2y = 1 \quad \textcircled{2}$$

To write an equivalent system, multiply the terms in each equation by the same constant.

For equation $\textcircled{1}$, multiply each term by 2:

$$2(2x) - 2(y) = 2(-4)$$

$$4x - 2y = -8 \quad \textcircled{3}$$

For equation $\textcircled{2}$, multiply each term by 3:

$$3(3x) + 3(2y) = 3(1)$$

$$9x + 6y = 3 \quad \textcircled{4}$$

An equivalent system is:

$$4x - 2y = -8 \quad \textcircled{3}$$

$$9x + 6y = 3 \quad \textcircled{4}$$

b) $2x - y = -4 \quad \textcircled{1}$

$$3x + 2y = 1 \quad \textcircled{2}$$

To solve this system:

Solve equation $\textcircled{1}$ for y .

$$2x - y = -4 \quad \textcircled{1}$$

$$-y = -4 - 2x$$

$$y = 4 + 2x$$

Substitute $y = 4 + 2x$ in equation $\textcircled{2}$.

$$\begin{aligned}
 3x + 2y &= 1 & \textcircled{2} \\
 3x + 2(4 + 2x) &= 1 & \text{Simplify, then solve for } x. \\
 3x + 8 + 4x &= 1 \\
 7x &= -7 \\
 x &= -1
 \end{aligned}$$

Substitute $x = -1$ into equation $\textcircled{1}$.

$$\begin{aligned}
 2x - y &= -4 & \textcircled{1} \\
 2(-1) - y &= -4 \\
 -2 - y &= -4 \\
 -y &= -2 \\
 y &= 2
 \end{aligned}$$

Verify the solution.

Substitute $x = -1$ and $y = 2$ into each equation.

$2x - y = -4$ $\textcircled{1}$	$3x + 2y = 1$ $\textcircled{2}$
L.S. $= 2x - y$	L.S. $= 3x + 2y$
$= 2(-1) - 2$	$= 3(-1) + 2(2)$
$= -2 - 2$	$= -3 + 4$
$= -4$	$= 1$
$= \text{R.S.}$	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is correct.

$$\begin{aligned}
 4x - 2y &= -8 & \textcircled{3} \\
 9x + 6y &= 3 & \textcircled{4}
 \end{aligned}$$

To solve this system:

Solve equation $\textcircled{3}$ for $2y$.

$$\begin{aligned}
 4x - 2y &= -8 & \textcircled{3} \\
 -2y &= -8 - 4x \\
 2y &= 8 + 4x
 \end{aligned}$$

Substitute $2y = 8 + 4x$ in equation $\textcircled{4}$.

$$\begin{aligned}
 9x + 6y &= 3 & \textcircled{4} \\
 9x + 3(8 + 4x) &= 3 & \text{Simplify, then solve for } x. \\
 9x + 24 + 12x &= 3 \\
 21x &= -21 \\
 x &= -1
 \end{aligned}$$

Substitute $x = -1$ into equation $\textcircled{3}$.

$$\begin{aligned}
 4x - 2y &= -8 & \textcircled{3} \\
 4(-1) - 2y &= -8 \\
 -4 - 2y &= -8 \\
 -2y &= -4 \\
 -y &= -2 \\
 y &= 2
 \end{aligned}$$

Verify the solution.

Substitute $x = -1$ and $y = 2$ into each equation.

$4x - 2y = -8$ $\textcircled{3}$	$9x + 6y = 3$ $\textcircled{4}$
L.S. $= 4x - 2y$	L.S. $= 9x + 6y$
$= 4(-1) - 2(2)$	$= 9(-1) + 6(2)$
$= -4 - 4$	$= -9 + 12$
$= -8$	$= 3$
$= \text{R.S.}$	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is correct.
Equivalent linear systems have the same solution; so, since the solutions are the same, the systems are equivalent.

C

- 23.** Write a linear system to represent the situation.

Let s kilometres per hour represent the usual average speed.

Let d kilometres represent the distance from Penticton to Chute Lake.

The cyclists' usual average speed was reduced by 6 km/h, so the new average speed is:

$(s - 6)$ km/h

The cyclists took 4 h to travel d kilometres at $(s - 6)$ km/h.

Use the formula:

distance travelled = average speed $\times$ time

$$d = (s - 6) \times 4 \quad \text{Simplify.}$$

$$d = 4s - 24$$

On the return trip, the usual average speed was increased by 4 km/h, so the new average speed is:

$(s + 4)$ km/h

The cyclists took 2 h to travel d kilometres at $(s + 4)$ km/h.

So, $d = (s + 4) \times 2$ Simplify.

$$d = 2s + 8$$

A linear system that represents this situation is:

$$d = 4s - 24 \quad \text{①}$$

$$d = 2s + 8 \quad \text{②}$$

- a)** Solve the linear system.

From equation ①, substitute $d = 4s - 24$ into equation ②.

$$d = 2s + 8 \quad \text{②}$$

$$4s - 24 = 2s + 8 \quad \text{Simplify, then solve for } s.$$

$$2s = 32$$

$$s = 16$$

The usual average speed is 16 km/h.

- b)** Substitute $s = 16$ into equation ②.

$$d = 2s + 8 \quad \text{②}$$

$$d = 2(16) + 8$$

$$d = 32 + 8$$

$$d = 40$$

The distance from Penticton to Chute Lake is 40 km.

Check the answers.

Uphill: the cyclists travel at: $(16 - 6)$ km/h = 10 km/h

They travel for 4 h.

So, the distance travelled is: $4 \text{ h} \times 10 \text{ km/h} = 40 \text{ km}$; this agrees with the given information.

Downhill: the cyclists travel at: $(16 + 4)$ km/h = 20 km/h

They travel for 2 h.

So, the distance travelled is: $2 \text{ h} \times 20 \text{ km/h} = 40 \text{ km}$; this agrees with the given information.

So, the answers are correct.

24. Write a linear system to represent the situation.

Let the mean mass of the female rattlesnakes be represented by f grams.

Let the mean mass of the male rattlesnakes be represented by m grams.

The mean mass of 45 female rattlesnakes and 100 male rattlesnakes is 194 g.

So, one equation is: $\frac{45f + 100m}{145} = 194$

The mean mass of the male rattlesnakes is 37.7 g greater than the mean mass of the female rattlesnakes.

So, another equation is: $m = 37.7 + f$

A linear system is:

$$\frac{45f + 100m}{145} = 194 \quad \text{①}$$

$$m = 37.7 + f \quad \text{②}$$

To solve the linear system:

From equation ②, substitute $m = 37.7 + f$ into equation ①.

$$\frac{45f + 100m}{145} = 194 \quad \text{①}$$

$$\frac{45f + 100(37.7 + f)}{145} = 194 \quad \text{Multiply each side by 145.}$$

$$45f + 3770 + 100f = 28\,130 \quad \text{Simplify, then solve for } f.$$

$$45f + 100f = 28\,130 - 3770$$

$$145f = 24\,360 \quad \text{Divide each side by 145.}$$

$$f = 168$$

Substitute $f = 168$ into equation ②.

$$m = 37.7 + f \quad \text{②}$$

$$m = 37.7 + 168 \quad \text{Solve for } m.$$

$$m = 205.7$$

The mean mass of the male rattlesnakes is 205.7 g and the mean mass of the female rattlesnakes is 168 g.

Check the answers.

The mean mass, in grams, of all the rattlesnakes is:

$$\frac{45(168) + 100(205.7)}{145} = \frac{28\,130}{145}, \text{ or } 194; \text{ this agrees with the given information.}$$

The difference between the mean mass of the male rattlesnakes and the mean mass of female rattlesnakes is: $205.7 \text{ g} - 168 \text{ g} = 37.7 \text{ g}$; this agrees with the given information.

So, the answers are correct.

25. Write a linear system to represent the situation.

Let the rate of climb be represented by c metres per minute.

Let the rate of descent be represented by d metres per minute.

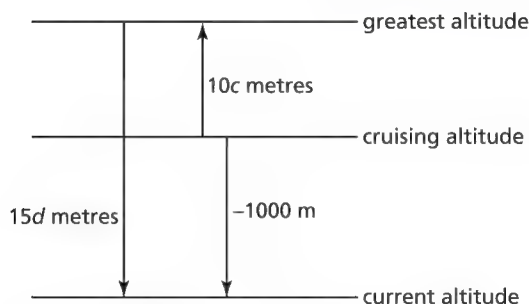
The difference between the rate of climb and rate of descent is 400 m/min.

So, one equation is: $c - d = 400$

The distance travelled in 10 min at a rate of c metres per minute is: $10c$ metres

The distance travelled in 15 min at a rate of d metres per minute is: $15d$ metres

Sketch a diagram:



Another equation is: $10c + 15d = -1000$

A linear system is:

$$c - d = 400 \quad \text{①}$$

$$10c + 15d = -1000 \quad \text{②}$$

To solve the system:

Solve equation ① for c .

$$c - d = 400 \quad \text{①}$$

$$c = d + 400$$

Substitute $c = d + 400$ into equation ②.

$$10c + 15d = -1000 \quad \text{②}$$

$$10(d + 400) + 15d = -1000 \quad \text{Simplify, then solve for } d.$$

$$10d + 4000 + 15d = -1000$$

$$25d = -5000$$

$$d = -200$$

Substitute $d = -200$ into equation ①.

$$c - d = 400 \quad \text{①}$$

$$c - (-200) = 400$$

$$c + 200 = 400$$

$$c = 200$$

The rate of climb is 200 m/min and the rate of descent is -200 m/min.

Check these answers.

The airplane climbed for 10 min at a rate of 200 m/min, so it travelled: $10 \times 200 \text{ m} = 2000 \text{ m}$

The airplane descended for 15 min at a rate of -200 m/min, so it travelled:

$$15 \times (-200 \text{ m}) = -3000 \text{ m}$$

The sum of these two distances is: $2000 \text{ m} + (-3000 \text{ m}) = -1000 \text{ m}$; this agrees with the given information.

The difference between the rate of climb and rate of descent was:

$$200 \text{ m/min} - (-200 \text{ m/min}) = 400 \text{ m/min}; \text{ this agrees with the given information.}$$

The solution is correct.

$$26. Ax + By = C \quad \text{①}$$

$$Bx + Ay = C \quad \text{②}$$

To solve the linear system:

Solve equation ① for x .

$$Ax + By = C$$

$$Ax = -By + C$$

$$x = \frac{-B}{A}y + \frac{C}{A}$$

Subtract By from each side.

Divide each side by A .

Substitute $x = \frac{-B}{A}y + \frac{C}{A}$ into equation ②.

$$Bx + Ay = C \quad \textcircled{2}$$

$$B\left(-\frac{B}{A}y + \frac{C}{A}\right) + Ay = C$$

Simplify.

$$-\frac{B^2}{A}y + \frac{BC}{A} + Ay = C$$

Multiply each side by A .

$$-B^2 + BC + A^2y = CA$$

Solve for y .

$$y(A^2 - B^2) = CA - BC$$

Divide each side by $(A^2 - B^2)$.

$$y = \frac{CA - BC}{A^2 - B^2}$$

Factor the numerator and denominator.

$$y = \frac{C(A - B)}{(A - B)(A + B)}$$

Since $A \neq B$, divide the numerator and

denominator by $(A - B)$.

$$y = \frac{C}{A + B}$$

Substitute for y in equation $\textcircled{1}$.

$$Ax + By = C \quad \textcircled{1}$$

$$Ax + B\left(\frac{C}{A + B}\right) = C$$

$$Ax + \frac{BC}{A + B} = C$$

Multiply each side by $(A + B)$.

$$Ax(A + B) + \frac{BC}{A + B}(A + B) = C(A + B)$$

$$Ax(A + B) + BC = CA + CB$$

$$Ax(A + B) = CA$$

Solve for x . Divide each side by $A(A + B)$.

Assume $A \neq -B$.

$$x = \frac{CA}{A(A + B)}$$

$$x = \frac{C}{A + B}$$

So, the values of x and y are equal for all given values of A , B , and C .

Check the solution.

In each equation, substitute: $x = \frac{C}{A + B}$ and $y = \frac{C}{A + B}$

$$Ax + By = C \quad \textcircled{1}$$

$$\text{L.S.} = Ax + By$$

$$= A\left(\frac{C}{A + B}\right) + B\left(\frac{C}{A + B}\right)$$

$$= \frac{AC + BC}{A + B}$$

$$= \frac{C(A + B)}{A + B}$$

$$= C$$

$$Bx + Ay = C \quad \textcircled{2}$$

$$\text{L.S.} = Bx + Ay$$

$$= B\left(\frac{C}{A + B}\right) + A\left(\frac{C}{A + B}\right)$$

$$= \frac{BC + AC}{A + B}$$

$$= \frac{C(B + A)}{A + B}$$

$$= C$$

= R.S.

= R.S.

For each equation, the left side is equal to the right side, so the solution is correct.

27. $Ax + By = -17$ ①

$Bx + Ay = 18$ ②

Substitute $x = -2$ and $y = 3$ into each equation.

In equation ①:

$$Ax + By = -17$$

$$A(-2) + B(3) = -17$$

$$-2A + 3B = -17 \quad \text{③}$$

In equation ②:

$$Bx + Ay = 18$$

$$B(-2) + A(3) = 18$$

$$-2B + 3A = 18 \quad \text{④}$$

Equations ③ and ④ form a linear system.

$$-2A + 3B = -17 \quad \text{③}$$

$$-2B + 3A = 18 \quad \text{④}$$

To solve the system:

Solve equation ③ for A .

$$-2A + 3B = -17 \quad \text{③}$$

$$-2A + 3B = -17 \quad \text{Subtract } 3B \text{ from each side.}$$

$$-2A = -3B - 17 \quad \text{Divide each side by } -2.$$

$$A = \frac{3}{2}B + \frac{17}{2}$$

Substitute $A = \frac{3}{2}B + \frac{17}{2}$ in equation ④.

$$-2B + 3A = 18 \quad \text{④}$$

$$-2B + 3\left(\frac{3}{2}B + \frac{17}{2}\right) = 18 \quad \text{Simplify.}$$

$$-2B + \frac{9}{2}B + \frac{51}{2} = 18$$

$$-\frac{4}{2}B + \frac{9}{2}B = -\frac{51}{2} + \frac{36}{2}$$

$$\frac{5}{2}B = -\frac{15}{2}$$

$$\frac{5}{2}B = -\frac{15}{2}$$

Multiply each side by $\frac{2}{5}$.

$$B = -3$$

Substitute $B = -3$ in equation ④.

$$-2B + 3A = 18 \quad \text{④}$$

$$-2(-3) + 3A = 18$$

$$6 + 3A = 18$$

$$3A = 12$$

$$A = 4$$

Check the solution.

In each equation, substitute: $A = 4$ and $B = -3$

$$-2A + 3B = -17 \quad \textcircled{3}$$

$$\text{L.S.} = -2A + 3B$$

$$= -2(4) + 3(-3)$$

$$= -8 - 9$$

$$= -17$$

$$= \text{R.S.}$$

$$-2B + 3A = 18 \quad \textcircled{4}$$

$$\text{L.S.} = -2B + 3A$$

$$= -2(-3) + 3(4)$$

$$= 6 + 12$$

$$= 18$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is correct.

Lesson 7.5 **Using an Elimination Strategy to Solve a System of Linear Equations**

Exercises (pages 437-439)

A

3. a) $x - 4y = 1$ ①
 $x - 2y = -1$ ②

Since the x -terms are equal, subtract the equations.

$$\begin{array}{r} x - 4y = 1 \quad \text{①} \\ -(x - 2y = -1) \quad \text{②} \\ \hline -4y + 2y = 1 + 1 \\ -2y = 2 \\ y = -1 \end{array}$$

Substitute $y = -1$ into equation ①.

$$\begin{array}{r} x - 4y = 1 \quad \text{①} \\ x - 4(-1) = 1 \\ x + 4 = 1 \\ x = -3 \end{array}$$

Verify the solution.

In each equation, substitute: $x = -3$ and $y = -1$

$x - 4y = 1$ ①	$x - 2y = -1$ ②
L.S. = $x - 4y$	L.S. = $x - 2y$
$= -3 - 4(-1)$	$= -3 - 2(-1)$
$= -3 + 4$	$= -3 + 2$
$= 1$	$= -1$
$= \text{R.S.}$	$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$x = -3$ and $y = -1$

b) $3a + b = 5$ ①
 $9a - b = 15$ ②

Since the b -terms are opposites, add the equations.

$$\begin{array}{r} 3a + b = 5 \quad \text{①} \\ +(9a - b = 15) \quad \text{②} \\ \hline 12a = 20 \\ a = \frac{20}{12}, \text{ or } \frac{5}{3} \end{array}$$

Substitute $a = \frac{5}{3}$ into equation ①.

$$\begin{array}{r} 3a + b = 5 \quad \text{①} \\ 3\left(\frac{5}{3}\right) + b = 5 \\ 5 + b = 5 \\ b = 0 \end{array}$$

Verify the solution.

In each equation, substitute: $a = \frac{5}{3}$ and $b = 0$

$$3a + b = 5 \quad \textcircled{1}$$

$$\text{L.S.} = 3a + b$$

$$= 3\left(\frac{5}{3}\right) + 0$$

$$= 5$$

$$= \text{R.S.}$$

$$9a - b = 15 \quad \textcircled{2}$$

$$\text{L.S.} = 9a - b$$

$$= 9\left(\frac{5}{3}\right) - 0$$

$$= 15$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$a = \frac{5}{3} \text{ and } b = 0$$

c) $3x - 4y = 1 \quad \textcircled{1}$

$$3x - 2y = -1 \quad \textcircled{2}$$

Since the x -terms are equal, subtract the equations.

$$3x - 4y = 1 \quad \textcircled{1}$$

$$-(3x - 2y = -1) \quad \textcircled{2}$$

$$-4y + 2y = 1 + 1$$

$$-2y = 2$$

$$y = -1$$

Substitute $y = -1$ into equation $\textcircled{1}$.

$$3x - 4y = 1 \quad \textcircled{1}$$

$$3x - 4(-1) = 1$$

$$3x + 4 = 1$$

$$3x = -3$$

$$x = -1$$

Verify the solution.

In each equation, substitute: $x = -1$ and $y = -1$

$$3x - 4y = 1 \quad \textcircled{1}$$

$$\text{L.S.} = 3x - 4y$$

$$= 3(-1) - 4(-1)$$

$$= -3 + 4$$

$$= 1$$

$$= \text{R.S.}$$

$$3x - 2y = -1 \quad \textcircled{2}$$

$$\text{L.S.} = 3x - 2y$$

$$= 3(-1) - 2(-1)$$

$$= -3 + 2$$

$$= -1$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = -1$$

d) $3x - 4y = 0 \quad \textcircled{1}$

$$5x - 4y = 8 \quad \textcircled{2}$$

Since the y -terms are equal, subtract the equations.

$$3x - 4y = 0 \quad \textcircled{1}$$

$$-(5x - 4y = 8) \quad \textcircled{2}$$

$$3x - 5x = 0 - 8$$

$$-2x = -8$$

$$x = 4$$

Substitute $x = 4$ into equation $\textcircled{1}$.

$$3x - 4y = 0 \quad \textcircled{1}$$

$$3(4) - 4y = 0$$

$$12 - 4y = 0$$

$$-4y = -12$$

$$y = 3$$

Verify the solution.

In each equation, substitute: $x = 4$ and $y = 3$

$$3x - 4y = 0 \quad \textcircled{1}$$

$$\begin{aligned} \text{L.S.} &= 3x - 4y \\ &= 3(4) - 4(3) \\ &= 12 - 12 \\ &= 0 \\ &= \text{R.S.} \end{aligned}$$

$$5x - 4y = 8 \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= 5x - 4y \\ &= 5(4) - 4(3) \\ &= 20 - 12 \\ &= 8 \\ &= \text{R.S.} \end{aligned}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = 4 \text{ and } y = 3$$

4. a) $x - 2y = -6 \quad \textcircled{1}$
 $3x - y = 2 \quad \textcircled{2}$

i) The coefficients of x are 1 and 3.

Multiply equation $\textcircled{1}$ by 3.

$$\begin{aligned} 3(x - 2y &= -6) \\ 3x - 6y &= -18 \end{aligned} \quad \textcircled{3}$$

An equivalent system is:

$$3x - y = 2 \quad \textcircled{2}$$

$$3x - 6y = -18 \quad \textcircled{3}$$

ii) The coefficients of y are -2 and -1 .

Multiply equation $\textcircled{2}$ by 2.

$$\begin{aligned} 2(3x - y &= 2) \\ 6x - 2y &= 4 \end{aligned} \quad \textcircled{4}$$

An equivalent system is:

$$x - 2y = -6 \quad \textcircled{1}$$

$$6x - 2y = 4 \quad \textcircled{4}$$

b) $15x - 2y = 9 \quad \textcircled{1}$
 $5x + 4y = 17 \quad \textcircled{2}$

i) The coefficients of x are 15 and 5.

Multiply equation $\textcircled{2}$ by 3.

$$\begin{aligned} 3(5x + 4y &= 17) \\ 15x + 12y &= 51 \end{aligned} \quad \begin{array}{l} \textcircled{2} \\ \textcircled{3} \end{array}$$

An equivalent system is:

$$15x - 2y = 9 \quad \textcircled{1}$$

$$15x + 12y = 51 \quad \textcircled{3}$$

ii) The coefficients of y are -2 and 4 .

Multiply equation $\textcircled{1}$ by -2 .

$$\begin{aligned} -2(15x - 2y &= 9) \\ -30x + 4y &= -18 \end{aligned} \quad \textcircled{4}$$

An equivalent system is:

$$5x + 4y = 17 \quad \textcircled{2}$$

$$-30x + 4y = -18 \quad \textcircled{4}$$

c) $7x + 3y = 9$ ①
 $5x + 2y = 7$ ②

- i) The coefficients of x are 7 and 5.

Multiply equation ① by 5.

$$5(7x + 3y = 9)$$

$$35x + 15y = 45 \quad \text{③}$$

Multiply equation ② by 7.

$$7(5x + 2y = 7)$$

$$35x + 14y = 49 \quad \text{④}$$

An equivalent system is:

$$35x + 15y = 45 \quad \text{③}$$

$$35x + 14y = 49 \quad \text{④}$$

- ii) The coefficients of y are 3 and 2.

Multiply equation ① by 2.

$$2(7x + 3y = 9)$$

$$14x + 6y = 18 \quad \text{⑤}$$

Multiply equation ② by 3.

$$3(5x + 2y = 7)$$

$$15x + 6y = 21 \quad \text{⑥}$$

An equivalent system is:

$$14x + 6y = 18 \quad \text{⑤}$$

$$15x + 6y = 21 \quad \text{⑥}$$

d) $14x + 15y = 16$ ①
 $21x + 10y = -1$ ②

- i) The coefficients of x are 14 and 21; their least common multiple is 42.

Multiply equation ① by 3.

$$3(14x + 15y = 16)$$

$$42x + 45y = 48 \quad \text{③}$$

Multiply equation ② by 2.

$$2(21x + 10y = -1)$$

$$42x + 20y = -2 \quad \text{④}$$

An equivalent system is:

$$42x + 45y = 48 \quad \text{③}$$

$$42x + 20y = -2 \quad \text{④}$$

- ii) The coefficients of y are 15 and 10; their least common multiple is 30.

Multiply equation ① by 2.

$$2(14x + 15y = 16)$$

$$28x + 30y = 32 \quad \text{⑤}$$

Multiply equation ② by 3.

$$3(21x + 10y = -1)$$

$$63x + 30y = -3 \quad \text{⑥}$$

An equivalent system is:

$$28x + 30y = 32 \quad \text{⑤}$$

$$63x + 30y = -3 \quad \text{⑥}$$

5. a) $x - 2y = -6$ ①

$3x - y = 2$ ②

Use the first equivalent system to eliminate x .

$3x - y = 2$ ②

$3x - 6y = -18$ ③

Subtract equation ③ from equation ②.

$3x - y = 2$ ②

$-(3x - 6y = -18)$ ③

$-y + 6y = 2 + 18$

$5y = 20$

$y = 4$

Use the second equivalent system to eliminate y .

$x - 2y = -6$ ①

$6x - 2y = 4$ ④

Subtract equation ④ from equation ①.

$x - 2y = -6$ ①

$-(6x - 2y = 4)$ ④

$x - 6x = -6 - 4$

$-5x = -10$

$x = 2$

Verify the solution.

In each original equation, substitute: $x = 2$ and $y = 4$

$x - 2y = -6$ ①

L.S. $= x - 2y$

$= 2 - 2(4)$

$= 2 - 8$

$= -6$

$= \text{R.S.}$

$3x - y = 2$ ②

L.S. $= 3x - y$

$= 3(2) - 4$

$= 6 - 4$

$= 2$

$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$x = 2$ and $y = 4$

b) $15x - 2y = 9$ ①

$5x + 4y = 17$ ②

Use the first equivalent system to eliminate x .

$15x - 2y = 9$ ①

$15x + 12y = 51$ ③

Subtract equation ③ from equation ①.

$15x - 2y = 9$ ①

$-(15x + 12y = 51)$ ③

$-2y - 12y = 9 - 51$

$-14y = -42$

$y = 3$

Use the second equivalent system to eliminate y .

$5x + 4y = 17$ ②

$-30x + 4y = -18$ ④

Subtract equation ④ from equation ②.

$5x + 4y = 17$ ②

$-(-30x + 4y = -18)$ ④

$5x + 30x = 17 + 18$

$35x = 35$

$$x = 1$$

Verify the solution.

In each original equation, substitute: $x = 1$ and $y = 3$

$$15x - 2y = 9 \quad \textcircled{1}$$

$$\text{L.S.} = 15x - 2y$$

$$= 15(1) - 2(3)$$

$$= 15 - 6$$

$$= 9$$

$$= \text{R.S.}$$

$$5x + 4y = 17 \quad \textcircled{2}$$

$$\text{L.S.} = 5x + 4y$$

$$= 5(1) + 4(3)$$

$$= 5 + 12$$

$$= 17$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = 1 \text{ and } y = 3$$

c) $7x + 3y = 9 \quad \textcircled{1}$

$$5x + 2y = 7 \quad \textcircled{2}$$

Use the first equivalent system to eliminate x .

$$35x + 15y = 45 \quad \textcircled{3}$$

$$35x + 14y = 49 \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{3}$.

$$35x + 15y = 45 \quad \textcircled{3}$$

$$-(35x + 14y = 49) \quad \textcircled{4}$$

$$15y - 14y = 45 - 49$$

$$y = -4$$

Use the second equivalent system to eliminate y .

$$14x + 6y = 18 \quad \textcircled{5}$$

$$15x + 6y = 21 \quad \textcircled{6}$$

Subtract equation $\textcircled{6}$ from equation $\textcircled{5}$.

$$14x + 6y = 18 \quad \textcircled{5}$$

$$-(15x + 6y = 21) \quad \textcircled{6}$$

$$14x - 15x = 18 - 21$$

$$-x = -3$$

$$x = 3$$

Verify the solution.

In each original equation, substitute: $x = 3$ and $y = -4$

$$7x + 3y = 9 \quad \textcircled{1}$$

$$\text{L.S.} = 7x + 3y$$

$$= 7(3) + 3(-4)$$

$$= 21 - 12$$

$$= 9$$

$$= \text{R.S.}$$

$$5x + 2y = 7 \quad \textcircled{2}$$

$$\text{L.S.} = 5x + 2y$$

$$= 5(3) + 2(-4)$$

$$= 15 - 8$$

$$= 7$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = 3 \text{ and } y = -4$$

d) $14x + 15y = 16 \quad \textcircled{1}$

$$21x + 10y = -1 \quad \textcircled{2}$$

Use the first equivalent system to eliminate x .

$$42x + 45y = 48 \quad \textcircled{3}$$

$$42x + 20y = -2 \quad \textcircled{4}$$

Subtract equation ④ from equation ③.

$$\begin{array}{rcl} 42x + 45y & = & 48 \quad \text{③} \\ -(42x + 20y = -2) & & \text{④} \\ \hline 45y - 20y & = & 48 + 2 \\ 25y & = & 50 \\ y & = & 2 \end{array}$$

Use the second equivalent system to eliminate y .

$$28x + 30y = 32 \quad \text{⑤}$$

$$63x + 30y = -3 \quad \text{⑥}$$

Subtract equation ⑥ from equation ⑤.

$$\begin{array}{rcl} 28x + 30y & = & 32 \quad \text{⑤} \\ -(63x + 30y = -3) & & \text{⑥} \\ \hline 28x - 63x & = & 32 + 3 \\ -35x & = & 35 \\ x & = & -1 \end{array}$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$$14x + 15y = 16 \quad \text{①}$$

$$\text{L.S.} = 14x + 15y$$

$$= 14(-1) + 15(2)$$

$$= -14 + 30$$

$$= 16$$

$$= \text{R.S.}$$

$$21x + 10y = -1 \quad \text{②}$$

$$\text{L.S.} = 21x + 10y$$

$$= 21(-1) + 10(2)$$

$$= -21 + 20$$

$$= -1$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = 2$$

B

6. a) $2x + y = -5 \quad \text{①}$

$$3x + 5y = 3 \quad \text{②}$$

Multiply equation ① by 5 so the y -terms are equal.

$$5(2x + y = -5)$$

$$10x + 5y = -25 \quad \text{③}$$

Subtract equation ② from equation ③ to eliminate y .

$$\begin{array}{rcl} 10x + 5y & = & -25 \quad \text{③} \\ -(3x + 5y = 3) & & \text{②} \\ \hline 10x - 3x & = & -25 - 3 \\ 7x & = & -28 \\ x & = & -4 \end{array}$$

Substitute $x = -4$ in equation ①.

$$2x + y = -5 \quad \text{①}$$

$$2(-4) + y = -5$$

$$-8 + y = -5$$

$$y = 3$$

Verify the solution.

In each equation, substitute: $x = -4$ and $y = 3$

$$2x + y = -5 \quad \textcircled{1}$$

$$\text{L.S.} = 2x + y$$

$$= 2(-4) + 3$$

$$= -8 + 3$$

$$= -5$$

$$= \text{R.S.}$$

$$3x + 5y = 3 \quad \textcircled{2}$$

$$\text{L.S.} = 3x + 5y$$

$$= 3(-4) + 5(3)$$

$$= -12 + 15$$

$$= 3$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -4 \text{ and } y = 3$$

b) $3m - 6n = 0 \quad \textcircled{1}$

$$9m + 3n = -7 \quad \textcircled{2}$$

Multiply equation $\textcircled{2}$ by 2 so the n -terms are opposites.

$$2(9m + 3n = -7)$$

$$18m + 6n = -14 \quad \textcircled{3}$$

Add equations $\textcircled{1}$ and $\textcircled{3}$ to eliminate n .

$$3m - 6n = 0 \quad \textcircled{1}$$

$$+(18m + 6n = -14) \quad \textcircled{3}$$

$$3m + 18m = -14$$

$$21m = -14$$

Divide each side by 21.

$$m = \frac{-14}{21}, \text{ or } -\frac{2}{3}$$

Multiply equation $\textcircled{1}$ by 3 so the m -terms are equal.

$$3(3m - 6n = 0)$$

$$9m - 18n = 0 \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{2}$ to eliminate m .

$$9m + 3n = -7 \quad \textcircled{2}$$

$$-(9m - 18n = 0) \quad \textcircled{4}$$

$$3n + 18n = -7$$

$$21n = -7$$

Divide each side by 21.

$$n = -\frac{1}{3}$$

Verify the solution.

In each equation, substitute: $m = -\frac{2}{3}$ and $n = -\frac{1}{3}$

$$3m - 6n = 0 \quad \textcircled{1}$$

$$\text{L.S.} = 3m - 6n$$

$$= 3\left(-\frac{2}{3}\right) - 6\left(-\frac{1}{3}\right)$$

$$= -2 + 2$$

$$= 0$$

$$= \text{R.S.}$$

$$9m + 3n = -7 \quad \textcircled{2}$$

$$\text{L.S.} = 9m + 3n$$

$$= 9\left(-\frac{2}{3}\right) + 3\left(-\frac{1}{3}\right)$$

$$= -6 - 1$$

$$= -7$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$m = -\frac{2}{3} \text{ and } n = -\frac{1}{3}$$

c) $2s + 3t = 6$ ①

$5s + 10t = 20$ ②

Multiply equation ① by 5 and equation ② by 2 so the s -terms are equal.

$5(2s + 3t = 6)$

$2(5s + 10t = 20)$

$10s + 15t = 30$ ③

$10s + 20t = 40$ ④

Subtract equation ③ from equation ④ to eliminate s .

$10s + 20t = 40$ ④

$-(10s + 15t = 30)$ ③

$20t - 15t = 40 - 30$

$5t = 10$

$t = 2$

Substitute $t = 2$ into equation ①.

$2s + 3t = 6$ ①

$2s + 3(2) = 6$

$2s + 6 = 6$

$2s = 0$

$s = 0$

Verify the solution.

In each equation, substitute: $s = 0$ and $t = 2$

$2s + 3t = 6$ ①

$5s + 10t = 20$ ②

L.S. = $2s + 3t$

L.S. = $5s + 10t$

$= 2(0) + 3(2)$

$= 5(0) + 10(2)$

$= 6$

$= 20$

$= \text{R.S.}$

$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$s = 0$ and $t = 2$

d) $3a + 2b = 5$ ①

$2a + 3b = 0$ ②

Multiply equation ① by 2 and equation ② by 3 so the a -terms are equal.

$2(3a + 2b = 5)$

$3(2a + 3b = 0)$

$6a + 4b = 10$ ③

$6a + 9b = 0$ ④

Subtract equation ④ from equation ③ to eliminate s .

$6a + 4b = 10$ ③

$-(6a + 9b = 0)$ ④

$4b - 9b = 10 - 0$

$-5b = 10$

$b = -2$

Substitute $b = -2$ into equation ②.

$2a + 3b = 0$ ②

$2a + 3(-2) = 0$

$2a = 6$

$a = 3$

Verify the solution.

In each equation, substitute: $a = 3$ and $b = -2$

$$3a + 2b = 5 \quad \textcircled{1}$$

$$2a + 3b = 0 \quad \textcircled{2}$$

$$\text{L.S.} = 3a + 2b$$

$$\text{L.S.} = 2a + 3b$$

$$= 3(3) + 2(-2)$$

$$= 2(3) + 3(-2)$$

$$= 9 - 4$$

$$= 6 - 6$$

$$= 5$$

$$= 0$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$a = 3 \text{ and } b = -2$$

7. a) $8x - 3y = 38 \quad \textcircled{1}$

$$3x - 2y = -1 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 2 and equation $\textcircled{2}$ by 3 so the y -terms are equal.

$$2(8x - 3y = 38)$$

$$3(3x - 2y = -1)$$

$$16x - 6y = 76 \quad \textcircled{3}$$

$$9x - 6y = -3 \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{3}$ to eliminate y .

$$16x - 6y = 76 \quad \textcircled{3}$$

$$-(9x - 6y = -3) \quad \textcircled{4}$$

$$16x - 9x = 76 - (-3)$$

$$7x = 79$$

$$x = \frac{79}{7}$$

Multiply equation $\textcircled{1}$ by 3 and equation $\textcircled{2}$ by 8 so the x -terms are equal.

$$3(8x - 3y = 38)$$

$$8(3x - 2y = -1)$$

$$24x - 9y = 114 \quad \textcircled{5}$$

$$24x - 16y = -8 \quad \textcircled{6}$$

Subtract equation $\textcircled{6}$ from equation $\textcircled{5}$ to eliminate x .

$$24x - 9y = 114 \quad \textcircled{5}$$

$$-(24x - 16y = -8) \quad \textcircled{6}$$

$$-9y - (-16y) = 114 - (-8)$$

$$-9y + 16y = 122$$

$$7y = 122$$

$$y = \frac{122}{7}$$

Verify the solution.

In each original equation, substitute: $x = \frac{79}{7}$ and $y = \frac{122}{7}$

$$8x - 3y = 38 \quad \textcircled{1}$$

$$3x - 2y = -1 \quad \textcircled{2}$$

$$\text{L.S.} = 8x - 3y$$

$$\text{L.S.} = 3x - 2y$$

$$= 8\left(\frac{79}{7}\right) - 3\left(\frac{122}{7}\right)$$

$$= 3\left(\frac{79}{7}\right) - 2\left(\frac{122}{7}\right)$$

$$= \frac{632}{7} - \frac{366}{7}$$

$$= \frac{237}{7} - \frac{244}{7}$$

$$= \frac{266}{7}$$

$$= -\frac{7}{7}$$

$$= 38$$

$$= -1$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = \frac{79}{7} \text{ and } y = \frac{122}{7}$$

b) $2a - 5b = 29$ ①

$7a - 3b = 0$ ②

Multiply equation ① by 7 and equation ② by 2 so the a -terms are equal.

$7(2a - 5b = 29) \qquad 2(7a - 3b = 0)$

$14a - 35b = 203$ ③ $14a - 6b = 0$ ④

Subtract equation ④ from equation ③ to eliminate a .

$14a - 35b = 203$ ③

$-(14a - 6b = 0)$ ④

$-35b - (-6b) = 203 - 0$

$-35b + 6b = 203$

$-29b = 203$

$b = -7$

Substitute $b = -7$ into equation ②.

$7a - 3b = 0$ ②

$7a - 3(-7) = 0$

$7a + 21 = 0$

$7a = -21$

$a = -3$

Verify the solution.

In each equation, substitute: $a = -3$ and $b = -7$

$2a - 5b = 29$ ①

$7a - 3b = 0$ ②

$\text{L.S.} = 2a - 5b$

$\text{L.S.} = 7a - 3b$

$= 2(-3) - 5(-7)$

$= 7(-3) - 3(-7)$

$= -6 + 35$

$= -21 + 21$

$= 29$

$= 0$

$= \text{R.S.}$

$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$a = -3 \text{ and } b = -7$

c) $18a - 15b = 4$ ①

$10a + 3b = 6$ ②

Multiply equation ② by 5 so the b -terms are opposites.

$5(10a + 3b = 6)$

$50a + 15b = 30$ ③

Add equations ① and ③ to eliminate b .

$18a - 15b = 4$ ①

$+(50a + 15b = 30)$ ③

$18a + 50a = 4 + 30$

$68a = 34$

$a = \frac{34}{68}, \text{ or } \frac{1}{2}$

Substitute $a = \frac{1}{2}$ into equation ②.

$$\begin{aligned}
 10a + 3b &= 6 & \textcircled{2} \\
 10\left(\frac{1}{2}\right) + 3b &= 6 \\
 5 + 3b &= 6 \\
 3b &= 1 \\
 b &= \frac{1}{3}
 \end{aligned}$$

Verify the solution.

In each equation, substitute: $a = \frac{1}{2}$ and $b = \frac{1}{3}$

$18a - 15b = 4$ $\textcircled{1}$	$10a + 3b = 6$ $\textcircled{2}$
L.S. = $18a - 15b$	L.S. = $10a + 3b$
$= 18\left(\frac{1}{2}\right) - 15\left(\frac{1}{3}\right)$	$= 10\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right)$
$= 9 - 5$	$= 5 + 1$
$= 4$	$= 6$
$= \text{R.S.}$	$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$$a = \frac{1}{2} \text{ and } b = \frac{1}{3}$$

d) $6x - 2y = 21$ $\textcircled{1}$
 $4x + 3y = 1$ $\textcircled{2}$

Multiply equation $\textcircled{1}$ by 3 and equation $\textcircled{2}$ by 2 so the y -terms are opposites.

$3(6x - 2y = 21)$	$2(4x + 3y = 1)$
$18x - 6y = 63$ $\textcircled{3}$	$8x + 6y = 2$ $\textcircled{4}$

Add equations $\textcircled{3}$ and $\textcircled{4}$ to eliminate y .

$$\begin{aligned}
 18x - 6y &= 63 & \textcircled{3} \\
 + (8x + 6y) &= 2 & \textcircled{4} \\
 \hline
 18x + 8x &= 63 + 2 \\
 26x &= 65 \\
 x &= \frac{65}{26}, \text{ or } \frac{5}{2}
 \end{aligned}$$

Substitute $x = \frac{5}{2}$ in equation $\textcircled{2}$.

$$\begin{aligned}
 4x + 3y &= 1 & \textcircled{2} \\
 4\left(\frac{5}{2}\right) + 3y &= 1 \\
 3y &= -9 \\
 y &= -3
 \end{aligned}$$

Verify the solution.

In each equation, substitute: $x = \frac{5}{2}$ and $y = -3$

$$6x - 2y = 21 \quad \textcircled{1}$$

$$\text{L.S.} = 6x - 2y$$

$$= 6\left(\frac{5}{2}\right) - 2(-3)$$

$$= 15 + 6$$

$$= 21$$

$$= \text{R.S.}$$

$$4x + 3y = 1 \quad \textcircled{2}$$

$$\text{L.S.} = 4x + 3y$$

$$= 4\left(\frac{5}{2}\right) + 3(-3)$$

$$= 10 - 9$$

$$= 1$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = \frac{5}{2} \text{ and } y = -3$$

8. Let x represent the attendance in 2006.

Let y represent the attendance in 2008.

The mean attendance was 45 265.

So, one equation is:

$$\frac{x + y}{2} = 45\,265$$

Multiply each side by 2.

$$x + y = 90\,530$$

The attendance in 2008 was 120 more than the attendance in 2006.

So, another equation is: $y - x = 120$

A linear system is:

$$x + y = 90\,530 \quad \textcircled{1}$$

$$y - x = 120 \quad \textcircled{2}$$

Add equations $\textcircled{1}$ and $\textcircled{2}$ to eliminate the x -terms.

$$x + y = 90\,530 \quad \textcircled{1}$$

$$+ (y - x = 120) \quad \textcircled{2}$$

$$y + y = 90\,530 + 120$$

$$2y = 90\,650$$

$$y = 45\,325$$

Substitute $y = 45\,325$ in equation $\textcircled{1}$.

$$x + y = 90\,530 \quad \textcircled{1}$$

$$x + 45\,325 = 90\,530$$

$$x = 45\,205$$

The attendance in 2006 was 45 205. The attendance in 2008 was 45 325.

Verify the solution.

The mean attendance is: $\frac{45\,205 + 45\,325}{2} = \frac{90\,530}{2}$, or 45 265; this agrees with the given

information.

The difference between the attendance in 2008 and 2006 was: $45\,325 - 45\,205 = 120$; this agrees with the given information.

So, the solution is correct.

9. Let t represent the number of cones on Talise's dress.

Let s represent the number of cones on her sister's dress.

The total number of cones is 545.

So, one equation is: $t + s = 545$

Talise's dress had 185 more cones than her sister's dress.

So, another equation is: $t - s = 185$

A linear system is:

$$t + s = 545 \quad \textcircled{1}$$

$$t - s = 185 \quad \textcircled{2}$$

Add equations $\textcircled{1}$ and $\textcircled{2}$ to eliminate s .

$$t + s = 545 \quad \textcircled{1}$$

$$+(t - s) = 185 \quad \textcircled{2}$$

$$t + t = 545 + 185$$

$$2t = 730$$

$$t = 365$$

Substitute $t = 365$ in equation $\textcircled{1}$.

$$t + s = 545 \quad \textcircled{1}$$

$$365 + s = 545$$

$$s = 545 - 365$$

$$s = 180$$

There were 365 cones on Talise's dress and 180 cones on her sister's dress.

Verify the solution.

The total number of cones is: $365 + 180 = 545$; this agrees with the given information.

The difference in the number of cones is: $365 - 180 = 185$; this agrees with the given information.

So, the solution is correct.

10. Let k represent the cost of a knife in beaver pelts.

Let b represent the cost of a blanket in beaver pelts.

10 knives + 20 blankets = 200 beaver pelts

So, one equation is: $10k + 20b = 200$

Divide both sides by 10 to simplify:

$$10k + 20b = 200$$

$$k + 2b = 20 \quad \textcircled{1}$$

15 knives + 25 blankets = 270 beaver pelts

So, another equation is: $15k + 25b = 270$

Divide both sides by 5 to simplify:

$$15k + 25b = 270$$

$$3k + 5b = 54 \quad \textcircled{2}$$

So, a linear system is:

$$k + 2b = 20 \quad \textcircled{1}$$

$$3k + 5b = 54 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 3.

$$3(k + 2b = 20)$$

$$3k + 6b = 60 \quad \textcircled{3}$$

Subtract equation $\textcircled{2}$ from equation $\textcircled{3}$ to eliminate k .

$$3k + 6b = 60 \quad \textcircled{3}$$

$$-(3k + 5b = 54) \quad \textcircled{2}$$

$$6b - 5b = 60 - 54$$

$$b = 6$$

Substitute $b = 6$ in equation ①.

$$k + 2b = 20 \quad \text{①}$$

$$k + 2(6) = 20$$

$$k + 12 = 20$$

$$k = 8$$

One knife cost 8 beaver pelts. One blanket cost 6 beaver pelts.

Verify the solution.

In beaver pelts, 10 knives + 20 blankets cost: $10(8) + 20(6) = 80 + 120$, or 200; this agrees with the given information.

In beaver pelts, 15 knives + 25 blankets cost: $15(8) + 25(6) = 120 + 150$, or 270; this agrees with the given information.

So, the solution is correct.

11. Let m beats per minute represent the moderate tempo.

Let f beats per minute represent the fast tempo.

A moderate tempo for 4.5 min and a fast tempo for 30 s, or 0.5 min produced 620 beats.

So, one equation is: $4.5m + 0.5f = 620$

The moderate tempo was 40 beats/min less than the fast tempo.

So, another equation is: $f - m = 40$

A linear system is:

$$4.5m + 0.5f = 620 \quad \text{①}$$

$$f - m = 40 \quad \text{②}$$

Multiply equation ① by 2.

$$2(4.5m + 0.5f = 620)$$

$$9m + f = 1240 \quad \text{③}$$

Subtract equation ② from equation ③.

$$9m + f = 1240 \quad \text{③}$$

$$-(f - m = 40) \quad \text{②}$$

$$9m - (-m) = 1240 - 40$$

$$10m = 1200$$

$$m = 120$$

Substitute $m = 120$ in equation ②.

$$f - m = 40 \quad \text{②}$$

$$f - 120 = 40$$

$$f = 160$$

The moderate tempo is 120 beats/min and the fast tempo is 160 beats/min.

Verify the solution.

The number of beats in a moderate tempo for 4.5 min and a fast tempo for 0.5 min is:

$4.5(120) + 0.5(160) = 540 + 80$, or 620; this agrees with the given information.

The difference in rates is: $160 \text{ beats/min} - 120 \text{ beats/min} = 40 \text{ beats/min}$; this agrees with the given information.

So, the solution is correct.

12. a) $\frac{a}{2} + \frac{b}{3} = 1$ ①

$$\frac{a}{4} - \frac{2b}{3} = -1 \quad ②$$

The lowest common denominator of the fractions in equation ① is 6, so multiply equation ① by 6.

$$6\left(\frac{a}{2} + \frac{b}{3} = 1\right)$$

$$3a + 2b = 6 \quad ③$$

The lowest common denominator of the fractions in equation ② is 12, so multiply equation ② by 12.

$$12\left(\frac{a}{4} - \frac{2b}{3} = -1\right)$$

$$3a - 8b = -12 \quad ④$$

An equivalent linear system is:

$$3a + 2b = 6 \quad ③$$

$$3a - 8b = -12 \quad ④$$

Subtract equation ④ from equation ③ to eliminate a .

$$3a + 2b = 6 \quad ③$$

$$-(3a - 8b = -12) \quad ④$$

$$2b + 8b = 6 + 12$$

$$10b = 18$$

$$b = \frac{18}{10}, \text{ or } \frac{9}{5}$$

Substitute $b = \frac{9}{5}$ in equation ③.

$$3a + 2b = 6 \quad ③$$

$$3a + 2\left(\frac{9}{5}\right) = 6$$

$$3a + \left(\frac{18}{5}\right) = 6$$

$$3a = 6 - \frac{18}{5}$$

$$3a = \frac{30}{5} - \frac{18}{5}$$

$$3a = \frac{12}{5}$$

$$a = \frac{4}{5}$$

Verify the solution.

In each original equation, substitute: $a = \frac{4}{5}$ and $b = \frac{9}{5}$

$$\frac{a}{2} + \frac{b}{3} = 1 \quad \textcircled{1}$$

$$\frac{a}{4} - \frac{2b}{3} = -1 \quad \textcircled{2}$$

$$\text{L.S.} = \frac{1}{2}a + \frac{1}{3}b$$

$$\text{L.S.} = \frac{1}{4}a - \frac{2}{3}b$$

$$= \frac{1}{2}\left(\frac{4}{5}\right) + \frac{1}{3}\left(\frac{9}{5}\right)$$

$$= \frac{1}{4}\left(\frac{4}{5}\right) - \frac{2}{3}\left(\frac{9}{5}\right)$$

$$= \frac{2}{5} + \frac{3}{5}$$

$$= \frac{1}{5} - \frac{6}{5}$$

$$= \frac{5}{5}$$

$$= -\frac{5}{5}$$

$$= 1$$

$$= -1$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$a = \frac{4}{5} \text{ and } b = \frac{9}{5}$$

b) $\frac{x}{2} + \frac{y}{2} = 7 \quad \textcircled{1}$

$$3x + 2y = 48 \quad \textcircled{2}$$

The lowest common denominator of the fractions in equation ① is 2, so multiply equation ① by 2.

$$2\left(\frac{x}{2} + \frac{y}{2} = 7\right)$$

$$x + y = 14 \quad \textcircled{3}$$

Multiply equation ③ by 3.

$$3(x + y = 14)$$

$$3x + 3y = 42 \quad \textcircled{4}$$

Subtract equation ② from equation ④ to eliminate x .

$$3x + 3y = 42 \quad \textcircled{4}$$

$$-(3x + 2y = 48) \quad \textcircled{2}$$

$$3y - 2y = 42 - 48$$

$$y = -6$$

Substitute $y = -6$ in equation ③.

$$x + y = 14 \quad \textcircled{3}$$

$$x + (-6) = 14$$

$$x = 14 + 6$$

$$x = 20$$

Verify the solution.

In each original equation, substitute: $x = 20$ and $y = -6$

$$\frac{x}{2} + \frac{y}{2} = 7 \quad \textcircled{1}$$

$$3x + 2y = 48 \quad \textcircled{2}$$

$$\text{L.S.} = \frac{1}{2}x + \frac{1}{2}y$$

$$\text{L.S.} = 3x + 2y$$

$$= \frac{1}{2}(20) + \frac{1}{2}(-6)$$

$$= 3(20) + 2(-6)$$

$$= 10 - 3$$

$$= 60 - 12$$

$$= 7$$

$$= 48$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:
 $x = 20$ and $y = -6$

c) $0.03x + 0.15y = 0.027 \quad \textcircled{1}$

$$-0.5x - 0.5y = 0.05 \quad \textcircled{2}$$

Multiply equation $\textcircled{2}$ by 0.3.

$$0.3(-0.5x - 0.5y = 0.05)$$

$$-0.15x - 0.15y = 0.015 \quad \textcircled{3}$$

Add equations $\textcircled{1}$ and $\textcircled{3}$ to eliminate y .

$$0.03x + 0.15y = 0.027 \quad \textcircled{1}$$

$$+(-0.15x - 0.15y = 0.015) \quad \textcircled{3}$$

$$0.03x - 0.15x = 0.027 + 0.015$$

$$-0.12x = 0.042$$

$$x = \frac{0.042}{-0.12}$$

$$x = -0.35$$

Substitute $x = -0.35$ in equation $\textcircled{2}$.

$$-0.5x - 0.5y = 0.05 \quad \textcircled{2}$$

$$-0.5(-0.35) - 0.5y = 0.05$$

$$0.175 - 0.5y = 0.05$$

$$-0.5y = 0.05 - 0.175$$

$$-0.5y = -0.125$$

$$y = \frac{-0.125}{-0.5}$$

$$y = 0.25$$

Verify the solution.

In each original equation, substitute: $x = -0.35$ and $y = 0.25$

$$0.03x + 0.15y = 0.027 \quad \textcircled{1}$$

$$-0.5x - 0.5y = 0.05 \quad \textcircled{2}$$

$$\text{L.S.} = 0.03x + 0.15y$$

$$\text{L.S.} = -0.5x - 0.5y$$

$$= 0.03(-0.35) + 0.15(0.25)$$

$$= -0.5(-0.35) - 0.5(0.25)$$

$$= -0.0105 + 0.0375$$

$$= 0.175 - 0.125$$

$$= 0.027$$

$$= 0.05$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:
 $x = -0.35$ and $y = 0.25$

d) $-1.5x + 2.5y = 0.5$ ①

$2x + y = 1.5$ ②

Multiply equation ② by 2.5.

$2.5(2x + y = 1.5)$

$5x + 2.5y = 3.75$ ③

Subtract equation ① from equation ③ to eliminate y .

$5x + 2.5y = 3.75$ ③

$\underline{-(-1.5x + 2.5y = 0.5)} \quad \text{①}$

$5x + 1.5x = 3.75 - 0.5$

$6.5x = 3.25$

$x = \frac{3.25}{6.5}$

$x = 0.5$

Substitute $x = 0.5$ in equation ②.

$2x + y = 1.5$ ②

$2(0.5) + y = 1.5$

$1 + y = 1.5$

$y = 0.5$

Verify the solution.

In each original equation, substitute: $x = 0.5$ and $y = 0.5$

$-1.5x + 2.5y = 0.5$ ①

$2x + y = 1.5$ ②

L.S. $= -1.5x + 2.5y$

L.S. $= 2(0.5) + (0.5)$

$= -1.5(0.5) + 2.5(0.5)$

$= 1 + 0.5$

$= -0.75 + 1.25$

$= 1.5$

$= 0.5$

$= \text{R.S.}$

$= \text{R.S.}$

Since the left side is equal to the right side for each equation, the solution is correct:

$x = 0.5$ and $y = 0.5$

13. Write a linear system to represent the situation.

Let c represent the number of Canadian players.

Let f represent the number of foreign players.

There are 25 players.

So, one equation is: $c + f = 25$

Seven-ninths of the Canadian players and three-sevenths of the foreign players are over 6 ft. tall. There are 17 players who are over 6 ft. tall.

So, another equation is: $\frac{7}{9}c + \frac{3}{7}f = 17$

A linear system is:

$c + f = 25$ ①

$\frac{7}{9}c + \frac{3}{7}f = 17$ ②

Solve the linear system.

Multiply equation ② by the least common denominator of the fractions, which is 63.

$63\left(\frac{7}{9}c + \frac{3}{7}f = 17\right)$

$49c + 27f = 1071$ ③

Multiply equation ① by 27.

$$27(c + f = 25)$$

$$27c + 27f = 675 \quad \text{④}$$

Subtract equation ④ from equation ③.

$$49c + 27f = 1071 \quad \text{③}$$

$$-(27c + 27f = 675) \quad \text{④}$$

$$49c - 27c = 1071 - 675$$

$$22c = 396$$

$$c = 18$$

Substitute $c = 18$ in equation ①.

$$c + f = 25 \quad \text{①}$$

$$18 + f = 25$$

$$f = 7$$

There are 18 Canadian players and 7 foreign players.

Verify the solution.

The total number of players is: $18 + 7 = 25$; this agrees with the given information.

Seven-ninths of the Canadian players and three-sevenths of the foreign players is:

$$\frac{7}{9}(18) + \frac{3}{7}(7) = 14 + 3, \text{ or } 17; \text{ this agrees with the given information.}$$

So, the solution is correct.

14. Write a linear system to represent the situation.

Let g represent the number of girls surveyed.

Let b represent the number of boys surveyed.

76 students were surveyed.

So, one equation is: $g + b = 76$

One-quarter of the girls and three-quarters of the boys played online games; this is 39 students.

So, another equation is: $\frac{1}{4}g + \frac{3}{4}b = 39$

A linear system is:

$$g + b = 76 \quad \text{①}$$

$$\frac{1}{4}g + \frac{3}{4}b = 39 \quad \text{②}$$

Solve the linear system.

Multiply equation ② by the least common denominator of the fractions, which is 4.

$$4\left(\frac{1}{4}g + \frac{3}{4}b = 39\right)$$

$$g + 3b = 156 \quad \text{③}$$

Subtract equation ① from equation ③.

$$g + 3b = 156 \quad \text{③}$$

$$-(g + b = 76) \quad \text{①}$$

$$3b - b = 156 - 76$$

$$2b = 80$$

$$b = 40$$

Substitute $b = 40$ in equation ①.

$$g + b = 76 \quad \textcircled{1}$$

$$g + 40 = 76$$

$$g = 36$$

36 girls and 40 boys were surveyed.

Verify the solution.

The total number of students is: $36 + 40 = 76$; this agrees with the given information.

One-quarter of the girls and three-quarters of the boys is: $\frac{1}{4}(36) + \frac{3}{4}(40) = 9 + 30$, or 39; this

agrees with the given information.

So, the solution is correct.

15. a) An equation that Balance scales 1 models is:

$$x + x + x + y = 10 + 5 + 1 + 1 \quad \text{Simplify.}$$

$$3x + y = 17$$

An equation that Balance scales 2 models is:

$$x + y = 5 + 1 + 1 \quad \text{Simplify.}$$

$$x + y = 7$$

A linear system is:

$$3x + y = 17 \quad \textcircled{1}$$

$$x + y = 7 \quad \textcircled{2}$$

- b) From Balance scales 2, the sum of mass x and mass y is 7 kg.

So, for Balance scales 1, if masses x and y are removed from the left side and 7 kg is removed from the right side, then the same mass has been removed from both sides, and the scales remain balanced.

- c) When masses x and y are removed from the left side of Balance scales 1, a mass of $2x$ remains.

When a mass of 7 kg is removed, a mass of 10 kg remains.

So, $2x = 20$ kg, and $x = 5$ kg

Since masses x and y equal 7 kg, and mass x is 5 kg, then mass y is 2 kg.

- d) Removing masses x , y , and 7 kg from Balance scales 1 is the same as subtracting equation $\textcircled{2}$ from equation $\textcircled{1}$, to eliminate y .

16. Let a dollars represent the cost of an adult's ticket.

Let c dollars represent the cost of a child's ticket.

Use the given information to write a linear system:

$$a + 3c = 27.75 \quad \textcircled{1}$$

$$2a + 2c = 27.50 \quad \textcircled{2}$$

Solve the linear system to determine the cost of each ticket.

Multiply equation $\textcircled{1}$ by 2.

$$2(a + 3c = 27.75)$$

$$2a + 6c = 55.50 \quad \textcircled{3}$$

Subtract equation $\textcircled{2}$ from equation $\textcircled{3}$ to eliminate a .

$$2a + 6c = 55.50 \quad \textcircled{3}$$

$$-(2a + 2c = 27.50) \quad \textcircled{2}$$

$$6c - 2c = 55.50 - 27.50$$

$$4c = 28$$

$$c = 7$$

Substitute $c = 7$ in equation ①.

$$a + 3c = 27.75 \quad \text{①}$$

$$a + 3(7) = 27.75$$

$$a + 21 = 27.75$$

$$a = 6.75$$

A child pays \$7.00 and an adult pays \$6.75, so a child's ticket is more expensive.

Verify the solution.

One adult and 3 children would pay: $\$6.75 + 3(\$7.00) = \$6.75 + \$21.00$, or \$27.75; this agrees with the given information.

Two adults and 2 children would pay: $2(\$6.75) + 2(\$7.00) = \$13.50 + \$14.00$, or \$27.50; this agrees with the given information.

So, the solution is correct.

17. Write a linear system to model the situation.

Let p kilograms represent the mass of peas.

Let l kilograms represent the mass of lentils.

The total mass of peas and lentils is 25 kg.

So, one equation is: $p + l = 25$

A mass of p kilograms of peas costs $5p$ dollars.

A mass of l kilograms of lentils costs $6.50l$ dollars.

The total cost is \$140.00.

So, another equation is: $5p + 6.50l = 140$

A linear system is:

$$p + l = 25 \quad \text{①}$$

$$5p + 6.50l = 140 \quad \text{②}$$

Multiply equation ① by 5.

$$5(p + l = 25)$$

$$5p + 5l = 125 \quad \text{③}$$

Subtract equation ③ from equation ② to eliminate p .

$$5p + 6.50l = 140 \quad \text{②}$$

$$-(5p + 5l = 125) \quad \text{③}$$

$$6.50l - 5l = 140 - 125 \quad \text{③}$$

$$1.50l = 15$$

$$l = 10$$

Substitute $l = 10$ in equation ①.

$$p + l = 25 \quad \text{①}$$

$$p + 10 = 25$$

$$p = 15$$

The mass of peas is 15 kg and the mass of lentils is 10 kg.

Verify the solution.

The total mass of peas and lentils is: $15 \text{ kg} + 10 \text{ kg} = 25 \text{ kg}$; this agrees with the given information.

The total cost of peas and lentils is: $15(\$5) + 10(\$6.50) = \$75 + \$65$, or \$140; this agrees with the given information.

So, the solution is correct.

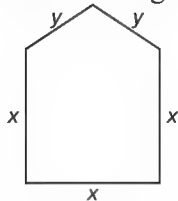
18. $3x + 2y = 21$ ①
 $x - y = 2$ ②

A pentagon has 5 sides.

From equation ①, the coefficients of x and y have a sum of 5. This suggests that x and y could be the lengths of the sides of the pentagon and the equation models the perimeter of the pentagon.

From equation ②, a side with length x units is 2 units longer than a side with length y units.

Sketch a diagram.



A problem could be:

A pentagon has 3 sides with one length and 2 sides with another length.

The perimeter of the pentagon is 21 cm.

A longer side is 2 cm longer than a shorter side.

What are the lengths of the sides of the pentagon?

To solve the problem, solve the linear system.

Multiply equation ② by 3.

$$3(x - y = 2)$$

$$3x - 3y = 6 \quad \text{③}$$

Subtract equation ③ from equation ①.

$$\begin{array}{r} 3x + 2y = 21 \quad \text{①} \\ -(3x - 3y = 6) \quad \text{③} \\ \hline 2y + 3y = 21 - 6 \\ 5y = 15 \\ y = 3 \end{array}$$

Substitute $y = 3$ in equation ②.

$$\begin{array}{r} x - y = 2 \quad \text{②} \\ x - 3 = 2 \\ x = 5 \end{array}$$

The polygon has 3 sides with length 5 cm and 2 sides with length 3 cm.

Verify the solution.

The perimeter of the polygon is: $5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} + 3 \text{ cm} = 21 \text{ cm}$; this agrees with the given information.

The difference between a longer side and a shorter side is: $5 \text{ cm} - 3 \text{ cm} = 2 \text{ cm}$; this agrees with the given information.

So, the solution is correct.

19. a) $3x + y = 17$ ①
 $x + y = 7$ ②

Equation ① has coefficients 3 and 1, which could be the numbers of items bought.

Equation ② has coefficients 1, so x and y could be the costs, in dollars, of the items bought.

A possible problem is:

Three binders and 1 pen cost \$17.00.

One binder and 1 pen cost \$7.00.

What is the cost of 1 binder and 1 pen?

- b) x dollars represents the cost of 1 binder.
 y dollars represents the cost of 1 pen.

To solve the linear system:

Subtract equation ② from equation ① to eliminate y .

$$3x + y = 17 \quad \text{①}$$

$$-(x + y = 7) \quad \text{②}$$

$$3x - x = 17 - 7$$

$$2x = 10$$

$$x = 5$$

Substitute $x = 5$ in equation ②.

$$x + y = 7 \quad \text{②}$$

$$5 + y = 7$$

$$y = 2$$

One binder costs \$5.00 and 1 pen costs \$2.00.

Verify the solution.

Three binders and one pen cost: $3(\$5.00) + \$2.00 = \$17.00$; this agrees with the given information.

One binder and one pen cost: $\$5.00 + \$2.00 = \$7.00$; this agrees with the given information.

So, the solution is correct.

20. a) $3x + 4y = 29 \quad \text{①}$

$$2x - 5y = -19 \quad \text{②}$$

One way to solve the system is to multiply equation ① by 2 and multiply equation ② by 3, then subtract the equations to eliminate x .

Another way to solve the system is to multiply equation ① by 5 and multiply equation ② by 4, then add the equations to eliminate y .

b) Solution 1:

Multiply equation ① by 2 and multiply equation ② by 3.

$$2(3x + 4y = 29)$$

$$3(2x - 5y = -19)$$

$$6x + 8y = 58 \quad \text{③}$$

$$6x - 15y = -57 \quad \text{④}$$

Subtract equation ④ from equation ③.

$$6x + 8y = 58 \quad \text{③}$$

$$-(6x - 15y = -57) \quad \text{④}$$

$$8y + 15y = 58 + 57$$

$$23y = 115$$

$$y = 5$$

Substitute $y = 5$ in equation ①.

$$3x + 4y = 29 \quad \text{①}$$

$$3x + 4(5) = 29$$

$$3x + 20 = 29$$

$$3x = 9$$

$$x = 3$$

Solution 2:

Multiply equation ① by 5 and multiply equation ② by 4.

$$5(3x + 4y = 29)$$

$$4(2x - 5y = -19)$$

$$15x + 20y = 145 \quad \text{③}$$

$$8x - 20y = -76 \quad \text{④}$$

Add equations ③ and ④.

$$15x + 20y = 145 \quad \textcircled{3}$$

$$+(8x - 20y = -76) \quad \textcircled{4}$$

$$15x + 8x = 145 - 76$$

$$23x = 69$$

$$x = 3$$

Substitute $x = 3$ in equation $\textcircled{1}$.

$$3x + 4y = 29 \quad \textcircled{1}$$

$$3(3) + 4y = 29$$

$$9 + 4y = 29$$

$$4y = 20$$

$$y = 5$$

Verify the solution.

In each equation, substitute: $x = 3$ and $y = 5$

$$3x + 4y = 29 \quad \textcircled{1}$$

$$2x - 5y = -19 \quad \textcircled{2}$$

$$\text{L.S.} = 3x + 4y$$

$$\text{L.S.} = 2x - 5y$$

$$= 3(3) + 4(5)$$

$$= 2(3) - 5(5)$$

$$= 9 + 20$$

$$= 6 - 25$$

$$= 29$$

$$= -19$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = 3 \text{ and } y = 5$$

21. a) A possible situation is:

Two hundred students were checked for colour blindness and 14 students were found to be colour blind. Two percent of the females and 12% of the males were colour blind.

Let f represent the number of females in the study.

Let m represent the number of males in the study.

The total number of students in the study is 200.

So, one equation is: $f + m = 200$

2% of females is: $0.02f$

12% of males is: $0.12m$

14 students were colour blind.

So, another equation is: $0.02f + 0.12m = 14$

A linear system is:

$$f + m = 200 \quad \textcircled{1}$$

$$0.02f + 0.12m = 14 \quad \textcircled{2}$$

- b) A related problem is: How many females and how many males were in the study?

To solve the linear system:

Multiply equation $\textcircled{1}$ by 0.02.

$$0.02(f + m = 200)$$

$$0.02f + 0.02m = 4 \quad \textcircled{3}$$

Subtract equation $\textcircled{3}$ from equation $\textcircled{2}$ to eliminate f .

$$0.02f + 0.12m = 14 \quad \textcircled{2}$$

$$-(0.02f + 0.02m = 4) \quad \textcircled{3}$$

$$0.12m - 0.02m = 14 - 4$$

$$0.10m = 10$$

$$m = 100$$

Substitute $m = 100$ in equation ①.

$$f + m = 200 \quad \text{①}$$

$$f + 100 = 200$$

$$f = 100$$

There were 100 females and 100 males in the study.

Verify the solution.

The total number of people in the study is: $100 + 100 = 200$; this agrees with the given information.

2% of 100 + 12% of 100 = 2 + 12, or 14; this agrees with the given information.

So, the solution is correct.

C

22. Write then solve a linear system.

Let s dollars represent the money invested in the stock.

Let b dollars represent the money invested in the bond.

The stock lost 10.5%, so the interest in dollars at the end of the year was: $-0.105s$

The bond gained 3.5%, so the interest in dollars at the end of the year was: $0.035b$

The total loss was \$84.00.

So, one equation is: $-0.105s + 0.035b = -84$

If s dollars had been invested in the bond, then the interest in dollars would have been: $0.035s$

If b dollars had been invested in the stock, then the interest in dollars would have been: $-0.105b$

The total loss would have been \$14.00.

So, another equation is:

$$0.035s - 0.105b = -14$$

A linear system is:

$$-0.105s + 0.035b = -84 \quad \text{①}$$

$$0.035s - 0.105b = -14 \quad \text{②}$$

Multiply equation ① by 3.

$$3(-0.105s + 0.035b = -84) \quad \text{①}$$

$$-0.315s + 0.105b = -252 \quad \text{③}$$

Add equations ② and ③ to eliminate b .

$$0.035s - 0.105b = -14 \quad \text{②}$$

$$+(-0.315s + 0.105b = -252) \quad \text{③}$$

$$0.035s - 0.315s = -14 - 252$$

$$-0.28s = -266$$

$$s = 950$$

Substitute $s = 950$ in equation ①.

$$-0.105s + 0.035b = -84 \quad \text{①}$$

$$-0.105(950) + 0.035b = -84$$

$$-99.75 + 0.035b = -84$$

$$0.035b = -84 + 99.75$$

$$0.035b = 15.75$$

$$b = 450$$

Cam invested \$950 in the stock and \$450 in the bond.

Verify the solution.

The stock lost 10.5% and the bond gained 3.5%, so the interest was:

$$-(0.105 \times \$950) + (0.035 \times \$450) = -\$99.75 + \$15.75$$

$$= -\$84.00; \text{ this agrees with the given information}$$

If the amounts had been interchanged, the interest would have been:

$$\begin{aligned} -(0.105 \times \$450) + (0.035 \times \$950) &= -\$47.25 + \$33.25 \\ &= -\$14; \text{ this agrees with the given information} \end{aligned}$$

So, the solution is correct.

23. a) $2x + 5y = 8$

Another equation whose coefficients and constant term differ by 3 is:

$$4x + 7y = 10$$

Solve this linear system:

$$2x + 5y = 8 \quad \textcircled{1}$$

$$4x + 7y = 10 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 2.

$$2(2x + 5y = 8)$$

$$4x + 10y = 16 \quad \textcircled{3}$$

Subtract equation $\textcircled{2}$ from equation $\textcircled{3}$ to eliminate x .

$$4x + 10y = 16 \quad \textcircled{3}$$

$$-(4x + 7y = 10) \quad \textcircled{2}$$

$$10y - 7y = 16 - 10$$

$$3y = 6$$

$$y = 2$$

Substitute $y = 2$ in equation $\textcircled{1}$.

$$2x + 5y = 8 \quad \textcircled{1}$$

$$2x + 5(2) = 8$$

$$2x + 10 = 8$$

$$2x = -2$$

$$x = -1$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$$2x + 5y = 8 \quad \textcircled{1}$$

$$4x + 7y = 10 \quad \textcircled{2}$$

$$\text{L.S.} = 2x + 5y$$

$$\text{L.S.} = 4x + 7y$$

$$= 2(-1) + 5(2)$$

$$= 4(-1) + 7(2)$$

$$= -2 + 10$$

$$= -4 + 14$$

$$= 8$$

$$= 10$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = 2$$

b) Another linear system is:

$$5x + 8y = 11 \quad \textcircled{1}$$

$$14x + 17y = 20 \quad \textcircled{2}$$

To solve this linear system:

Multiply equation $\textcircled{1}$ by 14 and equation $\textcircled{2}$ by 5.

$$14(5x + 8y = 11)$$

$$5(14x + 17y = 20)$$

$$70x + 112y = 154 \quad \textcircled{3}$$

$$70x + 85y = 100 \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{3}$ to eliminate x .

$$70x + 112y = 154 \quad \textcircled{3}$$

$$-(70x + 85y = 100) \quad \textcircled{4}$$

$$112y - 85y = 154 - 100$$

$$27y = 54$$

$$y = 2$$

Substitute $y = 2$ in equation ①.

$$5x + 8y = 11 \quad \text{①}$$

$$5x + 8(2) = 11$$

$$5x + 16 = 11$$

$$5x = -5$$

$$x = -1$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$$5x + 8y = 11 \quad \text{①}$$

$$14x + 17y = 20 \quad \text{②}$$

$$\text{L.S.} = 5x + 8y$$

$$\text{L.S.} = 14x + 17y$$

$$= 5(-1) + 8(2)$$

$$= 14(-1) + 17(2)$$

$$= -5 + 16$$

$$= -14 + 34$$

$$= 11$$

$$= 20$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = 2$$

Another linear system is:

$$-2x + y = 4 \quad \text{①}$$

$$-7x - 4y = -1 \quad \text{②}$$

To solve this linear system:

Multiply equation ① by 4.

$$4(-2x + y = 4)$$

$$-8x + 4y = 16 \quad \text{③}$$

Add equations ② and ③ to eliminate y .

$$-7x - 4y = -1 \quad \text{②}$$

$$+(-8x + 4y = 16) \quad \text{③}$$

$$-15x = 15$$

$$-15x = 15$$

$$x = -1$$

Substitute $x = -1$ in equation ①.

$$-2x + y = 4 \quad \text{①}$$

$$-2(-1) + y = 4$$

$$2 + y = 4$$

$$y = 2$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$$-2x + y = 4 \quad \text{①}$$

$$-7x - 4y = -1 \quad \text{②}$$

$$\text{L.S.} = -2x + y$$

$$\text{L.S.} = -7x - 4y$$

$$= -2(-1) + 2$$

$$= -7(-1) - 4(2)$$

$$= 2 + 2$$

$$= 7 - 8$$

$$= 4$$

$$= -1$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = 2$$

- c) For each linear system, the solution is: $x = -1$ and $y = 2$
- d) Suppose the coefficients differ by a constant d , no coefficient equals 0, and $a \neq b$. Then the linear system can be expressed as:

$$ax + (a + d)y = a + 2d \quad \textcircled{1}$$

$$bx + (b + d)y = b + 2d \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by b and equation $\textcircled{2}$ by a .

$$b(ax + (a + d)y = a + 2d)$$

$$a(bx + (b + d)y = b + 2d)$$

$$bax + b(a + d)y = ba + 2bd \quad \textcircled{3}$$

$$abx + a(b + d)y = ab + 2ad \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{3}$ to eliminate x .

$$bax + b(a + d)y = ba + 2bd \quad \textcircled{3}$$

$$- [abx + a(b + d)y = ab + 2ad] \quad \textcircled{4}$$

$$b(a + d)y - a(b + d)y = ba + 2bd - (ab + 2ad) \quad \text{Expand and simplify.}$$

$$aby + bdy - aby - ady = ba + 2bd - ab - 2ad$$

$$bdy - ady = 2bd - 2ad$$

Factor each side.

$$dy(b - a) = 2d(b - a)$$

Divide each side by $d(b - a)$.

$$y = 2$$

Substitute $y = 2$ in equation $\textcircled{1}$.

$$ax + (a + d)y = a + 2d \quad \textcircled{1}$$

$$ax + (a + d)2 = a + 2d \quad \text{Simplify.}$$

$$ax + 2a + 2d = a + 2d$$

$$ax = -a$$

$$x = -1$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 2$

$$ax + (a + d)y = a + 2d \quad \textcircled{1}$$

$$bx + (b + d)y = b + 2d \quad \textcircled{2}$$

$$\text{L.S.} = ax + (a + d)y$$

$$\text{L.S.} = bx + (b + d)y$$

$$= a(-1) + (a + d)2$$

$$= b(-1) + (b + d)2$$

$$= -a + 2a + 2d$$

$$= -b + 2b + 2d$$

$$= a + 2d$$

$$= b + 2d$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is correct:

$$x = -1 \text{ and } y = 2$$

24. a) Write a linear system to model the situation.

Let the yield of wheat be represented by w bushels.

Let the yield of barley be represented by b bushels.

The total yield was 99 840 bushels.

So, one equation is: $w + b = 99\,840$

The sale of w bushels of wheat at \$6.35/bushel earns: $6.35w$ dollars

The sale of b bushels of barley at \$2.70/bushel earns: $2.70b$ dollars

The total money earned was \$363 008.

So, another equation is: $6.35w + 2.70b = 363\,008$

A linear system is:

$$w + b = 99\,840 \quad \textcircled{1}$$

$$6.35w + 2.70b = 363\,008 \quad \textcircled{2}$$

To solve this linear system:

Multiply equation $\textcircled{1}$ by 6.35.

$$6.35(w + b = 99\,840)$$

$$6.35w + 6.35b = 633\,984 \quad \textcircled{3}$$

Subtract equation ② from equation ③ to eliminate w .

$$6.35w + 6.35b = 633\,984 \quad \text{③}$$

$$-(6.35w + 2.70b = 363\,008) \quad \text{②}$$

$$6.35b - 2.70b = 633\,984 - 363\,008$$

$$3.65b = 270\,976$$

$$b = 74\,240$$

Substitute $b = 74\,240$ in equation ①.

$$w + b = 99\,840 \quad \text{①}$$

$$w + 74\,240 = 99\,840$$

$$w = 25\,600$$

Verify the solution.

The total yield of grain, in bushels, is: $74\,240 + 25\,600 = 99\,840$; this agrees with the given information.

The total income from the sale of grain is: $25\,600 \times \$6.35 + 74\,240 \times \$2.70 = \$363\,008$; this agrees with the given information.

So, the solution is correct.

640 acres of wheat were harvested.

So, the yield in bushels per acre is: $\frac{25\,600}{640} = 40$

640 $\times$ 2, or 1280 acres of barley were harvested.

So, the yield in bushels per acre is: $\frac{74\,240}{1280} = 58$

There were 40 bushels/acre for wheat and 58 bushels/acre for barley.

- b)** No, I would not have to write then solve a different linear system.

1 acre is 0.4047 ha.

40 bushels per acre is the same as 40 bushels per 0.4047 ha.

In 1 ha, the yield is: $\frac{40}{0.4047} \doteq 98.8$

40 bushels/acre is approximately 98.8 bushels/ha.

Similarly, 58 bushels/acre is the same as 58 bushels per 0.4047 ha.

In 1 ha, the yield is: $\frac{58}{0.4047} \doteq 143.3$

58 bushels/acre is approximately 143.3 bushels/ha.

Checkpoint 2

Assess Your Understanding (page 441)

7.4

1. a) $5x + y = 4$ ①

$x + y = 2$ ②

Solve equation ② for y .

$x + y = 2$ ②

$y = 2 - x$

Substitute $y = 2 - x$ in equation ①.

$5x + y = 4$ ①

$5x + (2 - x) = 4$

$5x + 2 - x = 4$

$4x = 2$

$x = \frac{2}{4}, \text{ or } \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation ②.

$x + y = 2$ ②

$\frac{1}{2} + y = 2$

$y = 2 - \frac{1}{2}$

$y = \frac{3}{2}$

Verify the solution.

In each equation, substitute: $x = \frac{1}{2}$ and $y = \frac{3}{2}$

$5x + y = 4$ ①

L.S. = $5x + y$

$= 5\left(\frac{1}{2}\right) + \frac{3}{2}$

$= \frac{5}{2} + \frac{3}{2}$

$= \frac{8}{2}$

$= 4$

$= \text{R.S.}$

$x + y = 2$ ②

L.S. = $x + y$

$= \frac{1}{2} + \frac{3}{2}$

$= \frac{4}{2}$

$= 2$

$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:

$x = \frac{1}{2}$ and $y = \frac{3}{2}$

b) $3x - y = 1$ ①

$2x + y = -1$ ②

Solve equation ② for y .

$2x + y = -1$ ②

$y = -1 - 2x$

Substitute $y = -1 - 2x$ in equation ①.

$$\begin{aligned}
 3x - y &= 1 & \textcircled{1} \\
 3x - (-1 - 2x) &= 1 \\
 3x + 1 + 2x &= 1 \\
 5x &= 0 \\
 x &= 0 \\
 \text{Substitute } x = 0 &\text{ in equation } \textcircled{2}. \\
 2x + y &= -1 & \textcircled{2} \\
 2(0) + y &= -1 \\
 y &= -1
 \end{aligned}$$

Verify the solution.

In each equation, substitute: $x = 0$ and $y = -1$

$$\begin{array}{ll}
 3x - y = 1 & \textcircled{1} & 2x + y = -1 & \textcircled{2} \\
 \text{L.S.} = 3x - y & & \text{L.S.} = 2x + y & \\
 = 3(0) - (-1) & & = 2(0) + (-1) & \\
 = 1 & & = -1 & \\
 = \text{R.S.} & & = \text{R.S.} &
 \end{array}$$

For each equation, the left side is equal to the right side, so the solution is:
 $x = 0$ and $y = -1$

$$\begin{aligned}
 \text{c) } \frac{x}{3} + \frac{y}{4} &= -\frac{9}{4} & \textcircled{1} \\
 \frac{5x}{6} - \frac{3y}{4} &= -\frac{17}{4} & \textcircled{2}
 \end{aligned}$$

Write an equivalent system with integer coefficients.

For equation $\textcircled{1}$, the common denominator is the lowest common multiple of 3 and 4, which is 12:

$$\begin{aligned}
 \frac{x}{3} + \frac{y}{4} &= -\frac{9}{4} & \text{Multiply each term by 12.} \\
 12\left(\frac{1}{3}x\right) + 12\left(\frac{1}{4}y\right) &= 12\left(-\frac{9}{4}\right) & \text{Simplify.} \\
 4x + 3y &= -27 & \textcircled{3}
 \end{aligned}$$

For equation $\textcircled{2}$, the common denominator is the lowest common multiple of 6 and 4, which is 12:

$$\begin{aligned}
 \frac{5x}{6} - \frac{3y}{4} &= -\frac{17}{4} & \text{Multiply each term by 12.} \\
 12\left(\frac{5}{6}x\right) - 12\left(\frac{3}{4}y\right) &= 12\left(-\frac{17}{4}\right) & \text{Simplify.} \\
 10x - 9y &= -51 & \textcircled{4}
 \end{aligned}$$

Solve equation $\textcircled{3}$ for $3y$.

$$\begin{aligned}
 4x + 3y &= -27 & \textcircled{3} \\
 3y &= -27 - 4x
 \end{aligned}$$

Substitute for $3y$ in equation $\textcircled{4}$.

$$\begin{aligned}
 10x - 9y &= -51 & \textcircled{4} \\
 10x - 3(-27 - 4x) &= -51 & \text{Simplify, then solve for } x. \\
 10x + 81 + 12x &= -51 \\
 22x &= -51 - 81 \\
 22x &= -132 \\
 x &= -6
 \end{aligned}$$

Substitute $x = -6$ into equation ③.

$$\begin{aligned} 4x + 3y &= -27 & \text{③} \\ 4(-6) + 3y &= -27 & \text{Simplify, then solve for } y. \\ -24 + 3y &= -27 \\ 3y &= -27 + 24 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

Verify the solution.

In each original equation, substitute: $x = -6$ and $y = -1$

$$\begin{array}{ll} \frac{x}{3} + \frac{y}{4} = -\frac{9}{4} & \text{①} \\ \text{L.S.} = \frac{x}{3} + \frac{y}{4} & \\ = \frac{-6}{3} + \frac{-1}{4} & \\ = -2 - \frac{1}{4} & \\ = -\frac{9}{4} & \\ = \text{R.S.} & \end{array} \quad \begin{array}{ll} \frac{5x}{6} - \frac{3y}{4} = -\frac{17}{4} & \text{②} \\ \text{L.S.} = \frac{5x}{6} - \frac{3y}{4} & \\ = \frac{5(-6)}{6} - \frac{3(-1)}{4} & \\ = -5 + \frac{3}{4} & \\ = -\frac{20}{4} + \frac{3}{4} & \\ = -\frac{17}{4} & \\ = \text{R.S.} & \end{array}$$

For each equation, the left side is equal to the right side, so the solution is:
 $x = -6$ and $y = -1$

2. a) Let a represent the number of Inukshuit with 6 stones.
Let b represent the number of Inukshuit with 7 stones.
The total number of stones in all Inukshuit sold was 494.
So, one equation is: $6a + 7b = 494$
13 more Inukshuit with 6 stones were sold than with 7 stones.
So, another equation is: $a = b + 13$
A linear system is:
 $6a + 7b = 494$ ①
 $a = b + 13$ ②

- b) Solve this linear system:

$$\begin{aligned} 6a + 7b &= 494 & \text{①} \\ a &= b + 13 & \text{②} \end{aligned}$$

From equation ②, substitute $a = b + 13$ in equation ①.

$$\begin{aligned} 6a + 7b &= 494 & \text{①} \\ 6(b + 13) + 7b &= 494 \\ 6b + 78 + 7b &= 494 \\ 13b &= 494 - 78 \\ 13b &= 416 \\ b &= 32 \end{aligned}$$

Substitute $b = 32$ into equation ②.

$$a = b + 13 \quad \text{②}$$

$$a = 32 + 13$$

$$a = 45$$

45 Inukshuit with 6 stones were sold and 32 Inukshuit with 7 stones were sold.

Verify the solution.

The total number of stones is: $45(6) + 32(7) = 494$; this agrees with the given information.

The difference in number of types sold is: $45 - 32 = 13$; this agrees with the given information.

So, the solution is correct.

3. Write a linear system to model the situation.

Let a dollars represent the amount invested in a bond that earns 5.5% interest.

Let b dollars represent the amount invested in a bond that earns 4.5% interest.

The total amount invested was \$1000.

So, one equation is: $a + b = 1000$

The interest in dollars earned at 5.5% was: $0.055a$

The interest in dollars earned at 4.5% was: $0.045b$

The total interest earned was \$50.

So, another equation is: $0.055a + 0.045b = 50$

A linear system is:

$$a + b = 1000 \quad \text{①}$$

$$0.055a + 0.045b = 50 \quad \text{②}$$

Solve equation ① for a .

$$a + b = 1000 \quad \text{①}$$

$$a = 1000 - b$$

Substitute $a = 1000 - b$ in equation ②.

$$0.055a + 0.045b = 50 \quad \text{②}$$

$$0.055(1000 - b) + 0.045b = 50$$

$$55 - 0.055b + 0.045b = 50$$

$$-0.01b = -5$$

$$b = 500$$

Substitute $b = 500$ in equation ①.

$$a + b = 1000 \quad \text{①}$$

$$a + 500 = 1000$$

$$a = 500$$

\$500 was invested in each bond.

Verify the solution.

The total amount invested was: $\$500 + \$500 = \$1000$; this agrees with the given information.

The total interest was: $0.055(\$500) + 0.045(\$500) = \$27.50 + \$22.50$, or \$50; this agrees with the given information.

So, the solution is correct.

7.5

4. a) $3x - y = -11$ ①

$-x + y = -1$ ②

Since the y -terms are opposites, add the equations to eliminate y .

$3x - y = -11$ ①

$+(-x + y = -1)$ ②

$3x - x = -11 - 1$

$2x = -12$

$x = -6$

Substitute $x = -6$ into equation ②.

$-x + y = -1$ ②

$-(-6) + y = -1$

$6 + y = -1$

$y = -7$

Verify the solution.

In each equation, substitute: $x = -6$ and $y = -7$

$3x - y = -11$ ①

$-x + y = -1$ ②

L.S. = $3x - y$

L.S. = $-x + y$

$= 3(-6) - (-7)$

$= -(-6) + (-7)$

$= -18 + 7$

$= 6 - 7$

$= -11$

$= -1$

$= \text{R.S.}$

$= \text{R.S.}$

For each equation, the left side is equal to the right side; so the solution is:

$x = -6$ and $y = -7$

b) $\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3}$ ①

$\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8}$ ②

The lowest common denominator of the fractions in equation ① is 6, so multiply equation ① by 6.

$6\left(\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3}\right)$

$2x + 5y = 16$ ③

The lowest common denominator of the fractions in equation ② is 8, so multiply equation ② by 8.

$8\left(\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8}\right)$

$2x - 6y = -17$ ④

In equation ③ and ④, the coefficients of x are equal.

Subtract equation ④ from equation ③ to eliminate x .

$2x + 5y = 16$ ③

$-(2x - 6y = -17)$ ④

$5y + 6y = 16 + 17$

$11y = 33$

$y = 3$

Substitute $y = 3$ in equation ③.

$$2x + 5y = 16 \quad \text{③}$$

$$2x + 5(3) = 16$$

$$2x + 15 = 16$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Verify the solution.

In each original equation, substitute: $x = \frac{1}{2}$ and $y = 3$

$$\frac{1}{3}x + \frac{5}{6}y = \frac{8}{3} \quad \text{①}$$

$$\text{L.S.} = \frac{1}{3}x + \frac{5}{6}y$$

$$= \frac{1}{3}\left(\frac{1}{2}\right) + \frac{5}{6}(3)$$

$$= \frac{1}{6} + \frac{15}{6}$$

$$= \frac{16}{6}$$

$$= \frac{8}{3}$$

$$= \text{R.S.}$$

$$\frac{1}{4}x - \frac{3}{4}y = -\frac{17}{8} \quad \text{②}$$

$$\text{L.S.} = \frac{1}{4}x - \frac{3}{4}y$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) - \frac{3}{4}(3)$$

$$= \frac{1}{8} - \frac{9}{4}$$

$$= \frac{1}{8} - \frac{18}{8}$$

$$= -\frac{17}{8}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side; so the solution is:

$$x = \frac{1}{2} \text{ and } y = 3$$

c) $0.5x - 0.3y = 0.15 \quad \text{①}$

$$-0.3x + 0.5y = -0.65 \quad \text{②}$$

Multiply equation ① by 0.3 and equation ② by 0.5.

$$0.3(0.5x - 0.3y = 0.15)$$

$$0.5(-0.3x + 0.5y = -0.65)$$

$$0.15x - 0.09y = 0.045 \quad \text{③}$$

$$-0.15x + 0.25y = -0.325 \quad \text{④}$$

Add equations ③ and ④ to eliminate x .

$$0.15x - 0.09y = 0.045 \quad \text{③}$$

$$+(-0.15x + 0.25y = -0.325) \quad \text{④}$$

$$-0.09y + 0.25y = 0.045 - 0.325$$

$$0.16y = -0.28$$

$$y = \frac{-0.28}{0.16}$$

$$y = -1.75$$

Substitute $y = -1.75$ in equation ①.

$$0.5x - 0.3y = 0.15 \quad \text{①}$$

$$0.5x - 0.3(-1.75) = 0.15$$

$$0.5x + 0.525 = 0.15$$

$$0.5x = 0.15 - 0.525$$

$$0.5x = -0.375$$

$$x = \frac{-0.375}{0.5}$$

$$x = -0.75$$

Verify the solution.

In each original equation, substitute: $x = -0.75$ and $y = -1.75$

$$0.5x - 0.3y = 0.15 \quad \textcircled{1}$$

$$-0.3x + 0.5y = -0.65 \quad \textcircled{2}$$

$$\text{L.S.} = 0.5x - 0.3y$$

$$\text{L.S.} = -0.3x + 0.5y$$

$$= 0.5(-0.75) - 0.3(-1.75)$$

$$= -0.3(-0.75) + 0.5(-1.75)$$

$$= -0.375 + 0.525$$

$$= 0.225 - 0.875$$

$$= 0.15$$

$$= -0.65$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side; so the solution is:

$$x = -0.75 \text{ and } y = -1.75$$

d) $x + 2y = -2 \quad \textcircled{1}$

$$-2x + y = 6 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 2 so the x -terms are opposites.

$$2(x + 2y = -2)$$

$$2x + 4y = -4 \quad \textcircled{3}$$

Add equations $\textcircled{2}$ and $\textcircled{3}$ to eliminate x .

$$-2x + y = 6 \quad \textcircled{2}$$

$$+(2x + 4y = -4) \quad \textcircled{3}$$

$$y + 4y = 6 - 4$$

$$5y = 2$$

$$y = \frac{2}{5}$$

Multiply equation $\textcircled{2}$ by 2 so the y -terms are equal.

$$2(-2x + y = 6)$$

$$-4x + 2y = 12 \quad \textcircled{4}$$

Subtract equation $\textcircled{4}$ from equation $\textcircled{1}$ to eliminate y .

$$x + 2y = -2 \quad \textcircled{1}$$

$$-(-4x + 2y = 12) \quad \textcircled{4}$$

$$x + 4x = -2 - 12$$

$$5x = -14$$

$$x = -\frac{14}{5}$$

Verify the solution.

In each equation, substitute: $x = -\frac{14}{5}$ and $y = \frac{2}{5}$

$$x + 2y = -2 \quad \textcircled{1}$$

$$-2x + y = 6 \quad \textcircled{2}$$

$$\text{L.S.} = x + 2y$$

$$\text{L.S.} = -2x + y$$

$$= -\frac{14}{5} + 2\left(\frac{2}{5}\right)$$

$$= -2\left(-\frac{14}{5}\right) + \frac{2}{5}$$

$$= -\frac{14}{5} + \frac{4}{5}$$

$$= \frac{28}{5} + \frac{2}{5}$$

$$= -\frac{10}{5}$$

$$= -2$$

$$= \text{R.S.}$$

$$= \frac{30}{5}$$

$$= 6$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side; so the solution is:

$$x = -\frac{14}{5} \text{ and } y = \frac{2}{5}$$

5. Write a linear system to model the situation.

Let s represent the number of times Trish bought soup.

Let c represent the number of times Trish bought a main course.

Trish bought 160 food items.

So, one equation is: $s + c = 160$

The amount in dollars that Trish spent on soup is: $1.75s$

The amount in dollars that Trish spent on a main course is: $4.75c$

Trish spent a total of \$490.

So, another equation is: $1.75s + 4.75c = 490$

A linear system is:

$$s + c = 160 \quad \textcircled{1}$$

$$1.75s + 4.75c = 490 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 1.75.

$$1.75(s + c = 160)$$

$$1.75s + 1.75c = 280 \quad \textcircled{3}$$

Subtract equation $\textcircled{3}$ from equation $\textcircled{2}$ to eliminate s .

$$1.75s + 4.75c = 490 \quad \textcircled{2}$$

$$-(1.75s + 1.75c = 280) \quad \textcircled{3}$$

$$4.75c - 1.75c = 490 - 280 \quad \textcircled{3}$$

$$3c = 210$$

$$c = 70$$

Substitute $c = 70$ in equation $\textcircled{1}$.

$$s + c = 160 \quad \textcircled{1}$$

$$s + 70 = 160$$

$$s = 90$$

Trish bought soup 90 times and a main course 70 times.

Verify the solution.

The total number of meals Trish bought is: $90 + 70 = 160$; this agrees with the given information.

The cost of the food Trish bought is: $90(\$1.75) + 70(\$4.75) = \$157.50 + \$332.50$, or \$490.00; this agrees with the given information.

So, the solution is correct.

6. Write a linear system to model the situation.

Let l millilitres represent the larger volume.

Let s millilitres represent the smaller volume.

The difference in volumes is 1000 mL.

So, one equation is: $l - s = 1000$

The amount of acid, in millilitres, in the larger volume is: $0.055l$

The amount of acid, in millilitres, in the smaller volume is: $0.045s$

The total amount of acid is 100 mL.

So, another equation is: $0.055l + 0.045s = 100$

A linear system is:

$$l - s = 1000 \quad \text{①}$$

$$0.055l + 0.045s = 100 \quad \text{②}$$

Solve the linear system.

Multiply equation ① by 0.045.

$$0.045(l - s = 1000)$$

$$0.045l - 0.045s = 45 \quad \text{③}$$

Add equations ② and ③ to eliminate s .

$$0.055l + 0.045s = 100 \quad \text{②}$$

$$+(0.045l - 0.045s = 45) \quad \text{③}$$

$$0.055l + 0.045l = 100 + 45$$

$$0.1l = 145$$

$$l = 1450$$

Substitute $l = 1450$ in equation ①.

$$l - s = 1000 \quad \text{①}$$

$$1450 - s = 1000$$

$$-s = 1000 - 1450$$

$$s = 450$$

The larger volume of acid solution was 1450 mL and the smaller volume of the solution was 450 mL.

Verify the solution.

The difference in volumes is: $1450 \text{ mL} - 450 \text{ mL} = 1000 \text{ mL}$; this agrees with the given information.

The total amount of acid is: $0.055(1450) + 0.045(450 \text{ mL}) = 79.75 \text{ mL} + 20.25 \text{ mL}$, or 100 mL; this agrees with the given information.

So, the solution is correct.

7. Write a linear system to model the situation.

The sum of the angles in a triangle is 180° .

So, one equation is: $x + y + 60 = 180$, or $x + y = 120$

The sum of the angles in a quadrilateral is 360° .

So, another equation is: $2x + y + 95 + 90 = 360$, or $2x + y = 175$

A linear system is:

$$x + y = 120 \quad \text{①}$$

$$2x + y = 175 \quad \text{②}$$

The y -coefficients are the same, so subtract equation ① from equation ② to eliminate y .

$$2x + y = 175 \quad \text{②}$$

$$-(x + y = 120) \quad \text{①}$$

$$2x - x = 175 - 120$$

$$x = 55$$

Substitute $x = 55$ in equation ①.

$$x + y = 120 \quad \text{①}$$

$$55 + y = 120$$

$$y = 65$$

In the diagram, $x = 55$ and $y = 65$

Verify the solution.

The sum of the angles in the triangle is: $55^\circ + 65^\circ + 60^\circ = 180^\circ$; this agrees with the given information.

The sum of the angles in the quadrilateral is: $2(55^\circ) + 65^\circ + 95^\circ + 90^\circ = 360^\circ$; this agrees with the given information.
So, the solution is correct.

Lesson 7.6 Properties of Systems of Linear Equations Exercises (pages 448-449)

A

4. a) Write each equation in slope-intercept form to identify the slope.

i) $-x + y = 5$

$$y = x + 5$$

The coefficient of x is 1, so the slope is 1.

ii) $-x - y = 10$

$$-y = x + 10$$

$$y = -x - 10$$

The coefficient of x is -1 , so the slope is -1 .

iii) $-2x + 2y = 10$

$$2y = 2x + 10$$

$$y = x + 5$$

The coefficient of x is 1, so the slope is 1.

iv) $x + y = 5$

$$y = -x + 5$$

The coefficient of x is -1 , so the slope is -1 .

- b) Parallel lines have the same slope. These lines are parallel:

$$-x + y = 5 \text{ and } -2x + 2y = 10$$

$$-x - y = 10 \text{ and } x + y = 5$$

- c) Lines that are not parallel intersect. These lines intersect:

$$-x + y = 5 \text{ and } -x - y = 10$$

$$-x + y = 5 \text{ and } x + y = 5$$

$$-x - y = 10 \text{ and } -2x + 2y = 10$$

$$-2x + 2y = 10 \text{ and } x + y = 5$$

5. a) Two lines form a linear system with exactly one solution when they intersect in exactly one point. These are: lines A and C; and lines B and C

- b) Two lines form a linear system with no solution when they are parallel. These are lines A and B.

6. Write each equation in slope-intercept form to identify the slope and y -intercept.

$$4x + 2y = 20$$

$$2y = -4x + 20 \text{ Divide each side by 2.}$$

$$y = -2x + 10 \text{ The slope is } -2 \text{ and the } y\text{-intercept is } 10.$$

$$x - 3y = 12$$

$$-3y = -x + 12 \text{ Divide each side by } -3.$$

$$y = \frac{1}{3}x - 4 \text{ The slope is } \frac{1}{3} \text{ and the } y\text{-intercept is } -4.$$

$$5x - 15y = -60$$

$$-15y = -5x - 60$$

Divide each side by -15 .

$$y = \frac{1}{3}x + 4$$

The slope is $\frac{1}{3}$ and the y -intercept is 4.

$$2x + y = 10$$

$$y = -2x + 10$$

The slope is -2 and the y -intercept is 10.

$$6x + 3y = 5$$

$$3y = -6x + 5$$

Divide each side by 3.

$$y = -2x + \frac{5}{3}$$

The slope is -2 and the y -intercept is $\frac{5}{3}$.

$$2x - 6y = 24$$

$$-6y = -2x + 24$$

Divide each side by -6 .

$$y = \frac{1}{3}x - 4$$

The slope is $\frac{1}{3}$ and the y -intercept is -4 .

- a) A linear system with no solution has equations that represent parallel lines; that is, their slopes are equal but their y -intercepts are different. Here are these linear systems:

$$4x + 2y = 20 \text{ and } 6x + 3y = 5$$

$$2x + y = 10 \text{ and } 6x + 3y = 5$$

$$x - 3y = 12 \text{ and } 5x - 15y = -60$$

$$5x - 15y = -60 \text{ and } 2x - 6y = 24$$

- b) A linear system with exactly one solution has equations that do not represent parallel lines or coincident lines; that is, their slopes are different. Here are these linear systems:

$$4x + 2y = 20 \text{ and } x - 3y = 12$$

$$4x + 2y = 20 \text{ and } 5x - 15y = -60$$

$$4x + 2y = 20 \text{ and } 2x - 6y = 24$$

$$2x + y = 10 \text{ and } x - 3y = 12$$

$$2x + y = 10 \text{ and } 5x - 15y = -60$$

$$2x + y = 10 \text{ and } 2x - 6y = 24$$

$$6x + 3y = 5 \text{ and } x - 3y = 12$$

$$6x + 3y = 5 \text{ and } 5x - 15y = -60$$

$$6x + 3y = 5 \text{ and } 2x - 6y = 24$$

- c) A linear system with an infinite number of solutions has equations with lines that are coincident; that is, their slopes are equal and their y -intercepts are equal. Here are these linear systems:

$$4x + 2y = 20 \text{ and } 2x + y = 10$$

$$x - 3y = 12 \text{ and } 2x - 6y = 24$$

B

7. Write each equation in slope-intercept form to identify the slope and y -intercept.

- a) First equation: $x + 2y = 6$

$$2y = -x + 6$$

Divide each side by 2.

$$y = -\frac{1}{2}x + 3$$

The slope is $-\frac{1}{2}$ and the y -intercept is 3.

Second equation: $x + y = -2$

$$y = -x - 2$$

The slope is -1 and the y -intercept is -2 .

The slopes are different, so the lines intersect at one point, and there is exactly one solution.

b) First equation: $3x + 5y = 9$

$$5y = -3x + 9$$

Divide each side by 5.

$$y = -\frac{3}{5}x + \frac{9}{5}$$

The slope is $-\frac{3}{5}$ and the y -intercept is $\frac{9}{5}$.

Second equation: $6x + 10y = 18$

$$10y = -6x + 18$$

Divide each side by 10.

$$y = -\frac{6}{10}x + \frac{18}{10}$$

Simplify.

$$y = -\frac{3}{5}x + \frac{9}{5}$$

The slope is $-\frac{3}{5}$ and the y -intercept is $\frac{9}{5}$.

The slopes are equal and the y -intercepts are equal, so the lines are coincident and the linear system has infinite solutions.

c) First equation: $2x - 5y = 30$

$$-5y = -2x + 30$$

Divide each side by -5 .

$$y = \frac{2}{5}x - 6$$

The slope is $\frac{2}{5}$ and the y -intercept is -6 .

Second equation: $4x - 10y = 15$

$$-10y = -4x + 15$$

Divide each side by -10 .

$$y = \frac{4}{10}x + \frac{15}{-10}$$

Simplify.

$$y = \frac{2}{5}x - \frac{3}{2}$$

The slope is $\frac{2}{5}$ and the y -intercept is $-\frac{3}{2}$.

The slopes are equal but the y -intercepts are different, so the lines are parallel and there is no solution.

d) First equation: $\frac{x}{2} + \frac{y}{3} = \frac{1}{2}$

$$\frac{y}{3} = -\frac{x}{2} + \frac{1}{2}$$

Multiply each side by 3.

$$y = -\frac{3}{2}x + \frac{3}{2}$$

The slope is $-\frac{3}{2}$ and the y -intercept is $\frac{3}{2}$.

Second equation: $\frac{x}{2} + \frac{y}{3} = \frac{1}{4}$

$$\frac{y}{3} = -\frac{x}{2} + \frac{1}{4}$$

Multiply each side by 3.

$$y = -\frac{3}{2}x + \frac{3}{4}$$

The slope is $-\frac{3}{2}$ and the y -intercept is $\frac{3}{4}$.

The slopes are equal but the y -intercepts are different, so the lines are parallel and there is no solution.

8. a) One equation of a linear system is: $-2x + y = 1$

Solve for y .

$$-2x + y = 1$$

$$y = 2x + 1$$

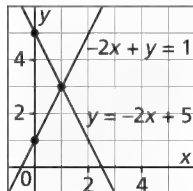
This line has slope 2 and y -intercept 1.

Sketch a graph of this line in the first quadrant.

For the second line, choose a y -intercept such as 5.

Choose a slope that is different from the slope of the first line, such as -2 .

Draw the second line on the grid.



Use the slope-intercept form to write the equation of the second line as: $y = -2x + 5$

A linear system is:

$$-2x + y = 1$$

$$y = -2x + 5$$

- b) One equation of a linear system is: $-2x + y = 1$

From part a, it can be written as: $y = 2x + 1$

This line has slope 2 and y -intercept 1.

The second line does not intersect this line, so it has the same slope but different y -intercept.

Let the y -intercept be -3 ; the slope is 2.

Use the slope-intercept form to write the equation of the second line as: $y = 2x - 3$

A linear system is:

$$-2x + y = 1$$

$$y = 2x - 3$$

- c) One equation of a linear system is: $-2x + y = 1$

To determine a coincident line, multiply the equation by a constant, such as 3.

$$3(-2x + y = 1)$$

$$-6x + 3y = 3$$

A linear system is:

$$-2x + y = 1$$

$$-6x + 3y = 3$$

9. a) Line A has slope -0.5 and y -intercept 4.

Line B has slope -0.5 and y -intercept 2.

Lines A and B have the same slope but different y -intercepts, so the lines are parallel and the linear system has no solution.

- b) Line A has slope -0.5 and y -intercept 4.

Line C has slope 0.5 and y -intercept 4.

Lines A and C have different slopes, so the lines intersect at exactly one point and the linear system has exactly one solution.

- c) Line B has slope -0.5 and y -intercept 2 .
Line C has slope 0.5 and y -intercept 4 .
Lines B and C have different slopes, so the lines intersect at exactly one point and the linear system has exactly one solution.

10. When the slopes of two lines are different, the lines intersect in exactly one point, so the linear system has exactly one solution.

11. When two lines have the same slope, they could be parallel or coincident.
I need to know the y -intercepts. If the y -intercepts are different, the lines are parallel and the linear system has not solution. If the y -intercepts are equal, then the lines are coincident and the linear system has infinite solutions.

12. One equation is: $3x - 4y = 12$

Write this equation in slope-intercept form.

$$3x - 4y = 12$$

$$-4y = -3x + 12 \quad \text{Divide each side by } -4.$$

$$y = \frac{3}{4}x - 3 \quad \text{The slope is } \frac{3}{4} \text{ and the } y\text{-intercept is } -3.$$

A linear system with exactly one solution will have a second equation with a different slope, such as 3 ; and may have a different y -intercept, such as 5 .

The second equation could be: $y = 3x + 5$

A linear system with no solution will have a second equation with the same slope of $\frac{3}{4}$, and different y -intercept, such as 4 .

The second equation could be: $y = \frac{3}{4}x + 4$

A linear system with infinite solutions will have a second equation that has the same slope and the same y -intercept. Multiply the first equation by a constant, such as 2 .

$$2(3x - 4y = 12)$$

$$6x - 8y = 24$$

The second equation could be: $6x - 8y = 24$

13. Write a linear system that is modelled by the balance scales.

Let the mass of the large container be represented by x kilograms.

Let the mass of the small container be represented by y kilograms.

For the first balance scales:

$$2x + y = 3 \quad \textcircled{1}$$

For the second balance scales:

$$6x + 3y = 9 \quad \text{Simplify; divide each term by } 3.$$

$$2x + y = 3 \quad \textcircled{2}$$

The two equations are equivalent, so their graphs are coincident and the linear system has infinite solutions.

So, the problem has infinite solutions.

14. Write a linear system to model the situation.

Let n represent the number of nickels.

Let d represent the number of dimes.

The total number of coins is 300.

So, one equation is: $n + d = 300$

Write this equation in slope-intercept form.

$$d = -n + 300 \quad \textcircled{1}$$

The value of the coins is \$23.25.

So, another equation is: $0.05n + 0.10d = 23.25$

Write this equation in slope-intercept form.

$$0.10d = -0.05n + 23.25 \quad \text{Divide each side by 0.10.}$$

$$d = -0.5n + 232.5 \quad \textcircled{2}$$

The slopes of the graphs of the lines are different, so the linear system has one solution.

So, the problem has one solution.

15. Write a linear system to model the situation.

Let s dollars represent the amount of money in the savings account.

Let c dollars represent the amount of money in the chequing account.

The total amount of money is \$85.

So, one equation is: $s + c = 85$

Write this equation in slope-intercept form.

$$c = -s + 85 \quad \textcircled{1}$$

The amount of money in each account is doubled and the total is then \$170.

So, another equation is: $2s + 2c = 170$

Write this equation in slope-intercept form.

$$2s + 2c = 170$$

$$2c = -2s + 170 \quad \text{Divide each side by 2.}$$

$$c = -s + 85 \quad \textcircled{2}$$

The equations are equivalent, so the linear system has infinite solutions.

However, the number of solutions to the problem is not infinite, because the possible sums of money are not infinite. There are many solutions to the problem.

16. Write a linear system to model the situation.

Let a represent the number of people on Saturday.

Let b represent the number of people on Sunday.

The total number of people is 568.

So, one equation is: $a + b = 568$

Write this equation in slope-intercept form.

$$b = -a + 568 \quad \textcircled{1}$$

There were 44 more people on Sunday than on Saturday.

So, another equation is: $b = a + 44 \quad \textcircled{2}$

The slopes of the graphs of the lines are different, so the linear system has one solution.

So, the problem has one solution.

17. Write a linear system to model the situation.

Let a represent the number of adults who visited the museum.

Let s represent the number of students who visited the museum.

The total number of people is 75.

So, one equation is: $a + s = 75$

Write this equation in slope-intercept form.

$$s = -a + 75 \quad \textcircled{1}$$

An adult's ticket cost \$5 and a student's ticket cost \$3. The total cost was \$275.

So, another equation is: $5a + 3s = 275$

Write this equation in slope-intercept form.

$$3s = -5a + 275 \quad \text{Divide each side by 3.}$$

$$s = -\frac{5}{3}a + \frac{275}{3} \quad \textcircled{2}$$

The slopes of the graphs of the lines are different, so the linear system has one solution.

So, the problem has one solution.

- 18.** For two lines to have 0 points of intersection, the lines are parallel, which means their slopes are equal and their y -intercepts are different.

For two lines to have 1 point of intersection, the lines are not parallel, which means that their slopes are different and their y -intercepts may or may not be equal. If the y -intercepts are equal, this point is the point of intersection of the lines.

For two lines to have infinite points of intersection, the lines are coincident, which means their slopes are equal and their y -intercepts are equal.

- 19. a)** For a linear system to have infinite solutions, the graphs are coincident.

Write any equation, such as: $3x + 4y = 5$

Multiply the equation by a constant, such as -2 .

$$-2(3x + 4y = 5)$$

$$-6x - 8y = -10$$

A linear system is:

$$3x + 4y = 5 \quad \textcircled{1}$$

$$-6x - 8y = -10 \quad \textcircled{2}$$

- b)** To solve this system by elimination, I would multiply equation $\textcircled{1}$ by -2 to make the x -coefficients the same. But when I multiply, I make the y -coefficients the same and the constants the same. When I then subtract to eliminate the x -coefficients, I also eliminate the y -coefficients, and the constants, and I am left with $0 = 0$.

- 20. a)** For a linear system with no solution, the graphs are parallel. The lines have the same slope.

Write any equation, such as: $y = 4x - 7$

Write another equation with the same slope but different y -intercept, such as: $y = 4x + 8$

A linear system is:

$$y = 4x - 7 \quad \textcircled{1}$$

$$y = 4x + 8 \quad \textcircled{2}$$

- b)** To solve this system by elimination, I subtract equation $\textcircled{2}$ from equation $\textcircled{1}$ to eliminate y . But, when I subtract, I also eliminate x , and I am left with $0 = -15$, which is not a true statement.

- 21.** If the graphs intersect at a point that is not on the grid, then I cannot determine the coordinates of the point of intersection. But if I know or can determine the equations, I can calculate the coordinates.

The graphs may look parallel and I might think there is no solution, but if I know or can determine the equations, then I know if the lines are parallel.

Sometimes the slopes of the graphs are very close in value, and I cannot identify the coordinates of the point of intersection from the graph. But if I know or can determine the equations, I can calculate the coordinates.

C

22. a) i) $2x + 3y = 4$ ①

$4x + 6y = 8$ ②

Write each equation in slope-intercept form.

For equation ①:

$2x + 3y = 4$

$3y = -2x + 4$

$y = -\frac{2}{3}x + \frac{4}{3}$

The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.

For equation ②:

$4x + 6y = 8$

$6y = -4x + 8$

$y = -\frac{2}{3}x + \frac{4}{3}$

The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.

Since the slopes are equal and the y-intercepts are equal, the lines are coincident and the linear system has infinite solutions.

ii) $2x + 3y = 4$ ①

$4x + 6y = 7$ ②

Write each equation in slope-intercept form.

From part a) i), equation ① is:

$y = -\frac{2}{3}x + \frac{4}{3}$

The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.

For equation ②:

$4x + 6y = 7$

$6y = -4x + 7$

$y = -\frac{2}{3}x + \frac{7}{6}$

The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{7}{6}$.

Since the slopes are equal and the y-intercepts are different, the lines are parallel and the linear system has no solution.

iii) $2x + 3y = 4$ ①

$4x + 5y = 8$ ②

Write each equation in slope-intercept form.

From part a) i), equation ① is:

$y = -\frac{2}{3}x + \frac{4}{3}$

The slope is $-\frac{2}{3}$ and the y-intercept is $\frac{4}{3}$.

For equation ②:

$4x + 5y = 8$

$5y = -4x + 8$

$y = -\frac{4}{5}x + \frac{8}{5}$

The slope is $-\frac{4}{5}$ and the y-intercept is $\frac{8}{5}$.

Since the slopes are different, the lines intersect and the linear system has one solution.

b) i) $2x + 3y = 4$ ①
 $4x + 6y = 8$ ②

Each coefficient and constant term in equation ② is 2 times the corresponding coefficient and constant term in equation ①.

So, to determine if a linear system has infinite solutions, I check to see if the two equations are equivalent; that is, if, when I divide each pair of coefficients and constant terms, I get the same quotient; for example, for the linear system above,

$$\frac{4}{2} = 2; \frac{6}{3} = 2, \text{ and } \frac{8}{4} = 2$$

ii) $2x + 3y = 4$ ①
 $4x + 6y = 7$ ②

Each coefficient in equation ② is 2 times the corresponding coefficient in equation ①.

So, to determine if a linear system has no solution, I check to see if, when I divide each pair of coefficients, I get the same quotient; for example, for the linear system

above, $\frac{4}{2} = 2$ and $\frac{6}{3} = 2$; there is no relationship between the pair of constant terms.

iii) $2x + 3y = 4$ ①
 $4x + 5y = 8$ ②

There is no relationship between the pairs of coefficients and the pair of constant

terms; for example, for the linear system above, $\frac{4}{2} \neq \frac{5}{3} \neq \frac{8}{4}$. So, if no relationship

exists, then the linear system has one solution.

23. Write each equation in slope-intercept form.

$$Ax + By = C \quad \text{①}$$

$$By = -Ax + C$$

$$y = -\frac{A}{B}x + \frac{C}{B} \quad \text{③} \quad \text{This line has slope } -\frac{A}{B}.$$

$$Dx + Ey = F \quad \text{②}$$

$$Ey = -Dx + F$$

$$y = -\frac{D}{E}x + \frac{F}{E} \quad \text{④} \quad \text{This line has slope } -\frac{D}{E}.$$

$AE - DB = 0$ can be written as:

$$AE = DB \quad \text{Divide each side by } E.$$

$$A = \frac{DB}{E} \quad \text{Divide each side by } B.$$

$$\frac{A}{B} = \frac{D}{E}$$

Since $\frac{A}{B} = \frac{D}{E}$, then $-\frac{A}{B} = -\frac{D}{E}$ and the slopes of the graphs of the equations are equal.

This means that the lines are parallel and the linear system does not have exactly one solution.

24. a) Write each equation in slope-intercept form.

For equation ①:

$$\frac{1}{2}x + \frac{5}{3}y = 2$$

$$\frac{5}{3}y = -\frac{1}{2}x + 2$$

Multiply each side by $\frac{3}{5}$.

$$y = -\frac{3}{10}x + \frac{6}{5}$$

For equation ②:

$$kx + \frac{5}{2}y = 3$$

$$\frac{5}{2}y = -kx + 3$$

Multiply each side by $\frac{2}{5}$.

$$y = \frac{-2k}{5}x + \frac{6}{5}$$

- i) For the linear system to have exactly one solution, the slopes must be different.

That is,

$$\frac{-2k}{5} \neq -\frac{3}{10}$$

Multiply each side by 10.

$$-4k \neq -3$$

Divide each side by -4 .

$$k \neq \frac{3}{4}$$

For exactly one solution, k can be any real number except $\frac{3}{4}$.

- ii) Since the y -intercepts are equal, for the linear system to have infinite solutions, the slopes must also be equal.

That is,

$$\frac{-2k}{5} = -\frac{3}{10}$$

Multiply each side by 10.

$$-4k = -3$$

Divide each side by -4 .

$$k = \frac{3}{4}$$

For infinite solutions, $k = \frac{3}{4}$

- b) For the system to have no solution, the lines must be parallel; that is, their slopes are equal and their y -intercepts are different. But the lines have the same y -intercept, so they cannot be parallel.

Review

(pages 452-454)

7.1

1. a) Let s represent the number of times that teams from Saskatchewan have won the BRIT.
Let o represent the number of times that teams from outside Saskatchewan have won the BRIT.
There have been 41 tournaments.
So, one equation is: $s + o = 41$
Teams from outside Saskatchewan have won the BRIT 17 more times than teams from Saskatchewan.
So, another equation is: $o - s = 17$
A linear system is:
 $s + o = 41$
 $o - s = 17$
 - b) Solution A is incorrect because it states that teams from Saskatchewan have won the BRIT more times than team from outside Saskatchewan; this disagrees with the given information.
Solution B is correct.
The difference in numbers of times (outside the province – inside the province) is:
 $29 - 12 = 17$; this agrees with the given information.
The total number of tournaments is: $12 + 29 = 41$; this agrees with the given information.
2. a) Let s represent the number of small driveways cleared.
Let l represent the number of large driveways cleared.
The cost to clear a small driveway is \$15 and the cost to clear a large driveway is \$25.
The total cost is \$475.
So, one equation is: $15s + 25l = 475$
25 driveways were cleared.
So, another equation is: $s + l = 25$
A linear system is:
 $15s + 25l = 475$
 $s + l = 25$
 - b) If solution A is correct, then:
The total number of driveways cleared is: $10 + 15 = 25$; this agrees with the given information.
The total cost is: $10(\$15) + 15(\$25) = \$150 + \$375$, or \$525; this disagrees with the given information, so solution A is not correct.
If solution B is correct, then:
The total number of driveways cleared is: $15 + 10 = 25$; this agrees with the given information.
The total cost is: $15(\$15) + 10(\$25) = \$225 + \$250$, or \$475; this agrees with the given information, so solution B is correct.
3. At a theatre, a ticket costs more than a box of popcorn, so let \$9.95 be the cost of a ticket and let \$5.50 be the cost of a box of popcorn.
Then t represents the number of tickets and p represents the number of boxes of popcorn purchased.
The first equation represents the total cost of popcorn and tickets.
The second equation represents the difference in the number of tickets purchased and the number of boxes of popcorn purchased.

A problem could be:

A group of people went to a movie theatre and spent \$76.20 on tickets and popcorn.

A ticket costs \$9.95 and a box of popcorn costs \$5.50.

There were 3 more tickets purchased than boxes of popcorn.

How many tickets were purchased? How many boxes of popcorn were purchased?

7.2

4. a) To determine the equation of each line, use the form of a linear equation that involves the coordinates of two points on the line:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

For the line that goes up to the right, the coordinates of two points are: (8, 5) and (-2, -1)

Substitute: $y_1 = 5$, $x_1 = 8$, $y_2 = -1$, and $x_2 = -2$

$$\frac{y - 5}{x - 8} = \frac{-1 - 5}{-2 - 8}$$

$$\frac{y - 5}{x - 8} = \frac{-6}{-10}$$

$$\frac{y - 5}{x - 8} = \frac{3}{5} \quad \text{Multiply each side by } (x - 8).$$

$$y - 5 = \frac{3}{5}(x - 8) \quad \text{Multiply each side by 5.}$$

$$5(y - 5) = 3(x - 8)$$

$$5y - 25 = 3x - 24$$

$$-1 = 3x - 5y$$

$$3x - 5y = -1 \quad \textcircled{1}$$

For the line that goes down to the right, the coordinates of two points are: (2, 5) and (4, -1)

Substitute: $y_1 = 5$, $x_1 = 2$, $y_2 = -1$, and $x_2 = 4$

$$\frac{y - 5}{x - 2} = \frac{-1 - 5}{4 - 2}$$

$$\frac{y - 5}{x - 2} = \frac{-6}{2}$$

$$\frac{y - 5}{x - 2} = -3 \quad \text{Multiply each side by } (x - 2).$$

$$y - 5 = -3(x - 2) \quad \text{Remove brackets.}$$

$$y - 5 = -3x + 6$$

$$3x + y = 6 + 5$$

$$3x + y = 11 \quad \textcircled{2}$$

A linear system is:

$$3x - 5y = -1 \quad \textcircled{1}$$

$$3x + y = 11 \quad \textcircled{2}$$

- b) From the graph, the solution appears to be: $x = 3$ and $y = 2$

Verify the solution.

Substitute the coordinates of the solution in each equation.

$$3x - 5y = -1 \quad \textcircled{1} \quad 3x + y = 11 \quad \textcircled{2}$$

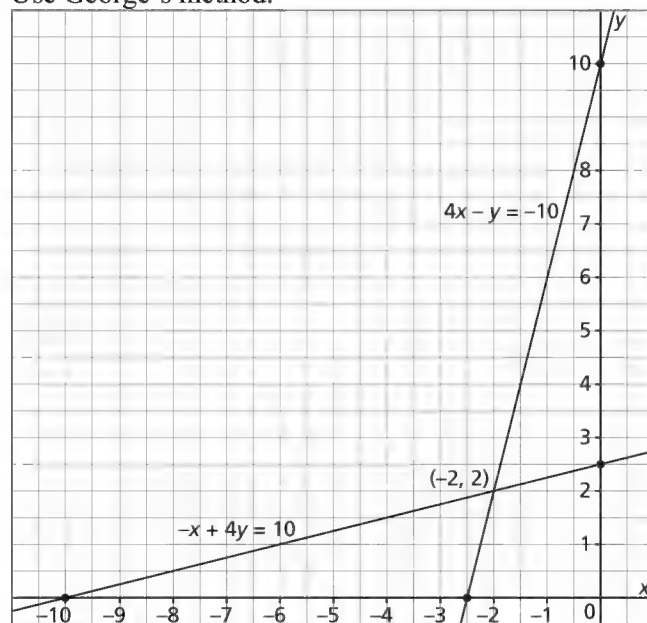
$$\begin{array}{rcl}
 \text{L.S.} & = & 3x - 5y \\
 & = & 3(3) - 5(2) \\
 & = & 9 - 10 \\
 & = & -1 \\
 & = & \text{R.S.}
 \end{array}
 \qquad
 \begin{array}{rcl}
 \text{L.S.} & = & 3x + y \\
 & = & 3(3) + 2 \\
 & = & 9 + 2 \\
 & = & 11 \\
 & = & \text{R.S.}
 \end{array}$$

For each equation, the left side is equal to the right side, so the solution is exact.

5. a) George plotted two points on each line, so he will draw a line through each pair of points, then identify the coordinates of the point where the lines intersect.

Sunita has each equation in slope-point form. For each line, she will mark a point at the y -intercept, then use the rise and run indicated by the slope to mark another point. Sunita will then draw a line through the points, and identify the coordinates of the point where the lines intersect.

- b) Use George's method.



From the graph, the solution appears to be: $x = -2$ and $y = 2$

Verify the solution.

Substitute the coordinates of the solution in each equation.

$$\begin{array}{rcl}
 -x + 4y = 10 & \text{①} & 4x - y = -10 & \text{②} \\
 \text{L.S.} = -x + 4y & & \text{L.S.} = 4x - y & \\
 = -(-2) + 4(2) & & = 4(-2) - 2 & \\
 = 2 + 8 & & = -8 - 2 & \\
 = 10 & & = -10 & \\
 = \text{R.S.} & & = \text{R.S.} &
 \end{array}$$

For each equation, the left side is equal to the right side, so the solution is exact.

6. $x - y = 15$
 $2x + y = 6$

To graph $x - y = 15$:

I determine the intercepts as two points through which to draw the line.

When $x = 0, y = -15$

When $y = 0, x = 15$

I mark points at $(0, -15)$ and $(15, 0)$, then draw a line through them.

To graph $2x + y = 6$:

I determine the intercepts as two points through which to draw the line.

When $x = 0, y = 6$

When $y = 0, x = 3$

I mark points at $(0, 6)$ and $(3, 0)$, then draw a line through them.

I choose a scale on the grid so that the point of intersection is shown.

I identify the coordinates of this point, then verify the solution by substituting these coordinates in each equation to make sure the left side is equal to the right side.

7. a) $4x - 2y = 1$ ①

$3x - 4y = 16$ ②

To graph $4x - 2y = 1$:

Determine the intercepts as two points through which to draw the line.

When $x = 0$:

$$4(0) - 2y = 1$$

$$-2y = 1$$

$$y = -\frac{1}{2}$$

When $y = 0$:

$$4x - 2(0) = 1$$

$$4x = 1$$

$$x = \frac{1}{4}$$

It may be difficult to plot a point at $\left(\frac{1}{4}, 0\right)$, so determine the coordinates of another point.

Substitute $x = 1$:

$$4(1) - 2y = 1$$

$$-2y = -3$$

$$y = \frac{3}{2}$$

On a grid, mark points at $\left(0, -\frac{1}{2}\right)$ and $\left(1, \frac{3}{2}\right)$, then draw a line through them.

To graph $3x - 4y = 16$:

The x -intercept will not be a whole number, so choose a value of x that is a multiple of 4, since the y -coefficient and the constant are multiples of 4.

Substitute $x = 4$:

$$3(4) - 4y = 16$$

$$-4y = 16 - 12$$

$$-4y = 4$$

$$y = -1$$

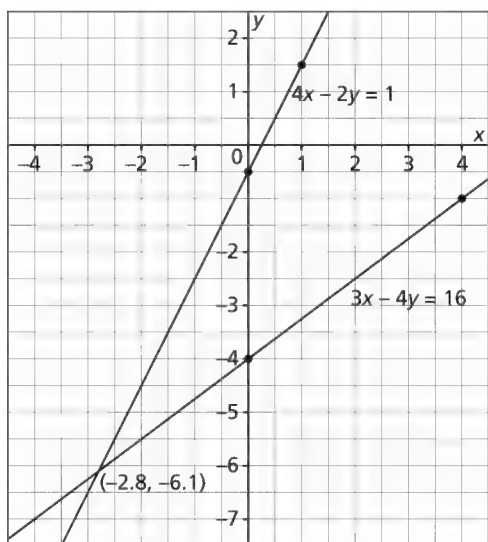
Substitute $x = 0$:

$$3(0) - 4y = 16$$

$$-4y = 16$$

$$y = -4$$

On a grid, mark points at $(4, -1)$ and $(0, -4)$, then draw a line through them.



From the graph, the point of intersection appears to be: $(-2.8, -6.1)$

- b)** Verify the solution. In each equation, substitute: $x = -2.8$ and $y = -6.1$

$$4x - 2y = 1 \quad \textcircled{1}$$

$$\begin{aligned} \text{L.S.} &= 4x - 2y \\ &= 4(-2.8) - 2(-6.1) \\ &= -11.2 + 12.2 \\ &= 1 \\ &= \text{R.S.} \end{aligned}$$

$$3x - 4y = 16 \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= 3x - 4y \\ &= 3(-2.8) - 4(-6.1) \\ &= -8.4 + 24.4 \\ &= 16 \\ &= \text{R.S.} \end{aligned}$$

In each equation, the left side is equal to the right side, so the solution is exact.

7.3

- 8. a)** Let c milligrams represent the mass of sodium in a bowl of cereal.
Let b milligrams represent the mass of sodium in a slice of bacon.
Owen's breakfast of 2 bowls of cereal and 4 slices of bacon contained 940 mg of sodium.
So, one equation is: $2c + 4b = 940$
Natalie's breakfast of 1 bowl of cereal and 3 slices of bacon contained 620 mg of sodium.
So, another equation is: $c + 3b = 620$
A linear system is:
 $2c + 4b = 940 \quad \textcircled{1}$
 $c + 3b = 620 \quad \textcircled{2}$

- b)** Each line represents one of the equations in the linear system.
Write each equation in slope-intercept form.

$$2c + 4b = 940 \quad \textcircled{1}$$

$$4b = -2c + 940$$

$$b = -\frac{1}{2}c + 235 \quad \text{The } b\text{-intercept is } 235.$$

$$c + 3b = 620 \quad \textcircled{2}$$

$$3b = -c + 620$$

$$b = -\frac{1}{3}c + \frac{620}{3} \quad \text{The } b\text{-intercept is } 206.\bar{6}.$$

Equation ① has the greater b -intercept, so it is represented by the pink line.

Equation ② has the lesser b -intercept, so it is represented by the blue line.

- c) From the graph, the solution is: (170, 150)

This means that 1 bowl of cereal contains 170 mg of sodium and 1 slice of bacon contains 150 mg.

Verify the solution.

The sodium in Owen's breakfast is: $2(170 \text{ mg}) + 4(150 \text{ mg}) = 940 \text{ mg}$; this agrees with the given information.

The sodium in Natalie's breakfast is: $170 \text{ mg} + 3(150 \text{ mg}) = 620 \text{ mg}$; this agrees with the given information.

So, the solution is exact.

9. a) $2x + 3y = 13$ ①

$5x - 2y = 1$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

$$2x + 3y = 13$$

$$3y = -2x + 13$$

Divide each side by 3.

$$y = -\frac{2}{3}x + \frac{13}{3}$$

For equation ②:

$$5x - 2y = 1$$

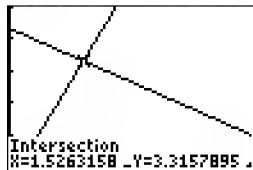
$$-2y = -5x + 1$$

Divide each side by -2 .

$$y = \frac{5}{2}x - \frac{1}{2}$$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(-2/3)X + 13/3$. Move the cursor down to $Y2 =$ and input the expression $(5/2)X - 1/2$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 5$, $Y_{\min} = 0$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, press $\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC , then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x \doteq 1.526$ and $y \doteq 3.316$

Verify the solution.

Substitute $x = 1.526$ and $y = 3.316$ into each equation.

$$2x + 3y = 13 \quad \text{①}$$

$$5x - 2y = 1 \quad \text{②}$$

$$\text{L.S.} = 2x + 3y$$

$$\text{L.S.} = 5x - 2y$$

$$\text{R.S.} = 1$$

$$= 2(1.526) + 3(3.316)$$

$$= 5(1.526) - 2(3.316)$$

$$= 13$$

$$= 0.998$$

$$= \text{R.S.}$$

The left side is equal to the right side in equation ①, but the left side is only approximately equal to the right side in equation ②, so the solution is approximate.

b) $y = \frac{1}{6}x - 2$ ①

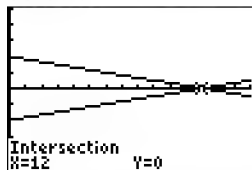
$y = -\frac{1}{6}x + 2$ ②

Each equation is written in $y = mx + b$ form.

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(1/6)X-2$. Move the cursor down to $Y2=$ and input the expression $(-1/6)X+2$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 15$, $Y_{\min} = -5$, and $Y_{\max} = 5$. To show the coordinates of the point of intersection, press

$\boxed{2\text{nd}}\boxed{\text{TRACE}}$ for CALC, then selected 5:intersect. Press $\boxed{\text{ENTER}}$ 3 times to get the screen below.



From the calculator screen, the solution is: $x = 12$ and $y = 0$

Verify the solution.

Substitute $x = 12$ and $y = 0$ into each equation.

$y = \frac{1}{6}x - 2$ ①

$y = -\frac{1}{6}x + 2$ ②

L.S. = y

R.S. = $\frac{1}{6}x - 2$

L.S. = y

R.S. = $-\frac{1}{6}x + 2$

= 0

= $\frac{1}{6}(12) - 2$
= $2 - 2$
= 0

= 0

= $-\frac{1}{6}(12) + 2$
= $-2 + 2$
= 0

In each equation, the left side is equal to the right side, so the solution is correct.

c) $4x - 5y = 20$ ①

$8x + 5y = 19$ ②

Write each equation in $y = mx + b$ form.

For equation ①:

$4x - 5y = 20$

$-5y = -4x + 20$ Divide each side by -5 .

$y = \frac{4}{5}x - 4$

For equation ②:

$8x + 5y = 19$

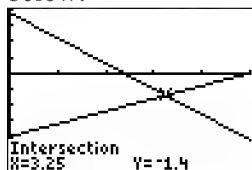
$5y = -8x + 19$ Divide each side by 5.

$y = -\frac{8}{5}x + \frac{19}{5}$

On a TI-83 graphing calculator, press $\boxed{Y=}$, then next to $Y1 =$ input the expression $(4/5)X-4$. Move the cursor down to $Y2=$ and input the expression $(-8/5)X+19/5$.

Press $\boxed{\text{GRAPH}}$. To see the point of intersection, set the $\boxed{\text{WINDOW}}$ to $X_{\min} = 0$, $X_{\max} = 5$, $Y_{\min} = -6$, and $Y_{\max} = 4$. To show the coordinates of the point of intersection, press

2nd**TRACE** for CALC, then selected 5:intersect. Press **ENTER** 3 times to get the screen below.



From the calculator screen, the solution is: $x = 3.25$ and $y = -1.4$

Verify the solution.

Substitute $x = 3.25$ and $y = -1.4$ into each equation.

$$4x - 5y = 20 \quad \textcircled{1}$$

$$8x + 5y = 19 \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= 4x - 5y \\ &= 4(3.25) - 5(-1.4) \\ &= 20 \\ &= \text{R.S.} \end{aligned}$$

$$\begin{aligned} \text{L.S.} &= 8x + 5y \\ &= 8(3.25) + 5(-1.4) \\ &= 19 \\ &= \text{R.S.} \end{aligned}$$

In each equation, the left side is equal to the right side, so the solution is correct.

d) $\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16} \quad \textcircled{1}$

$$-2x + 4y = 20 \quad \textcircled{2}$$

Write each equation in $y = mx + b$ form.

For equation $\textcircled{1}$:

$$\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16}$$

$$\frac{3y}{4} = -\frac{x}{2} - \frac{25}{16}$$

Multiply each side by $\frac{4}{3}$.

$$\frac{4}{3} \left(\frac{3y}{4} \right) = \frac{4}{3} \left(-\frac{x}{2} \right) - \frac{4}{3} \left(\frac{25}{16} \right)$$

$$y = -\frac{2}{3}x - \frac{25}{12}$$

For equation $\textcircled{2}$:

$$-2x + 4y = 20$$

$$4y = 2x + 20$$

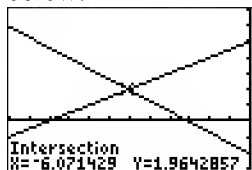
Divide each side by 4.

$$y = \frac{1}{2}x + 5$$

On a TI-83 graphing calculator, press **Y=**, then next to $Y1 =$ input the expression $(-2/3)X - 25/12$. Move the cursor down to $Y2 =$ and input the expression $(1/2)X + 5$.

Press **GRAPH**. To see the point of intersection, set the **WINDOW** to $X_{\min} = -12$, $X_{\max} = 0$, $Y_{\min} = -3$, and $Y_{\max} = 7$. To show the coordinates of the point of intersection, press

2nd**TRACE** for CALC, then selected 5:intersect. Press **ENTER** 3 times to get the screen below.



From the calculator screen, the solution is: $x \doteq -6.071$ and $y \doteq 1.964$

Verify the solution.

Substitute $x = -6.071$ and $y = 1.964$ into each equation.

$$\frac{x}{2} + \frac{3y}{4} = -\frac{25}{16} \quad \textcircled{1}$$

$$-2x + 4y = 20 \quad \textcircled{2}$$

$$\text{L.S.} = \frac{x}{2} + \frac{3y}{4}$$

$$\text{L.S.} = -2x + 4y$$

$$\text{R.S.} = 20$$

$$= \frac{-6.071}{2} + \frac{3(1.964)}{4}$$

$$= -2(-6.071) + 4(1.964)$$

$$= -1.5625$$

$$= 19.998$$

$$= \text{R.S.}$$

The left side is equal to the right side in equation $\textcircled{1}$, but the left side is only approximately equal to the right side in equation $\textcircled{2}$, so the solution is approximate.

7.4

10. a) $x + y = -5 \quad \textcircled{1}$

$$x + 3y = -15 \quad \textcircled{2}$$

Solve equation $\textcircled{1}$ for x .

$$x + y = -5 \quad \textcircled{1}$$

$$x = -5 - y$$

Substitute $x = -5 - y$ in equation $\textcircled{2}$.

$$x + 3y = -15 \quad \textcircled{2}$$

$$-5 - y + 3y = -15$$

$$2y = -10$$

$$y = -5$$

Substitute $y = -5$ in equation $\textcircled{1}$.

$$x + y = -5 \quad \textcircled{1}$$

$$x + (-5) = -5$$

$$x = 0$$

Verify the solution.

In each equation, substitute: $x = 0$ and $y = -5$

$$x + y = -5 \quad \textcircled{1}$$

$$x + 3y = -15 \quad \textcircled{2}$$

$$\text{L.S.} = x + y$$

$$\text{L.S.} = x + 3y$$

$$= 0 + (-5)$$

$$= 0 + 3(-5)$$

$$= -5$$

$$= -15$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = 0 \text{ and } y = -5$$

b) $7x + y = 10 \quad \textcircled{1}$

$$3x - 2y = -3 \quad \textcircled{2}$$

Solve equation $\textcircled{1}$ for y .

$$7x + y = 10 \quad \textcircled{1}$$

$$y = -7x + 10$$

Substitute $y = -7x + 10$ in equation $\textcircled{2}$.

$$3x - 2y = -3 \quad \textcircled{2}$$

$$3x - 2(-7x + 10) = -3$$

$$3x + 14x - 20 = -3$$

$$17x = 17$$

$$x = 1$$

Substitute $x = 1$ in equation ①.

$$7x + y = 10 \quad \text{①}$$

$$7(1) + y = 10$$

$$7 + y = 10$$

$$y = 3$$

Verify the solution.

In each equation, substitute: $x = 1$ and $y = 3$

$$7x + y = 10 \quad \text{①}$$

$$\text{L.S.} = 7x + y$$

$$= 7(1) + 3$$

$$= 10$$

$$= \text{R.S.}$$

$$3x - 2y = -3 \quad \text{②}$$

$$\text{L.S.} = 3x - 2y$$

$$= 3(1) - 2(3)$$

$$= 3 - 6$$

$$= -3$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is: $x = 1$ and $y = 3$

c) $\frac{1}{2}x + 3y = \frac{5}{6} \quad \text{①}$

$$\frac{1}{3}x - 5y = \frac{16}{9} \quad \text{②}$$

Write an equivalent system with integer coefficients.

For equation ①, the common denominator is the lowest common multiple of 2 and 6, which is 6:

$$\frac{1}{2}x + 3y = \frac{5}{6}$$

Multiply each term by 6.

$$6\left(\frac{1}{2}x\right) + 6(3y) = 6\left(\frac{5}{6}\right)$$

Simplify.

$$3x + 18y = 5 \quad \text{③}$$

For equation ②, the common denominator is the lowest common multiple of 3 and 9, which is 9:

$$\frac{1}{3}x - 5y = \frac{16}{9}$$

Multiply each term by 9.

$$9\left(\frac{1}{3}x\right) - 9(5y) = 9\left(\frac{16}{9}\right)$$

Simplify.

$$3x - 45y = 16 \quad \text{④}$$

Solve equation ④ for $3x$.

$$3x - 45y = 16 \quad \text{④}$$

$$3x = 16 + 45y$$

Substitute for $3x$ in equation ③.

$$3x + 18y = 5 \quad \text{③}$$

$$(16 + 45y) + 18y = 5$$

$$16 + 45y + 18y = 5$$

$$63y = -11$$

$$y = -\frac{11}{63}$$

To determine the value of x , solve equation ③ for $9y$.

$$3x + 18y = 5 \quad \text{③}$$

$$18y = 5 - 3x$$

$$9y = \frac{5}{2} - \frac{3}{2}x$$

Substitute for $9y = \frac{5}{2} - \frac{3}{2}x$ in equation ④.

$$3x - 45y = 16 \quad \text{④}$$

$$3x - 5\left(\frac{5}{2} - \frac{3}{2}x\right) = 16$$

$$3x - \frac{25}{2} + \frac{15}{2}x = 16$$

$$3x + \frac{15}{2}x = 16 + \frac{25}{2}$$

$$\frac{6}{2}x + \frac{15}{2}x = \frac{32}{2} + \frac{25}{2}$$

$$\frac{21}{2}x = \frac{57}{2}$$

$$21x = 57$$

$$x = \frac{57}{21}, \text{ or } \frac{19}{7}$$

Verify the solution.

In each original equation, substitute: $x = \frac{19}{7}$ and $y = -\frac{11}{63}$

$$\frac{1}{2}x + 3y = \frac{5}{6} \quad \text{①}$$

$$\text{L.S.} = \frac{1}{2}x + 3y$$

$$= \frac{1}{2}\left(\frac{19}{7}\right) + 3\left(-\frac{11}{63}\right)$$

$$= \frac{1}{2}\left(\frac{19}{7}\right) + 3\left(-\frac{11}{63}\right)$$

$$= \frac{19}{14} - \frac{11}{21}$$

$$= \frac{57}{42} - \frac{22}{42}$$

$$= \frac{35}{42}, \text{ or } \frac{5}{6}$$

$$= \text{R.S.}$$

$$\frac{1}{3}x - 5y = \frac{16}{9} \quad \text{②}$$

$$\text{L.S.} = \frac{1}{3}x - 5y$$

$$= \frac{1}{3}\left(\frac{19}{7}\right) - 5\left(-\frac{11}{63}\right)$$

$$= \frac{1}{3}\left(\frac{19}{7}\right) - 5\left(-\frac{11}{63}\right)$$

$$= \frac{19}{21} + \frac{55}{63}$$

$$= \frac{57}{63} + \frac{55}{63}$$

$$= \frac{112}{63}, \text{ or } \frac{16}{9}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:

$$x = \frac{19}{7} \text{ and } y = -\frac{11}{63}$$

d) $0.6x - 0.2y = -0.2 \quad \text{①}$

$-0.03x - 0.07y = 0.17 \quad \text{②}$

Consider the x -coefficients: $0.6 = 0.03 \times 20$

Solve equation ② for $0.03x$.

$$\begin{aligned}-0.03x - 0.07y &= 0.17 & \text{②} \\ -0.03x &= 0.17 + 0.07y \\ 0.03x &= -0.17 - 0.07y\end{aligned}$$

Write equation ① as:

$$\begin{aligned}20(0.03x) - 0.2y &= -0.2 \\ \text{Substitute } 0.03x &= -0.17 - 0.07y \\ 20(-0.17 - 0.07y) - 0.2y &= -0.2 \\ -3.4 - 1.4y - 0.2y &= -0.2 \\ -1.6y &= 3.2 \\ y &= -2\end{aligned}$$

Substitute $y = -2$ in equation ①.

$$\begin{aligned}0.6x - 0.2y &= -0.2 & \text{①} \\ 0.6x - 0.2(-2) &= -0.2 \\ 0.6x + 0.4 &= -0.2 \\ 0.6x &= -0.6 \\ x &= -1\end{aligned}$$

Verify the solution.

In each equation, substitute: $x = -1$ and $y = -2$

$0.6x - 0.2y = -0.2$ ①	$-0.03x - 0.07y = 0.17$ ②
L.S. = $0.6x - 0.2y$	L.S. = $-0.03x - 0.07y$
$= 0.6(-1) - 0.2(-2)$	$= -0.03(-1) - 0.07(-2)$
$= -0.6 + 0.4$	$= 0.03 + 0.14$
$= -0.2$	$= 0.17$
$= \text{R.S.}$	$= \text{R.S.}$

For each equation, the left side is equal to the right side, so the solution is:

$$x = -1 \text{ and } y = -2$$

- 11. a)** Laura multiplied equation ① by 4 because that is the lowest common denominator of the fractions in that equation, and this multiplication will remove the fractions from the equation.
Laura multiplied equation ② by 6 because that is the lowest common denominator of the fractions in that equation.
- b)** The new linear system will have the same solution as the original equation because multiplying the equations in a linear system by a constant does not change their graphs, so the point of intersection of the graphs in both linear systems will be the same.

c) $-\frac{3}{2}x - \frac{1}{4}y = -\frac{1}{2}$ ①

$$\frac{1}{3}x + \frac{5}{6}y = \frac{19}{3} \quad \text{②}$$

Write an equivalent system with integer coefficients.

From part a, multiply equation ① by 4.

$$4\left(-\frac{3}{2}x\right) - 4\left(\frac{1}{4}y\right) = 4\left(-\frac{1}{2}\right) \quad \text{Simplify.}$$

$$-6x - y = -2$$

$$6x + y = 2 \quad \text{③}$$

Multiply each term by -1 .

From part a, multiply equation ② by 6.

$$6\left(\frac{1}{3}x\right) + 6\left(\frac{5}{6}y\right) = 6\left(\frac{19}{3}\right) \quad \text{Simplify.}$$

$$2x + 5y = 38 \quad \textcircled{4}$$

Solve equation $\textcircled{3}$ for y .

$$6x + y = 2 \quad \textcircled{3}$$

$$y = -6x + 2$$

Substitute for $y = -6x + 2$ in equation $\textcircled{4}$.

$$2x + 5y = 38 \quad \textcircled{4}$$

$$2x + 5(-6x + 2) = 38$$

$$2x - 30x + 10 = 38$$

$$-28x = 28$$

$$x = -1$$

Substitute $x = -1$ in equation $\textcircled{3}$.

$$6x + y = 2 \quad \textcircled{3}$$

$$6(-1) + y = 2$$

$$-6 + y = 2$$

$$y = 8$$

Verify the solution.

In each original equation, substitute: $x = -1$ and $y = 8$

$$-\frac{3}{2}x - \frac{1}{4}y = -\frac{1}{2} \quad \textcircled{1} \qquad \frac{1}{3}x + \frac{5}{6}y = \frac{19}{3} \quad \textcircled{2}$$

$$\text{L.S.} = -\frac{3}{2}x - \frac{1}{4}y$$

$$\text{L.S.} = \frac{1}{3}x + \frac{5}{6}y$$

$$= -\frac{3}{2}(-1) - \frac{1}{4}(8)$$

$$= \frac{1}{3}(-1) + \frac{5}{6}(8)$$

$$= \frac{3}{2} - \frac{4}{2}$$

$$= -\frac{1}{3} + \frac{20}{3}$$

$$= -\frac{1}{2}$$

$$= \frac{19}{3}$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

For each equation, the left side is equal to the right side, so the solution is:
 $x = -1$ and $y = 8$

12. a) Let q represent the number of quarter cup measures.

Let t represent the number of two-thirds cup measures.

$5\frac{3}{4}$ cups were measured using both cups.

$$\text{So, one equation is: } \frac{1}{4}q + \frac{2}{3}t = 5\frac{3}{4}$$

One more quarter cup measure was used than two-thirds cup measure.

$$\text{So, another equation is: } q - t = 1$$

A linear system is:

$$\frac{1}{4}q + \frac{2}{3}t = 5\frac{3}{4} \quad \textcircled{1}$$

$$q - t = 1 \quad \textcircled{2}$$

- b) Solve the linear system.

Solve equation ② for q .

$$q - t = 1 \quad \text{②}$$

$$q = t + 1$$

Substitute $q = t + 1$ in equation ①.

$$\frac{1}{4}q + \frac{2}{3}t = 5\frac{3}{4} \quad \text{①}$$

$$\frac{1}{4}(t + 1) + \frac{2}{3}t = 5\frac{3}{4}$$

$$\frac{1}{4}t + \frac{1}{4} + \frac{2}{3}t = 5\frac{3}{4}$$

$$\frac{1}{4}t + \frac{2}{3}t = 5\frac{3}{4} - \frac{1}{4}$$

Write the fractions on the left side with a common denominator.

$$\frac{3}{12}t + \frac{8}{12}t = 5\frac{1}{2}$$

Write the mixed number as an improper fraction.

$$\frac{11}{12}t = \frac{11}{2}$$

Multiply each side by $\frac{12}{11}$.

$$t = \frac{12}{2}, \text{ or } 6$$

Substitute $t = 6$ in equation ②.

$$q - t = 1 \quad \text{②}$$

$$q - 6 = 1$$

$$q = 7$$

7 quarter cup measures were used and 6 two-thirds cup measured were used.

Verify the solution.

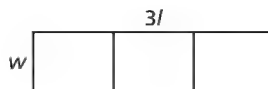
7 quarter cups and 6 two-thirds cups measure: $7\left(\frac{1}{4}\right) + 6\left(\frac{2}{3}\right) = 1\frac{3}{4} + 4$, or $5\frac{3}{4}$; this

agrees with the given information.

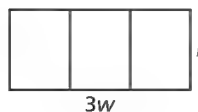
The difference in numbers of cups used is: $7 - 6 = 1$; this agrees with the given information. So the solution is correct.

13. a) Let l feet represent the length of a table.
Let w feet represent the width of a table.

First 3 tables placed end to end:



First 3 tables placed side by side:



- b) From the first diagram above, when the tables are placed end to end, the total length is $30l$ feet and the width is w feet. The perimeter is 306 ft.
So, one equation is:

$$30l + 30l + w + w = 306 \quad \text{Simplify.}$$

$$60l + 2w = 306$$

From the second diagram above, when the tables are placed side by side, the total length is $30w$ feet and the width is l feet. The perimeter is 190 ft.

So, one equation is:

$$30w + 30w + l + l = 190 \quad \text{Simplify.}$$

$$60w + 2l = 190$$

A linear system is:

$$60l + 2w = 306 \quad \textcircled{1}$$

$$2l + 60w = 190 \quad \textcircled{2}$$

- c) To solve the linear system:

Solve equation $\textcircled{2}$ for $2l$.

$$2l + 60w = 190 \quad \textcircled{2}$$

$$2l = -60w + 190$$

Substitute $2l = -60w + 190$ into equation $\textcircled{1}$.

$$60l + 2w = 306 \quad \textcircled{1}$$

$$30(-60w + 190) + 2w = 306$$

$$-1800w + 5700 + 2w = 306$$

$$-1798w = 306 - 5700$$

$$-1798w = -5394$$

$$w = 3$$

Substitute $w = 3$ in equation $\textcircled{2}$.

$$2l + 60w = 190 \quad \textcircled{2}$$

$$2l + 60(3) = 190$$

$$2l + 180 = 190$$

$$2l = 10$$

$$l = 5$$

Each table is 5 ft. long and 3 ft. wide.

Verify the solution.

When the tables are placed end to end:

The length is: $30(5 \text{ ft.}) = 150 \text{ ft.}$

The width is: 3 ft.

So, the perimeter is: $2(150 \text{ ft.}) + 2(3 \text{ ft.}) = 306 \text{ ft.}$; this agrees with the given information.

When the tables are placed side by side:

The length is: $30(3 \text{ ft.}) = 90 \text{ ft.}$

The width is: 5 ft.

So, the perimeter is: $2(90 \text{ ft.}) + 2(5 \text{ ft.}) = 190 \text{ ft.}$; this agrees with the given information.

So, the solution is correct.

14. Write a linear system to model the problem.

Let t represent the number of triangles.

Let s represent the number of squares.

The total number of shapes is 150.

So, one equation is: $t + s = 150$

40% of the triangles were blue; this is: $0.4t$

60% of the squares were blue; this is: $0.6s$

83 shapes were blue.

So, another equation is: $0.4t + 0.6s = 83$

A linear system is:

$$t + s = 150 \quad \textcircled{1}$$

$$0.4t + 0.6s = 83 \quad \textcircled{2}$$

Solve equation $\textcircled{1}$ for t .

$$t + s = 150 \quad \textcircled{1}$$

$$t = -s + 150$$

Substitute $t = -s + 150$ in equation $\textcircled{2}$.

$$0.4t + 0.6s = 83 \quad \textcircled{2}$$

$$0.4(-s + 150) + 0.6s = 83$$

$$-0.4s + 60 + 0.6s = 83$$

$$0.2s = 23$$

Divide each side by 0.2.

$$s = 115$$

Substitute $s = 115$ in equation $\textcircled{1}$.

$$t + s = 150 \quad \textcircled{1}$$

$$t + 115 = 150$$

$$t = 35$$

In the design, there were 35 triangles and 115 squares.

Verify the solution.

The total number of shapes was: $115 + 35 = 150$; this agrees with the given information.

40% of the triangles and 60% of the squares are: $0.4(35) + 0.6(115) = 83$; this agrees with the given information.

So, the solution is correct.

7.5

15. a) $-3x - y = 5 \quad \textcircled{1}$

$$2x + y = -5 \quad \textcircled{2}$$

Since the y -terms are opposites, add the equations to eliminate y .

$$\begin{array}{r} -3x - y = 5 \quad \textcircled{1} \\ + (2x + y = -5) \quad \textcircled{2} \\ \hline -3x + 2x = 5 - 5 \end{array}$$

$$-x = 0$$

$$-x = 0$$

$$x = 0$$

Substitute $x = 0$ into equation $\textcircled{2}$.

$$2x + y = -5 \quad \textcircled{2}$$

$$2(0) + y = -5$$

$$y = -5$$

Verify the solution.

In each equation, substitute: $x = 0$ and $y = -5$

$$-3x - y = 5 \quad \textcircled{1}$$

$$2x + y = -5 \quad \textcircled{2}$$

$$\text{L.S.} = -3x - y$$

$$\text{L.S.} = 2x + y$$

$$= -3(0) - (-5)$$

$$= 2(0) + (-5)$$

$$= 5$$

$$= -5$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is:

$$x = 0 \text{ and } y = -5$$

b) $2x - 4y = 13 \quad \textcircled{1}$

$$4x - 5y = 8 \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 2.

$$2(2x - 4y = 13)$$

$$4x - 8y = 26 \quad \textcircled{3}$$

Subtract equation $\textcircled{3}$ from equation $\textcircled{2}$ to eliminate x .

$$4x - 5y = 8 \quad \textcircled{2}$$

$$-(4x - 8y = 26) \quad \textcircled{3}$$

$$-5y + 8y = 8 - 26$$

$$3y = -18$$

$$y = -6$$

Substitute $y = -6$ into equation $\textcircled{2}$.

$$4x - 5y = 8 \quad \textcircled{2}$$

$$4x - 5(-6) = 8$$

$$4x + 30 = 8$$

$$4x = -22$$

$$x = \frac{-22}{4}, \text{ or } -\frac{11}{2}$$

Verify the solution.

In each original equation, substitute: $x = -\frac{11}{2}$ and $y = -6$

$$2x - 4y = 13 \quad \textcircled{1}$$

$$\text{L.S.} = 2x - 4y$$

$$= 2\left(-\frac{11}{2}\right) - 4(-6)$$

$$= -11 + 24$$

$$= 13$$

$$= \text{R.S.}$$

$$4x - 5y = 8 \quad \textcircled{2}$$

$$\text{L.S.} = 4x - 5y$$

$$= 4\left(-\frac{11}{2}\right) - 5(-6)$$

$$= -22 + 30$$

$$= 8$$

$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is:

$$x = -\frac{11}{2} \text{ and } y = -6$$

16. a) $3x - 4y = 8.5 \quad \textcircled{1}$

$$4x + 2y = 9.5 \quad \textcircled{2}$$

Since the y -coefficient in equation $\textcircled{1}$ is a multiple of the y -coefficient in equation $\textcircled{2}$, I would multiply equation $\textcircled{2}$ by 2 so the coefficients have the same numerical value.

b) Since the y -coefficients would then be opposites, I would add the new equation to equation $\textcircled{1}$ to eliminate y .

c) $3x - 4y = 8.5 \quad \textcircled{1}$

$$4x + 2y = 9.5 \quad \textcircled{2}$$

Multiply equation $\textcircled{2}$ by 2.

$$2(4x + 2y = 9.5)$$

$$8x + 4y = 19 \quad \textcircled{3}$$

Add equations $\textcircled{1}$ and $\textcircled{3}$ to eliminate y .

$$3x - 4y = 8.5 \quad \textcircled{1}$$

$$+(8x + 4y = 19) \quad \textcircled{3}$$

$$3x + 8x = 8.5 + 19$$

$$11x = 27.5$$

$$x = 2.5$$

Substitute $x = 2.5$ into equation ②.

$$4x + 2y = 9.5 \quad \text{②}$$

$$4(2.5) + 2y = 9.5$$

$$10 + 2y = 9.5$$

$$2y = -0.5$$

$$y = -0.25$$

Verify the solution.

In each original equation, substitute: $x = 2.5$ and $y = -0.25$

$$3x - 4y = 8.5 \quad \text{①}$$

$$4x + 2y = 9.5 \quad \text{②}$$

$$\text{L.S.} = 3x - 4y$$

$$\text{L.S.} = 4x + 2y$$

$$= 3(2.5) - 4(-0.25)$$

$$= 4(2.5) + 2(-0.25)$$

$$= 7.5 + 1$$

$$= 10 - 0.5$$

$$= 8.5$$

$$= 9.5$$

$$= \text{R.S.}$$

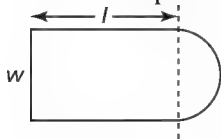
$$= \text{R.S.}$$

Since the left side is equal to the right side for each equation, the solution is:

$$x = 2.5 \text{ and } y = -0.25$$

17. a) Let l feet represent the length of the rectangular part.

Let w feet represent the width of the rectangular part.



The length of the rectangular part is 7 ft. longer than the width.

So, one equation is: $l = 7 + w$

The perimeter is approximately $68\frac{5}{6}$ ft.

The perimeter is also: $2l + w + \text{length of semicircle}$

The diameter of the semicircle is the width, w feet.

The length of the semicircle is: $\frac{1}{2} \pi \times \text{diameter} = \frac{1}{2} \pi w$

Another equation is: $2l + w + \frac{1}{2} \pi w = 68\frac{5}{6}$

A linear system is:

$$l = 7 + w \quad \text{①}$$

$$2l + w + \frac{1}{2} \pi w = 68\frac{5}{6} \quad \text{②}$$

- b) Since equation ① is solved for l , use substitution to solve the linear system.

Substitute $l = 7 + w$ in equation ②.

$$2l + w + \frac{1}{2} \pi w = 68\frac{5}{6} \quad \text{②}$$

$$2(7 + w) + w + \frac{1}{2} \pi w = 68\frac{5}{6}$$

$$14 + 2w + w + \frac{1}{2} \pi w = 68\frac{5}{6}$$

Write the mixed number as an improper fraction.

$$14 + 3w + \frac{1}{2} \pi w = \frac{413}{6}$$

Remove w as a common factor.

$$w(3 + \frac{1}{2} \pi) = \frac{413}{6} - 14$$

$$w(3 + \frac{1}{2} \pi) = \frac{413}{6} - \frac{84}{6}$$

$$w(3 + \frac{1}{2} \pi) = \frac{329}{6}$$

Divide each side by $(3 + \frac{1}{2} \pi)$.

$$w = \frac{329}{6} \div (3 + \frac{1}{2} \pi)$$

$$w = 11.9964\dots$$

$$w \doteq 12$$

Substitute $w = 12$ in equation ①.

$$l = 7 + w \quad \text{①}$$

$$l = 7 + 12$$

$$l = 19$$

The length of the rectangular part is approximately 19 ft. and the width is approximately 12 ft.

Verify the solution.

The difference between the length and width is approximately: $19 \text{ ft.} - 12 \text{ ft.} = 7 \text{ ft.}$; this agrees with the given information.

The perimeter, in feet, is: $2(19) + 12 + \frac{1}{2} \pi (12) = 50 + 6\pi$, or approximately 68.85; this agrees approximately with the given information.

So, the lengths are correct to the nearest foot.

7.6

18. a) For a linear system with infinite solutions, one equation is multiplied by a constant to get the other equation. For example,

$$5x - 2y = 12 \quad \text{Multiply this equation by 3.}$$

$$15x - 6y = 36$$

For a linear system with no solution, the x -coefficients can be the same, the y -coefficients can be the same, and the constant terms are different. For example,

$$5x - 2y = 12$$

$$5x - 2y = 7$$

- b) When I graph the equations of the linear system with infinite solutions, the graphs coincide.
When I graph the equations of the linear system with no solution, the lines are parallel.
- c) When I write each equation in slope-intercept form:
The equations of the linear system with infinite solutions are the same.
The equations of the linear system with no solution have the same x -coefficients, which are the slopes of the graphs; but different constant terms, which are the y -intercepts of the graphs.

19. Let g and o represent the numbers on the shirts.

Write a linear system for each set of clues.

a) Clue 1:

The difference between the numbers is 33.

So, one equation is: $g - o = 33$

The difference between triple the numbers is 99.

So, another equation is: $3g - 3o = 99$

A linear system is:

$$g - o = 33$$

$$3g - 3o = 99$$

These equations are equivalent. There are infinite solutions to the linear system, and there are many possible solutions to the problem. There is not sufficient information to determine one solution.

Clue 2:

The sum of the numbers is 57.

So, one equation is: $g + o = 57$

When you divide each number by 3, then add the quotients, the sum is 20.

So, another equation is: $\frac{g}{3} + \frac{o}{3} = 20$

Multiply each side by 3.

$$g + o = 60$$

A linear system is:

$$g + o = 57$$

$$g + o = 60$$

This linear system has no solution.

There must be an error in the clues because the sum of the numbers cannot be 57 and 60.

So, there is not enough information to determine the numbers.

Clue 3:

The sum of the numbers is 57.

So, one equation is: $g + o = 57$

The difference of the numbers is 33.

So, another equation is: $g - o = 33$

A linear system is:

$$g + o = 57$$

$$g - o = 33$$

b) Solve the linear system in Clue 3 in part a.

$$g + o = 57 \quad \text{①}$$

$$g - o = 33 \quad \text{②}$$

Add the equations to eliminate o .

$$g + o = 57 \quad \text{①}$$

$$+(g - o = 33) \quad \text{②}$$

$$2g = 90$$

$$g = 45$$

Substitute $g = 45$ in equation ①.

$$g + o = 57 \quad \text{①}$$

$$45 + o = 57$$

$$o = 12$$

The numbers on the shirts are 45 and 12.

Verify the solution.

The sum of the numbers is: $45 + 12 = 57$; this agrees with the information in Clue 3.

The difference of the numbers is: $45 - 12 = 33$; this agrees with the information in Clue 3.

So, the solution is correct.

20. a) $-x + 5y = 8$ ①

$2x - 10y = 7$ ②

If I multiply the x -coefficient in equation ① by -2 , I get the x -coefficient in equation ②.

If I multiply the y -coefficient in equation ① by -2 , I get the y -coefficient in equation ②.

When two equations have their coefficients related in this way, the slopes of the graphs of the lines are equal, the lines are parallel, and the linear system has no solution.

b) $-\frac{3}{2}x + \frac{1}{4}y = -\frac{1}{4}$ ①

$\frac{3}{4}x - \frac{y}{8} = \frac{1}{8}$ ②

Multiply each equation by the lowest common denominator of its fractions to write the equation with integer coefficients.

Multiply equation ① by 4.

$-6x + y = -1$ ③

Multiply equation ② by 8.

$6x - y = 1$ ④

If I multiply equation ① by -1 , I get equation ②. So, the equations are equivalent, their graphs coincide and the linear system has infinite solutions.

c) $0.5x + y = 0.3$

$-x + 2y = 0.6$

There is no relationship between corresponding coefficients, so the equations do not represent graphs that are parallel or coincident, and the linear system has one solution.

d) $2x - y = -5$ ①

$6x - 3y = 15$ ②

If I multiply the x -coefficient in equation ① by 3, I get the x -coefficient in equation ②.

If I multiply the y -coefficient in equation ① by 3, I get the y -coefficient in equation ②.

So, the slopes of the graphs of the lines are equal, the lines are parallel, and the linear system has no solution.

21. a) In question 20c, the linear system is:

$0.5x + y = 0.3$ ①

$-x + 2y = 0.6$ ②

Write each equation in slope-intercept form.

For equation ①:

$0.5x + y = 0.3$

$y = -0.5x + 0.3$

The slope of the graph is -0.5 .

For equation ②:

$-x + 2y = 0.6$

$2y = x + 0.6$

Divide each side by 2.

$y = 0.5x + 0.3$

The slope of the graph is 0.5 .

Since the slopes are different, the graphs intersect at one point, and there is only 1 solution.

So, when two lines in a linear system have different slopes, the linear system has only 1 solution.

- b) The slopes of the graphs are not sufficient information to distinguish between a linear system that has no solution and a linear system that has infinite solutions.

In question 20a, the linear system is:

$$-x + 5y = 8 \quad \textcircled{1}$$

$$2x - 10y = 7 \quad \textcircled{2}$$

Write each equation in slope-intercept form.

For equation $\textcircled{1}$:

$$-x + 5y = 8$$

$$5y = x + 8$$

Divide each side by 5.

$$y = \frac{1}{5}x + \frac{8}{5}$$

The slope of the graph is $\frac{1}{5}$.

For equation $\textcircled{2}$:

$$2x - 10y = 7$$

$$-10y = -2x + 7$$

Divide each side by -10 .

$$y = \frac{1}{5}x - \frac{7}{10}$$

The slope of the graph is $\frac{1}{5}$.

The graphs have the same slope so I have to look at their y -intercepts to determine whether the graphs are parallel (then the linear system would have no solution), or coincident (then the linear system would have infinite solutions). In the linear system above, the y -intercepts are different, so the lines are parallel and the linear system has no solution.

Practice Test

(page 455)

1. $3x - 2y = 4.5$ ①

$-x + \frac{y}{2} = -1.25$ ②

A. If I multiply equation ① by 3, it becomes:

$$3(3x - 2y = 4.5)$$

$$9x - 6y = 13.5$$
 ③

If I add equation ③ to equation ②, I get:

$$9x - 6y = 13.5$$
 ③

$$+(-x + \frac{y}{2} = -1.25)$$
 ②

This will not eliminate x because the x -coefficients are not opposites.

So, A is not the correct answer.

B. Solve each equation for y to determine the slope of its graph.

For equation ①:

$$3x - 2y = 4.5$$

$$-2y = -3x + 4.5$$

$$y = 1.5x - 2.25$$

Divide each side by -2 .

The slope of the line is 1.5.

For equation ②:

$$-x + \frac{y}{2} = -1.25$$

$$\frac{y}{2} = x - 1.25$$

Multiply each side by 2.

$$y = 2x - 2.5$$

The slope of the line is 2.

Since the slopes are different, the lines intersect at one point and there is one solution.

So, B is the correct answer.

C. The new system will be equivalent if the equation $4x - 2y = -5$ is equivalent to

$$-x + \frac{y}{2} = -1.25$$
 ②

Multiply ② by -4 so the x -coefficient is equal to the x -coefficient of $4x - 2y = -5$:

$$-4(-x + \frac{y}{2} = -1.25)$$

$$4x - 2y = 5$$
 ③

Since the constant terms in equations ③ and $4x - 2y = -5$ are different, the new system will not have the same solution as the original system. So, C is not the correct answer.

D. Verify the solution.

Substitute $x = 1$ and $y = -0.75$ in both equations.

$$3x - 2y = 4.5$$
 ①

$$-x + \frac{y}{2} = -1.25$$
 ②

$$\text{L.S.} = 3x - 2y$$

$$\text{L.S.} = -x + \frac{y}{2}$$

$$\begin{array}{lcl}
 & = 3(1) - 2(-0.75) & = -1 + \frac{(-0.75)}{2} \\
 & = 3 + 1.5 & = -1 - 0.375 \\
 & = 4.5 & = -1.375 \\
 & = \text{R.S.} & \neq \text{R.S.}
 \end{array}$$

Since the left side is not equal to the right side in both equations, D is not the correct answer.

2. In A, the slopes of the lines are different so the linear system has exactly one solution. So, A is the correct answer.
 In B, the equations are equivalent and the linear system has infinite solutions. So, B is not the correct answer.
 In C, the graphs have the same slope, and different y -intercepts. The linear system has no solution. So, C is not the correct answer.
 In D, only the left sides of the equations are equivalent. The linear system has no solution. So, D is not the correct answer.

3. The linear system in question 2, A is:

$$y = 3x - 2$$

$$y = -4x + 5$$

The slopes of the graphs are 3 and -4 . The graphs have different slopes so they intersect in exactly one point, and the linear system has exactly one solution.

The linear system in question 2, B is:

$$4x - 2y = -0.2 \quad \textcircled{1}$$

$$-x + 0.5y = 0.05 \quad \textcircled{2}$$

Multiply equation $\textcircled{2}$ by -4 .

$$-4(-x + 0.5y = 0.05)$$

$$4x - 2y = -0.2 \quad \textcircled{3}$$

Equations $\textcircled{1}$ and $\textcircled{3}$ are the same, so equations $\textcircled{1}$ and $\textcircled{2}$ are equivalent. This means that their graphs coincide, so they intersect at an infinite number of points, and the linear system has infinite solutions.

The linear system in question 2, C is:

$$y = 3x - 2$$

$$y = 3x + 2$$

The slopes of both graphs is 3. The graphs have different y -intercepts, so the graphs are parallel. A linear system has no solution when its graphs are parallel because the lines do not intersect.

The linear system in question 2, D is:

$$\frac{1}{3}x + \frac{1}{2}y = \frac{1}{6} \quad \textcircled{1}$$

$$\frac{1}{6}x + \frac{1}{4}y = \frac{1}{6} \quad \textcircled{2}$$

Multiply equation $\textcircled{1}$ by 6.

$$\frac{1}{3}x + \frac{1}{2}y = \frac{1}{6} \quad \textcircled{1}$$

Multiply equation $\textcircled{2}$ by 12.

$$\frac{1}{6}x + \frac{1}{4}y = \frac{1}{6} \quad \textcircled{2}$$

$$6\left(\frac{1}{3}\right)x + 6\left(\frac{1}{2}\right)y = 6\left(\frac{1}{6}\right)$$

$$2x + 3y = 1 \quad \textcircled{3}$$

$$12\left(\frac{1}{6}\right)x + 12\left(\frac{1}{4}\right)y = 12\left(\frac{1}{6}\right)$$

$$2x + 3y = 1 \quad \textcircled{4}$$

Equations $\textcircled{3}$ and $\textcircled{4}$ have the same x -coefficients and y -coefficients, but different constant terms. So, the linear system has no solution.

4. a) A linear system is:

$$1.75s + 2.50a = 15.50 \quad \textcircled{1}$$

$$s - a = 4 \quad \textcircled{2}$$

Since the situation involves adults and students and their fares on a transit system, then s could represent the number of students who travel and a could represent the number of adults who travel.

Since the coefficients of s and a in equation $\textcircled{1}$ are decimals, then these coefficients could represent the fares for a students and an adult.

Equation $\textcircled{1}$ represents the total cost for s students and a adults.

Equation $\textcircled{2}$ represents the difference in the numbers of students and adults who travel.

A problem could be:

A group of students and adults travel for a total cost of \$15.50.

A student ticket costs \$1.75 and an adult ticket costs \$2.50.

There are 4 more students than adults.

How many students travelled? How many adults travelled?

- b) $1.75s + 2.50a = 15.50 \quad \textcircled{1}$

$$s - a = 4 \quad \textcircled{2}$$

To solve the linear system:

Solve equation $\textcircled{2}$ for s .

$$s - a = 4 \quad \textcircled{2}$$

$$s = a + 4$$

Substitute $s = a + 4$ in equation $\textcircled{1}$.

$$1.75s + 2.50a = 15.50 \quad \textcircled{1}$$

$$1.75(a + 4) + 2.50a = 15.50$$

$$1.75a + 7 + 2.50a = 15.50$$

$$4.25a = 8.50$$

$$a = 2$$

Collect like terms, then solve for a .

Collect like terms, then solve for a .

Substitute $a = 2$ in equation $\textcircled{2}$.

$$s - a = 4 \quad \textcircled{2}$$

$$s - 2 = 4$$

$$s = 6$$

Six students and 2 adults travelled.

Verify the solution.

The difference in numbers of students and adults is: $6 - 2 = 4$; this agrees with the given information.

The cost for 6 students and 2 adults is: $6(\$1.75) + 2(\$2.50) = \$15.50$; this agrees with the given information.

So, the solution is correct.

5. a) i) $-3x - 4y = -2$ ①

$x + 2y = 3$ ②

Solve equation ② for x .

$x + 2y = 3$ ②

$x = -2y + 3$

Substitute $x = -2y + 3$ in equation ①.

$-3x - 4y = -2$ ①

$-3(-2y + 3) - 4y = -2$ Simplify, then solve for y .

$6y - 9 - 4y = -2$

$2y = 7$

$y = \frac{7}{2}$

Substitute $y = \frac{7}{2}$ in equation ②.

$x + 2y = 3$ ②

$x + 2\left(\frac{7}{2}\right) = 3$

$x + 7 = 3$

$x = -4$

Verify the solution.

In each equation, substitute: $x = -4$ and $y = \frac{7}{2}$

$-3x - 4y = -2$ ①

$x + 2y = 3$ ②

L.S. $= -3x - 4y$

L.S. $= x + 2y$

$= -3(-4) - 4\left(\frac{7}{2}\right)$

$= -4 + 2\left(\frac{7}{2}\right)$

$= 12 - 14$

$= -4 + 7$

$= -2$

$= 3$

$= \text{R.S.}$

$= \text{R.S.}$

In each equation, the left side is equal to the right side, so the solution is:

$x = -4$ and $y = \frac{7}{2}$

ii) $-0.5x + 0.2y = -1$ ①

$0.3x - 0.6y = -1.8$ ②

Multiply equation ① by 3.

$3(-0.5x + 0.2y = -1)$

$-1.5x + 0.6y = -3$ ③

Add equations ② and ③ to eliminate y .

$0.3x - 0.6y = -1.8$ ②

$+(-1.5x + 0.6y = -3)$ ③

$0.3x - 1.5x = -1.8 - 3$

$-1.2x = -4.8$

$x = 4$

Substitute $x = 4$ in equation ①.

$-0.5x + 0.2y = -1$ ①

$-0.5(4) + 0.2y = -1$

$-2 + 0.2y = -1$

Simplify, then solve for y .

$$\begin{aligned} 0.2y &= 1 \\ y &= 5 \end{aligned}$$

Divide each side by 0.2.

Verify the solution.

In each equation, substitute: $x = 4$ and $y = 5$

$$-0.5x + 0.2y = -1 \quad \textcircled{1}$$

$$0.3x - 0.6y = -1.8 \quad \textcircled{2}$$

$$\text{L.S.} = -0.5x + 0.2y$$

$$\text{L.S.} = 0.3x - 0.6y$$

$$= -0.5(4) + 0.2(5)$$

$$= 0.3(4) - 0.6(5)$$

$$= -2 + 1$$

$$= 1.2 - 3$$

$$= -1$$

$$= -1.8$$

$$= \text{R.S.}$$

$$= \text{R.S.}$$

In each equation, the left side is equal to the right side, so the solution is:

$$x = 4 \text{ and } y = 5$$

$$\text{iii) } x - \frac{1}{3}y = \frac{4}{3} \quad \textcircled{1}$$

$$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2} \quad \textcircled{2}$$

Multiply each equation by a common denominator to produce a linear system with integer coefficients.

Multiply equation $\textcircled{1}$ by 3.

$$3x - 3\left(\frac{1}{3}y\right) = 3\left(\frac{4}{3}\right)$$

$$3x - y = 4 \quad \textcircled{3}$$

Multiply equation $\textcircled{2}$ by 6.

$$6\left(\frac{5}{6}x\right) + 6\left(\frac{1}{2}y\right) = 6\left(\frac{3}{2}\right)$$

$$5x + 3y = 9 \quad \textcircled{4}$$

Solve equation $\textcircled{3}$ for y .

$$3x - y = 4 \quad \textcircled{3}$$

$$-y = -3x + 4$$

$$y = 3x - 4$$

Substitute $y = 3x - 4$ in equation $\textcircled{4}$.

$$5x + 3y = 9 \quad \textcircled{4}$$

$5x + 3(3x - 4) = 9$ Simplify, then solve for x .

$$5x + 9x - 12 = 9$$

$$14x = 21$$

$$x = \frac{21}{14}, \text{ or } \frac{3}{2}$$

Substitute $x = \frac{3}{2}$ in equation $\textcircled{3}$.

$$3x - y = 4 \quad \textcircled{3}$$

$$3\left(\frac{3}{2}\right) - y = 4$$

$$\frac{9}{2} - y = 4$$

$$-y = \frac{8}{2} - \frac{9}{2}$$

$$-y = -\frac{1}{2}$$

$$y = \frac{1}{2}$$

Verify the solution.

In each equation, substitute: $x = \frac{3}{2}$ and $y = \frac{1}{2}$

$$x - \frac{1}{3}y = \frac{4}{3} \quad \textcircled{1}$$

$$\begin{aligned} \text{L.S.} &= x - \frac{1}{3}y \\ &= \frac{3}{2} - \frac{1}{3}\left(\frac{1}{2}\right) \\ &= \frac{3}{2} - \frac{1}{6} \\ &= \frac{9}{6} - \frac{1}{6} \\ &= \frac{8}{6} \\ &= \frac{4}{3} \\ &= \text{R.S.} \end{aligned}$$

$$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2} \quad \textcircled{2}$$

$$\begin{aligned} \text{L.S.} &= \frac{5}{6}x + \frac{1}{2}y \\ &= \frac{5}{6}\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) \\ &= \frac{15}{12} + \frac{1}{4} \\ &= \frac{15}{12} + \frac{3}{12} \\ &= \frac{18}{12} \\ &= \frac{3}{2} \\ &= \text{R.S.} \end{aligned}$$

In each equation, the left side is equal to the right side, so the solution is:

$$x = \frac{3}{2} \text{ and } y = \frac{1}{2}$$

- b)** For the linear system in part a) ii), the solution is: $x = 4$ and $y = 5$

This means that if the equations below are graphed on the same grid, they will intersect at the point with coordinates (4, 5); that is, the solution of a linear system is the coordinates of the point of intersection of the graphs of the system.

$$-0.5x + 0.2y = -1$$

$$0.3x - 0.6y = -1.8$$

- 6. a)** Let s represent the number of squares used in the design.
Let t represent the number of triangles used in the design.

The design contains 90 shapes.

So, one equation is: $s + t = 90$

The area of a square is 25 cm^2 .

So, the area of s squares is: $25s$ square centimetres

The area of a triangle is 12.5 cm^2 .

So, the area of t triangles is: $12.5t$ square centimetres

The total area of the design is 1500 cm^2 .

So, another equation is: $25s + 12.5t = 1500$

A linear system is:

$$s + t = 90 \quad \textcircled{1}$$

$$25s + 12.5t = 1500 \quad \textcircled{2}$$

- b) Solve the linear system to solve the problem.

Solve equation $\textcircled{1}$ for s .

$$s + t = 90 \quad \textcircled{1}$$

$$s = -t + 90$$

Substitute $s = -t + 90$ in equation $\textcircled{2}$.

$$25s + 12.5t = 1500 \quad \textcircled{2}$$

$$25(-t + 90) + 12.5t = 1500$$

$$-25t + 2250 + 12.5t = 1500 \quad \text{Simplify, then solve for } t.$$

$$-12.5t = -750$$

$$t = \frac{-750}{-12.5}$$

$$t = 60$$

Substitute $t = 60$ in equation $\textcircled{1}$.

$$s + t = 90 \quad \textcircled{1}$$

$$s + 60 = 90$$

$$s = 30$$

Thirty squares and 60 triangles were used.

Verify the solution.

The total number of shapes used is: $30 + 60 = 90$; this agrees with the given information.

The total area is: $30(25 \text{ cm}^2) + 60(12.5 \text{ cm}^2) = 1500 \text{ cm}^2$; this agrees with the given information.

So, the solution is correct.